



---

## **PUBLISHED BY PITMAN**

---

### **THERMODYNAMICS APPLIED TO HEAT ENGINES**

By E. H. LEWITT, B.Sc., A.M.I.Mech.E.

In demy 8vo, cloth gilt, 360 pp. **12s. 6d.** net.

### **UNIFLOW, BACK-PRESSURE, AND STEAM EXTRACTION ENGINES**

By Eng. Lieut.-Com. T. ALLEN, R.N. (S.R.),  
M.Eng., M.I.Mech.E.

In medium 8vo, cloth gilt, 680 pp. **42s.** net.

### **DIESEL ENGINES : MARINE, LOCOMOTIVE, AND STATIONARY**

By DAVID LOUIS JONES, *Commissioned Chief  
Machinist, U.S. Navy; Instructor, Diesel Engine  
Department, U.S. Navy Submarine Department.*

In medium 8vo, cloth gilt, 565 pp., with 341  
illustrations. **21s.** net.

### **STEAM TURBINE OPERATION**

By W. J. KEARTON, M.Eng., A.M.I.Mech.E.,  
A.M.Inst.N.A.

In demy 8vo, 312 pp. **12s. 6d.** net.

### **TURBO-BLOWERS AND COMPRESSORS**

By W. J. KEARTON, M.Eng., A.M.I.Mech.E.,  
A.M.I.N.A.

In demy 8vo, cloth gilt, 348 pp., with 153 illus-  
trations and numerous worked examples. **21s.** net.

---

**Sir Isaac Pitman & Sons, Ltd., Parker St., London, W.C.2**



# TECHNICAL THERMODYNAMICS

BY  
PROFESSOR DIPL.-ING. W. SCHÜLE

TRANSLATED FROM THE  
GERMAN "TECHNISCHE THERMODYNAMIK"

BY  
E. W. GEYER, B.Sc.  
LECTURER ON HEAT ENGINES IN THE UNIVERSITY OF GLASGOW



LONDON  
SIR ISAAC PITMAN & SONS, LTD.  
1933

SIR ISAAC PITMAN & SONS, LTD.  
PARKER STREET, KINGSWAY, LONDON, W.C.2  
THE PITMAN PRESS, BATH  
THE RIALTO, COLLINS STREET, MELBOURNE  
ASSOCIATED COMPANIES  
PITMAN PUBLISHING CORPORATION  
2 WEST 45TH STREET, NEW YORK  
SIR ISAAC PITMAN & SONS (CANADA), LTD.  
(INCORPORATING THE COMMERCIAL TEXT BOOK COMPANY)  
381-383 CHURCH STREET, TORONTO

PRINTED IN GREAT BRITAIN  
AT THE PITMAN PRESS, BATH

## TRANSLATOR'S PREFACE

SCHÜLE'S *Technische Thermodynamik* is extensively used in the technical schools throughout central Europe. This volume is a translation of Volume I of his work, and was undertaken in order to present, to British readers, not only the thermodynamic problems from a new angle, but also to give a survey of the results of a considerable amount of recent German experimental work. It is hoped that the latter, together with the numerous references found throughout the book, may prove of special use to research students.

The first six chapters are devoted mainly to the laws of gases, and are applicable, therefore, to internal combustion engine problems. The remaining four chapters deal with the properties of steam and other vapours. In practical steam work the Fahrenheit scale is almost always employed in this country, and, hence, in these chapters, the original Centigrade temperatures have been changed to Fahrenheit, while, in the earlier chapters, the Centigrade values have been retained.

In the original German text the pressures are given in  $\text{kg./cm.}^2$  or, in Continental nomenclature, "technical atmospheres." In practically all cases these have been converted to British units, i.e.  $\text{lb./in.}^2$ . The heat units have also been expressed in British units (C.H.U./lb. or B.Th.U./lb.). As a considerable amount of arithmetical work has been required in these conversions it is possible that slips may have occurred. Notification of these will be gratefully acknowledged by the translator.

Prior to Professor Schüle's lamented demise in 1931, the translator was in communication with him regarding several corrections to the original German text. These corrections, accepted by Professor Schüle, are incorporated in this translation.

The translator desires to express his gratitude to Professor W. J. Goudie for his encouragement and helpful criticisms, and for the large amount of work he has undertaken in reading through the proofs.

E. W. GEYER.

JAMES WATT ENGINEERING LABORATORIES,  
UNIVERSITY OF GLASGOW.  
June, 1933.



# CONTENTS

TRANSLATOR'S PREFACE	PAGE V
INTRODUCTION . . . . .	IX

## CHAPTER I

### GASES

Gas laws—Characteristic equation—Gas mixtures—Boyle's Law—Gay-Lussac's Law—Gas analysis—The combustion triangle

## CHAPTER II

THERMAL PROPERTIES OF GASES . . . . .	41
---------------------------------------	----

Specific heat—Steam—Calorific value—Heat loss in flue gases—Mixture pressure—Change of state—Expansion and compression—Adiabatic expansion—Entropy of gas

## CHAPTER III

FIRST AND SECOND LAWS OF THERMODYNAMICS . . . . .	149
---	-----

Heat and mechanical work—Heat engine efficiency—Carnot cycle—Absolute temperature—Internal energy—Refrigeration—Irreversible processes

## CHAPTER IV

FLOW OF GASES . . . . .	215
-------------------------	-----

The Pitot tube—Gas discharge from nozzles—Expansion nozzles—Diaphragm orifices—Pressure losses in pipes—Limiting pressures—Equations of flow

## CHAPTER V

DEVIATIONS FROM THE IDEAL GAS STATE . . . . .	317
---	-----

Gases at very high pressures—Relations between different gases—Van der Waal's equation—Kinetic theory of gases

## CHAPTER VI

SPECIAL APPLICATIONS OF THE GAS LAWS . . . . .	325
--	-----

Air compressors—Compressed air transmission—The Otto cycle—The Diesel cycle—Explosion turbines

## CHAPTER VII

STEAM AT NORMAL PRESSURES . . . . .	354
-------------------------------------	-----

Saturated steam—Latent heat—Dryness fraction—Entropy of steam—Adiabatic change of state

## CHAPTER VIII

STEAM AT HIGH PRESSURES	PAGE . 395
Saturated steam—Pressure-temperature curves—Specific volume—Total heat—Steam charts	

## CHAPTER IX

GENERAL RELATIONS FOR VAPOURS . . .	. 419
The Clausius equation—Entropy diagrams—Superheated vapours—Flow resistances—Total heat diagrams—Liquefaction of gases	

## CHAPTER X

APPLICATIONS	. 484
Thermodynamic efficiency—Back pressure machines—Extraction turbines—Indicator diagrams—Superheated steam—Uniflow engines—Steam accumulators—Heat pumps—The reaction wheel—Steam turbines	

INDEX	. 625
-------	-------

## INSET PLATES

## CHART

No. 1.

No. 2.  $T$ - $\phi$  and  $T$ - $v$  Diagram for Steam.

No. 3. Total Heat—Entropy Diagram for Steam. } In Pocket at end of Book

# INTRODUCTION

## GENERAL CLASSIFICATION OF GASES, VAPOURS AND LIQUIDS

**Gases.** Permanent gases are distinguished from vapours, in that they are not readily condensed at normal temperatures, even when subjected to high pressures.

The following are examples of the commoner simple gases of technical importance (diatomic): oxygen ( $O_2$ ), nitrogen ( $N_2$ ), hydrogen ( $H_2$ ), carbon monoxide (CO), nitric oxide (NO), and the polyatomic gases: methane or marsh gas ( $CH_4$ ), ethylene ( $C_2H_4$ ), and acetylene ( $C_2H_2$ ).

The following substances, under the conditions stated, can also be regarded as gases: carbon dioxide ( $CO_2$ ) at high temperatures, and steam ( $H_2O$ ) at flue gas temperatures or very low pressures.

The following gas mixtures are of technical importance: atmospheric air, town gas, producer gas, blast furnace gas, coke oven gas, and the products of combustion in the heated state at exit from boilers and internal combustion engines.

**Saturated vapours and liquids.** Contrary to the case with gases, the aggregate state of saturated vapours is quite unstable. Small changes in temperature, pressure, or volume can result in a partial change from the vapour to the liquid state (mist formation) or *vice versa*. Many vapours exist simultaneously with the liquids. Vapours and liquids of technical importance are: water ( $H_2O$ ), ammonia ( $NH_3$ ), sulphur dioxide ( $SO_2$ ), carbon dioxide ( $CO_2$ ), and, at normal temperatures (below  $32^\circ C.$ ), chlorine ( $Cl_2$ ). In addition, there are the mineral oil distillates used in internal combustion engines (such as petrol, paraffin, and gas oil), which consist of hydrocarbons in varying proportions, as well as the heavy oils such as naphtha, etc., the residuals of mineral oil distillation, and spirit alcohol ( $C_2H_6O$  and water).

Further, there are the important oils obtained from the distillation of hard and bituminous coal. These are called hard or bituminous coal tar oils. Benzene ( $C_6H_6$ ) belongs to the first group, but in its commercial form benzene is mixed with toluene ( $C_7H_8$ ) and xylene ( $C_8H_{10}$ ). The bituminous coal tar oils, such as paraffin crude oil and the oils of various composition derived from this oil, are known generally as "paraffins."

An example of the properties of vapours is furnished by the water vapour present in the atmosphere. Due to relatively small fluctuations of pressure and temperature, the vapour continuously changes into the wet state (forming mist or clouds) or into the liquid state (rain) or the solid state (snow, hoarfrost), or *vice versa*.

**Wet and dry vapour.** If vapours are in contact with a liquid surface (e.g. in a boiler) they contain liquid drops in the form of mist and are said to be wet. This is the usual condition of saturated vapour, and occurs even without the liquid surface being present. Complete dryness

is a limiting state (between wet saturated and superheated) and is very unstable.

The state of dry saturation is, however, important, since wet vapour is to be regarded as a mixture of dry saturated vapour and liquid at the same temperature.

**Superheated or unsaturated vapours.** As in the case of gases, the state of aggregation of these substances remains constant during changes of temperature, pressure, and volume, but only within definite yet moderately wide limits. Contrary to the case with gases, they can be reduced to the vapour state by even a moderate change in pressure or temperature.

Conversely, all saturated vapours can be superheated by the addition of heat.

For the same substance, e.g. steam, the saturated and superheated states are distinctly different, since the substance is governed by different laws in these states. For the same reason a distinction has to be made between gases and superheated vapours.

If the weight of vapour (formed, for example, by the evaporation of liquid) contained in a given space is less than the maximum possible at the existing temperature, the vapour is superheated. This accounts for the use of the term "unsaturated," which is sometimes applied to superheated vapour. The space will only be saturated with vapour when it can absorb no more vapour.

Atmospheric vapour can again be cited as illustrating the properties of superheated vapour. Condensation of this (unsaturated) vapour can only occur when the temperature has dropped to a certain value, depending on the degree of saturation. In steam pipes, so long as the steam is superheated, no condensation occurs, whereas with saturated steam, condensation occurs continuously, due to loss of heat through the walls.

By means of sufficient cooling and simultaneous compression, gases can be reduced artificially to vapours or liquids. In the process they pass through the superheat region. Conversely, substances which normally are in the liquid or vapour state can, by heating or by a reduction in pressure, be brought to the gaseous state.

In this way therefore, according to the properties possessed under normal temperatures and pressures, a substance is said to be a gas, a superheated vapour, a saturated vapour, or a liquid.

It is not always possible to effect the change from the solid or liquid state of a substance to the vapour or gaseous state without having, at the same time, an accompanying chemical change (dissociation).

**Magnitudes which determine the state of gases and vapours and their usual technical units.** The state of a gas or vapour is determined by its specific volume (or specific density), its pressure, and its temperature. These will now be considered separately.

THE VOLUME occupied by unit weight is called the "specific volume" ( $v$ ), while the weight of unit volume is known as the "specific density," or simply density ( $d$ ).

It follows that 
$$v = \frac{1}{d}$$



Instead of  $v$  or  $d$  the weight  $W$  of a volume  $V$  may be given, in which case

$$d = \frac{W}{V} \text{ or } W = dV$$

and

$$v = \frac{V}{W} \text{ or } V = vW$$

The ratio of the density of a substance to some other standard substance, such as air or water, at the same state is called the specific density with respect to that substance (air or water). In place of specific density the term specific weight is also used, particularly for liquids and solids, with respect to water at 4° C. In textbooks on physics the specific weight is generally defined as the ratio of the weight of a body to the weight of water occupying the same volume.

THE PRESSURE ( $p$ ) exerted by the gas or vapour on unit area of the enclosing vessel is sometimes called the "specific pressure," but more generally "pressure" or "vapour tension."

THE TEMPERATURE is expressed in degrees Centigrade or degrees Fahrenheit, and in both cases may be either "ordinary" or "absolute" ( $t$  or  $T$ ).

For gases and superheated vapours, two of these magnitudes are always sufficient to give the third. The relation connecting the three is called the "characteristic equation" of the gas or superheated vapour.

For dry saturated vapours the temperature alone suffices to determine the pressure or the volume, each according to a special law. These give, therefore, two equations of state, one connecting the pressure and temperature and the other connecting the temperature and volume. A third equation connecting pressure and volume follows from these.

In the case of wet vapours the proportions of liquid and dry vapour affect the specific weight and volume.

The volume of a liquid depends chiefly on the temperature and but slightly on the pressure.

UNITS. The British unit of weight employed is the pound, and of the volume the cubic foot. Hence the specific weight is the weight of 1 ft.<sup>3</sup> in pounds ( $d = \text{lb./ft.}^3$ ), and the specific volume is the volume of 1 lb. in ft.<sup>3</sup> ( $v = \text{ft.}^3/\text{lb.}$ ). Since, in this, the unit surface considered is 1 ft.<sup>2</sup>, the pressure is expressed as pounds on 1 ft.<sup>2</sup> ( $P = \text{lb./ft.}^2$ ), and is introduced thus in the characteristic equation.

In practice it is usual to express the pressure in pounds per square inch and not per square foot. A pressure of 14.7 lb./in.<sup>2</sup> is called an atmosphere, i.e. 14.7 lb./in.<sup>2</sup> = 1 at. = 2116.8 lb./ft.<sup>2</sup> On the Continent the pressure is measured in kilograms per square metre or per square centimetre and 1 kg./cm.<sup>2</sup> = 14.223 lb./in.<sup>2</sup>, and is called an atmosphere there. It should be noted that approximately 100 lb./in.<sup>2</sup> = 7 kg./cm.<sup>2</sup>

Pressure is frequently measured by the height of a column of liquid (water, mercury, alcohol, etc.), in which case the following relations are of use—

$$\begin{aligned} 1 \text{ in. H}_2\text{O} &= 0.036 \text{ lb./in.}^2 \\ 1 \text{ in. Hg} &= 0.49 \text{ lb./in.}^2 = 13.595 \text{ in. H}_2\text{O.} \\ 30 \text{ in. Hg} &= 1 \text{ at.} = 14.7 \text{ lb./in.}^2 \end{aligned}$$

**Gauge, vacuum, and absolute pressure.** The usual commercial instruments used for pressure measurements (pressure and vacuum gauges, and liquid columns) give the pressure of the gas or vapour above or below the existing atmospheric pressure, and therefore measure a pressure difference. The true or absolute pressure of the gas is obtained from the readings by adding the gauge pressure to the atmospheric pressure in the case of pressure gauges, and by subtracting the vacuum reading from the air pressure in the case of vacuum gauges.

The barometer, on the other hand, measures absolute pressures. Gauge and absolute pressures can thus be conveniently distinguished as "manometric" and "barometric" pressures. Absolute pressures must, of course, always be used in the characteristic equation.

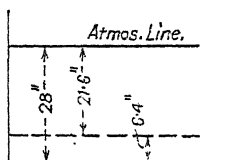


FIG. 1

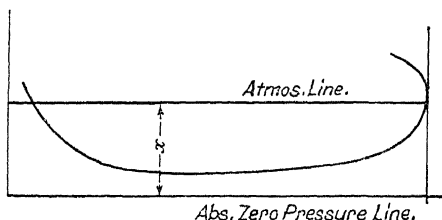


FIG. 2

The same absolute or true pressure can show widely different manometric readings, depending on the barometer reading, which, in turn, depends on the weather and height above sea level.

**Example. 1.** The reading on a boiler pressure gauge is 75 lb./in.<sup>2</sup> Find the absolute steam pressure  $p$  if the barometer reading is 28 in. Hg.

Since 28 in. Hg =  $0.491 \times 28 = 13.75$  lb./in.<sup>2</sup>, the absolute pressure is  $p = 75 + 13.75 = 88.75$  lb./in.<sup>2</sup> abs.

**2.** A vacuum gauge on a steam condenser shows a reading of 21.6 in. Hg, while the barometer reading is 28 in. Hg. What is the absolute pressure in the condenser?

The required pressure is  $28 - 21.6 = 6.4$  in. Hg, giving

$$p = 0.491 \times 6.4 = 3.14 \text{ lb./in.}^2 \text{ abs. (See Fig. 1.)}$$

**3.** At what distance below the atmospheric line on an indicator card should the absolute zero pressure line be drawn if the spring scale is  $1'' = 36$  lb./in.<sup>2</sup> and the barometer reading at the same time is 27.5 in. Hg? (See Fig. 2.)

The atmospheric pressure is  $27.5 \times 0.491 = 13.5$  lb./in.<sup>2</sup> abs., hence the zero pressure line lies at a distance

$$x = 13.5/36 = 0.375 \text{ in. below the atmospheric line.}$$

**4.** The vacuum gauge of a steam engine shows a higher reading when the barometric pressure is increased and other conditions remain the same. Account for this.

The absolute pressure in the condenser, when the conditions (load, dryness, water temperature, and density) remain steady, is the same, hence the vacuum, shown by the gauge, increases as the absolute atmospheric pressure increases (Fig. 1). Measuring from the absolute zero pressure line, the atmospheric line will be raised or lowered as the barometric pressure rises or falls.

Vacuum gauges showing absolute pressures are frequently used for measuring condenser pressures, and are independent of the barometric pressure.

5. The instrument illustrated in Fig. 3 shows a reading of  $h$  in. Hg in a steam turbine condenser. Express the vacuum in the condenser as a percentage of the barometric pressure.

$$\% \text{ vacuum} = \frac{B - h}{B} \times 100$$

where  $B$  = barometric reading in inches Hg; e.g. if  $h = 1.97$  in. and  $B = 29.5$  in.

$$\% \text{ vacuum} = \left( \frac{29.5 - 1.97}{29.5} \right) 100 = 93.3\%$$

6. The water standing in the gauge glass of a boiler is usually colder than the water in the boiler, due to external cooling and bad heat

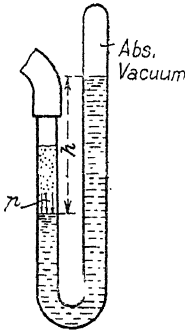


FIG. 3

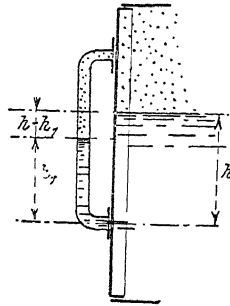


FIG. 4

conduction of the water. This means that the level of the water in the glass will be lower than that of the water in the boiler. The difference  $h - h_1$ , shown in Fig. 4, is determined as follows.

On both water surfaces the pressure of the steam is the same, provided the weight of the steam be neglected. Actually the pressure on the water surface in the glass is greater, due to the weight of the column of steam  $h - h_1$ . Denoting the density of the steam by  $d_s$ , the density of the water in the glass by  $d_1$ , and the density of the water in the boiler by  $d$ , we have

$$d_s(h - h_1) + d_1 h_1 = dh$$

from which 
$$h - h_1 = h \frac{d_1 - d}{d_1 - d_s}$$

and therefore depends on the height of the water level  $h$ .

Thus for steam at 150 lb./in.<sup>2</sup> abs.,  $d_s = 0.333$ ,  $d = 55.4$  lb./ft.<sup>3</sup>, we have

$$h - h_1 = h \frac{d_1 - 55.4}{d_1 - 0.333}$$

If now the temperature in the glass is  $35^{\circ}$  F. less than that in the boiler, and is thus  $323.5^{\circ}$  F. instead of  $358.5^{\circ}$  F., then  $d_1 = 56.6$  lb./ft.<sup>3</sup> and  $h - h_1 = \frac{h}{47}$ . If  $h = 4$  in. the water level difference is only  $0.085''$ .

For a water temperature in the glass of  $212^{\circ}$  F. the equation is  $h - h_1 = \frac{h}{13.5}$ , and thus with  $h = 6$  in.,  $h - h_1 = 0.45$  in.

Recent tests on the possible temperature differences between the glass and boiler have shown that at a boiler pressure of 115 lb./in.<sup>2</sup> gauge ( $347.4^{\circ}$  F.), for example, the temperature in the gauge glass was only  $155^{\circ}$  F.\*

**Reduction of barometer readings to  $0^{\circ}$  C.** The density of mercury at temperatures above  $0^{\circ}$  C. is less than the value assumed above. Hence when the temperature of a barometer is higher than  $0^{\circ}$  C. the reading has to be corrected by subtracting the following values per 1000 mm. of the mercury column—

$0^{\circ}$	5	10	15	20	25	$30^{\circ}$ C.
0.00	0.87	1.73	2.59	3.45	4.31	5.17 mm. Hg.

Below  $0^{\circ}$  C. corresponding quantities have to be added.

Thus if the barometer reading at  $20^{\circ}$  C. is 755 mm., the barometer reading at  $0^{\circ}$  C. is  $0.755 \times 3.45 = 2.6$  mm. less, giving

$$755 - 2.6 = 752.4 \text{ mm. Hg.}$$

TEMPERATURE ( $t$ ) is measured in degrees centigrade or Fahrenheit. The fixed points of the centigrade thermometer are given at the melting point of ice ( $0^{\circ}$  C.) and the boiling point of water under a pressure of 760 mm. Hg ( $100^{\circ}$  C.).  $1^{\circ}$  C. on the mercury thermometer is the hundredth part of the distance between these points.

In Great Britain and America the scale usually employed is the Fahrenheit ( $^{\circ}$  F.). This is divided into 180 parts between the ice ( $32^{\circ}$  F.) and the boiling ( $212^{\circ}$  F.) points. Its zero is therefore  $32^{\circ}$  F. below the ice point.

The following relations hold between the two scales—

$$^{\circ}\text{F.} = 32 + \frac{18}{10}^{\circ}\text{C.}; \quad ^{\circ}\text{C.} = \frac{10}{18} (^{\circ}\text{F.} - 32); \quad 0^{\circ}\text{F.} = -17.8^{\circ}\text{C.}$$

$$0^{\circ}\text{C.} = 32^{\circ}\text{F.}$$

The same fixed points are used in the hydrogen thermometer scale. Nitrogen and carbon dioxide gas thermometers do not show complete agreement with the hydrogen gas thermometer. Gas thermometers are too unwieldy for use in technical work.

Mercury thermometer readings deviate slightly from those given by the gas thermometer, depending on the type of glass. The deviation is greatest between  $40^{\circ}$  and  $60^{\circ}$  C., but for the glass usually employed it is not more than  $0.1^{\circ}$  C. and hence is of no great importance in ordinary technical measurements.

The thermodynamic scale was adopted as the standard by the German Government in 1924. This is obtained from a gas thermometer, which is

\* *Siemens Zeitschr.* Bd. 9. No. 10. Schaack und Lohmann. "Wasserstandsmessung auf hydrostatischer Grundlage."

considered as being filled with an ideal gas. Between  $0^{\circ}\text{C.}$  and  $450^{\circ}\text{C.}$  this scale is practically the same as that of the hydrogen thermometer.

In addition to mercury-in-glass thermometers, electrical resistance thermometers and thermo-electric pyrometers are used. For flue gas temperatures the optical pyrometer is employed.\*

\* On correct temperature measurements, reference should be made to O. Knoblauch und Hencky, *Anleitung zu genauen technischen Temperaturmessungen.*



# TECHNICAL THERMODYNAMICS

## CHAPTER I

### GASES

**Boyle's Law.** This law deals with the changes in pressure and volume of gases caused by mechanical compression or expansion at constant temperature. If a volume  $V_0$  of any gas at temperature  $t_0$ , absolute pressure  $p_0$ , and having specific volume  $v_0$ , be changed to  $V$ , so that at the end of the process the temperature is still  $t_0$ , then the absolute pressure is inversely proportional to the volume,

i.e.

$$p_0 \propto \frac{1}{V}$$

or

$$pV = p_0V_0 \text{ and } pv = p_0v_0.$$

The product of the pressure and volume is constant for different states of the gas, provided no change occurs in the temperature.

With respect to the specific densities of the gas, the law states that, at constant temperature, the specific densities are proportional to the absolute pressures, since

$$\frac{v_0}{v} \propto \frac{p}{p_0} = \frac{d}{d_0}$$

**Example 1.** Atmospheric air at 1.5 lb./in.<sup>2</sup> below atmospheric pressure and at 20° C. is compressed to 105 lb./in.<sup>2</sup> gauge pressure, without change in temperature. Find the volume compression-ratio. The barometer reading is 28.4" Hg.

The absolute initial pressure is  $0.491 \times 28.4 - 1.5 = 13.93 - 1.5 = 13.43$  lb./in.<sup>2</sup> abs.

The absolute final pressure is  $13.93 + 105 = 118.93$  lb./in.<sup>2</sup> abs.

Hence the volume ratio is

$$\frac{V}{V_0} = \frac{p_0}{p} = \frac{13.43}{118.93} = \frac{1}{9.56}$$

or the compression ratio is 9.56. The temperature level has no effect on this result.

**Example 2.** Find the density of air at 0° C. and at a pressure of 23.6" Hg, if its density at 0° C. and 30" Hg is 0.0808 lb./ft.<sup>3</sup>

$$\text{Density at 23.6" Hg} = \frac{23.6}{30} \times 0.0808 = 0.0635 \text{ lb./ft.}^3$$

## TECHNICAL THERMODYNAMICS

**Example 3.** A mass of air at 15 lb./in.<sup>2</sup> gauge pressure expands at constant temperature to three times its original volume. Find the final pressure. Barometer reading 21.6" Hg.

Absolute initial pressure =  $0.491 \times 21.6 + 15 = 10.6 + 15 = 25.6$  lb./in.<sup>2</sup> abs.

Hence the absolute final pressure is

$$p = \frac{25.6}{3} = 8.53 \text{ lb./in.}^2 \text{ abs.}$$

i.e.  $10.6 - 8.53 = 2.07$  lb./in.<sup>2</sup> below atmospheric pressure.

**Gay-Lussac's Law.** This law deals with the change in volume of gases when heated or cooled at constant pressure. If a mass of gas at any constant pressure be heated, its volume increases, for each degree centigrade rise in temperature, by  $\frac{1}{273}$  part of the volume occupied by the gas at 0° C. and at the same pressure. This law is valid for all gases, but only rigidly for the ideal gas state, from which the actual gases deviate more or less, the deviation depending on the pressure and temperature. The value of the expansion coefficient of an ideal gas is  $\frac{1}{273.09} = 0.0036618$ .

**FIRST FORM OF THE LAW.** If  $v_0$  is the volume at 0° C. the volume  $v$  at  $t_1$ ° C. is

$$v_1 = v_0 + v_0 \frac{t_1}{273} = v_0 \left( 1 + \frac{t_1}{273} \right)$$

For another temperature  $t_2 > t_1$  the volume would be

$$v_2 = v_0 + v_0 \frac{t_2}{273}$$

Subtraction gives

$$v_2 - v_1 = v_0 \left( \frac{t_2 - t_1}{273} \right)$$

(which is the absolute increase in volume in heating from  $t_1$  to  $t_2$ ).

**SECOND FORM.** The ratio of the volumes  $v_2$  to  $v_1$  gives the proportional volume change as

$$\frac{v_2}{v_1} = \frac{1 + \frac{t_2}{273}}{1 + \frac{t_1}{273}}$$

or

$$\frac{v_2}{v_1} = \frac{273 + t_2}{273 + t_1}$$

In this equation  $t_1$  and  $t_2$  are in degrees centigrade, hence the numbers 273, which appear in the numerator and denominator and which are added to  $t_1$  and  $t_2$ , must also represent degrees centigrade.

If the zero of the centigrade scale is chosen 273 degrees below the normal zero mark, a new scale ( $T$ ) is obtained in which the same temperature is expressed by adding 273.

Therefore,  $T_1 = 273 + t_1$   
 $T_2 = 273 + t_2$



This new scale is called the absolute temperature scale and  $T_1$ ,  $T_2$ ,  $T_3$  are called absolute temperatures.

We now have 
$$\frac{v_2}{v_1} = \frac{T_2}{T_1}$$

that is, during a constant pressure change the volumes of a given mass of gas are proportional to the absolute temperatures.

The term "absolute temperature" appears at first sight to be arbitrary. For calculations involving the use of Gay-Lussac's Law, however, this quantity is of the same significance as the absolute pressure when used in Boyle's Law. It is rather remarkable that the number 273 should apply to such widely different substances as  $H_2$ ,  $N_2$ ,  $O_2$ , and CO as well

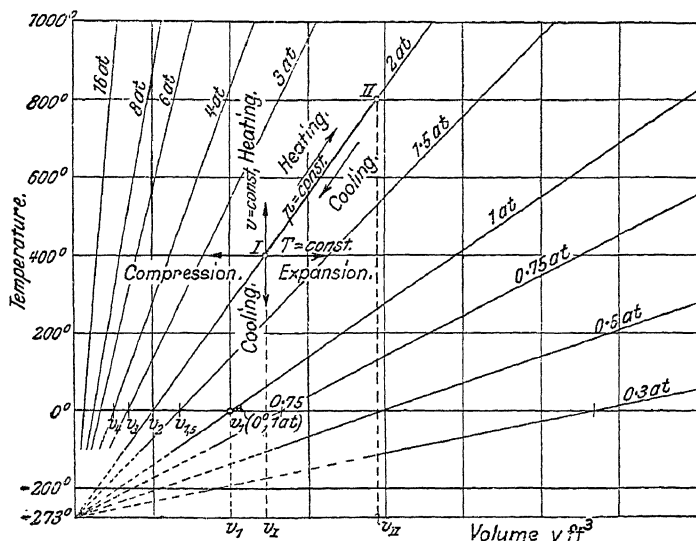


FIG. 5

as gas mixtures such as air. The fundamental basis of the conception of absolute temperature is explained on page 164.

Gay-Lussac's Law cannot hold good down to temperatures near absolute zero, since at  $T_2 = 0$  ( $t = -273$ ) the volume is  $v_2 = 0$ , as shown by this law. It is inconceivable that any substance can have a zero volume.

The law can also be expressed in terms of the densities; thus, for different temperatures but the same pressure, the densities of a given gas vary inversely as the absolute temperatures.

Since

$$\frac{v_2}{v_1} = \frac{d_1}{d_2}$$

we have

$$\frac{d_1}{d_2} = \frac{T_2}{T_1} \quad (p = \text{constant}).$$

If the temperatures ( $t$  or  $T$ ) are plotted to a base of volumes (Fig. 5), Gay-Lussac's Law will be represented by a straight line I-II passing through

the origin. For any other pressure the straight line has a different slope, but again passes through the origin. The slope increases as the pressure increases, since the volume occupied by the gas at increased pressures decreases when the temperature is the same. The law in its entirety is represented by an infinite number of straight lines, all radiating from the origin. Fig. 5 represents one of the simplest forms of state diagrams for gases, each point in the field corresponding to a definite gas state.

**Example 4.** Find the reduction in volume of a mass of gas when it is cooled from  $20^{\circ}\text{C.}$  to  $-20^{\circ}\text{C.}$ , the pressure remaining constant.

$$\text{Here} \quad \frac{v_{+20}}{v_{-20}} = \frac{273 + 20}{273 - 20} = \frac{295}{253} = \frac{1.16}{1} = \frac{1}{0.862}$$

Hence the volume is 13.8% less or the density 16% greater when the gas is cooled from  $20^{\circ}\text{C.}$  to  $-20^{\circ}\text{C.}$

**Example 5.** If the flue gases from a furnace are cooled from  $1200^{\circ}\text{C.}$  to  $250^{\circ}\text{C.}$ , find the reduction in volume.

$$\text{Here} \quad \frac{v_{1200}}{v_{250}} = \frac{273 + 1200}{273 + 250} = \frac{1473}{523} = 2.82$$

The final volume is 65.5% less than the initial. This gives a measure of the permissible reduction in area of the flues towards the chimney.

**Example 6.** Find the proportional increase in weight of air, contained in a vessel, when the air temperature is reduced from  $50^{\circ}\text{C.}$  to  $10^{\circ}\text{C.}$

The densities at  $10^{\circ}$  and  $50^{\circ}$  are in the ratio

$$\frac{273 + 50}{273 + 10} = 1.14$$

which is therefore the proportional increase by weight. The percentage increase is 14%.

**Combined Boyle-Gay-Lussac Law.\*** The density of a gas is, from Boyle's Law, directly proportional to the pressure, and, from Gay-Lussac's Law, inversely proportional to the absolute temperature. Hence if  $d_1$  is the density at  $t_1^{\circ}\text{C.}$  and  $p_1$  lb./in.<sup>2</sup> abs., then, from Boyle's Law, the density at  $t_1^{\circ}\text{C.}$  and  $p$  lb./in.<sup>2</sup> abs. is  $d_1 \frac{p}{p_1}$ . From Gay-Lussac's Law it is greater than  $d_1 \frac{p}{p_1}$  when the temperature is changed to  $t^{\circ}\text{C.}$  (keeping  $p$  constant), in the ratio

$$\frac{t_1 + 273}{t + 273} \text{ or } \frac{T_1}{T}$$

hence

$$d = d_1 \frac{p}{p_1} \frac{T_1}{T}$$

This result is valid, since Boyle's Law holds good at any temperature and Gay-Lussac's Law holds good at any pressure. See page 317 for deviations from these laws.

$$\text{Since} \quad d = \frac{1}{v} \text{ and } d_1 = \frac{1}{v_1}$$

\* Termed Boyle Charles Law by English writers.

## GASES

we have also

$$v = v_1 \frac{p_1}{p} \frac{T}{T_1}$$

and since

$$v = \frac{V}{W} \text{ and } v_1 = \frac{V_1}{W}$$

$$V = V_1 \frac{p_1}{p} \frac{T}{T_1}$$

**Example 7.** The density of dry air at  $0^\circ \text{C.}$  and  $30'' \text{ Hg}$  is  $0.0808 \text{ lb./ft.}^3$ . Find the value at  $20^\circ \text{C.}$  and  $28'' \text{ Hg}$ .

$$\text{Here } d = \frac{0.0808 \times 28 (273 + 0)}{30(273 + 20)} = 0.0702 \text{ lb./ft.}^3$$

**Example 8.**  $32 \text{ ft.}^3$  of air are contained in a vessel in which the vacuum gauge reading is  $23'' \text{ Hg}$  (bar. =  $29'' \text{ Hg}$ ). The temperature of the air is  $17^\circ \text{C.}$  Find the weight of air in the vessel.

$$\text{Density is } \sim - \frac{0.0808(29 - 23) 273}{30 \times 290} = 0.01523 \text{ lb./ft.}^3$$

Hence, weight of air =  $W = dV = 0.01523 \times 32 = 0.487 \text{ lb.}$

**Example 9.** A gas engine uses  $19.4 \text{ ft.}^3$  of town gas per h.p. hour. Pressure of gas  $27.5'' \text{ Hg}$  and temperature  $20^\circ \text{C.}$  Find the corresponding volume of this amount of gas when reduced to  $0^\circ \text{C.}$  and  $30'' \text{ Hg}$ . (S.T.P. conditions.)

Volume of gas at  $0^\circ \text{C.}$  and  $30'' \text{ Hg}$  is given by

$$V_0 = V \frac{p}{p_0} \frac{T_0}{T} = \frac{19.4 \times 27.5}{30} \times \frac{273}{293} = 16.58 \text{ ft.}^3$$

The change in volume and density of gases is of very great importance in airship navigation, both for balloons and dirigibles. The volume of gas in the balloon must correspond to the free air volume so that the gas pressure in the balloon is always equal to, or slightly greater than, that of the surrounding air. This air pressure varies with the altitude, the temperature, and state of the weather. The balloon structure is not capable of withstanding any considerable difference in pressure.

Any cause which produces a rise in the inner pressure of the balloon above the outer means, therefore, a reduction in the quantity of gas carried. Causes which effect this rise are: reduction of the outer air pressure as the balloon rises, heating of the contents of the balloon by radiation, and heating by its passage through warmer zones.

The reverse effect is produced when the pressure inside tends to become less than that outside, such as occurs when the altitude of the balloon is decreased or when the gas in the balloon is cooled. If no gas is supplied in these changes the balloon tends to collapse, due to the reduced internal volume.

**Example 10.** How many cubic feet of gas must escape from a stretched balloon containing  $35,320 \text{ ft.}^3$  if the outer air pressure falls from  $27.5'' \text{ Hg}$  to  $19.7'' \text{ Hg}$  and the temperature rises from  $10^\circ \text{C.}$  to  $20^\circ \text{C.}$

The volume of gas in the final state would be

$$\text{i.e.} \quad V = \frac{35320 \times 27.5 \times 293}{19.7 \times 283} = 51,000 \text{ ft.}^3$$

Since the capacity of the balloon is only 35,320 ft.<sup>3</sup> the volume of gas (at the end conditions) which escapes is 51,000 - 35,320 = 15,680 ft.<sup>3</sup> Taking the initial conditions the volume lost is

**Characteristic equation of gases.** The relation between the pressure, volume, and temperature shown on page 5,

$$v_1 = v \frac{p}{p_1} \frac{T_1}{T}$$

can be written in the form

$$\frac{pv}{T} = \frac{pv_1}{T_1}$$

The value of  $\frac{pv}{T}$  is therefore constant, for the same gas, whatever the values of  $p$ ,  $v$ , and  $T$ . If this value be denoted by  $R$ , then

$$\frac{pv}{T} = R = \text{constant}$$

or

$$pv = RT \quad (p \text{ in lb./ft.}^2 \text{ and } v \text{ in ft.}^3)$$

This is called the "characteristic equation" of the gas. It establishes the general relation between the three quantities  $p$ ,  $v$ , and  $T$  which determine the state of the gas. This equation is always valid no matter what the pressure, volume, or temperature may be. It will be recognized that if two of these quantities are known the third can be determined.

The value of the constant  $R$  depends on the particular gas dealt with.

It is given by  $R = \frac{pv}{T} = \frac{p}{Td}$  and can be calculated if the density ( $d_0$ ) at any pressure ( $p_0$ ) and any temperature ( $T_0 = 273 + t_0$ ) is known. If, for example,  $t_0 = 0^\circ \text{C.}$  and  $p_0 = 14.7 \times 144 \text{ lb./ft.}^2 \text{ abs.}$ , then

$$R = \frac{14.7 \times 144}{7.75} = 27.3 \text{ ft.-lb./lb.}$$

For dry air at  $0^\circ \text{C.}$  and 30" Hg,  $d_0 = 0.0808 \text{ lb./ft.}^3$

$$\text{hence} \quad \frac{7.75}{0.0808} = 95.8 \text{ ft.-lb./lb.}$$

Hence if the gas constant is known, the density can be found for any state. See page 92 for the mechanical significance of the gas constants, and page 12 for a tabulated list of constants for different gases, as well as further discussions on the value of  $R$ .

It is important to note that in the characteristic equation, and in all relations deduced from it later, the pressure  $p$  must be expressed as an absolute pressure (in lb./ft.<sup>2</sup>). Only in cases involving a pressure ratio can other units, such as lb./in.<sup>2</sup> abs. and mercury column readings, be used without conversion.

The characteristic equation in the form given above holds for 1 lb. of gas, since it contains the volume  $v$  of 1 lb. If  $v$  is replaced by any volume  $V$  weighing  $W$  lb., then, since  $v = \frac{V}{W}$ ,

we have

$$pV = WRT$$

In this general form, the equation holds for any weight. It is, however, not correct to say that the condition of a gas is given when two of the

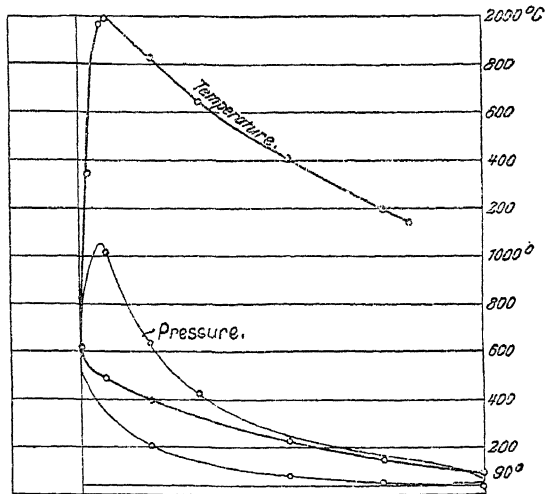


FIG. 6

three magnitudes—pressure, volume, and temperature—are known; in place of volume, the term “specific volume” should be used. If the pressure, temperature, and gas constant are known, the specific volume is, of course, found from  $pv = RT$ . If, on the other hand, only the pressure and total volume  $V$  are given, for example, by the indicator diagram of an internal combustion engine, then neither the temperature nor the specific volume can be determined. Only the product  $WRT$  is known, or, if  $R$  is known, the product  $WT$ . Hence, to find  $T$  and  $v$  or  $d$  the weight of the working gases has to be found. The determination of this weight is laborious and not usually obtained by any simple direct method.

If, however, the temperature at any point on the diagram is known, approximately, the temperatures at all other points can be found from

provided no change occurs in the weight of the gas and in the value of the gas constant  $R$ . The percentage error in  $T$  is the same as that in the assumed value  $T_1$ .

This method has been adopted in plotting the temperature changes on the indicator diagram of a gas engine (shown in Fig. 6) during compression, explosion, and expansion. The temperature at the beginning of compression is assumed to be  $90^{\circ}\text{C}$ . Since, in the expression for  $T$  given above, only ratios of pressure and volume are required, any scales may be used for these quantities. The gas constant is assumed to suffer no change during the complete process (in order to simplify the calculations) and hence its value is not required.

Actually the gas constant  $R_e$  during expansion is smaller than the constant  $R_c$  during compression.

For compression we have  $p_1 V_1 = W R_c T_1$   
and for expansion  $p V = W R_e T$

hence 
$$T = T_1 \frac{R_c}{R_e} \times \frac{p}{p_1} \times \frac{V}{V_1}$$

giving a slightly higher value for  $T$  than that given by assuming  $R_e = R_c$ , since  $R_e < R_c$  (see page 24).

**Graphical representation of temperature changes.** If the ordinate at  $A$  be chosen to represent the initial temperature  $T_1$  on the given  $pv$  curve, the temperature at any other point  $B$  on this curve can be conveniently found (as shown in Fig. 7) by drawing an horizontal through  $B$  to cut the vertical  $AE$  in  $F$ . By projecting the radial line  $OF$  to cut the vertical  $DB$  the point  $C$  is obtained, and  $DC$  gives the required temperature.

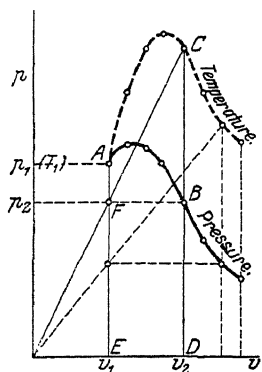


FIG. 7

PROOF.  $\frac{CD}{FE} = \frac{v_2}{v_1}$ , hence  $CD = \frac{p_2 v_2}{v_1}$  and

since we also have  $CD =$

therefore  $\frac{CD}{p_1} = \frac{T_2}{T_1}$  or  $\frac{CD}{AE} = \frac{T_2}{T_1}$

**Analyses of gas mixtures by weight and by volume.** The proportions in which the constituents of a gas mixture exist can be given by weight or by volume. While the former requires no explanation, the latter should be regarded as follows. The individual gases are considered as being separated and reduced to equal pressures and temperatures. The ratio  $v$  of the volume occupied by any single gas to the sum of all the individual volumes then gives the proportional volume occupied by this gas.

The proportions by weight  $w_1, w_2, w_3 \dots$  can be calculated from the proportions by volume  $v_1, v_2, v_3 \dots$  (and *vice versa*), if the densities or the molecular weights of all the constituents are known.

Thus the total weight of unit volume, i.e. the density of the mixture, is

$$d = d_1 v_1 + d_2 v_2 + d_3 v_3 \dots \quad (1)$$

hence the proportional weights of the individual gases are

$$w_1 = \frac{d_1 v_1}{d} \quad \text{and} \quad w_2 = \frac{d_2 v_2}{d} \quad (2)$$

or, if the values  $d_1, d_2 \dots$  are replaced by the proportionally equal molecular weights  $m_1, m_2 \dots$  (page 11),

$$w_1 = \frac{m_1 v_1}{m_1 v_1 + m_2 v_2 + \dots}, w_2 = \frac{m_2 v_2}{m_1 v_1 + m_2 v_2 + \dots} \quad (3)$$

**Mean or apparent molecular weight.** From equation (1) it follows immediately if  $d_1, d_2, d_3 \dots$  and  $d$  are replaced by the proportionally equal molecular weights  $m_1, m_2, m_3 \dots$  that the mean or apparent molecular weight of the mixture is

$$m = m_1 v_1 + m_2 v_2 + m_3 v_3 + \dots \quad (4)$$

The density (page 11) is then given by

$$d = \frac{m}{359} \text{ (at } 0^\circ \text{ C. and } 14.7 \text{ lb./in.}^2 \text{ abs.)} \quad (5)$$

**General relations for all gases.** When two substances combine chemically the reaction always occurs according to the law of fixed weight proportions discovered by Dalton. If different elements be denoted by  $A, B, C, D \dots$ , and the invariable proportions by weight with which they unite be denoted by  $a, b, c, d \dots$ , Dalton's Law states that when  $A$  and  $B$  combine to form a new substance,  $a$  parts by weight of  $A$  unite with  $b$  parts by weight of  $B$  to form  $a + b$  parts by weight of the new substance. This always occurs, even if the substances are mixed in other proportions by weight before uniting. In this case a proportion of the substance which is in excess of that demanded by the ratio  $a/b$  remains unchanged (e.g. combustion with excess air). In splitting up a substance composed of  $A$  and  $B$ , the separated products always have the ratio  $a/b$ , whether the whole or only part of the mass be split up.

It follows also from Dalton's Law that if chemical combination occurs between  $A$  and  $C$  or  $D$ , then  $a$  parts by weight of  $A$  unite with  $c$  parts by weight of  $C$  or  $d$  parts by weight of  $D$ . New substances are thus obtained in which  $A$  is contained in the ratios  $\frac{a}{b}, \frac{a}{c}, \frac{a}{d} \dots$ . Further, if  $B$  unites with  $C$ , the new substance contains the original substances in the proportion  $b/c$ . If, for any arbitrarily chosen substance, the fundamental number  $a$  be adopted, a series of different numbers of combining weights  $a, b, c, d$ , etc. (equivalent weights), are obtained for the elements, from which two or more can combine to form new substances. It has also been found that whole multiples of  $a, b, c, d \dots$  can combine, i.e.  $ma, nb, \dots$  parts by weight of  $A, B, \dots$  where  $m$  and  $n = 1, 2, 3$ , or other whole number. (Law of constant and multiple proportions.)

The fundamental comparative number now chosen is  $a = 16$  for oxygen, and is called the atomic weight of oxygen. Every other element has its own definite constant atomic weight ( $b, c, d, \dots$ , etc.).

If, along with the usual chemical notation for a compound substance, the atomic weights are also considered, the proportions by weight are immediately determined. Thus, for carbon monoxide,

since  $\text{CO} = \text{C} + \text{O}$

it follows that, since the atomic weight of C is 12 and of O 16, there are 12 parts by weight of carbon and 16 parts by weight of oxygen in 28 parts by weight of carbon monoxide.

In carbon dioxide according to



or

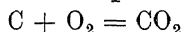


i.e. 44 parts by weight of carbon dioxide are made up of 12 parts of carbon and 32 parts of oxygen. Hence, in burning 1 lb. of carbon,  $\frac{32}{12} = 2.667$  lb. of oxygen are necessary, and the weight of  $\text{CO}_2$  formed is  $\frac{44}{12} = 3.667$  lb.

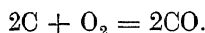
While the smallest parts of the elements are called atoms, the smallest parts of compounds of the elements are called molecules. The molecular weight of a compound is that given by the sum of the atomic weights contained in the compound. From this it follows that the molecular weight of CO is  $12 + 16 = 28$ , of  $\text{CO}_2$   $12 + 2 \times 16 = 44$ , and of water  $\text{H}_2\text{O}$ ,  $2 \times 1.008 + 16 = 18.016$ .

Double atoms, such as those of hydrogen in water or oxygen in carbon dioxide, are found also in the case of certain elements in their free gaseous state, and are then called molecules. Examples of these are the diatomic gases hydrogen, oxygen, and nitrogen and, as such, are symbolized by  $\text{H}_2$ ,  $\text{O}_2$ , and  $\text{N}_2$ . On the other hand, argon, helium, and mercury are examples of monatomics, even when in the gaseous state.

Hence, in the combustion of solid carbon with gaseous oxygen, it is preferable to write the reaction equation as



or in the formation of carbon monoxide

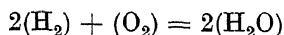


In the first example one atom of carbon unites with one molecule of oxygen, i.e.  $\text{O}_2$ , to form one molecule of  $\text{CO}_2$ . In the second case two atoms of carbon unite with one molecule of oxygen to form two molecules of CO.

For the proportions by weight, it is, of course, immaterial whether the first or second method be adopted, but, as shown by the following, the second method must be adopted when considering the proportions by volume.

A second fundamental law, discovered by Gay-Lussac, deals with the proportions by volume when two or more gases unite to form a new substance. Gay-Lussac found that gases unite in the simplest proportions by volume. In every reaction either equal volumes or whole multiples of these volumes unite, and the volume of the new gas formed can be equal to, or a whole multiple of, one of the original gas volumes. Thus, 2 volumes of hydrogen and 1 volume of oxygen always unite to form 2 volumes of steam, all three substances being considered at the same pressure and temperature.

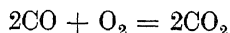
The reaction equation



therefore indicates the proportions by volume if the bracketed letters represent equal volumes of the substances. The equation has exactly the same construction as the second form given above, and this rule, according to which the weight equation can also be regarded as showing volumetric proportions, is universally applicable to gaseous substances. If, therefore, the reaction equation for the weight proportions is so written that it shows the number of molecules of the individual gases, then it



also serves to give the volume proportions. Thus, when carbon monoxide burns with oxygen to form carbon dioxide, then, from the equation



we can say that 2 units by volume of carbon monoxide burn with 1 unit by volume of oxygen to form a volume of carbon dioxide equal to the original volume of carbon monoxide. In this reaction, therefore, there is a reduction from 3 units to 2 units by volume.

By combining Dalton's and Gay-Lussac's Laws along with the atomic and molecular hypotheses, Avogadro's principle is established.

According to this the number of molecules found in equal volumes of different gases, at the same temperature and pressure, is the same. The densities  $d$  of gases are therefore proportional to their molecular weights.

For any two gases 1 and 2

$$\overline{d_2}$$

and since

$$d_1 = \frac{1}{v_1} \text{ and } d_2 = \frac{1}{v_2}$$

it follows that

$$m_1 v_1 =$$

$v_1$  and  $v_2$  are the volumes occupied by 1 lb., and hence  $m_1 v_1$  and  $m_2 v_2$  can be regarded as the volumes occupied by  $m_1$  and  $m_2$  lb. respectively of these gases. If  $m$  is the molecular weight in general of any gas, then a weight of  $m$  lb. of this gas is called the "pound molecule" (or "mol."). Hence  $m_1 v_1$ ,  $m_2 v_2$ , . . . are the volumes of 1 mol., and these are equal for all gases at the same temperature and pressure.

We can now state that different gases whose weights are in proportion to their molecular weights occupy equal volumes. Thus 32 lb. of oxygen occupy the same volume as 28.08 lb. of  $\text{N}_2$  or 28 lb. of CO or 44 lb. of  $\text{CO}_2$  provided they are at the same temperature and pressure. This volume is determined from the weight of 1 ft.<sup>3</sup> of oxygen at 0° C. and 14.7 lb./in.<sup>2</sup> abs. With  $d_{\text{O}_2} = 0.0891842$  lb./ft.<sup>3</sup> the volume of 1 lb. of  $\text{O}_2$  is

$$v = 0.0891842$$

The molecular weight of oxygen is  $m = 32$ , and hence 1 mol. of oxygen = 32 lb. and at 0° C. and 14.7 lb./in.<sup>2</sup> abs. this occupies a volume

$$\text{of } \frac{32}{0.0891842} = 359 \text{ ft.}^3$$

The volume of 1 mol. of any other gas has the same value.

The densities of all gases can thus be determined from

$$m \qquad \qquad \qquad m \qquad \qquad \qquad (6)$$

at 0° C. and 14.7 lb./in.<sup>2</sup> abs. If, on the other hand, the density of a gas is known, its molecular weight can be found. This is of special use in the case of gas mixtures, since  $m = 359d$  and, when the analysis by volume is known,  $m = 359 (v_1 d_1 + v_2 d_2 + \dots)$ .

The following table gives the molecular weights and densities of the more important technical gases—

Gas	Symbol	Mol. Weight $m$	Density lb./ft. <sup>3</sup> at 0° C. and 14.7 lb./in. <sup>2</sup> abs. $d$	Gas Constant $R$
Oxygen . . .	O <sub>2</sub>	32	0.0891	86.9
Hydrogen . . .	H <sub>2</sub>	2.016(2)	0.00562	1380
Nitrogen . . .	N <sub>2</sub>	28.08	0.0783	98.9
Carbon monoxide . . .	CO	28	0.0781	99.3
Carbon dioxide . . .	CO <sub>2</sub>	44	0.1226	63.2
Steam . . .	H <sub>2</sub> O	18.016	—	154.2
Methane . . .	CH <sub>4</sub>	16.03	0.0447	173.5
Air . . .	—	29(28.95)	0.0808	96.0
Town gas . . .	—	11.5	0.0322	242.0
Generator gas . . .	—	22.4-26.9	0.0624-0.0749	103-124

For a given weight  $W$  of a gas the characteristic equation is (from page 7)

$$pV = WRT$$

If  $W = m$  (weight of 1 mol.) then for  $t = 0^\circ \text{C.}$  or  $T = 273^\circ \text{C. abs.}$  and  $p = 144 \times 14.7 \text{ lb./ft.}^2 \text{ abs.}$  the volume is  $V = 359 \text{ ft.}^3$

Hence the equation becomes

$$144 \times 14.7 \times 359 = mR \times 273$$

giving

$$mR = 2780 = \bar{R}$$

Hence the gas constant of any gas having a molecular weight  $m$  or of a gas mixture having an apparent molecular weight  $m$  (page 9) can be found from

$$R = \frac{2780}{m} \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

The specific density of a gas with respect to air at the same pressure and temperature is

$$\delta = \frac{d}{d_A}$$

and since

$$d = \frac{p}{RT} \text{ and } d_A = \frac{p}{R_A T}$$

$$\delta = \frac{R_A}{R} = \frac{96}{R} \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

This specific density (with respect to air) is thus independent of the temperature and pressure and is a constant for any given gas. From the relation between  $R$  and the molecular weight we also have

$$\delta = \frac{m}{28.95}$$

which is also given directly by Avogadro's principle.

Thus the specific density of steam is

$$\frac{96}{154.2} = 0.622 \text{ or } \frac{18.016}{28.95} = 0.622$$

In this way the specific density of a substance having the same molecular weight in the gaseous as in the solid, liquid, or other state, is found. Conversely, the molecular weight in the gaseous state can be found when the specific density is known from an experimental determination.

Denoting the volume of 1 mol. at a pressure  $p$  lb./ft.<sup>2</sup> abs. and absolute temperature  $T$  by  $\bar{v}$  gives

$$p\bar{v} = mRT$$

or

$$p\bar{v} = 2780T$$

This equation holds for any gas or mixture of gases when the weight is  $m$  lb. For a weight of  $N$  mols. it becomes

$$pN\bar{v} = NmRT$$

or

$$pV = N\bar{R}T \text{ with } V \text{ in ft.}^3 \text{ and } p \text{ in lb./ft.}^2 \text{ abs.}$$

In heat engine calculations this form of the characteristic equation is not much used, but it is preferable in physical-chemical calculations. The number 2780 is called the universal gas constant, since it is valid for all gases (see page 93).

**Equation for gas mixtures. (Dalton's Law.)** The laws of Boyle and Gay-Lussac hold for mixtures of gases just as for single gases. Hence the characteristic equation of a gas mixture has the form

$$pv = RT \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

The method of finding the gas constant for a mixture having a definite analysis (e.g. atmospheric air or a mixture of air and fuel gas or a flue gas mixture) from the gas constants of the constituents will now be considered.

For this, two additional fundamental principles are required—

1. In a mixture of gases each gas obeys its own characteristic equation as if the other constituents were absent.

2. The pressure  $p$  of the mixture is to be regarded as equal to the sum of the pressures ( $p_1, p_2, p_3 \dots$ ) of the constituents, so that

$$p = p_1 + p_2 + p_3 + \dots \quad . \quad . \quad . \quad (10)$$

$p_1, p_2, p_3 \dots$  are called the partial pressures. They are those pressures which would be exerted by the constituents if they could be separated from one another, while keeping the volume and temperature constant (e.g. by chemical absorption). In air, for example, the oxygen and nitrogen have different pressures, each being smaller than the atmospheric pressure. On the other hand, all the constituents of a mixture occupy the same volume (i.e. the whole volume) and are at the same temperature. The specific volumes of the constituents are thus different.

If now the weights of the constituents in a mixture of gases weighing  $W$  lb. are  $W_1, W_2, W_3$  lb., so that

$$W = W_1 + W_2 + W_3 \dots$$

then for the individual gases (from the first principle given above)

$$p_1 V = W_1 R_1 T$$

$$p_2 V = W_2 R_2 T$$

$$p_3 V = W_3 R_3 T$$

Summation gives

$$(p_1 + p_2 + p_3)V = (W_1 R_1 + W_2 R_2 + W_3 R_3)T$$

and since

$$p_1 + p_2 + p_3 = p$$

$$pV = (W_1 R_1 + W_2 R_2 + W_3 R_3)T \quad . \quad . \quad . \quad (11)$$

But, for the mixture as a whole, the general characteristic equation

$$pV = WR_m T$$

with the still unknown mixture constant  $R_m$ , is valid.

By equating the two expressions it follows that

$$WR_m = W_1 R_1 + W_2 R_2 + W_3 R_3$$

$$\text{or} \quad R_m = \frac{W_1 R_1}{W} + \frac{W_2 R_2}{W} + \frac{W_3 R_3}{W} \quad . \quad . \quad . \quad (12)$$

Hence if the individual gas constants are multiplied by the corresponding proportional weights and are then added together, the result gives the constant for the mixture, i.e. if

$$\frac{W_1}{W} = w_1 \quad \frac{W_2}{W} = w_2 \dots \text{etc.}$$

$$\text{we have} \quad R_m = w_1 R_1 + w_2 R_2 + w_3 R_3 + \dots \quad . \quad . \quad . \quad (13)$$

From page 12, however, the value of  $R_m$  can also be found from the known analysis by volume or weight of the gas without using the individual gas constants. In this form

$$R_m = \frac{2780}{m} = \frac{2780}{m_1 v_1 + m_2 v_2 + m_3 v_3} \quad . \quad . \quad . \quad (14)$$

The density of the gas mixture, expressed in terms of the densities of the constituents, is found from

$$d_m = \frac{R_m T}{p}$$

and

$$R_1 = \frac{p}{d_1 T}, \quad R_2 = \frac{p}{d_2 T}$$

giving

$$d_m = \frac{1}{\frac{w_1}{d_1} + \frac{w_2}{d_2} + \frac{w_3}{d_3}} + \dots \quad . \quad . \quad . \quad (15)$$

In this, all the densities must be reduced to the same pressure and temperature, e.g. 0° C. and 14.7 lb./in.<sup>2</sup> abs. From page 11, however, the value is also given by

$$d_m = \frac{m}{359} = \frac{m_1 v_1 + m_2 v_2 + m_3 v_3 + \dots}{359} \quad . \quad . \quad (16)$$

for which a knowledge of the densities of the constituents is not required.

**Partial pressure values.** From the characteristic equations

$$p_1 = \frac{W_1 R_1 T}{V}, \quad p_2 = \frac{W_2 R_2 T}{V} \quad \text{etc.}$$

from which the partial pressures can be found. They can, however, be expressed as fractions of the total pressure, which is given by

$$p = \frac{WR_m T}{V}$$

Hence, by division, 
$$\frac{p_1}{p} = \frac{W_1 R_1}{W R_m}$$

or 
$$p_1 = w_1 \frac{R_1}{R_m} p \quad . \quad . \quad . \quad . \quad . \quad (17)$$

and in the same way 
$$p_2 = w_2 \frac{R_2}{R_m} p$$

$$p_3 = w_3 \frac{R_3}{R_m} p$$

where  $w_1, w_2, w_3 \dots$  are the proportions by weight of the separate gases. Hence the constituents of a gas mixture exert pressures which vary as their fractional weights and as their gas constants.

The corresponding expressions for the partial pressures in terms of the analysis by volume are simpler than those just found. If the constituents are considered as being brought to the total pressure ( $p$ ) of the mixture, then, for the first gas,

$$pV_1 = W_1 R_1 T$$

and, for the same gas in the mixture,

$$p_1 V = W_1 R_1 T$$

hence

$$pV_1 = p_1 V$$

or

$$p_1 = \frac{pV_1}{V} \quad . \quad . \quad . \quad . \quad . \quad (18)$$

Similarly

$$p_2 = p \frac{V_2}{V} \text{ and } p_3 = p \frac{V_3}{V}, \text{ etc.}$$

The proportional partial pressures  $p_1/p, p_2/p \dots$  etc., are thus the same as the corresponding proportional volumes.

**Example 11.** By weight, air consists of 23.2 parts of oxygen and 76.8 parts of nitrogen in 100 parts. Express the pressures exerted by oxygen and nitrogen in mm. of Hg when the air pressure is 760 mm.

$$\text{From equation (7) } R_{O_2} = \frac{2780}{32} = 86.9 \text{ and } R_{N_2} = \frac{2780}{28.08} = 98.9$$

and hence the gas constant for air is

$$\begin{aligned} R_A &= 0.232 \times 86.9 + 0.768 \times 98.9 \\ &= 20.0 + 76.0 = 96.0 \end{aligned}$$

hence

$$p_{O_2} = \frac{0.232 \times 86.9}{96} p = 0.21p$$

and

$$p_{N_2} = \frac{0.768 \times 98.9}{96} p = 0.79p$$

Hence in the example the pressure of the oxygen is  $0.21 \times 760 = 159.8$  mm. Hg, and of the nitrogen  $0.79 \times 760 = 600.2$  mm. Hg, and the volumetric analysis of the air is  $V_{O_2} = 0.21$   $V_{N_2} = 0.79$ .

**Example 12.** Find the partial pressures and the gas constant for a mixture of  $1.3 \text{ ft.}^3$  of air and  $1 \text{ ft.}^3$  of generator gas if the latter has a density of  $0.0749 \text{ lb./ft.}^3$  at  $0^\circ \text{C.}$  and  $14.7 \text{ lb./in.}^2$  abs. (giving a gas constant,  $R_g = 103.3$ ).

For the generator gas the partial pressure expressed as a fraction of the total pressure is  $\frac{1}{2.3} = 0.435$ , and for the air  $\frac{1.3}{2.3} = 0.565$ .

The mean molecular weight of the generator gas is

$$m_1 = 359d_1 = 359 \times 0.0749$$

and of the air

$$359 \times 0.0808$$

hence, for the mixture,

$$\begin{aligned} m &= 359 \times 0.0749 \times 0.435 + 359 \times 0.0808 \times 0.565 \\ &= 11.7 + 16.4 = 28.1 \text{ (see page 11)} \end{aligned}$$

The gas constant is therefore

**Example 13.** Find the partial pressure in mm. of Hg exerted by water vapour in moist air at 760 mm. Hg pressure when the amount of vapour per lb. is  $0.005 \text{ lb.}$  (assuming that the air temperature is not less than  $6^\circ \text{C.}$ ). Gas constant for superheated water vapour  $R_w = 154.2$ .

The vapour pressure, by equation (17), is

$$p' = \frac{0.005}{1} \times \frac{154.2}{96} p = 0.008p$$

or, in mm. Hg  $0.008 \times 760 = 6.08$

(Strictly speaking, in place of 96, the slightly larger constant for moist air should be taken.)

**Moist Air.** Atmospheric air always contains a certain amount of water vapour, which is unsaturated (superheated) when the atmosphere is clear. It is allowable and customary to regard this vapour, which exerts a very small pressure, as a gaseous addition, provided it does not become saturated or wet, in which case mist or clouds are formed.

The border state of saturation—in which not only the vapour but also the air is said to be saturated (with vapour)—must occur, in accordance with the properties of water vapour, as soon as  $1 \text{ ft.}^3$  of the air contains as much vapour by weight ( $d$ ,  $\text{lb./ft.}^3$  from steam tables) as corresponds either to the temperature of the air or of the vapour, since these are the same. In this state the vapour exerts the greatest pressure possible at the existing temperature. The existence of more vapour in the air could only be imagined if the vapour pressure rose above this value. Saturated air, or any other gas saturated with water vapour, contains, therefore, a definite weight of vapour in  $1 \text{ ft.}^3$ , this weight depending only on the temperature and not on the pressure of the air, and is identical with the weight of  $1 \text{ ft.}^3$  of saturated vapour at the temperature of the air.

If the water vapour in the air is wet, its weight per  $\text{ft.}^3$  is greater than



The characteristic equation holds for the unsaturated state of the vapour

$$p'v' = R_v(273 + t)$$

and in the saturated state at the same temperature

$$R_v(273 + t)$$

hence

$$p'v' = p_s v_s$$

where  $v'$  and  $v_s$  are the volumes of 1 lb. of vapour in the unsaturated and saturated states.

But

$$v' = \frac{1}{d'} \text{ and } v_s = \frac{1}{d_s}$$

hence

$$\frac{d'}{d_s} = \frac{p'}{p_s} = \phi \quad . \quad . \quad . \quad (20)$$

It is only necessary therefore, when  $p'$  is known, to find  $p_s$  corresponding to  $t$  from steam tables in order to determine the relative humidity.

The weight  $d'$  of vapour in 1 ft.<sup>3</sup> is given by

$$d' = \phi d_s = \frac{p'}{p_s} d_s \quad . \quad . \quad . \quad (21)$$

in which  $d_s$  is taken from the tables.

The pressure of the air alone contained in the same space is  $p - p'$  and hence its weight is

$$d_a = 0.0808 \frac{p - p'}{760} \times \frac{273}{273 + t}$$

where  $p - p'$  is in mm. of Hg.

From this it follows finally that the weight of 1 ft.<sup>3</sup> of moist air (density) is

$$d = d' + d_a = \phi d_s + 0.0808 \frac{p - p'}{760} \times \frac{273}{T} \quad . \quad (22)$$

$$\text{or} \quad d = \phi d_s + 0.029 \frac{p - p'}{T} \quad . \quad (23)$$

This gives the gas constant for moist air as

$$R = \frac{p}{d(273 + t)}$$

with  $p$  in lb./ft.<sup>2</sup>

$R$  is obtained more simply as follows. By volume the moist air consists of  $\frac{p'}{p}$  parts of water vapour and  $1 - \frac{p'}{p}$  parts of dry air. Hence its mean molecular weight is

$$m = \frac{p'}{p} m_{H_2O} + \left(1 - \frac{p'}{p}\right) m_A \quad . \quad . \quad . \quad (24)$$

$$= 18.016 \frac{p'}{p} + \left(1 - \frac{p'}{p}\right) 28.95$$

$$= 28.95 - 10.93 \frac{p'}{p} \quad . \quad (25)$$



Hence moist air is always lighter than dry air at the same pressure and temperature.

From  $m$  the gas constant is given by

$$28.95 - 10.93 \frac{\bar{p}'}{p}$$

and the density (at  $0^\circ \text{C.}$  and  $14.7 \text{ lb./in.}^2 \text{ abs.}$ ) by

$$d_0 = \frac{m}{359} = 0.0808 - 0.0304 \frac{p'}{p} \quad (27)$$

and for  $t^\circ \text{C.}$  and  $B$  inches mercury,

$$d = d_0 \frac{273}{273 + t} \times \frac{B}{30} \quad (28)$$

The weight of vapour in 1 lb. of moist air is, from equation (17),

$$w = \frac{p'}{p} \times \frac{R}{R'}$$

Hence, from equation (26), and with  $R' = \frac{2780}{18.02}$

$$w = \frac{18.02}{28.95 - 10.93 \frac{p'}{p}} \times \frac{p'}{p}$$

or

$$w = \frac{0.622}{1 - 0.378 \frac{p'}{p}} \times \frac{p'}{p} \quad (29)$$

For small values of  $\frac{p'}{p}$  this is, approximately,

$$w = 0.622 \frac{p'}{p} \quad (30)$$

The ratio of the weights of vapour and air in moist air is given by the ratio of the two terms on the right of equation (24),

$$x = \frac{m_{\text{H}_2\text{O}} p'}{m_{\text{a}}(p - p')} = \frac{0.622 p'}{p - p'}$$

*The dew point* is that temperature to which the unsaturated air must be cooled in order to make it saturated, and is found from steam tables by reading the temperature corresponding to the measured vapour pressure  $p'$ .

**Example 14.** The vapour pressure found by means of a wet and dry bulb hygrometer was  $0.374'' \text{ Hg}$ , while the air temperature was  $15^\circ \text{C.}$  and the barometer reading  $29.8'' \text{ Hg}$ . Find the relative humidity, the weight of vapour per  $\text{ft.}^3$  of air, and the dew point temperature.

The vapour pressure at  $15^\circ \text{C.}$  is found from steam tables to be  $0.503'' \text{ Hg}$ , so that the relative humidity is

$$\phi = \frac{0.374}{0.503} = 0.746$$

From steam tables,  $d_s = 0.0008 \text{ lb./ft.}^3$ , hence the air contains  $0.746 \times 0.0008 = 0.000597 \text{ lb./ft.}^3$  of vapour (absolute vapour content).

The weight of pure air in  $1 \text{ ft.}^3$  is

$$\begin{aligned} &= \frac{0.0808 (29.8 - 0.374)}{30} \times \frac{273}{273 + 15} \\ &= \frac{0.0808 \times 29.426 \times 273}{30 \times 288} = 0.0752 \text{ lb.} \end{aligned}$$

The density of the moist air is

$$\begin{aligned} d &= 0.0752 + 0.000597 \\ &= 0.075797 \text{ lb./ft.}^3 \text{ (at } 15^\circ \text{ C. and } 30'' \text{ Hg)} \end{aligned}$$

The gas constant is

$$R = \frac{29.8 \times 0.491 \times 144}{0.075797 \times 288} = 96.4$$

The saturation temperature corresponding to the pressure of  $0.374'' \text{ Hg}$  is  $10.5^\circ \text{ C.}$  This is the dew point temperature.

Alternative method for finding  $R$ ,

$$m = 28.95 - \frac{10.93 \times 0.374}{29.8} = 28.85$$

$$\begin{aligned} &m \\ &- \frac{2780}{28.85} = 96.4 \end{aligned}$$

The weight of vapour in  $1 \text{ lb.}$  of moist air is

$$\frac{0.622 \times 0.374}{29.8} = 0.0078 \text{ lb.}$$

The ratio by weight of vapour to air is

$$\frac{0.622 \times 0.374}{29.426} = \frac{1}{127}$$

## FUELS AND FUEL COMPOSITIONS

The combustible constituents of the technically important fuels are carbon and hydrogen, which are usually present as compounds called "hydrocarbons." A mixture of these (which may have widely different compositions) along with other elements forms the fuel. In gaseous fuels small quantities of sulphur are found, in addition to carbon monoxide. The following non-combustible constituents are also found in fuels: oxygen and nitrogen compounds, moisture and mineral constituents (ash).

The important solid fuels are: wood, peat, lignite, bituminous coal, anthracite, briquettes—made from lignite and bituminous coal—charcoal and coke; while the following are the more important liquid fuels: raw mineral oil (naphtha) and its distillates—gasoline, petrol, paraffin, gas oil, and the residuals of distillation, anthracite tar and its distillates in varying proportions. The last are distinguished as "light" and "heavy" tar oils. Benzene belongs to the light oils and in the commercial form is

AVERAGE COMPOSITIONS OF SOLID AND LIQUID FUELS (PER CENT BY WEIGHT)

FUEL	C	H	S	N	O	H <sub>2</sub> O	Ash	Remarks
Wood . . . . .	49	6	—	—	44	Not more than 20	< 0.8	Refers to air dried condition
Peat . . . . .	48	4.5	0.5-0.9	1.1-2.7	20-34	20	2-30	Refers to air dried condition
Lignite . . . . .	52	4	1-4	0.6-1.7	15-20	12-14	4-11	Refers to air dried condition
Lignite Briquettes . . . . .	54	4.4	1	0.5	22	12	6	Refers to air dried condition
Bituminous Coal . . . . .	80	4.7	0.5-1.5	0.6-1.7	6-11	1.3	6-5	Refers to air dried condition
Anthracite . . . . .	92	3.2	0.8	0.7	2.3	—	1-2	Refers to air dried condition
Coke . . . . .	88	0.7	0.86	—	1.4	2	7	From Ruhr coal
Mineral Oil and Distillates . . . . .	85	14	—	—	1	—	—	Petrol = Hexane + Heptane = C <sub>6</sub> H <sub>14</sub> + C <sub>7</sub> H <sub>16</sub>
Heavy Bituminous Coal Tar Oil, . . . . .	85-90	7-13	0.4-1	—	—	—	—	Aromatics
Light Bituminous Coal Tar Oil . . . . .								0.59C <sub>8</sub> H <sub>8</sub> + 0.11C <sub>7</sub> H <sub>8</sub> + 0.09C <sub>8</sub> H <sub>10</sub> (Benzene) (Toluene) (Xylene) + 0.06 C <sub>6</sub> H <sub>12</sub> + 0.15 (Petrol) (Residuals)
Benzene (C <sub>6</sub> H <sub>6</sub> ) . . . . .	92.3	7.7	—	—	—	—	—	
Lignite Tar Oil (Paraffin) . . . . .	85	12	—	—	—	—	—	C <sub>n</sub> H <sub>n-2n</sub> + 2, C <sub>n</sub> H <sub>n-2n</sub> , C <sub>n</sub> H <sub>n-2</sub>
Alcohol (Ethyl) . . . . .	52.2	13.0	—	—	34.8	—	—	C <sub>2</sub> H <sub>6</sub> O
Naphthalene . . . . .	93.7	6.3	—	—	—	—	—	C <sub>10</sub> H <sub>8</sub>
Bituminous Coal Tar . . . . .	87-93	2-6	0.2-1	—	—	—	—	

mixed with toluene and xylene in varying proportions. The heavy oils form 40 per cent of the tar and the light oils 10 per cent, while the remaining 50 per cent is pitch. Anthracene oil, creosote oil, solar oil, and gas oil are examples of the heavy oils.

The benzene hydrocarbons are obtained in Germany chiefly from coking gases and only to a very limited extent from tar. Solid naphthalene is also obtained from tar or coke gases. There are also the bituminous coal tar oils of varying quality, such as solar oil, gas oil, and paraffin oil. Alcohol can also be used as a fuel.

In addition to petrol, benzol is now being extensively used, particularly in road transport. This benzol is cleaned (i.e. unsuitable constituents are removed) and then mixed with tar oils of high hydrogen content. Mixtures of benzol and petrol and of benzol and alcohol are also available, having their own trade names.

**Gaseous fuels.** Illuminating gas is a distilled product obtained by heating of coal with air excluded. The composition does not vary greatly with the coal used, nor at different instants during distillation.

#### MEAN ANALYSIS OF ILLUMINATING GAS

	H <sub>2</sub>	CH <sub>4</sub>	C <sub>n</sub> H <sub>2n</sub>	CO	CO <sub>2</sub>	O <sub>2</sub>	N <sub>2</sub>
By volume . . .	48.5	35.0	4.56	7.18	1.82	0.25	2.70
By weight . . .	8.4	48.7	10.9	17.0	7.6	0.7	6.7
Density . . .	0.0322 lb./ft. <sup>3</sup> at S.T.P.    Gas constant, $R = 241 \frac{\text{ft. lb.}}{\text{lb. } ^\circ\text{C}}$						

A rather more complete analysis by volume of two gases is—

H <sub>2</sub>	CH <sub>4</sub>	C <sub>2</sub> H <sub>4</sub>	C <sub>3</sub> H <sub>6</sub>	C <sub>6</sub> H <sub>6</sub>	CO	CO <sub>2</sub>	O <sub>2</sub>	N <sub>2</sub>
46.2	34.02	2.55	1.21	1.33	8.88	3.01	0.65	2.15
49.0	27.0	3.0	—	0.3	10.0	3.0	1.0	6.7

Town gas, as now supplied, is a mixture of distilled gases obtained from coal, and water gas obtained from coke. It contains about 50 per cent of CO and H<sub>2</sub>. Town gas thus contains less of the rich gas methane than illuminating gas.

**COKE OVEN GAS.** The composition changes continuously during the distillation. Thus the two analyses given here were made after 6 and 19 hours' distillation—

H <sub>2</sub>	CH <sub>4</sub>	C <sub>m</sub> H <sub>n</sub>	CO	CO <sub>2</sub>	N <sub>2</sub>
35.88	35.95	6.47	7.44	4.42	9.84
43.03	29.49	2.76	8.38	2.23	14.11

The density at S.T.P. after 2 hours was 0.0352, after 19 hours 0.0292, and after 34 hours 0.0256.

Most coke ovens are operated to produce by-products from the coke oven gases. The gas, with the by-products removed, can then be used as a fuel in a steam boiler or gas engine, and has a somewhat different composition from distilled gas, since benzene, ammonia, and tar have been separated out. Greiner gives the following as an average sample of the analysis by volume of coke oven gas from which the by-products have been removed—

H <sub>2</sub>	CH <sub>4</sub>	CO	CO <sub>2</sub>	N <sub>2</sub>
57	23	6	2	12

**PRODUCER GAS.** This is obtained from coke, coal, anthracite, lignite, briquettes, or peat. From the first three fuels the gas is produced by

drawing in air and steam under the grate of the generator. The analysis varies according to the fuel and amount of water used.

#### EXAMPLES OF PRODUCER GAS FROM DIFFERENT FUELS

Fuel Used	H <sub>2</sub>	CH <sub>4</sub>	CO	CO <sub>2</sub>	N <sub>2</sub>	
Coke . . .	7.0	2.0	27.6	4.8	58.6	Mean of many analyses
Bituminous Coal	11.5	1.1	23.3	5.7	58.4	Mean of 12 analyses
Anthracite . .	11.0	2.3	24.83	2.43	58.74	Mean of 5 analyses
Briquettes . .	25.9	2.1	17.10	10.50	44.3	Mean of 4 analyses
Moist peat . .	16.75	3.0	9.1	16.0	53.4	Mean of 2 analyses

**BLAST FURNACE GAS.** Tests, extending over a continuous period of two years, were carried out on a large number of American furnaces and showed that the analysis and calorific value of the blast furnace gas depended on the percentage coke used. The results are given in the following table—

Coke Used	75	100	140%
H <sub>2</sub>	2.7	2.78	2.9
CO	24	26.7	31
CO <sub>2</sub>	15	12.3	8
CH <sub>4</sub>	0.2	0.2	0.2
N <sub>2</sub>	58	58	58
Calorific Value C.H.U./ft. <sup>3</sup> at S.T.P.	46.8	51.7	59.2

#### PRODUCTS OF COMBUSTION AND THEIR VOLUMETRIC ANALYSES

In practically all technical combustion processes, atmospheric air is used. At sufficiently high temperatures the oxygen of this air unites with the carbon and hydrogen of the fuel to form carbon dioxide and steam respectively. Provided complete combustion is effected, the products of combustion are thus gaseous substances, consisting of carbon dioxide, steam, oxygen, and nitrogen in varying proportions, and, in their heated state, the laws of gas mixtures can be applied.

**Air required for complete combustion.** By weight, the percentage proportions of oxygen and nitrogen present in the atmosphere are 23.2 and 76.8, small traces of other gases being neglected. The corresponding percentages by volume are oxygen 21 and nitrogen 79.

If the weight of oxygen required per pound of fuel is  $k_{O_2}$  the weight of air necessary is

$$A_0 = \frac{k_{O_2} \text{ lb. air}}{0.232 \text{ lb. fuel}}$$

For complete combustion, however, this minimum theoretical weight is insufficient. Both in furnaces and in engines 25 to 100 per cent more air has to be supplied, so that the actual air is  $A = nA_0$  where  $n = 1.25$  to 2.

Of this air, however, only the oxygen in  $A_0$  lb. takes part in the combustion. The remaining  $(n-1)A_0$  lb. of air is merely heated with the other products and with the nitrogen in  $A_0$  lb. of air, i.e.  $0.768A_0$  lb. of nitrogen. The excess oxygen in  $(n-1)A_0$  lb. of excess air weighs  $0.232(n-1)A_0$  lb. If the composition of the original fuel be known (solid or liquid), then  $k_{O_2}$  can be calculated from

$$k_{O_2} = \frac{8}{3} C + 8 H - O$$

where C, H, and O represent the proportional parts by weight of 1 lb. of the fuel.

**The products of combustion.** Flue gases consist of  $CO_2$  and  $H_2O$  as well as  $(n-1)A_0$  lb. of air,  $0.768A_0$  lb. of nitrogen, and any incombustible gases in the original fuel, such as  $CO_2$ ,  $O_2$ , and  $N_2$ , which may be denoted by  $w_r$ .

The weight of flue gas per pound of fuel is  $(1 + nA_0)$  lb., so that its analysis by weight is

$$w_{CO_2} = \frac{1}{1 + nA_0}, \quad w_{H_2O} = \frac{1}{1 + nA_0}, \quad w_{(N_2 + O_2)} = \frac{1}{1 + nA_0}$$

where (for solid and liquid fuels)

$$k_{CO_2} = \frac{1}{11} C; \quad k_{H_2O} = 9 H; \quad + O_2 = (n - 0.232)A_0 + w_r$$

Thus, in the case of distilled mineral oil, with  $C = 0.85$ ,  $H = 0.14$ ,  $A_0 = 14.5$  lb./lb.;  $k_{CO_2} = 3.12$  and  $k_{H_2O} = 1.26$ .

The ratio of the weight of carbon dioxide formed to the weight of fuel is, for pure carbon, 3.33; for illuminating gas, about 2; for producer gas, about 0.5; and for mineral oil, about 3; the corresponding steam weight ratios are 0, 2.2, 0.07, and 1.26. The ratio of the total weight of the products of combustion to the weight of fuel is 12.5 for carbon, 14 for illuminating gas, 2 for producer gas, and 15.5 for mineral oil.

**Gas constants.** The proportional weight of the fuel gas before combustion is

$$\frac{1}{1 + nA_0}$$

and of the air

$$w_A = \frac{nA_0}{1 + nA_0}$$

The gas constant before combustion is thus (page 14)

$$R_0 = w_g R_g + w_A R_A$$

and, for the products of combustion,

×

$R_p$  is always slightly different from  $R_0$ , and, as shown by the characteristic equation, the volumes of different gases at the same pressure and temperature, when no change in weight occurs, are proportional to their gas constants, so that

$$\frac{R_p}{R_0} = \frac{V}{V_0}$$

This means that if  $R_p$  is less than  $R_0$  the volume of the products is less than that of the original mixture when both are at the same pressure

## GASES

and temperature. This volume contraction occurs in the combustion of most fuels, but for certain hydrocarbons a volume increase occurs.

**Example 15.** Taking an average sample of illuminating gas burned with the excess air factor  $n = 1.25$ , gives the proportional weights of gas and air as

$$w_g = 0.057 \text{ and } w_a = 0.943$$

so that, before combustion,

$$\begin{aligned} R_0 &= 0.057 \times 242 + 0.943 \times 96 \\ &= 13.8 + 90.5 = 104.3 \end{aligned}$$

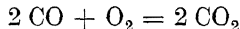
After combustion with

$$\begin{aligned} w_{\text{CO}_2} &= 0.11, \quad w_{\text{H}_2\text{O}} = 0.13, \text{ and } w_{(\text{N}_2 + \text{O}_2)} = 0.76 \\ R_p &= 0.11 \times 63.2 + 0.13 \times 154.2 + 0.76 \times 98.4 \\ &= 6.95 + 20.05 + 74.8 = 101.8 \end{aligned}$$

The volume contraction is thus  $101.8/104.3 = 0.975$ , which is relatively small. With more excess air it is still smaller.

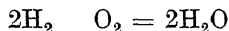
**The volumetric analysis** can be found by first finding the analysis by weight, from which the analysis by volume is found, as shown on page 8. Since, however, the laws governing the volumes in gaseous reactions are very simple, it is more convenient to deal with the volumes without reference to their weights.

**Gaseous fuels.** The combustion of gaseous carbon monoxide to carbon dioxide proceeds in accordance with the reaction equation



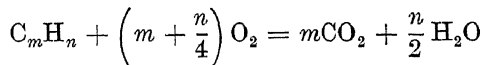
This means that 2 ft.<sup>3</sup> of CO can unite with 1 ft.<sup>3</sup> of O<sub>2</sub> to form 2 ft.<sup>3</sup> of CO<sub>2</sub>, provided these volumes are measured at the same pressure and temperature, and combustion is complete. A reduction in volume of 1 ft.<sup>3</sup> thus takes place, while the amount of oxygen required by volume is half that of the carbon monoxide.

The reaction equation for the combustion of gaseous hydrogen to steam is



and shows that the volume of steam formed is  $\frac{2}{3}$  that of the original mixture, and that the volume of oxygen required is half that of the hydrogen.

Hydrocarbons having the structure  $\text{C}_m\text{H}_n$  burn in accordance with the equation



The volume of oxygen required per ft.<sup>3</sup> of fuel is thus  $\left(m + \frac{n}{4}\right)$  ft.<sup>3</sup>, while the volume of products formed from  $\left(1 + m + \frac{n}{4}\right)$  ft.<sup>3</sup> of gaseous fuel is  $\left(m + \frac{n}{2}\right)$  ft.<sup>3</sup>. The change of volume, therefore, is

$$\left(m + \frac{n}{2}\right) - \left(1 + m + \frac{n}{4}\right) = \left(\frac{n}{4} - 1\right) \text{ ft.}^3 \quad . \quad . \quad . \quad (31)$$

Applying this, for example, to the combustion of methane  $\text{CH}_4$ , the change in volume is found to be zero and the volume of oxygen required is twice that of the methane. For hydrocarbons with more than 4 atoms of hydrogen, the volume of the products is greater than that of the fuel mixture. For example, in the combustion of 1 ft.<sup>3</sup> of gaseous benzene ( $\text{C}_6\text{H}_6$ ) the volume of the products, from equation (31) is  $\frac{4}{3} - 1 = \frac{1}{3}$  ft.<sup>3</sup> larger than the mixture volume. Similarly a volume increase occurs in the combustion of paraffin containing hexane ( $\text{C}_6\text{H}_{14}$ ) and heptane ( $\text{C}_7\text{H}_{16}$ ).

Taking account of the volume proportions in the reactions just described, the necessary volume of oxygen and the proportions by volume of the products formed, can readily be determined for any gaseous fuel when its analysis is known.

Let the analysis be

$$+ v_{\text{H}_2\text{O}} = 1 \quad (32)$$

then the minimum theoretical volume of oxygen required per ft.<sup>3</sup> of fuel is

$$\text{O}_{2\min} = \left[ \frac{v_{\text{CO}} + v_{\text{H}_2}}{2} + 2v_{\text{CH}_4} + 3v_{\text{C}_2\text{H}_4} + 2.5v_{\text{C}_2\text{H}_2} \right. \\ \left. + \left( m + \frac{n}{4} \right) v_{\text{C}_m\text{H}_n} - v_{\text{O}_2} \right] \text{ft.}^3 \quad (33)$$

$$\text{and } A_{\min} = \frac{\text{O}_{2\min}}{0.21} \text{ft.}^3 \text{ air} \quad (34)$$

The volumes of the constituents, in the products of combustion per ft.<sup>3</sup> of fuel gas, are

Carbon dioxide—

$$v_{\text{CO}} + v_{\text{CH}_4} + 2v_{\text{C}_2\text{H}_4} + 2v_{\text{C}_2\text{H}_2} + v_{\text{CO}_2} + mv_{\text{C}_m\text{H}_n} = V_{\text{CO}_2} \quad (35)$$

Steam—

$$v_{\text{H}_2} + 2v_{\text{CH}_4} + 2v_{\text{C}_2\text{H}_4} + v_{\text{C}_2\text{H}_2} + v_{\text{H}_2\text{O}} + \frac{n}{2} v_{\text{C}_m\text{H}_n} = V_{\text{H}_2\text{O}} \quad (36)$$

Oxygen—

$$(n-1) \text{O}_{2\min} = V_{\text{O}_2} \quad (37)$$

where  $n$  is the excess air factor.

Nitrogen—

$$0.79nA_{\min} + v_{\text{N}_2} = V_{\text{N}_2} \quad (38)$$

The sum of these four constituents gives the total volume of products per ft.<sup>3</sup> of fuel. If the volume of the initial mixture  $(1 + nA_{\min})$  ft.<sup>3</sup> be subtracted from this, the result gives the change in volume  $\Delta V$  per ft.<sup>3</sup> of fuel gas,

$$\text{i.e. } \Delta V = -\frac{1}{2}v_{\text{CO}} - \frac{1}{2}v_{\text{H}_2} - \frac{1}{2}v_{\text{C}_2\text{H}_2} + \left( \frac{n}{4} - 1 \right) v_{\text{C}_m\text{H}_n} \quad (39)$$



which can also be obtained directly. A reduction in volume occurs therefore when the combustible constituents are carbon monoxide, free hydrogen, and acetylene ( $C_2H_2$ ). In the case of the combustion of power and illuminating gas a reduction in volume always occurs, due to the high percentage of CO, but the reduction is smaller with illuminating gas on account of the presence of hydrocarbons containing more than 4 atoms of hydrogen.

For a fuel which contains several hydrocarbons having the structure  $C_mH_n$  in addition to CO and  $H_2$ , the general expression for the change in volume is

$$\Delta V = -\frac{1}{2}v_{CO} - \frac{1}{2}v_{H_2} + \Sigma \left( \frac{n}{4} - 1 \right) v_{C_mH_n} \quad (40)$$

if  $n$  is greater than 4,  $\Delta V$  may be positive. If, after combustion, the products are so far cooled that practically all the steam is condensed, the change in volume is greater by the amount  $V_{H_2O}$  and then

$$\left( \frac{n}{4} + 1 \right) v_{C_mH_n} \quad (41)$$

and is therefore negative in all cases.

The volume of the gaseous mixture before combustion per ft.<sup>3</sup> of fuel gas is

$$V_0 = 1 + nA_{min}$$

while the volume of the products at the same pressure and temperature is

$$V_p = 1 + nA_{min} + \Delta V \quad (42)$$

in which  $\Delta V$  is usually negative, so that  $V_p < V_0$ .

The ratio of the gas constants  $R_0$  and  $R_p$  is

$$\frac{R_0}{R_p} = \frac{V_0}{V_p} = \frac{1}{\alpha}$$

or

$$R_p = R_0 \frac{1 + nA_{min} + \Delta V}{1 + nA_{min}}$$

giving

$$\alpha = 1 + \frac{\Delta V}{1 + nA_{min}} \quad (43)$$

In most cases  $R_p$  is less than  $R_0$  and  $\alpha$  is less than 1, since  $\Delta V$  is generally negative.

**Example 16.** From the average analysis of producer gas,  $v_{H_2} = 7.0$ ;  $v_{CH_4} = 2.0$ ;  $v_{CO} = 27.6$ ;  $v_{CO_2} = 4.8$ ; and  $v_{N_2} = 58.6$  per cent, the following values are obtained—

$$O_{2min} = \frac{1}{2} \times 0.276 + \frac{1}{2} \times 0.07 + 2 \times 0.02 = 0.213$$

$$A_{min} = \frac{0.213}{0.21} = 1.014 \text{ ft.}^3/\text{ft.}^3 \text{ of gas}$$

$$\Delta V = -\frac{1}{2} \times 0.276 - \frac{1}{2} \times 0.07 = -0.173 \text{ ft.}^3/\text{ft.}^3 \text{ of gas.}$$

With theoretical minimum air supply, the original air fuel mixture volume is  $1 + 1.014 = 2.014 \text{ ft.}^3$  and the volume of the products is  $2.014 - 0.173 = 1.841 \text{ ft.}^3$ . The ratio of the gas constants is thus

$$R_p/R_0 = \frac{1.841}{2.014} = 0.914$$

The ratio of the density of the products to that of the air fuel mixture is  $\frac{1}{0.914} = 1.1$ . The volumes of the products are  $V_{\text{CO}_2} = 0.344 \text{ ft.}^3$ ;  $V_{\text{H}_2\text{O}} = 0.11 \text{ ft.}^3$ ;  $V_{\text{O}_2} = (n-1)0.213 \text{ ft.}^3$ ;  $V_{\text{N}_2} = (0.802n + 0.586) \text{ ft.}^3$ . With  $n = 1$ ,  $V_{\text{N}_2} = 1.388 \text{ ft.}^3$ , and  $V_{\text{O}_2} = 0$ , while with  $n = 1.5$ ,  $V_{\text{N}_2} = 1.789 \text{ ft.}^3$ , and  $V_{\text{O}_2} = 0.1065 \text{ ft.}^3$ . The total volume of the products is thus  $0.344 + 0.11 + 1.388 = 1.842 \text{ ft.}^3$  with  $n = 1$ , and  $2.3495 \text{ ft.}^3$  with  $n = 1.5$ . The volumetric analysis of the wet products, with  $n = 1.5$ , is

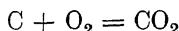
$$v_{\text{CO}_2} = 0.1465; v_{\text{O}_2} = 0.0454; v_{\text{N}_2} = 0.7613; v_{\text{H}_2\text{O}} = 0.0468$$

and for dry products

$$v_{\text{CO}_2} = 0.153; v_{\text{O}_2} = 0.048; v_{\text{N}_2} = 0.799; v_{\text{H}_2\text{O}} = 0$$

**Liquid and solid fuels.** Let the composition of a liquid or solid fuel be  $C$  lb. of carbon,  $H$  lb. of hydrogen, and  $O$  lb. of oxygen.

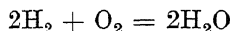
The reaction equation for carbon



shows that 1 lb. mol. of oxygen is required per lb. mol., i.e. per 12 lb. of carbon. The volume of this weight of oxygen is  $\bar{v} = 359 \text{ ft.}^3$  at S.T.P. (page 11).

In burning 1 lb. of carbon, therefore,  $\frac{v_0}{12} \text{ ft.}^3$  of oxygen are necessary, and, for  $C$  lb. of carbon,  $\frac{C\bar{v}}{12} \text{ ft.}^3$  of oxygen.

The reaction equation for hydrogen



shows that 1 lb. mol., i.e.  $\bar{v} = 359 \text{ ft.}^3$  of oxygen has to be supplied per 2 lb. mols., i.e.  $2 \times 2.016 \text{ lb.}$  of hydrogen. Hence, for  $H$  lb. of hydrogen

the volume of oxygen required is  $\frac{\bar{v}H}{2 \times 2.016} \text{ ft.}^3$

The volume of oxygen occupied by  $O$  lb. (weight of oxygen in the fuel composition) is  $\frac{\bar{v}O}{32}$ , since the volume occupied by 32 lb. is  $\bar{v}_0$ .

The oxygen supplied by the combustion air is thus

$$O_{min} = \bar{v} \left( \frac{C}{12} + \frac{H}{4} - \frac{O}{32} \right) \text{ ft.}^3 \quad . \quad . \quad . \quad (44)$$

$$\text{and} \quad A_{min} = 0.21 \left( \frac{C}{12} + \frac{H}{4} - \frac{O}{32} \right) \text{ ft.}^3 \quad . \quad . \quad . \quad (45)$$

per lb. of fuel.

In the same way it is found that the volume of  $\text{CO}_2$  produced from  $C$  lb. of carbon is  $\frac{\bar{v}C}{12}$ , and of  $\text{H}_2\text{O}$  from  $H$  lb. of hydrogen  $\frac{\bar{v}H}{2} \text{ ft.}^3$

If now the actual air supply is  $nA_{min}$ , the amount of oxygen present after combustion is

$$V_{\text{O}_2} = (n-1) O_{min} \text{ ft.}^3$$



the specific volume of 0.00945 lb./ft.<sup>3</sup> is 64° C. Products with a smaller vapour content become saturated at still lower temperatures.

When a flue gas is analysed by means of the Orsat apparatus, the gases are always saturated, due to the presence of the trap water. Now in any saturated gas at 20° C. the partial pressure of the water vapour is only 17.4 mm. of Hg, so that, when the total pressure is 760 mm. Hg, the fractional vapour pressure is  $\frac{17.4}{760} = 0.023$ . This means (page 18)

that the vapour volume in the Orsat apparatus cannot exceed 2.3 per cent, as compared with 4.68 per cent for the products in Example 16 of this chapter (page 27). In any case the analysis, as given by the Orsat apparatus, is the same as would be obtained if no vapour were present, since the water vapour condenses in the same ratio with each absorption.

Returning now to equation (47), the volume  $V'_p$  of the vapour-free, or dry flue gas, is smaller than that of the total flue gas volume by the amount of water vapour present in the latter. This gives

$$V_p = V'_p + \frac{2}{4} \times 32 = V'_p + 16$$

If moisture, weighing ( $H_2O$ ) lb. be contained in 1 lb. of the fuel, then this, as water vapour, occupies the volume  $\frac{\bar{v}(H_2O)}{18}$  ft.<sup>3</sup> at S.T.P.  $V_p$  is

then increased by this amount, but  $V'_p$  remains unaltered.

The volumetric analysis of flue gases with water vapour, when the fuel composition is  $C$ ,  $H$ , and  $O$ , is as follows—

$$v_{CO_2} = \frac{\bar{v}}{12} \frac{C}{V_p} \quad (\text{where } \bar{v} = 359) \quad (50)$$

$$v_{H_2O} = \frac{\bar{v}}{2} \frac{H}{V_p} \quad (51)$$

$$v_{O_2} = \frac{(n-1)O_{min}}{V_p} \quad (52)$$

$$v_{N_2} = \frac{79}{21} \frac{nO_{min}}{V_p} \quad (53)$$

Writing the bracketed expression in equation (47) as

$$\mu^* = \frac{H}{4} + \frac{n}{0.21} \left( \frac{C}{12} + \frac{H}{4} - \frac{O}{32} \right) + \frac{O}{32} \quad (54)$$

gives  $V_p = \mu \bar{v} \quad (55)$

and  $v_{CO_2} = \frac{C}{12\mu} \quad (56)$

$$v_{H_2O} = \frac{H}{2\mu} \quad (57)$$

$$v_{O_2} = \frac{n-1}{\mu} \left( \frac{C}{12} + \frac{H}{4} - \frac{O}{32} \right) \quad (58)$$

\*  $\mu$  is the number of mols. produced by the combustion of 1 lb. of fuel.

$$v_{N_2} = \frac{79n}{21\mu} \left( \frac{C}{12} + \frac{H}{4} - \frac{O}{32} \right) \quad (59)$$

These give the volumetric analysis when the air supply is  $n$  times the theoretical minimum and when the composition  $C$ ,  $H$ ,  $O$  by weight of the solid or liquid fuel is known.

For a fuel containing  $N$  lb. of nitrogen, equation (54) becomes

$$\mu = \frac{H}{4} + \frac{n}{0.21} \left( \frac{C}{12} + \frac{H}{4} - \frac{O}{32} \right) + \frac{O}{32} + \frac{N}{28.08} \quad (60)$$

and equation (59) becomes

$$v_{N_2} = \frac{79n}{21\mu} \left( \frac{C}{12} + \frac{H}{4} - \frac{O}{32} \right) + \frac{N}{28.08\mu} \quad (61)$$

For fuels containing  $H_2O$  lb. of moisture (in coal, wood, spirit alcohol, etc.)  $\mu$  is increased by  $\frac{H_2O}{18}$ , so that equation (60) becomes

$$\mu = \frac{H}{4} + \frac{n}{0.21} \left( \frac{C}{12} + \frac{H}{4} - \frac{O}{32} \right) + \frac{O}{32} + \frac{N}{28.08} + \frac{H_2O}{18} \quad (62)$$

and (57) becomes

$$v_{H_2O} = \frac{H}{2\mu} + \frac{H_2O}{18\mu} \quad (63)$$

**Example 17.** The composition of a bituminous coal is  $C = 0.80$ ,  $H = 0.047$ , and  $O = 0.06$ . Find the flue gas analysis by volume (a) with  $n = 1$ , and (b) with  $n = 1.7$ .

(a) With  $n = 1$  the necessary air per lb. of fuel is

$$A_{min} = \frac{359}{0.21} \left( \frac{0.8}{12} + \frac{0.047}{4} - \frac{0.06}{32} \right)$$

The flue gas volume is

$$\begin{aligned} V_p &= 131 + 89.75 \times 0.047 + 11.22 \times 0.06 \\ &= 131 + 4.22 + 0.673 = 135.9 \text{ ft.}^3 \text{ at S.T.P.} \end{aligned}$$

which, at  $273^\circ \text{C}$ . and  $14.7 \text{ lb./in.}^2$  abs., becomes

$$2 \times 135.9 = 271.8 \text{ ft.}^3/\text{lb. of fuel.}$$

With  $\mu = 0.379$  (for  $n = 1$ )

$$v_{CO_2} = \frac{0.80}{12 \times 0.379} = 0.175 \text{ or } 17.5 \text{ per cent}$$

$$v_{H_2O} = \frac{0.047}{2 \times 0.379} = 0.062 \text{ or } 6.2 \text{ per cent}$$

$$v_{O_2} = 0$$

$$v_{N_2} = \frac{79}{21} \times \frac{1}{0.379} \times 0.0766 = 0.758 \text{ or } 75.8 \text{ per cent.}$$

(b) With  $n = 1.7$  the air supply is  $1.7 \times 131 = 223 \text{ ft.}^3$  and the flue gas volume is  $223 + 4.22 + 0.673 = 227.9 \text{ ft.}^3$ , which at  $273^\circ \text{C}$ . becomes

## TECHNICAL THERMODYNAMICS

$2 \times 227.9 = 455.9$  ft.<sup>3</sup>/lb. of fuel. The factor  $\mu$  is  $\frac{227.9}{359} = 0.634$ , so that

$$v_{\text{CO}_2} = \frac{0.8}{12 \times 0.634} = 0.104 \text{ (10.4 per cent)}$$

$$v_{\text{H}_2\text{O}} = \frac{0.047}{2 \times 0.634} = 0.037 \text{ (3.7 per cent)}$$

$$v_{\text{O}_2} = \frac{0.7 \times 0.0766}{0.634} = 0.085 \text{ (8.5 per cent)}$$

$$v_{\text{N}_2} = \frac{79 \times 1.7 \times 0.0766}{21 \times 0.634} = 0.774 \text{ (77.4 per cent).}$$

### THE GAS ANALYSIS AND THE COMBUSTION TRIANGLE

**Analysis of dry flue gas.** In analysing flue gases, the proportions of the dry flue gases are determined, so that the following is valid for this state.

$$v_{\text{CO}_2} + v_{\text{O}_2} + v_{\text{N}_2} = 1$$

and, in place of equations (56) to (59), the following are substituted—

$$v_{\text{CO}_2} = \frac{C}{12\mu'} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (64)$$

$$v_{\text{H}_2\text{O}} = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (65)$$

$$v_{\text{O}_2} = \frac{n-1}{\mu'} \left( \frac{C}{12} + \frac{H}{4} - \frac{O}{32} \right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (66)$$

$$v_{\text{N}_2} = \frac{79}{21} \frac{n}{\mu'} \left( \frac{C}{12} + \frac{H}{4} - \frac{O}{32} \right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (67)$$

From equations (66) and (67),

$$n = \frac{21}{21 - 79 \frac{v_{\text{O}_2}}{v_{\text{N}_2}}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (68)$$

This equation thus enables the amount of excess air to be calculated when  $v_{\text{O}_2}$  and  $v_{\text{CO}_2}$  are known, since  $v_{\text{N}_2}$  is then also known. The value of  $v_{\text{N}_2}$  is always about 0.79, so that, very closely,

$$n = \frac{0.21}{0.21 - v_{\text{O}_2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (69)$$

For fuels without hydrogen,

$$v_{\text{O}_2} + v_{\text{CO}_2} = 0.21$$

$$\text{giving} \quad n = \frac{0.21}{v_{\text{CO}_2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (70)$$

so that the determination of the carbon dioxide is sufficient to give  $n$ .

Further, from equations (64) and (66),

$$\frac{v_{\text{CO}_2}}{v_{\text{O}_2}} = \frac{1}{C} \quad (71)$$

and, inserting the value of  $n$  given by equation (68),

$$0.79 \frac{1}{C} = 0.21 \quad (72)$$

This shows that, for any given fuel, a definite relationship exists between  $v_{O_2}$  and  $v_{CO_2}$ . If the  $v_{CO_2}$  values are plotted to a base of  $v_{O_2}$  values, a straight line is obtained (Fig. 8) which has its greatest possible  $v_{CO_2}$  value on the ordinate axis, i.e. where  $v_{O_2} = 0$ . This is

$$0.21 \frac{1}{C} \quad (73)$$

and thus depends on the composition of the fuel.

Thus in the case of pure carbon with  $H = 0$  and  $O = 0$ ,  $v_{CO_2 \text{ max}} = 0.21$ , while for mineral oil distillate, with  $C = 0.85$ ,  $N = 0.14$ , and  $O = 0$ ,  $v_{CO_2 \text{ max}} = 0.151$ . These straight lines all cut the abscissa axis at  $v_{O_2} = 0.21$  ( $v_{CO_2} = 0$ ).

The combustion lines for different fuels thus run as shown in Fig. 8, which is called the combustion triangle.

The excess air factors  $n$  vary from  $n = 1$  at the top to  $n = \infty$  at the bottom, as shown by equation (71), from which, for different values of  $\frac{v_{CO_2}}{v_{O_2}}$ , the corresponding  $n$  values can be determined and plotted on the straight lines, as shown in Fig. 8.

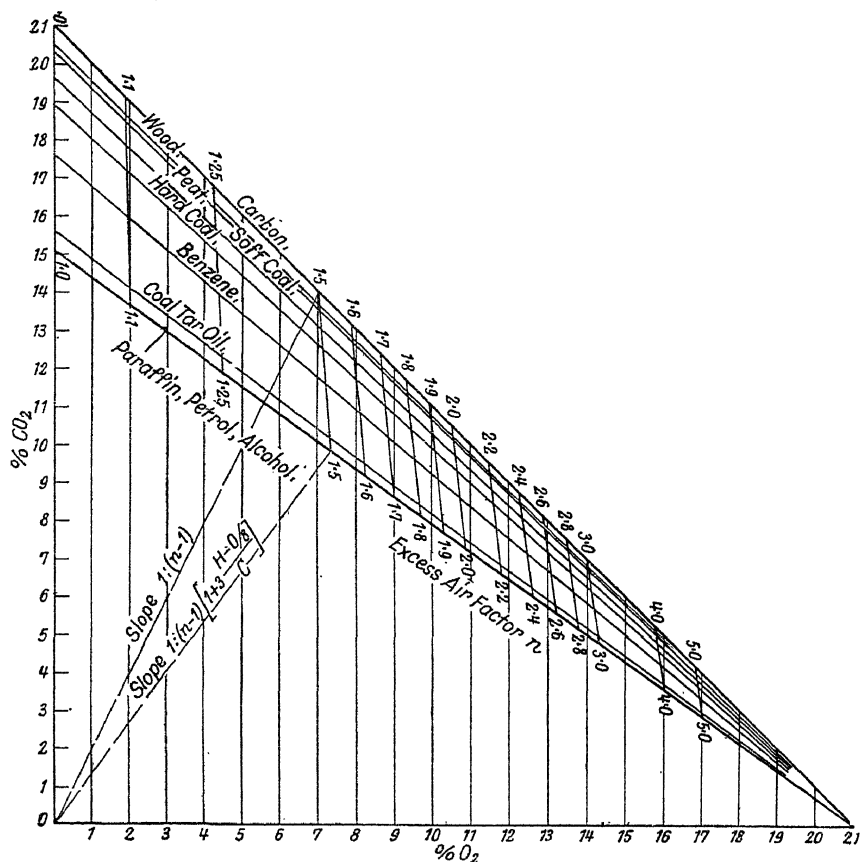
If the volumes of  $v_{CO_2}$  and  $v_{O_2}$ , as found by the flue gas analysis, be plotted on the combustion triangle, the point should lie on the straight line corresponding to the fuel. If it lies below this line, combustion is incomplete, and if above, an error is indicated either in the flue gas analysis or in the assumed composition of the fuel. The value of  $n$  is also read at the same time.

The value  $v_{CO_2} + v_{O_2}$  given by the flue gas analysis must lie within the limits  $v_{CO_2 \text{ max}}$  and 0.21 if the excess air factor lies between 1 and  $\infty$ . If this sum be plotted to the same base, a straight line is again obtained, as shown in Fig. 9. Should the sum given by the flue gas analysis be less than that given by this line, the combustion is incomplete, showing that the flue gases probably contain some CO as well as  $CO_2$  and  $O_2$ .

**Incomplete combustion.** Incomplete combustion occurs when less air is supplied than that chemically necessary for complete combustion, i.e. when the excess air factor is less than 1. Even when excess air is supplied, however, ( $n > 1$ ) incomplete combustion can occur, due either to incomplete mixing of the air and fuel, or to rapid cooling during the combustion process.

The presence of combustibles in the cooled flue gases is an indication of incomplete combustion. Of these, the most important is CO, but  $H_2$ ,  $CH_4$ , and other gaseous hydrocarbons are sometimes found, as well as solid carbon, in the form of dust. It is obvious, of course, that the flue gases formed with excess air when the combustion is incomplete will contain  $O_2$ , but it is also found that  $O_2$  may be present even with a deficient air supply, particularly when the air and fuel are not properly mixed. In the following, only flue gases containing CO in addition to

$\text{CO}_2$ ,  $\text{H}_2\text{O}$ ,  $\text{O}_2$ , and  $\text{N}_2$  will be considered. The carbon monoxide found in the flue gases, formed by burning a gaseous fuel, is accounted for by the presence of this gas in the original fuel gas, but in the case of a liquid or solid fuel containing  $C$  parts by weight of carbon, we have to regard the  $\text{CO}$  as being produced by the combustion of  $(1-x)C$  lb. of carbon, while the remaining  $x$  lb. burns to  $\text{CO}_2$ .







from equations (77) and (78) and then eliminating the quantities  $C$ ,  $H$ , and  $O$  by means of equations (76) and (77). The result is

$$n = \frac{21}{21 - \frac{79}{v_{N_2}} \{v_{O_2} - \frac{1}{2}v_{CO}\}} \quad . \quad . \quad . \quad (79)$$

which, with  $v_{CO} = 0$ , is the same as equation (68).

Equation (79) thus enables the excess air factor to be determined, when the flue gas analysis is known.

**Combustion triangle.** The relation between  $v_{CO_2}$ ,  $v_{CO}$ , and  $v_{O_2}$  is found as follows—

Dividing equation (77) by (75) gives

$$\frac{v_{O_2}}{v_{CO_2}} = \frac{n-1}{x} \left( 1 + \frac{3H}{C} - \frac{3}{8} \frac{O}{C} \right) + \frac{1}{2} \frac{(1-x)}{x} \quad . \quad . \quad (80)$$

Dividing equation (75) by equation (76) gives

$$\frac{v_{CO_2}}{v_{CO}} = \frac{x}{1-x} \quad . \quad . \quad . \quad . \quad . \quad . \quad (81)$$

hence 
$$x = \frac{v_{CO_2}}{v_{CO_2} + v_{CO}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (82)$$

Inserting this value in equation (80) gives, finally,

$$v_{O_2} = (n-1) [v_{CO_2} + v_{CO}] \left( 1 + \frac{3H}{C} - \frac{3}{8} \frac{O}{C} \right) + \frac{1}{2} v_{CO} \quad (83)$$

By putting  $v_{CO} = 0$  this equation, which relates the quantities  $CO_2$ ,  $O_2$ ,  $CO$ , and  $n$ , becomes the same as the simpler one (71) for complete combustion. If  $n-1$  be eliminated by means of equation (79), an equation relating  $CO_2$ ,  $O_2$ , and  $CO$  alone could be established, the graphical representation of which, on rectangular co-ordinates, would be a curved surface. It is preferable, however, to represent the relations on a plane, as in the case of the combustion triangle with complete combustion. To do this,  $v_{CO}$  has to be eliminated from equation (83) by expressing it in terms of  $v_{CO_2}$  and  $v_{O_2}$  as follows.

From equations (74) and (81)

$$\mu' = \left( \frac{C}{12} + \frac{H}{4} - \frac{O}{32} \right) \frac{(n-0.21)}{0.21} + \frac{C}{12} \left( 1 + \frac{1}{2} \frac{v_{CO}}{v_{CO_2} + v_{CO_2}} \right)$$

and this, along with equations (76) and (82), gives

$$v_{CO} = \frac{1 - B v_{CO_2}}{0.5 + B} \quad . \quad . \quad . \quad . \quad . \quad . \quad (84)$$

in which

$$B = \left( 1 + \frac{3H}{C} - \frac{3}{8} \frac{O}{C} \right) \frac{n-0.21}{0.21} + 1 \quad . \quad . \quad . \quad (85)$$

From (84) we have

$$v_{CO} + v_{CO_2} = \frac{2 + v_{CO_2}}{1 + 2B} \quad . \quad . \quad . \quad . \quad . \quad . \quad (86)$$

This, along with equation (84), gives, for equation (83),

$$(1 + 2B) v_{O_2} = (n - 1) (2 + v_{CO} - 1 - B v_{CO_2}) \quad (87)$$

in which  $(n - 1)$  is found from equation (79). For the graphical representation in the combustion triangle, these relations give the following—  
with  $n = 1$ , equation (83) gives

$$v_{O_2} = \frac{1}{2} v_{CO} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (88)$$

and hence, with equation (84),

$$v_{O_2} = \frac{1 - B v_{CO_2}}{1 + 2B} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (89)$$

In equation (85), with  $n = 1$ ,

$$B = \frac{1 + 0.79 \left( 3 \frac{H}{C} - \frac{3}{8} \frac{O}{C} \right)}{0.21} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (90)$$

hence, from equation (73),

$$B_{n=1} = \frac{1}{v_{CO_2 \max}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (91)$$

where  $v_{CO_2 \max}$  is the greatest carbon dioxide content possible with complete combustion. For  $n = 1$ , therefore,  $B$  has a constant value, and equation (89) is represented by a straight line in the combustion triangle, which cuts the ordinate axis at the value  $V_{CO_2 \max}$  and the abscissa axis at

$$v_{O_2 \max} = \frac{1}{1 + 2B} = \frac{v_{CO_2 \max}}{2 + v_{CO_2 \max}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (92)$$

as given by equations (89) and (91).

Thus, with  $v_{CO_2 \max} = 0.21$  (pure carbon),  $v_{O_2 \max} = \frac{0.21}{2 + 0.21} = 0.095$

while for oil with  $v_{CO_2 \max} = 0.151$ ,  $v_{O_2 \max} = \frac{0.151}{2.151} = 0.0702$ .

In general, the slope of this straight line is

$$\frac{v_{O_2 \max}}{v_{CO_2 \max}} = \frac{1}{2 + v_{CO_2 \max}}$$

and separates the region of incomplete combustion with excess air (on the right) from that in which insufficient air is supplied (Fig. 9). The area enclosed by it and the straight line for complete combustion gives the complete region of incomplete combustion with excess air, while the region between the straight line for  $n = 1$  and the ordinate axis serves for all possible cases with insufficient air.

Further, as shown by equation (88), with  $n = 1$ ,

$$v_{CO} = 2v_{O_2}$$



from equation (84), the difference in the carbon monoxide content on any line  $n = \text{constant}$  is, for the points 1 and 2,

$$v_{2\text{CO}} - v_{1\text{CO}} = (v_{2\text{CO}_2} - v_{1\text{CO}_2}) \frac{B}{0.5 + B}$$

in which  $B$  is constant. For very small differences this is

$$\Delta v_{\text{CO}} = \Delta v_{\text{CO}_2} \frac{B}{0.5 + B}$$

P. Meyer, in *Z.V.d.I.* (1929), page 824, shows the same result. ("Mischungsverhältnis und Verbrennungsvorgänge im Ölmotor.")

But

$$\frac{\Delta v_{\text{O}_2}}{\Delta v_{\text{CO}_2}}$$

is also constant, since the line  $n = \text{constant}$ , is straight.

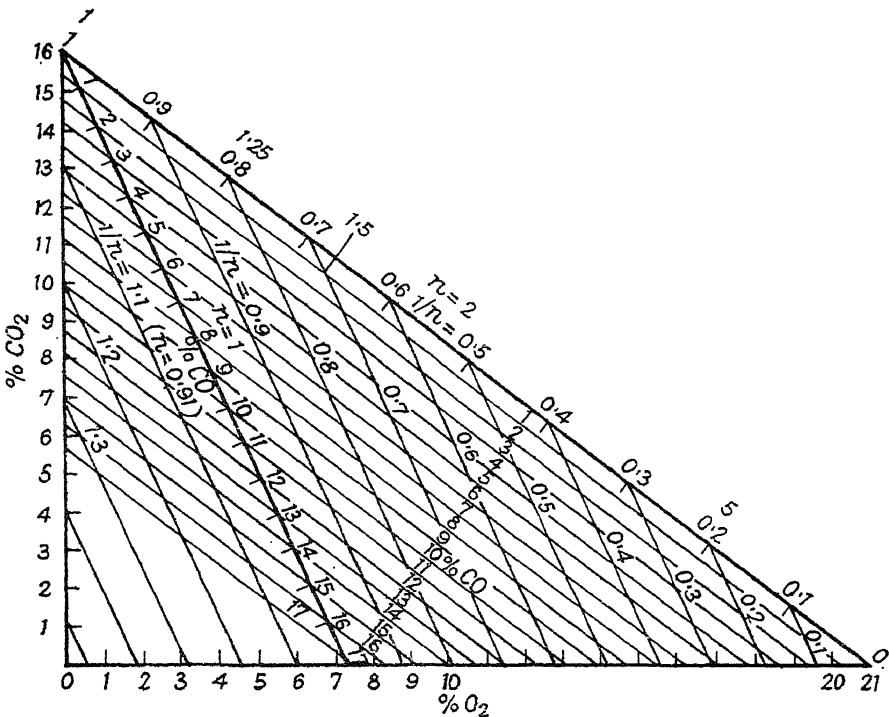


FIG. 10

Hence equal differences in the values of  $v_{\text{CO}_2}$  or of  $v_{\text{O}_2}$  give equal differences in the values of  $v_{\text{CO}}$ , so that the distances between the consecutive points 1, 2, 3, . . . giving the percentage carbon monoxide on the line  $n = \text{constant}$  are equal, just as on the line  $n = 1$ . From this it follows that the lines of constant CO content are curved. They could only be straight if the curves of constant  $n$  values were parallel. Actually the lines of constant CO content are quadratics, as shown by equation (87) when  $n - 1$  is replaced by its value given by equation (79) and  $v_{\text{O}_2}$  is

replaced by  $v_{\text{CO}_2}$ , as given by equation (80). The deviations from parallel straight lines are, however, very slight.

The percentage CO divisions are found from

$$+ \frac{1}{2B}$$

by making the vertical intercepts between the CO points equal to

times those of the  $\text{CO}_2$  divisions. On the various lines  $n = \text{constant}$  these differences are unequal, since  $B$  depends on  $n$ . Fig. 10 represents the complete combustion triangle for oil.

This representation is of little practical value in the case of combustion with insufficient air, since there is always  $\text{H}_2$  along with CO under this condition, as shown in the author's book *Neue Tabellen und Diagramme*. The diagram is of use, however, in the case of incomplete combustion with excess air, particularly when the analysis of the gases shows only the  $\text{CO}_2$  and  $\text{O}_2$  proportions, because the diagram then serves to determine the unknown CO value.

## CHAPTER II

### THERMAL PROPERTIES OF GASES

#### HEAT QUANTITIES

It is now known that heat is a form of energy, due to the motion of the molecules of a substance. This is known as the kinetic theory, but the subject matter of this chapter holds, whether or not the kinetic theory be true.

When heat is supplied to a substance, it spreads uniformly throughout it. Equal parts by weight of a substance, which is isolated from other substances, contain, under conditions of thermal equilibrium, equal fractions of the whole thermal energy contained in the body.

Experience shows that the same substances are capable of storing greatly different amounts of thermal energy. Varying amounts of heat can be supplied to a body by a flame or other heat source, while a hot substance can give up heat to other substances.

Even if nothing be known of the manner by which a substance receives its heat, it is possible to decide, by the sense of touch, whether a substance contains more or less heat than another, so that a distinction is made between "hotter" and "colder" bodies. This degree of heat is known as temperature.

The same substances, having the same weight, contain more heat at a high than at a low temperature, but experience shows that different substances having the same weight (such as 1 lb. of iron and 1 lb. of water) may contain very different quantities of heat even if they be at the same temperature. Thus water requires, in order to heat it through the same temperature range, about nine times more heat than the same weight of wrought iron.

Bodies at different temperature levels, when brought in contact with one another for a sufficient length of time, assume the same temperature, even if they have widely different properties. Temperature is thus an indication of the state of thermal energy, but is independent of the amount of heat contained in a substance. It is somewhat analogous to "potential" in electrical energy.

It is possible to decide with some certainty, by the sense of touch, in the case of two substances, whether and by how much, the one is warmer or colder than the other. In bodies of different material, however, this method in general fails when the temperature difference is not high. Naturally the sense of touch conveys no quantitative measure of these differences.

A measure of temperature, which is independent of the nature of heat, is afforded by the changes in the state of bodies when receiving or giving up heat. Almost all the physical properties of a body are varied by heat.

The most important of these is the change in volume, and it is this change which is generally used to measure temperature changes. Two different or similar substances are at the same temperature if they, on

being brought in contact with one another, experience no change in volume. The temperatures will also be equal if these bodies effect the same change in volume of a third substance, with which they are brought in contact, provided the weight of this third substance is negligibly small compared with the other two. If, for example, it is desired to find whether water and air are at the same temperature, a tube partly filled with mercury (and called a thermometer) is first placed in the air and then in the water. If no change in the volume of the mercury occurs, the air and water are at the same temperature. If, however, the mercury rises in the tube when this is placed in the water, then the water is warmer than the air. The temperature is thus decided by the position of the mercury thread.

The basic scale of temperatures is obtained by means of the expansion of gases, since, of all substances, these show the greatest change in volume when heated. Amongst gases, hydrogen is distinct as showing regularity in the scale.

Other changes, due to heat, made use of in thermometry are the variation in electrical resistance and the thermo-electric current. In the latter, two metal wires of different materials are soldered together at the ends. The one junction is placed in the region where the temperature is to be measured, and the other is placed in the atmosphere or in melting ice. The difference in potential at the two junctions is nearly proportional to the difference in temperature. For measuring flue gas temperatures, one wire is of platinum and the other is an alloy of platinum and rhodium. For steam temperatures, copper and constantan wires are used. In optical pyrometers, which are used for temperatures above  $1000^{\circ}\text{C.}$ , the temperature of glowing bodies is determined by their brightness.

The total radiation pyrometer measures the temperature by the intensity of the total, i.e. visible and invisible, radiation.

**Specific heat.** If the quantity of heat contained in a body be increased in any manner, the temperature rises. For equal weights of different bodies, however, equal temperature changes can correspond to very different quantities of heat. The capacity for taking up heat depends on the nature of the body.

The amount of heat received by 1 lb. of water when its temperature is raised from  $14.5^{\circ}\text{C.}$  to  $15.5^{\circ}\text{C.}$  is taken as the unit of heat and is called the "Centigrade heat unit," or C.H.U.

If, in place of 1 lb., the weight 1 kg. is used, the unit of heat is then called the "large calorie," and one-thousandth part of this gives the gram calorie.

The term "specific heat" of any body means the number of heat units required to raise 1 lb. of the body through  $1^{\circ}\text{C.}$

In the case of solids and liquids, the specific heat is constant, apart from slight variations with temperature, but in the case of gases or vapours,  $c$  varies considerably, depending on the external conditions under which the heating proceeds.

Heat quantities are generally measured by bringing the substances containing the heat into conducting communication with water. The heat transferred is then determined from the weight and rise in temperature of the water (calorimeter).

The specific heat of solids and liquids is not quite independent of



# THERMAL PROPERTIES OF GASES

## MEAN SPECIFIC HEAT OF SOLIDS AND LIQUIDS

(For the specific heat of gases, see pages 48 and 53)

SUBSTANCE	SPECIFIC HEAT C.H.U./lb.
Aluminium . . . . .	0.17-0.22
Lead . . . . .	0.03
Iron (various) . . . . .	0.11 (up to 100° C.)
Copper . . . . .	0.09
Zinc . . . . .	0.09
Tin . . . . .	0.056
Bronze . . . . .	0.09
Stones (various) . . . . .	About 0.20 when dry
Bituminous coal . . . . .	0.31
Glass . . . . .	0.11-0.22
Water. . . . .	1 at 15° C.
Ice . . . . .	0.502 (between -1° and -21° C.)
Ammonia . . . . .	0.93 (0° to 20° C.) 0.86 (0° to -20° C.)
Sulphur dioxide } Liquid	0.33 (0 to 20° C.) 0.31 (0° to -20° C.)
Carbon dioxide }	0.64 (0 to 20° C.) 0.48 (0° to -20° C.)
Alcohol . . . . .	0.56 (0° to 15° C.)
Olive oil . . . . .	0.47
Petrol and Paraffin . . . . .	0.50

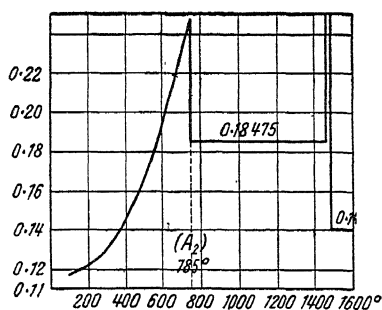


FIG. 11

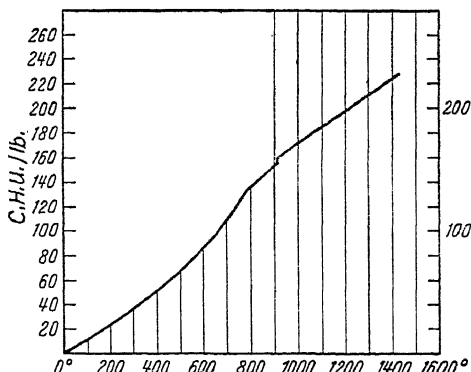


FIG. 12

temperature, even when far removed from the melting or boiling points. The differences, within moderate temperature limits, are, however, not serious. Thus the specific heat of mercury decreases continuously from 0.0335 at 0° C. to about 0.0327 at 100° C., i.e. a variation of 2.5 per cent. The variation of  $c$  for water within these limits is less than 1 per cent.

The peculiar dependence of the specific heat of iron on temperature is shown in Figs. 11, 12, and 13.\*

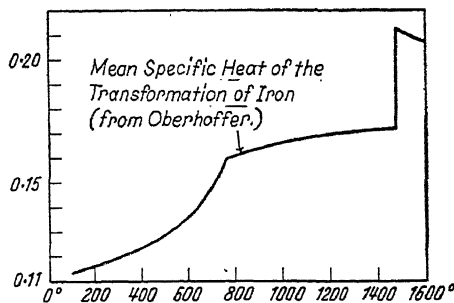


FIG. 13

\* From Stahl and Eisen, Oberhoffer (1927), page 582. See also *Forsch. Arb.*, No. 204. Wüst, Meuthen, and Durrer. "Die Temperatur-Wärmeinhaltskurven der technisch wichtigen Metalle."

value of  $c$  at  $785^{\circ}\text{C.}$  is due to the internal change in the iron (transformation point).

At very low temperatures, however, the  $c$  values are governed by totally different conditions. From Nernst's experiments on the specific heats at temperatures near absolute zero, new data have been obtained, which are not only of the greatest importance in connection with the specific heats themselves, but also for the general properties of substances and the nature of heat, particularly with regard to the laws of chemical reaction. It is only by means of these results that the gap between thermal and chemical processes can be bridged.

It was formerly assumed that the specific heat of solids and under-cooled liquids continuously decreased as the temperature decreased and that, at  $T = 0^{\circ}\text{C.}$ , its value could be found by extrapolation of the curve given by points near this region, as shown by the dotted curve

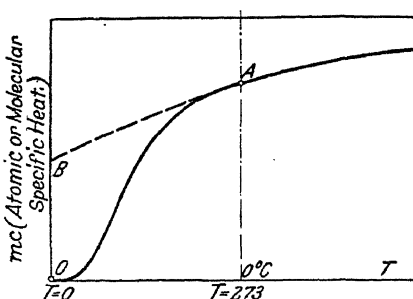


FIG. 14

$AB$  in Fig. 14.\* This assumption is now known to be wrong, as a result of experimental work carried out by Nernst (since 1911) at temperatures as low as that of liquid hydrogen. The tests proved that the specific heats of widely different simple and compound substances drop very rapidly below a certain temperature level, and actually become zero at  $T = 0$ . Near the absolute zero point, even at finite temperatures, the decrease in the specific heat values is immeasurably small. This is well shown by carbon in the diamond modification (and also by amorphous and graphitic carbon), for which the specific heat is exceedingly small at as high a temperature as  $40^{\circ}\text{C. abs.}$  For metals, the corresponding temperature is considerably lower.

Fig. 15 shows a part of the results of Nernst's measurements. The atomic or molecular specific heats† are plotted to a base of absolute temperatures. At temperatures above  $273^{\circ}\text{C. abs.}$  ( $0^{\circ}\text{C.}$ ) the different metals have approximately the same specific heats, amounting to about 6 C.H.U./mol. This relationship is known as the "Dulong-Petit Law." The accelerated drop for the different metals commences, however, at very different temperatures, the latest being for lead and the earliest for

\* Nernst also assumed this in establishing his heat theorem in 1906, but with the proviso that the specific heats of one and the same body, in different modifications, would be equal at  $T = 0$ .

† That is the specific heats per pound multiplied by the atomic or molecular weights  $m$ .

aluminium. At low temperatures, therefore, the atomic heats vary widely and the Dulong-Petit Law breaks down entirely. The greatest deviation from the atomic heats of the metals is shown by the diamond. The approach to a maximum value for this substance can only be expected at a high temperature.

The actual effect of these very small specific heat values with a

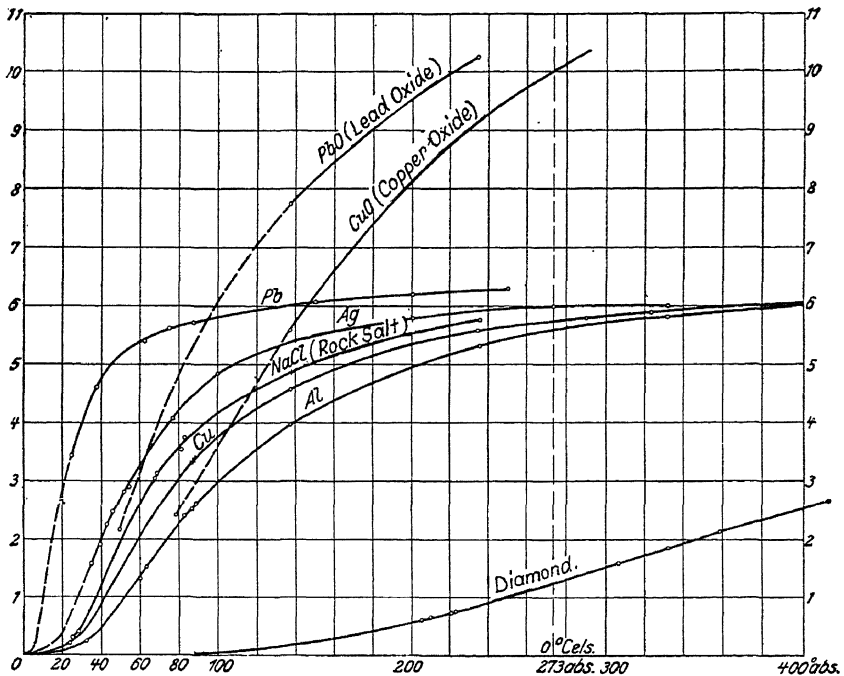


FIG. 15

supply or removal of heat is as follows. From the definition of specific heat we have

$$\Delta Q = (mc) \Delta T$$

where  $(mc)$  is the mean molecular specific heat during a small change in temperature  $\Delta T$ .

Hence 
$$\frac{\Delta Q}{mc}$$

If now  $mc$  is very small, for example one thousandth (diamond at 55° C.), then

$$\Delta T = 1000 \Delta Q$$

At about 100° C., however,  $mc$  becomes equal to about 2, so that

The same heat supply  $\Delta Q$  at low temperatures thus causes a temperature rise 2000 times greater than that at the higher temperature.

If, for example,  $\Delta Q = \frac{1}{1000}$  C.H.U., then, at the high temperature,

$$\Delta T = \frac{1}{2000}^{\circ}\text{C.}$$

and, at the lower temperature,

$$\Delta T = 1^{\circ}\text{C.}$$

**Gases.** Provided the temperature differences are small, the specific heats of gases ( $c_p$  and  $c_v$ ) may be regarded as practically constant. Tests at the high temperatures between  $1000^{\circ}\text{C.}$  and  $3000^{\circ}\text{C.}$  show, however, that the specific heats are considerably in excess of those between  $0^{\circ}$  and  $100^{\circ}\text{C.}$  This is of importance in problems connected with internal combustion engines where temperatures about  $2000^{\circ}\text{C.}$  are found.

In the case of superheated vapours which are near the saturated state (e.g.  $\text{CO}_2$  at room temperature) the variation, even for small temperature differences, is considerable. In the case of superheated steam, not only the temperature but also the pressure seriously affect the specific heat values, particularly in close proximity to the saturated state. The pressure has no effect, however, on the specific heat values of gases far removed from the saturated state.

**Mean and true specific heat.** If the amount of heat, received by 1 lb. of any body when its temperature is raised from  $t_0^{\circ}$  to  $t_1^{\circ}$ , be  $Q$ , the value

$$\frac{Q}{t_1 - t_0} = c_m$$

is called the mean specific heat. This is the average amount of heat required per degree rise in temperature.

If it is found that this quotient has the same value for any other temperature interval within  $t_1 - t_0$ , then it is the same as the true specific heat. In this case the same heat  $c$  is required for each degree rise. The mean and true specific heats are then constant.

If, however, the quantities of heat required for equal temperature differences at different temperature levels be unequal, the specific heats are also unequal. Even within the chosen intervals,  $c_m$  may be unequal. As the intervals are reduced, however, the difference between the true and mean specific heat tends to disappear until, for an indefinitely small interval, the specific heats become equal. The true specific heat is, therefore,

$$c = \frac{\Delta Q}{\Delta t}$$

where  $\Delta Q$  is the heat supplied for the small temperature rise  $\Delta t$ .

If the quantities of heat  $Q$  required to raise the temperature from  $t_0^{\circ}$  to  $t_1^{\circ}$  be plotted to a base of temperature (Fig. 16), the resulting curve is straight when the specific heat is constant and curved when the specific heat is variable, since it is only in the former case that the quantity of heat is proportional to the temperature change.

When  $c$  is variable, its true value is given by the ratio  $\Delta Q/\Delta t$ , and

## THERMAL PROPERTIES OF GASES

this is different at every temperature, since the tangent to the curve changes continuously. If the  $Q$  line be given, i.e. determined by experiment,  $c$  is found from

$$c = \frac{\Delta Q}{\Delta t} = \frac{Q}{t - t'} \quad (\text{true specific heat at } t)$$

and

$$c_m = \frac{Q}{t - t_0} \quad (\text{mean specific heat between } t_0 \text{ and } t)$$

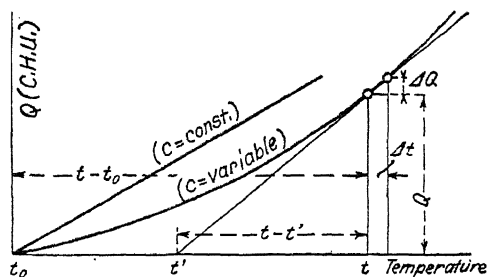


FIG. 16

If the true specific heats be plotted to a base of temperature (Fig. 17) the narrow shaded area is  $c \Delta t = \Delta Q$ . The total heat required to raise the temperature from  $t_0^\circ$  to  $t^\circ$  C. is therefore given by the area  $OA A_1 A'_1$ . The mean specific heat  $c_m$  is the mean height of this area. If the  $c_m$  values are plotted to a temperature base, the variation in their values is

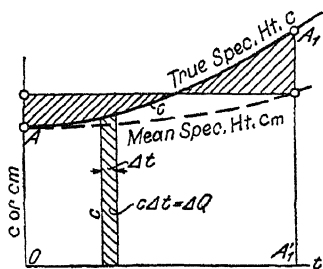


FIG. 17

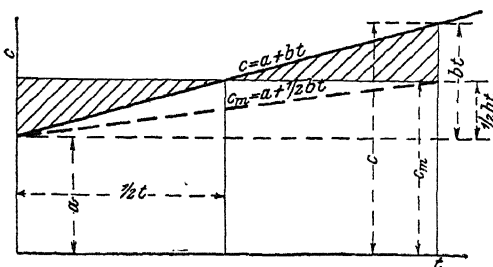


FIG. 18

obtained, and Fig. 17 shows that their values are all smaller than the true values.

In the special case of the specific heat varying as a linear function of the temperature, both the  $c$  and  $c_m$  lines are straight. The mean specific heat  $c_m$  between  $t_0$  and  $t$  is then the same as the true specific heat midway

between  $t_0$  and  $t$ , i.e. at  $\frac{t_0 + t}{2}$  (Fig. 18).

If, in general,  $c = a + bt$

then  $c_m = a + \frac{b}{2}t$

between 0 and  $t^\circ$ .

If the initial temperature is not  $0^\circ$  but  $t_0$ , the mean specific heat for the temperature interval  $t - t_0$  is

$$(c_m)_{t_0}^t = a -$$

**Relation between the mean specific heats  $c_{pm}$  and  $c_{vm}$  between the same temperature limits.** In almost all tests on specific heats it is the mean values  $c_{pm}$  or  $c_{vm}$  which are determined. A definite relationship, however, exists between the two, so that one can be found from the other.

The relationship between the true specific heats is (page 94)

$$+ 1.985$$

and, multiplying both sides by  $dt$ , gives

$$c_p dt = c_v dt + \frac{1.985 dt}{m}$$

Integrating between the limits  $t_0$  and  $t$  gives

$m$

$$\text{or} \quad c_{pm} = c_{vm} + \frac{1.985}{m}$$

$$\text{and also} \quad C_{pm} = C_{vm} + 1.985 \quad (\text{values per mol.}).$$

### SPECIFIC HEATS OF GASES

The external conditions under which the heating of solids or liquids occurs have little or no effect on the specific heat of these substances. The case is, however, entirely different with gases, since their specific heats are affected considerably by these conditions.

There are two fundamentally important cases, namely, heating at constant volume and heating at constant pressure. In the first case, the gas is contained in a closed vessel of constant volume, and the quantity of heat required to raise 1 lb. of the gas through  $1^\circ \text{C.}$  is then called the specific heat at constant volume  $c_v$ . In the second case the gas is heated in a vessel, sealed with a movable piston, so that the volume may change, while the pressure is kept constant. Under these conditions the amount of heat required to change the temperature of 1 lb. of the gas by  $1^\circ \text{C.}$  is called the specific heat at constant pressure  $c_p$ .

All measurements of the specific heats of gases can be related to these two cases, i.e. if  $c_v$  or  $c_p$  is known, the specific heat under any other conditions can be determined.  $c_v$  and  $c_p$  themselves are mutually determinate.

With regard to the specific heats  $c_v$  and  $c_p$ , all the different types of gases can be divided into three clearly defined groups, namely, the monatomics, the diatomics, or mixture of diatomics, and the polyatomics.

For the first and second groups the following general law is valid, with fair accuracy.

If the pressure, volume, and temperature of two different gases, having the same number of atoms in the molecule, are the same, the

amounts of heat required to increase their temperature through the same range are equal.\*

If the unit of weight chosen is that contained in 1 ft.<sup>3</sup> of gas at S.T.P., the specific heats  $C'_v$  and  $C'_p$  for this unit of weight are the same for all diatomic gases, i.e.

$$C'_v(\text{O}_2) = C'_v(\text{N}_2) = C'_v(\text{CO}) = C'_v(\text{H}_2) = C'_v(\text{air})$$

and similarly for the  $C'_p$  values.

The same law holds for the monatomic gases, but the actual values of  $C'_v$  and  $C'_p$  are then smaller than those for the diatomics in the ratio of 3 to 5.

The usual value of the specific heat, referred to 1 lb., is found from the above values as follows. The weights of equal volumes of different gases are proportional to their molecular weights  $m_1, m_2$ , etc. Now to heat  $m_1, m_2$ , etc., lb. of gases at constant pressure through 1° C., the heats necessary are  $m_1 c_{p1}, m_2 c_{p2}$ , etc., C.H.U.s. From the law given above these quantities of heat are equal, for diatomic gases, since  $m_1, m_2 \dots$  lb. of each gas occupies the same volume, namely, 359 ft.<sup>3</sup> at S.T.P. Hence

On the other hand, the specific heat referred to the mass of 359 ft.<sup>3</sup> at S.T.P. is 359  $C'_p$ , which gives

$$C_p = m_1 c_{p1} = m_2 c_{p2} = m_3 c_{p3} \dots = 359 C'_p$$

The product  $mc_p$ , i.e. the specific heat referred to a mass of  $m$  lb. of a gas, is called the molecular specific heat  $C_p$ . The diatomic gases have therefore the same molecular specific heats, as also the monatomic gases.

The average value for the diatomic gases at 0° C. is

$$C_p = mc_p = 6.86 \text{ C.H.U./lb. mol.}^\dagger$$

$$C_v = mc_v = 4.88 \text{ C.H.U./lb. mol.}$$

so that

$$C'_p = 0.0191 \text{ C.H.U./ft.}^3 \text{ at S.T.P.}$$

$$C'_v = 0.0136 \text{ C.H.U./ft.}^3 \text{ at S.T.P.}$$

The calculated values for  $c_p$ , using the respective molecular weights, are given below, along with the results obtained by direct measurement at temperatures between 15 and 20° C.

	O <sub>2</sub>	H <sub>2</sub>	N <sub>2</sub>	CO	Air	
$m =$	32	2.016	28.08	28	28.95	
$c_p =$	0.214	3.40	0.244	0.245	0.237	Calculated
$c_p =$	0.218	3.408	0.249	0.250	0.241	By test

and for  $c_v$ ,

$c_v =$	0.152	2.42	0.174	0.174	0.168	Calculated
$c_v =$	0.156	2.42	0.178	0.179	0.172	By test

\* Excepting the gases with a high molecular weight (halogens), such as chlorine. From recent determinations in the Nernst Laboratory the values for chlorine gas at 16° C. and 1 atmosphere are  $mc_v = 6.39$ ,  $mc_p = 8.49$ , and  $\frac{c_p}{c_v} = 1.33$ .

† On recently determined values for the separate gases, see below and also p. 131. As shown on p. 131, the law is only approximately true at very high temperatures.

**Variation with temperature.** It is now known that the specific heats  $c_v$  and  $c_p$  of all gases, apart from the monatomics, increase with increasing temperature. For most gases the increase is directly proportional to the temperature increase, so that

$$c_p = c_{p_0} + bt$$

and

$$c_v = c_{v_0} + bt$$

The factor  $b$  is the same for both  $c_p$  and  $c_v$ , since the difference between  $C_p$  and  $C_v$  must be the same at all temperatures.

From tests carried out by Holborn and Henning, in 1907, the specific heat of nitrogen between 0° C. and 1400° C. is given by

$$c_p = 0.235 + 0.000038t$$

and from tests by Langen, in 1903,

$$c_v = c_{v_0} + 0.0000378t$$

These results show almost complete agreement with regard to the variation with temperature, in spite of the widely different methods adopted in their determination.

The law mentioned above, which states that the specific heats of all diatomic gases are about equal, is valid also at high temperatures, as proved by Langen, who carried out tests on different mixtures of these

From more recent tests in the German Imperial Institute, the following has been obtained for nitrogen—

$$c_p = 0.2491 + 0.000019t$$

which gives an increase in  $c_p$  with temperature only about half that given by the equation above. At 800° C. both equations give about the same result.

The mean specific heat from this is

$$c_{pm} = 0.2491 + 0.0000095t$$

and the true molecular specific heat for nitrogen with  $m = 28.02$

$$C_p = mc_p = 6.98 + 0.000532t$$

and

$$C_v = mc_v = 4.99 + 0.000532t$$

The mean molecular specific heat between 0° and  $t^\circ$  is

$$C_{pm} = (mc_p)_m = 6.98 + 0.000266t$$

and

$$C_{vm} = (mc_v)_m = 4.99 + 0.000266t$$

Up to about 1000° C. these molecular specific heats can also be applied to the other diatomic gases.

At higher temperatures the specific heats increase more rapidly, and between 1200° C. and 2500° C. the following applies\*—

$$C_p = 7.0 + 0.000713t$$

These equations have been used for the results given in this table.

$t$	=	0	100	200	300	500	1000	1200	1400	2000° C.
$(mc_p)_m$	=	6.98	7.01	7.03	7.06	7.11	7.35	7.44	7.51	7.72
$mc_p$	=	6.98	7.03	7.09	7.14	7.25	7.64	7.82	8.00	8.43
$mc_v$	=	4.99	5.04	5.10	5.15	5.26	5.64	5.83	6.00	6.44

\* W. Schüle, *Neue Tabellen und Diagramme*, page 32.



The values given by Nernst\* for  $N_2$ ,  $O_2$ ,  $HCl$ , and  $CO$  are

$$mc_v = 4.90 \quad 4.93 \quad 5.17 \quad - \quad 5.35 \quad - \quad 5.75 \quad - \quad 6.22$$

and, for hydrogen,

$$mc_v = 4.75 \quad 4.78 \quad 5.02 \quad - \quad 5.20 \quad - \quad 5.60 \quad - \quad 6.00$$

The most recent experimental work by Nernst and Wohl† shows, however, that the variation in specific heats is not proportional to the temperature change, but is a totally different function of the temperature,

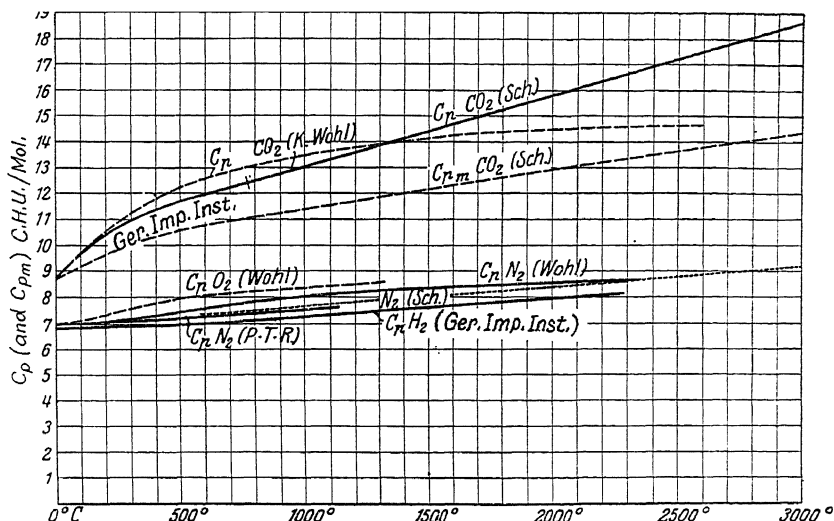


FIG. 19

as determined optically from the internal vibratory energy of the atoms in the molecule.

In Fig. 19 the values of the molecular specific heats, given by the above equations, have been plotted for  $N_2$ ,  $H_2$ , and  $O_2$  and show that the curves rise with temperature by varying amounts. The values for  $N_2$  from the German Imperial Institute have also been plotted, as well as those calculated by the author above  $1000^{\circ}C$ . from test results of various investigators.‡ This curve coincides with the German Imperial Institute curve at the lower temperatures.

**Relation between  $c_p$  and  $c_v$ . Ratio  $\gamma = c_p/c_v$ .** On page 94 it is shown that the relationship between  $c_p$  and  $c_v$  is expressed by

so that  $c_p$  is always greater than  $c_v$ .

\* W. Nernst, *Die theoretischen und experimentellen Grundlagen des neuen Wärme-satzes*.

† W. Nernst and K. Wohl, "Spez. Wärme bei hohen Temperaturen," *Zeit. f. Tech. Phys.*, Bd. 10 (1929), No. 12. Also K. Wohl and G. v. Elbe, "Die spez. Wärme des Wasserdampfes bei hohen Temperaturen," *Zeit. f. Phys. Chem.*, Abt. B., Bd. 5 (1929) 3/4 Heft.

‡ W. Schüle, *Neue Tabellen und Diagramme*.

## TECHNICAL THERMODYNAMICS

Now, from page 12, we have

$$R = \frac{2780}{m}$$

and hence

$$\frac{2780}{m} = 1.985$$

The difference between the molecular specific heats is thus constant for all gases and equal to 1.985.

If the specific heat  $c_p$  and the molecular weight  $m$  are known for any gas,  $c_v$  is given by

$$c_v = c_p - \frac{1.985}{m}$$

Further, with

$$= c_p - \frac{1.985}{m}$$

the ratio of the two specific heats is

$$mc_v$$

Alternatively,

$$\gamma = \frac{c_p}{mc_p - 1.985}$$

**Example 1.** Find the value of  $c_p$  for nitrogen if  $\gamma = 1.41$ .

$$mc_v = \frac{1.985 \times 1.41}{0.41} = 6.824 \text{ (mol. specific heat)}$$

$$\frac{6.824}{28.08} = 0.243$$

As shown by the expression for  $\gamma$ , all the diatomics must have the same  $\gamma$  value, since their molecular specific heats are equal. If  $c_v$  is assumed to be 4.99, as given above, then

The following values of  $\gamma$  have been determined by test: for  $H_2$ , 1.408,  $N_2$ ,  $\gamma = 1.41$ ,  $O_2$ ,  $\gamma = 1.398$ ; air,  $\gamma = 1.401$  to 1.406;  $CO$ , 1.403. The mean of these is 1.404.

**Effect of temperature on  $\gamma$ .** The relation

$$\gamma = 1 + \frac{1.985}{mc_v}$$

is valid for all temperatures. But, for diatomic gases,

$$mc_v = 4.99 + 0.000532t$$

therefore

$$\gamma = 1 + \frac{1.985}{4.99 + 0.000532t}$$

From this, it will be seen that  $\gamma$  decreases with increase in temperature, although the variation is small within small temperature changes. Thus, if  $\gamma = 1.398$  at  $0^\circ \text{C.}$ , its values at  $100^\circ$  and  $200^\circ \text{C.}$  are  $1.395$  and  $1.390$ . It is permissible therefore to accept the value  $\gamma = 1.40$  for temperatures below  $200^\circ \text{C.}$

At very high temperatures, however,  $\gamma$  becomes considerably smaller. Thus, at  $1000^\circ \text{C.}$ ,  $\gamma = 1.360$ . Fig. 23 shows how  $\gamma$  varies with temperature. Between  $0^\circ$  and  $2000^\circ \text{C.}$  we have, approximately,

$$\gamma = 1.40 - \frac{0.5}{10000} t \quad (\text{for } \text{O}_2, \text{N}_2, \text{H}_2, \text{CO, and air})$$

### SPECIFIC HEATS OF POLYATOMIC GASES

**Carbon dioxide.** The most reliable value at  $0^\circ \text{C.}$  is\* (for atmospheric pressure)  $c_p = 0.1971$ .

A close agreement to this is given by Partington, as shown by the following table—

At	$0^\circ$	$20^\circ$	$100^\circ \text{C.}$
	$= 0.1989$	$0.1991$	$0.2132$
	$= 0.1518$	$0.1527$	$0.1675$
	$= 1.3084$	$1.3017$	$1.2727$

Holborn and Henning† gave the following mean specific heat values between  $20^\circ$  and  $t^\circ \text{C.}$ —

$t$	$= 200$	$440$	$630$	$800$	$847$	$1000$	$1200$	$1360^\circ \text{C.}$
	$= 0.2168$	$0.2306$	$0.2423$	$0.2493$	$0.2491$	$0.2602$	$0.2654$	$0.2678$

These values were obtained by cooling a previously electrically heated mass of gas in an oil calorimeter.

Explosion tests, in which CO was burned to  $\text{CO}_2$  in a bomb, and the maximum pressure determined, were carried out by A. Langen,‡ and by Pier and Bjerrum.§ These gave the mean specific heat at constant volume. According to Schreber, Langen's tests gave between  $17^\circ$  and  $1500^\circ \text{C.}$ ,  $(mc_v)_m = 10.45$ , so that  $(mc_p)_m = 12.43$ , and  $(c_p)_m = 0.283$ ; and between  $17^\circ$  and  $1700^\circ \text{C.}$ ,  $(mc_v)_m = 11.20$ , so that  $(mc_p)_m = 13.18$ , and  $(c_p)_m = 0.300$ .

The mean value given by Bjerrum and Pier between  $17^\circ$  and  $2714^\circ \text{C.}$  is  $(mc_v)_m = 10.9$ , so that  $(mc_p)_m = 12.88$ , and  $c_{p,m} = 0.293$ .

The results of recent calculations by the author, based on various explosion tests, are given below. The values below  $800^\circ \text{C.}$  are those supplied by the German Imperial Institute. Between  $800^\circ$  and  $3000^\circ \text{C.}$  these results can be expressed by

$$C_p = 12.525 + 2.75 \frac{t - 800}{1000}$$

$$\text{or} \quad = 0.284 + 0.0625 \frac{t - 800}{1000}$$

\* Holborn, Scheel, and Henning, *Wärmetabellen d. Physik-Techn. Reichsanstalt.*

† *Annalen d. Physik* (1907), Bd. 22. (Phys.-Tech. Reichsanst.)

‡ *Z.V.d.I.* (1903), No. 18.

§ *Phys. Zeit.* (1913), page 973.

## MEAN AND TRUE SPECIFIC HEAT VALUES FOR CARBON DIOXIDE AT CONSTANT PRESSURE

$t^{\circ}\text{C.}$	0	100	200	300	400	500	600	700	800	900	1000
$c_p$	0.197	0.209	0.219	0.227	0.234	0.240	0.245	0.249	0.252	0.256	0.260
$c_v$	0.197	0.220	0.237	0.251	0.260	0.267	0.272	0.279	0.284	0.290	0.297
$\gamma$	1.297	1.258	1.235	1.219	1.210	1.203	1.198	1.193	1.188	1.184	1.179
$t$	1200	1400	1600	1800	2000	2200	2400	2600	2800	3000	
$c_m$	0.267	0.274	0.280	0.287	0.293	0.300	0.307	0.314	0.320	0.326	
	0.309	0.322	0.335	0.347	0.360	0.372	0.385	0.397	0.410	0.422	
	1.170	1.163	1.155	1.150	1.143	1.137	1.132	1.127	1.123	1.118	

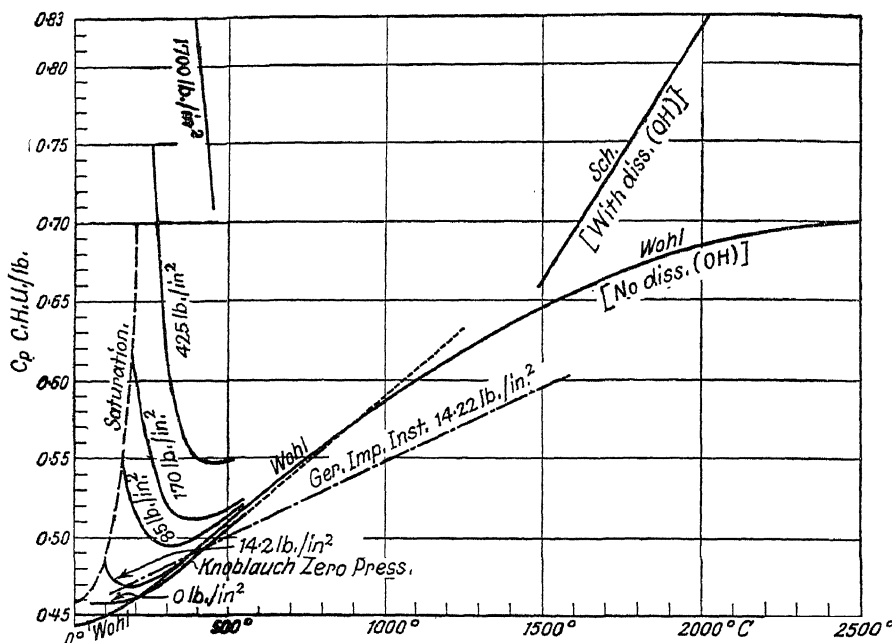


FIG. 20

**Steam (gaseous).** Steam can also be regarded as a gas, if it be sufficiently remote from the saturation state. This occurs in the case of the steam in flue gases, or in internal combustion engines, where, as a constituent, it exerts a low pressure and may have a temperature as high as  $2000^{\circ}\text{C.}$  At a pressure of one atmosphere and temperature of a few hundred degrees Centigrade, it behaves almost exactly as a gas. It may also be regarded as a gas at temperatures well below  $100^{\circ}\text{C.}$  when it exerts a very low pressure (for example, as vapour in moist air). Its specific heats  $c_p$  and  $c_v$  can, under these conditions, be regarded as dependent on the temperature, but not on the pressure. In Fig. 20, the values of  $c_p$ , as given by the Munich\* tests, have been plotted for gauge pressures of 0, 14.2 and 85, 170, 425 and 1700 lb./in.<sup>2</sup> The values for the latter group are very high, owing to the close proximity of the steam to the saturated state.

\* Knoblauch, Raisch, and Hausen. *Tabellen und Diagramme für Wasserdampf* (1923).

## THERMAL PROPERTIES OF GASES

Calorimetric tests, on steam at 1 atmosphere and between 110° C. and 1350° C., were carried out by Holborn and Henning, in the same way as described above for CO<sub>2</sub> and N<sub>2</sub>. Their results, as found in the heat tables of the German Technical Physical Institute (Holborn, Scheel, and Heuse), have also been plotted in Fig. 20. At the higher temperatures, between 1400° and 2700° C., the specific heats were determined by explosion tests with hydrogen and oxygen mixtures. The first tests of this nature were carried out by Mallard and Le Chatelier (1883) with a cylindrical bomb, but spherical bombs were used in the later tests by A. Langen.\*

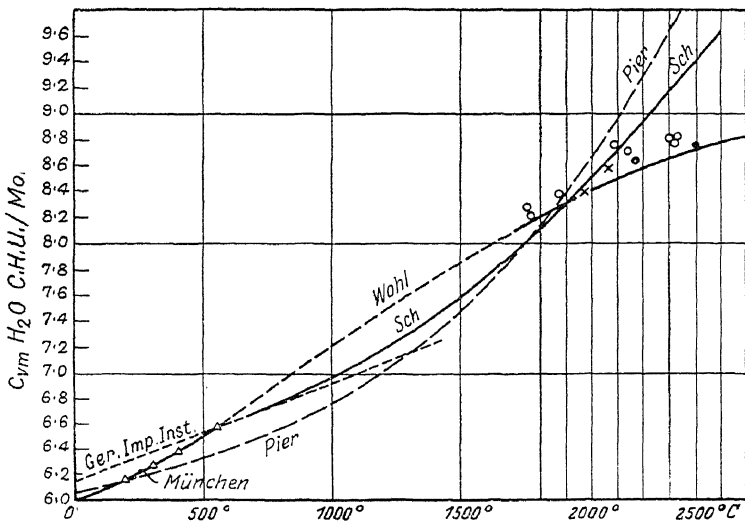


FIG. 21

Numerous explosion tests have been carried out (mostly in the Nernst Laboratories) by Pier (1909), Bjerrum (1912), Siegel (1914), K. Wohl (1924), and K. Wohl and G. v. Elbe (1929). The work of these investigators has been of use, not only in determining the specific heats of steam, but also in connection with dissociation. In these tests, the mean specific heats at constant volume, between the temperature before ignition (10 to 20° C.) and the maximum temperature of combustion, have been determined. In Fig. 21 the mean molecular specific heats  $(C_v)_m$  found recently by K. Wohl, as well as the earlier results due to Pier, have been plotted. Wohl's curve below 550° C. has been plotted from the results of the Munich tests, and, above, from the results of his energy values obtained by optical measurement (page 133). The results obtained by the author, from a comprehensive survey of a large number of explosion tests, are also shown. The lower portion of the curve is supplied by Holborn's and Henning's readings. This curve (marked Sch.) agrees closely with the test results and with Wohl's curve between 1800° C. and 2100° C., but above this, its slope, like that of Pier's, is considerably steeper. The cause of this deviation is probably due to dissociation of some of the H<sub>2</sub>O to OH and  $\frac{1}{2}$  H<sub>2</sub>, the effect of which

\* *Forsch. Arb.*, Heft. 8, 1903.

## TECHNICAL THERMODYNAMICS

causes an apparent increase in the specific heat. Since the determination of this dissociation is involved, it is preferable to use the steeper curve for which this effect (which can also occur with excess  $O_2$ ) is included.

### MEAN AND TRUE SPECIFIC HEATS OF HIGHLY SUPERHEATED STEAM AT CONSTANT PRESSURE\*

$t^\circ \text{C.}$	0	100	200	300	400	500	600	700	800	900	1000
	0.453	0.459	0.463	0.468	0.471	0.476	0.480	0.485	0.489	0.493	0.499
	0.453	0.462	0.471	0.481	0.490	0.499	0.508	0.517	0.527	0.537	0.550
$\gamma$	1.322	1.314	1.306	1.298	1.290	1.284	1.277	1.271	1.264	1.258	1.251
$t^\circ \text{C.}$	1100	1200	1400	1600	1800	2000	2200	2400	2600	2800	3000
	0.503	0.508	0.520	0.538	0.559	0.583	0.606	0.632	0.659	0.685	0.710
	0.564	0.581	0.632	0.695	0.758	0.823	0.886	0.948	1.012	1.073	1.135
	1.243	1.234	1.211	1.189	1.170	1.154	1.142	1.131	1.122	1.114	1.107

See the later chapters on vapours regarding the specific heats of steam in the saturation region and at high pressures.

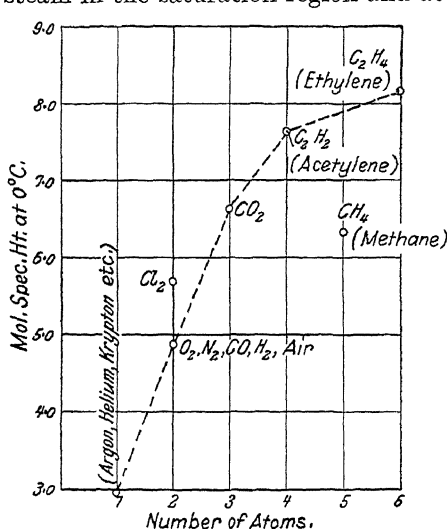


FIG. 22

**Ethylene ( $C_2H_4$ ).** The measured value of  $\gamma$  between  $0^\circ$  and  $40^\circ \text{C.}$  is 1.244 and of  $(c_p)_m$  between  $10^\circ$  and  $202^\circ \text{C.}$  0.404.

From  $\gamma$  we have

$$\frac{1.985}{0.244} = 8.13$$

so that with  $m = 28.032$

$$c_{v_0} = 0.290 \text{ and } c_{p_0} = 0.361$$

Comparing  $(c_p)_m$  with  $c_{p_0}$  again shows an increase in  $c_p$  with temperature.

**Acetylene ( $C_2H_2$ ).** The measured value of  $\gamma$  is 1.26, hence

$$mc_v = \frac{1.985}{0.26} = 7.64 \text{ and with } m = 26.016$$

$$c_v = 0.294 \text{ and } c_p = 0.370$$

\* From W. Schüle's *Neue Tabellen*. Wohl's values are given on page 135.

**Methane ( $CH_4$ ).** By measurement  $\gamma = 1.316$  and 1.313 (between  $11^\circ \text{C.}$  and  $30^\circ \text{C.}$ ), which gives

$$\frac{1.985}{\gamma} = 6.32$$

This is considerably higher than that of the diatomic gases (4.9). With the molecular weight 16.032 we have

$$= 0.394 \text{ and } c_{p_0} = 0.518$$

The value of  $c_{pm}$  between  $18^\circ \text{C.}$  and  $208^\circ \text{C.}$ , as given by test, is 0.593. This indicates a considerable rise in  $c_p$  with temperature, but the two values are insufficient to determine the relation between  $c_p$  and  $t$  with any certainty.

## THERMAL PROPERTIES OF GASES

A general survey of these results, as shown in Fig. 22, shows that the molecular specific heat of a gas increases as the number of atoms in the molecule of the gas increases. There is, however, no general law regarding this, and the specific heats depend on other conditions as well as the number of atoms. See also p. 131.

### SPECIFIC HEATS OF THE PRODUCTS OF COMBUSTION

When the combustion of a fuel with air is complete, the products formed consist of a mixture of carbon dioxide, steam, nitrogen, and oxygen. The oxygen, however, is absent when combustion occurs without excess air. The analysis of the products is usually given by volume,

$$\text{i.e.} \quad v_{\text{CO}_2} + v_{\text{H}_2\text{O}} + v_{\text{N}_2} + v_{\text{O}_2} = 1 \quad (93)$$

Now since in 1 mol. =  $m$  lb. giving 359 ft.<sup>3</sup> of products, there are  $m_{\text{CO}_2} v_{\text{CO}_2}$  lb. of carbon dioxide,  $m_{\text{H}_2\text{O}} v_{\text{H}_2\text{O}}$  lb. of steam, etc., it follows that in heating 1 mol. of products through 1° C.,

$$mc_p = v_{\text{CO}_2} (mc_p)_{\text{CO}_2} + v_{\text{H}_2\text{O}} (mc_p)_{\text{H}_2\text{O}} + [v_{\text{N}_2} + v_{\text{O}_2}] (mc_p)_{\text{diat.}} \quad (94)$$

In this the value of molecular weight of the products  $m$  is given by

$$m = 44v_{\text{CO}_2} + 18v_{\text{H}_2\text{O}} + 28.08v_{\text{N}_2} + 32v_{\text{O}_2} \quad (95)$$

The molecular specific heats  $(mc_p)_{\text{CO}_2}$ ,  $(mc_p)_{\text{H}_2\text{O}}$ ,  $(mc_p)_{\text{diat.}}$  are found from page 53 *et seq.*, and inserting these in equation (94) gives the molecular specific heat of products having any given analysis. Equation (94) serves also for the mean specific heat.

For a given fuel the specific heat of the products formed will depend on the amount of excess air supplied. Since the molecular specific heats of CO<sub>2</sub> and H<sub>2</sub>O are higher than that of air, it follows that the molecular specific heat of the products is greatest when no excess air is supplied. As the quantity of excess air is increased, the value of  $mc_p$  approaches that for diatomic gases. As shown in Fig. 23, the straight line for the diatomic gases forms a lower limit for all products, while the upper limit is given by the pure products (i.e. the products formed without excess air) corresponding to the fuel under consideration.

In this figure, the curves of the mean molecular specific heats, formed by the combustion of the more important fuels, have been plotted to a base of temperatures. These fuels and the analyses of the pure products formed in burning them are given in the following table—

1. Carbon . . . . .	$v_{\text{CO}_2} = 0.21$	$v_{\text{H}_2\text{O}} = 0$	$v_{\text{N}_2} = 0.79$	$m = 31.4$
2. Bituminous coal . . . . .	0.17	0.05	0.78	30.3
3. Lignite briquettes . . . . .	0.17	0.12	0.71	29.5
4. Mineral oil and distillates . . . . .	0.13	0.14	0.73	28.7
5. Illuminant gas . . . . .	0.10	0.25	0.65	27.2
6. Producer gas from coal . . . . .	0.13	0.14	0.73	28.7
Producer gas from coke . . . . .	0.19	0.06	0.75	30.5
7. Blast furnace gas . . . . .	0.24	0.02	0.74	31.8
8. 90% Spirit alcohol . . . . .	0.12	0.20	0.68	28.0

As shown in Fig. 23, the curves for all the pure products are in close proximity to one another over the entire range from 0° to 2300° C., and deviate but slightly from the mean straight line drawn through them.\*

\* The deviation at 400° C. is about  $\pm 0.05$  units in 7.8, and at 1500° C. about  $\pm 0.08$  units in 8.5, or about  $\pm 0.6$  to 1%. The blast furnace gas curve (not plotted in Fig. 23) shows the somewhat greater deviation of 0.2 units in 8.5, or 2.4%.





Thus, if  $v_{g_0}$  is the fractional volume of the products, and  $v_a$  that of the air, or

$$v_{g_0} + v_a = 1$$

then

$$mc_p = v_{g_0}(mc_p)_p + v_a(mc_p)_a \quad (98)$$

or

$$mc_p = (mc_p)_p - v_a [(mc_p)_p - (mc_p)_a]$$

The quantity in the square brackets is the vertical distance between the pure products line and the pure air line (in Fig. 23), so that if this distance be multiplied by the fractional air volume  $v_a$ , the resulting product gives the distance of a point on the excess air curve from the pure products curve.

The relation between the excess air factor  $n$  and the fractional volume  $v_a$  is found as follows—

The volume of excess air in the products of combustion per lb. (or, in the case of gases, per ft.<sup>3</sup>) of fuel is

$$V_a = (n - 1) A_{min}$$

If now the volume of the pure products is  $V_{g_0}$ , the total volume of products and excess air is

$$(n - 1) A_{min} \quad (99)$$

so that

$$(100)$$

Such fuels as bituminous coal, mineral oil, distillates, and spirit alcohol give values of  $V_{g_0}$  differing but slightly from  $A_{min}$ , so that, for these fuels,

$$n \doteq 1 + \frac{v_a}{\frac{1}{n}} \text{ or } v_a \doteq \frac{n-1}{\frac{1}{n}} \text{ and } v_{g_0} \doteq \frac{1}{\frac{1}{n}} \quad (101)$$

In the case of gaseous fuels, however,  $V_{g_0}$  is considerably greater than  $A_{min}$  and per ft.<sup>3</sup> of fuel is given by

$$V_{g_0} = 1 + A_{min} + \Delta V$$

where  $\Delta V$  represents the change in volume due to combustion (page 26).

Hence

$$A_{\text{excess}} = A_{min}$$

If, for example,

$$n = 1 \text{ (power gas) then } \frac{V_{g_0}}{A_{min}} \doteq 2$$

so that equation (101) is not valid and equation (100) must be used.

Hence, in these cases, the relation between  $n$  and  $\frac{v_a}{v_g}$  or  $v_a$  depends on the particular fuel used, since  $\frac{V_{g_0}}{A_{min}}$  depends on these fuels. Thus, the value of  $\frac{V_{g_0}}{A}$  is, for bituminous coal, 1.03; for mineral oil distillates, 1.07; for

illuminating gas, 1.13; for producer gas, 1.815; and for blast furnace gas, 2.24. As shown by equation (100),  $n$  is a linear function of  $\frac{v_a}{v_{g_0}}$  and has been plotted, for various fuels, in the small auxiliary diagram of Chart I. For a given value of  $n$ , the value of  $v_a$  is read on the abscissa scale.

The true molecular specific heats, found from the mean pure products line, are

and 
$$C_v = 5.51 +$$

while their ratio 
$$\frac{C_p}{C_v} = \gamma = 1 + \frac{1.985}{C_v}$$

The following are obtained for

$t =$	0	500	1000	1500	2000° C.
$\gamma =$	1.360	1.322	1.291	1.265	1.244

and, approximately,

$$\frac{0.55}{10000}$$

or 
$$\gamma = 1.365 \cdot 0.55$$

### CALORIFIC VALUE OF FUELS

Heat used in technical processes (and particularly in those processes used for the production of power) is obtained by means of the combustion of a fuel. Due to the chemical combination of oxygen with the carbon, carbon monoxide, hydrogen, and hydrocarbons contained in fuels, considerable quantities of energy, in the form of heat, are liberated. The term "calorific value" is used to denote the number of heat units which are liberated by the combustion of 1 lb. of a simple or compound fuel or of 1 ft.<sup>3</sup> of a gaseous fuel.

In chemistry, the term "heat tone" denotes the sum of the free and bound energies involved in any chemical reaction between two substances or in the chemical (i.e. molecular) change in a substance. The heat of combustion or calorific value is a special case of heat tone.

The determination of the calorific value of gaseous and easily vaporized fuels can be effected in free air, i.e. at constant atmospheric pressure. The heat absorbed by the products of combustion can be entirely removed by cooling water. The Junker calorimeter is designed on this principle and consists essentially of a tubular cooler, from which the exhaust gases leave at the room temperature. The heat developed (i.e. the heat of combustion) is determined from the amount, and temperature rise, of the water flowing continuously through the vessel.

This means cannot be adopted, however, in the case of solid fuels. For these, burning is accomplished in a closed, stout-walled vessel, called

a bomb calorimeter. Since solid fuels are more difficult to burn than gases or vapours (when the surroundings are cold), it is found necessary to supply pure oxygen, and not air, for combustion. On account of the small volume of the bomb, compressed oxygen is used.

The calorific values, determined at constant pressure and at constant volume, will not be the same for a given fuel, unless the combustion occurs without change in volume. The difference in the calorific values is found by considering the work done by the pressure, due to a change in volume  $\Delta V$ , when the fuel is burned under constant pressure conditions, i.e. at atmospheric pressure in the case of the Junker calorimeter. The work done by the air is  $144 \times 14.7 \Delta V$  ft. lb., which in heat units is

$$\frac{144 \times 14.7 \Delta V}{1400} = 1.512 \Delta V \text{ C.H.U.}$$

This quantity of heat does not appear in the case of combustion at constant volume.

The difference is relatively small. Thus, in the combustion of 2 ft.<sup>3</sup> of carbon monoxide, a reduction in volume of 1 ft.<sup>3</sup> occurs, so that the difference in the calorific values per 2 ft.<sup>3</sup> of fuel is 1.512 C.H.U. The calorific value of 1 ft.<sup>3</sup> of CO is 190 C.H.U./ft.<sup>3</sup>, and hence the difference is only

$$\frac{1.512 \times 100}{90 \times 0.4}$$

which is negligible.

The amount of moisture contained originally in the air and fuel has an effect on the calorific value, as determined by a calorimeter, since the steam formed by combustion will only be completely condensed if the cooled products are saturated with water vapour independently of this steam.\*

**Higher and lower calorific value.** This difference only occurs in the case of fuels containing hydrogen and moisture, i.e. when steam is formed during combustion. This steam is first reduced to the saturated state and then condensed to water in being cooled to the initial temperature in the calorimeter, so that it gives up its superheat, latent heat, and part of its sensible heat to the cooling water of the calorimeter.

The calorific value thus determined is known as the "higher" calorific value. In the useful application of combustion heat, however, as, for example, in furnaces and internal combustion engines, the hot products are never cooled down to room temperature, so that the H<sub>2</sub>O, formed by combustion, is found as vapour, and only in extreme cases as dry saturated vapour. Normally the steam is highly superheated, and hence it never gives up its latent nor sensible heat. There is no question, therefore, of being able to use this heat in steam boilers or in internal combustion engines.

If the higher calorific value, which is independent of the amount of moisture contained in a fuel, be used to determine the efficiency of a boiler, the result obtained is an unfair estimate of the boiler performance. If moist fuel is stoked, the sensible and latent heats are, in all cases,

\* Regarding this, see *Glückauf* (1929). W. Wunsch, "Die Bestimmung der Verbrennungswärme mit dem Junkers Calorimeter."

retained and carried away by the steam, the fault of which is due, not to the boiler, but to the fuel. It is only by neglecting this heat that a proper estimate can be made of the boiler performance.

The useful or "lower" calorific value is thus obtained by subtracting the sum of the total heats of the steam formed by the moisture and hydrogen in the original fuel from the higher calorific value. This sum amounts to about 600 C.H.U./lb., reckoned from an initial temperature of 20° C. Hence if  $w_m$  lb. of steam are formed per pound of fuel, the lower calorific value is

In the Junker calorimeter the water vapour in the products is condensed and collected, so that both the lower and higher calorific values can be found.

In the combustion of 1 lb. of hydrogen, 9 lb. of water are formed, having a total heat of  $9 \times 600 = 5400$  C.H.U. Now the higher calorific value of hydrogen is 34,200 C.H.U./lb., so that the lower calorific value is

$$H_L = 34200 - 5400 = 28,800 \text{ C.H.U./lb.}$$

Hence, in the case of illuminating gas, which contains a large amount of hydrogen, the difference between the lower and higher calorific values is considerable. Thus the values given by a Junker calorimeter test, for a particular illuminating gas, were

$$H_H = 338 \text{ C.H.U./ft.}^3 \text{ at S.T.P.}$$

and

$$H_L = 302 \text{ C.H.U./ft.}^3 \text{ at S.T.P.}$$

For bituminous coals, the lower calorific value is about 300 C.H.U. per pound of coal less than the higher.

The following general expression holds for solid fuels,

$$H_H - H_L = 9 \times 600 \times H + 600 H_2O = 600 (9H + H_2O)$$

where  $H$  and  $H_2O$  are the proportional weights of hydrogen and moisture in 1 lb. of the fuel.

Bituminous coal, lignite, peat, and wood can contain widely different quantities of moisture (expressed as, say,  $w_m$  lb. of moisture per pound of moist fuel). The analysis and calorific value of these fuels is usually given for the moisture free state, but, as actually used, they always contain a certain amount of moisture, even though they may be air dried. If  $H_0$  is the lower calorific value of moisture-free fuel, the lower calorific value with  $w_m$  lb. of moisture is

$$H_m = [(1 - w_m) H_0 - 600w_m] \text{ C.H.U./lb.}$$

since 1 lb. of fuel contains  $(1 - w_m)$  lb. of dry material and  $m$  lb. of moisture.\*

**Calorific values of gas mixtures.** If the proportions by weight or by volume of the constituents ( $\text{CO}$ ,  $\text{H}_2$ ,  $\text{CH}_4$ , etc.) of a fuel gas mixture be known, its calorific value can be determined by taking the sum of the calorific values of the constituents measured according to their fractional

\* See *Mitteil. aus d. Kgl. Materialprüfungsamt zu Berlin-Lichterfelde West* (1914); *Verfahren und Ergebnisse der Prüfung von Brennstoffen*, I. u. II.

proportions. The direct determination, by means of the Junker calorimeter, is, however, simpler than that of the gas analysis.

**Calorific value from the fuel composition.** For bituminous coal, the calorific value is given approximately by the sum of the calorific values of the combustible constituents, multiplied by their proportional weights. The fraction of hydrogen, which is united with oxygen, has, however, to be deducted from the proportional weight of hydrogen in the fuel. This gives\*

$$H_L = 8100 C + 29000 \left( \quad \quad \quad 2500S - \right.$$

where  $C$ ,  $H$ ,  $O$ ,  $S$ , and  $w_m$  are the proportions by weight of the constituents in 1 lb. of the fuel ( $w_m$  = weight of moisture).

## CALORIFIC VALUES OF FUELS

FUEL	$H_H$		$H_L$	
	C.H.U./lb.	C.H.U. 1 ft. <sup>3</sup> at S.T.P.	C.H.U./lb.	C.H.U. 1 ft. <sup>3</sup> at S.T.P.
Carbon (C) . . . . .	8,080	—	—	—
Carbon monoxide (CO) . . . . .	2,440	190	—	—
Hydrogen (H <sub>2</sub> ) . . . . .	34,200	192	28,800	160
Marsh gas (CH <sub>4</sub> ) . . . . .	13,240	590	11,910	530
Town gas (†) . . . . .	—	280	—	250
Coal gas . . . . .	—	260–310	—	224–280
Power gas . . . . .	—	—	—	74
Blast furnace gas . . . . .	—	—	—	47–60
Acetylene . . . . .	12,000	790	11,600	765
Paraffin . . . . .	—	—	10,500	—
Gas oil . . . . .	—	—	9,800–10,150	—
Petrol . . . . .	—	—	10,500	—
Alcohol . . . . .	7,184	—	6,480	—
Bituminous coal (‡) . . . . .	—	—	6,800–7,700	—
Coke . . . . .	—	—	6,900	—
Lignite (‡) . . . . .	—	—	4,500–5,000	—
Lignite briquettes (‡) . . . . .	—	—	4,800–5,100	—
Peat (‡) . . . . .	—	—	3,600–4,600	—
Wood . . . . .	—	—	4,500	—
Naphthalene . . . . .	9,700	—	9,370	—
Benzene . . . . .	10,000	—	9,590	—
Lignite tar oil . . . . .	—	—	9,000–9,800	—
Bituminous coal tar oil . . . . .	—	—	8,800–9,200	—
Crude tar . . . . .	—	—	8,000–8,800	—

\* A considerable amount of discussion has arisen over the accuracy and validity of this and similar equations. It gives, of course, a measure of the possible calorific value, but the calorimeter still offers the most reliable method of finding calorific values.

† Town gas, as now supplied, is a distilled gas mixed with water gas.

‡ The calorific values of these natural fuels show considerable variations, according to their origin and ash and moisture content. The usually accepted values (and given above) refer to the air dried state, in which the amounts of moisture by weight are 2 to 5 per cent for coal and 10 to 14 per cent for lignite briquettes. The weights of ash are about 5 to 12 per cent for coal, 5 to 20 per cent for lignite, and 6 to 10 per cent for lignite briquettes. The calorific value can also be given for pure coal, i.e. per unit weight of combustible material freed from ash and moisture. The usual values of these are 7950 C.H.U./lb. for bituminous coal, 6050 C.H.U./lb. for lignite, and 5200 C.H.U./lb. for peat.

**Calorific value of air fuel mixtures.** If the volume of air supplied per ft.<sup>3</sup> of fuel gas is  $nA_m$  ft.<sup>3</sup>, the lower calorific value of the mixture is

$$H_m = 1 + nA_m$$

It is this value, rather than  $H_u$ , which determines the available energy of the charge of an internal combustion engine.

Thus, with the excess air factor  $n = 1$ , the maximum  $H_m$  values for different fuels are as follows—

	CO	H <sub>2</sub>	CH <sub>4</sub>	Illum. Gas	Power Gas	Blast Furnace Gas
$(H_m)_{max} =$	56.2	56.5	50.6	50.0	37.4	33 C.H.U./ft. <sup>3</sup>

With excess air  $H_m$  becomes correspondingly smaller.

The heat of combustion per ft.<sup>3</sup> at S.T.P. of the products is greater than the above values in the ratio of  $\frac{1}{\alpha}$  (page 43), since the volume of the products at S.T.P. is  $\alpha$  times that of the unburned mixture.

### TOTAL HEAT AND INTERNAL ENERGY OF GASES AND PRODUCTS OF COMBUSTION (Heat Chart)

The quantity of heat which has to be supplied to unit mass of a gas in raising its temperature from 0° to  $t^\circ$  C., while the pressure is kept constant, is called the “total heat”  $H$  of the gas. If, in place of the pressure, the volume be kept constant, the corresponding heat is called the “internal energy”  $I$  of the gas.

Hence, for 1 lb. of the gas,

$$H = c_{pm}t \quad . \quad . \quad . \quad . \quad . \quad . \quad (102)$$

and

$$I = c_{vm}t \quad . \quad . \quad . \quad . \quad . \quad . \quad (103)$$

If the values of  $H$  and  $I$  are plotted as abscissae against  $t$  as ordinates, straight lines are obtained, provided  $c_{pm}$  and  $c_{vm}$  are independent of temperature. This is only true, however, in the case of monatomic gases. For diatomic gases there is a considerable increase in the values of  $c_{pm}$  and  $c_{vm}$  at high temperatures, so that their total heats and internal energies are represented by curves having the concave sides facing the  $H$  and  $I$  axes. The curvature is still more marked in the case of the polyatomic gas carbon dioxide. It should be noted in the case of steam, for pressures below 14.7 lb./in.<sup>2</sup> abs., the steam is condensed at some temperature below 100° C. The  $H$  and  $I$  curves can, however, be continued down to 0° C., if the pressures at the lower temperatures be assumed to be so small that the water vapour can be regarded as a gas. In this sense, equations (102) and (103) represent the total heat and internal energy, reckoned from water vapour at 0° C.

Since the values of  $c_{pm}$  and  $c_{vm}$  are widely different for different gases (including the diatomics), each gas has its own particular  $H$  and  $I$  curve. In the case of the important diatomic gases, however, the conditions can be simplified, since the mean molecular specific heats of these gases (N<sub>2</sub>, O<sub>2</sub>, CO, and air) are nearly the same, even at high temperatures, although an exact equality is impossible, according to the latest physical deductions. Hence, if the unit weight chosen is the mol. (i.e.  $m$  lb.,

where  $m$  is the molecular weight of the gas), one  $H$  line and one  $I$  line serve for all the diatomics. The same simplification can be adopted if the mass of 1 ft.<sup>3</sup> at S.T.P. be chosen as the unit, since these masses are proportional to the molecular weights. The specific heats referred to this mass are then

$$C'_{nm} = \frac{C}{359}$$

$$\text{and} \quad C'_{vm} = \frac{C_{vm}}{359}$$

and these are the same for all diatomic gases, so that the values given by

$$H' = C'_{pm}t$$

$$\text{and} \quad I' = C'_{vm}t$$

are also the same for all diatomics.

In addition, a simple relationship can be established between  $H'$  and  $I'$  from the relationship established on page 48 connecting  $C_{pm}$  and

$$\text{Since} \quad \quad \quad = C_{vm} + 1.985$$

it follows that

$$359 \quad - \quad 359 \quad + \quad 359$$

or

$$H' = I' + 0.00554t$$

Hence, if the values of  $0.00554t$  be plotted as abscissae against  $t$  as ordinates, the result gives a straight line, passing through the origin, and the values of  $I$  are given by the horizontal intercepts between this line and the  $H$  curve. The heat chart (given in the pocket at the end of the book), marked Chart I, has been constructed in this way. The curves for  $\text{CO}_2$  and  $\text{H}_2\text{O}$  are also added.

The chart also enables the total heat and internal energy of any gas mixture, consisting of diatomics,  $\text{CO}_2$  and  $\text{H}_2\text{O}$ , to be determined. Thus, if the analysis of the mixture is

$\text{CO}_2$	$\text{O}_2$	$\text{N}_2$	
10	8	72	10 % by volume

the total heat per ft.<sup>3</sup> at S.T.P. is

$$H = 0.10H_{\text{CO}_2} + (0.08 + 0.72)H_{\text{dia}}$$

so that the  $H$  curve for this flue gas is obtained by taking 10 per cent of the total heat of the  $\text{CO}_2$ , 80 per cent of the total heat of the diatomics, and 10 per cent of the total heat of the steam from the diagram and summing.

**Common curves for pure and air-diluted products.** As shown in the previous section, the differences between the mean molecular specific heats of the products formed from different fuels is small, so that, in most cases, it is sufficiently accurate to assume a mean curve of molecular specific heats for pure products. This enables a common curve of total heat to be drawn which suits all products formed with the theoretical

minimum air supply. Hence, using the value of specific heat given in equation (96), the value of  $H$  is given by

$$H = \left( 7.5 + \frac{0.66t}{1000} \right) \frac{t}{359} \text{ C.H.U./ft.}^3 \text{ at S.T.P.}$$

From this at

$t =$	0	500	1000	1500	2000 °C.
$H =$	0	10.9	22.8	35.6	49.2 C.H.U./ft. <sup>3</sup> at S.T.P.

Compared with this, the theoretical products from carbon, having the analysis  $\text{CO}_2 = 21$  per cent,  $\text{O}_2 = 0$ , and  $\text{N}_2 = 79$  per cent, give

$H =$	0	10.88	22.9	35.7	49.1
-------	---	-------	------	------	------

for the same temperatures, so that, for practical purposes, the deviations may be neglected, particularly in view of the uncertainty regarding the basic specific heat values.

This mean line for  $H$  has been plotted in Chart I. The lines for the products with excess air lie between this line and the line for pure air, and approach the latter as the excess air is increased.

When the fractional volumes of excess air in the products are

$$v_a = 0.1, 0.2, 0.3 \dots 0.9, 1.0$$

the corresponding  $H$  values are found by dividing the horizontal intercepts between the pure air and pure products curves into 10 equal parts and joining up the same division points. This method was adopted in drawing the dotted curves on the chart so that the total heat of the products of any fuel having a particular  $v_a$  value may be found. The relation between the excess air factor  $n$  and  $v_a$  is given by equation (100),

$$n = 1 + \left( \frac{v_a}{1 - v_a} \right) \frac{V_{g0}}{A_{min}}$$

Hence  $n$  is a linear function of  $\frac{v_a}{1 - v_a}$  and is represented in the auxiliary

diagram on the chart. The base, however, is marked in values of  $v_a$ , so that the relation between  $v_a$  and  $n$  is read directly.

**Amount of combustion heat developed in the products.** In any combustion process the heat developed depends on the combustion heat per unit mass of the products produced. Here again, it is preferable to use, as the unit of mass, 1 mol. or 1 ft.<sup>3</sup> at S.T.P. rather than 1 lb.

If, in the combustion of 1 ft.<sup>3</sup> of a gaseous fuel, having a calorific (lower) value  $H_L$ , the volume of the products formed is  $V_g$  ft.<sup>3</sup> at S.T.P., the heat contained in 1 ft.<sup>3</sup> of the products is

$$Q_p = \frac{H_L}{V_g}$$

The same equation holds for liquid and solid fuels, if  $V_g$  represents the volume at S.T.P. of products formed in burning 1 lb. of fuel having the lower calorific value  $H_L$  C.H.U./lb.

$Q_p$  has its greatest value when combustion occurs with the theoretical air supply  $A_{min}$  because  $V_g$  has then its smallest value  $V_{g0}$ . In this case

$$Q_{p \min} = \frac{H_L}{V_{g0}}$$



If, however, the air supply is  $n$  times greater, then

$$V_g = V_{g_0} + (n-1)A_{min}$$

and

$$Q_p = \frac{H_L}{V_{g_0} + (n-1)A_{min}} = \frac{H_L}{V_{g_0}} \left\{ \frac{1}{1 + (n-1) \frac{A_{min}}{V_{g_0}}} \right\}$$

As shown on page 28, the values of  $A_{min}$  and  $V_{g_0}$  can be calculated for a fuel of known composition, and a list of these values is given below. Along with these, the values of  $\frac{H_L}{V_{g_0}}$  \* for  $n = 1$  are shown.

The greatest development of heat per unit quantity of products is obtained from acetylene and carbon monoxide and the least from blast furnace gas.

TABLE OF THERMAL CONCENTRATIONS

FUEL	$A_{min}$ ft. <sup>3</sup>	$V_{g_0}$ ft. <sup>3</sup>	$H_L$ C.H.U.	$\frac{H_L}{V_{g_0}}$ C.H.U./ft. <sup>3</sup>	$\frac{A_{min}}{V_{g_0}}$
Acetylene . . . . .	11.9	12.4	840	67.8	0.960
Carbon monoxide . . . . .	2.38	2.88	190	65.9	0.825
Water gas . . . . .	2.24	2.78	164	59.0	0.806
Carbon . . . . .	142.5	142.5	8,100	56.8	1.00
Methane . . . . .	9.52	10.52	590	56.1	0.904
Benzene (C <sub>6</sub> H <sub>6</sub> ) . . . . .	165	171.0	9,590	56.1	0.957
Hydrogen . . . . .	2.38	2.88	160.5	55.7	0.827
Coal . . . . .	136	141.0	7,500	53.2	0.966
Lignite briquettes . . . . .	76.5	85.5	4,720	55.3	0.894
Mineral oil distillates . . . . .	186	198.5	10,500	53.0	0.936
Coke oven gas . . . . .	3.69	4.38	224	51.4	0.843
Alcohol . . . . .	111	126.5	6,480	51.2	0.877
Illuminant gas . . . . .	5.57	6.28	312	49.7	0.889
Generator gas from coke . . . . .	1.00	1.83	81	44.2	0.547
Generator gas from lignite briquettes . . . . .	1.22	2.00	85.1	42.6	0.610
Blast furnace gas . . . . .	0.69	1.55	54.3	35.0	0.445

In the above table the values of  $A_{min}$ ,  $V_{g_0}$ , and  $H_L$  are per ft.<sup>3</sup> of gaseous fuel and per lb. of liquid or solid fuel.

As the excess air is increased,  $Q_p$  decreases. In Fig. 24 the values of  $Q_p$ , for various fuels, have been plotted to a base of  $n$  values. For  $n = 2$  the value of  $Q_p$  for all the usual fuels, except blast furnace gas and carbon monoxide, lie within 26.5 and 31 C.H.U./ft.<sup>3</sup>, while the mean value is 28 C.H.U./ft.<sup>3</sup>. Hence the considerable differences which exist between the various fuels when burned with the theoretical minimum air disappear almost entirely when double this air is supplied.

\*  $\frac{H_L}{V_{g_0}}$  can be called the "thermal concentration" of the products, although in the above it has been called the "products heat." The term "total heat of the products" is not so suitable, since this has been used in a different sense, as explained on page 64.

**Dependence of combustion space and air supply on the calorific value.**

In the discussion above, and in Fig. 24, only a few characteristic examples of the coals, lignites, and oils have been considered. In addition to these, however, there are many others, of the same nature, distinguished mainly by their calorific values. In order to make a comprehensive survey of

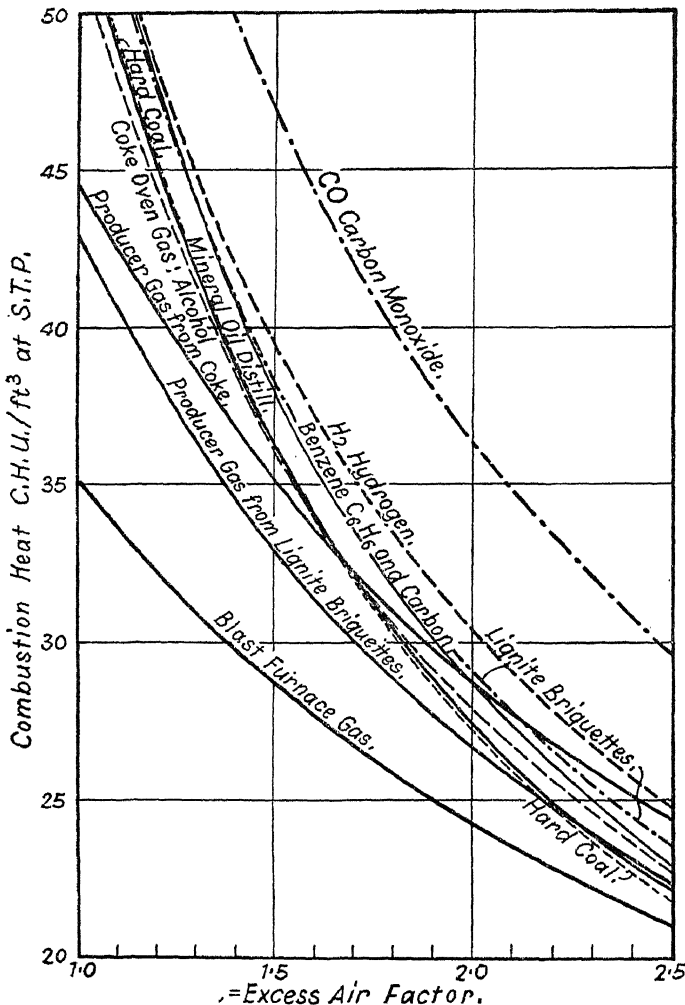


FIG. 24

the fuels used in practice, Rosin\* calculated the values of  $V_{g_0}$  and  $A_{min}$  for a large number of solid, liquid, and gaseous fuels, and plotted them to a base of calorific values  $\bar{H}_L$ . The results, as shown in Fig. 25, are

\* Z.V.d.I. (1927), page 383, P. Rosin, "Das I.t. Diagramm der Feuergase und der Wirkungsgrad von Öfen." Also P. Rosin and R. Fehling, *Das I.t. Diagramm der Verbrennung*. (Berlin, 1929.)

represented by straight lines in all the fuel groups, and can be expressed by the following equations—

For solid fuels,

$$V_{g_0} = \quad + 26.40; \quad A_{min} = \quad + 8.0 \text{ (ft.}^3 \text{ at S.T.P.)}$$

for oils,

$$V_{g_0} = \frac{\quad}{100}, \quad A_{min} = \quad + 32 \text{ (ft.}^3 \text{ at S.T.P.)}$$

for weak gases,

$$V_{g_0} = \frac{\quad}{100} \quad \frac{1.4H}{100}$$

and for rich gases,

$$0.25; \quad - 0.25 \text{ (ft.}^3 \text{ at S.T.P.)}$$

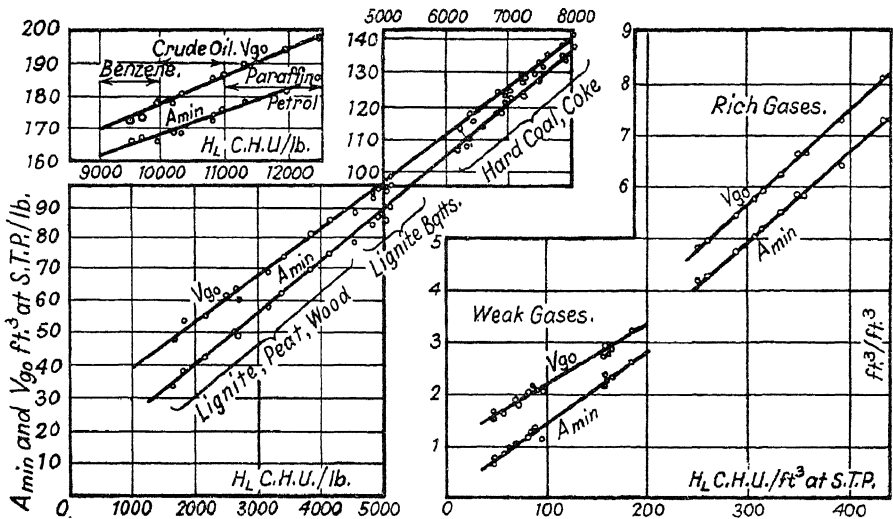


FIG. 25

Even if the composition of a fuel is not known, therefore, these equations enable the minimum air volume and products volume to be determined for the fuel from its calorific value alone. If, further, the products heats are plotted to a base of  $H_L$  it will be found that a common line serves for each of the four fuel groups, so that this enables the products heat to be determined from the calorific value above when  $n = 1$ . When  $n$  is greater than 1 a common set of curves will again serve for all fuels in the same group.

**Combustion temperature (at constant pressure).** The term "combustion temperature" means the temperature attained in complete uncooled combination of a fuel with air. Its value, when other conditions remain the same, will depend on whether combustion is effected at

constant pressure (usually atmospheric) as in furnaces, or at constant volume, as in explosion processes. In this section the first case will be considered.

For a given fuel and given initial temperature  $t_0$  of the air-fuel mixture the combustion temperature will depend primarily on the excess air factor  $n$ . Its greatest value occurs with the theoretical minimum air supply, i.e. when  $n = 1$ , since the minimum amount of products has then to be heated. This case, however, cannot be considered here, as a considerable amount of dissociation of carbon dioxide and steam occurs with most fuels at these high temperatures,\* and the calculations involved are beyond the scope of this text.

With values of  $n$  equal to, or greater than, 1.5, however, the dissociation effects can be neglected, so that the temperature is found merely from the specific heats of the constituents of the products or from their total heats.

The amount of heat absorbed by the products is determined from the products heats per unit mass given on page 66. Here, again, it is preferable to express these heats per ft.<sup>3</sup> at S.T.P. As shown below, for products containing steam, only the lower calorific value should be used.

When the products are heated from  $t_0^\circ$  to  $t_1^\circ$  C., at constant pressure, the heat absorbed per ft.<sup>3</sup> at S.T.P. is

where  $C'_{pm}$  is the mean specific heat between  $t_0$  and  $t_1^\circ$  C. This then gives

$$\frac{H_L}{V_p} = C'_{pm} (t_1 - t_0) \quad . \quad . \quad . \quad . \quad . \quad (104)$$

If  $C'_{pm}$  were independent of temperature, we could find  $t_1$  from

$$H_L \quad . \quad . \quad . \quad . \quad . \quad (105)$$

Since, however,  $C'_{pm}$  is a function of temperature for all products, equation (105) enables only an approximate value of  $t_1 - t_0$  to be found by assuming a possible value for  $C'_{pm}$ , as given, say, by Fig. 23. A more exact value would be found by using the expressions given on page 60 for the mean specific heats of the products, and solving the resulting quadratic equation for  $t_1$ . This method, however, is laborious, particularly since, for every excess air factor, different values have to be inserted in the equation for  $C'_{pm}$ . It is therefore preferable to use the following graphical method.

The method is based on the equation

The total heat line, corresponding to the analysis of the particular products dealt with, is first located on Chart I. This can be limited to

\* See, however, W. Schüle on combustion processes with dissociation, in his *Neue Tabellen und Diagramme*.

the curve between  $t_0$  and a guessed higher temperature  $t_1$ . From this curve (Fig. 26)  $t_1$  is found by adding the distance  $\frac{H_L}{V_{g_0}}$  to the abscissae at  $A$ ,

which lies on the curve at  $t_0$ , to give the abscissa of the point  $B$ , which is then at the required temperature  $t_1$ ;  $t_0$  is the initial air temperature. If, due to preheating, the initial temperature is  $t'_0$ , the value of  $t'_1$  will only be correct if no dissociation occurs. With dissociation its value will be lowered as shown by the curves, since heat is absorbed by dissociation.

Instead of determining the total heat curve for the particular products, the mean products curve for the excess air factor  $n$ , shown on the chart, may be used in finding  $t_1$ . The combustion temperatures for various fuels and various excess air factors, were found in this way from the chart with the appropriate  $\frac{H_L}{V}$  values. These temperatures have been

plotted to a base of  $n$  values in Fig. 27. The highest values (in the neighbourhood of  $n = 1$ ) have been taken from the author's *Neue Tabellen und Diagramme*, because, except in the case of coke oven gas, a considerable amount of dissociation occurs in this region. It will be seen that for  $n = 1.6$  the values of  $t_1$ , for all the fuels shown, do not vary more than  $\pm 20^\circ \text{C}$ . from the mean value of  $1520^\circ \text{C}$ . At temperatures above and below this, however, the deviations are greater. A considerable difference is found between the pure gases  $\text{CO}$  and  $\text{H}_2$ , the former values being, on the whole, considerably higher, and equal to  $1875^\circ \text{C}$ . at  $n = 1.6$ . For coke oven gas at  $n = 1.6$  the temperature is only  $1265^\circ \text{C}$ .

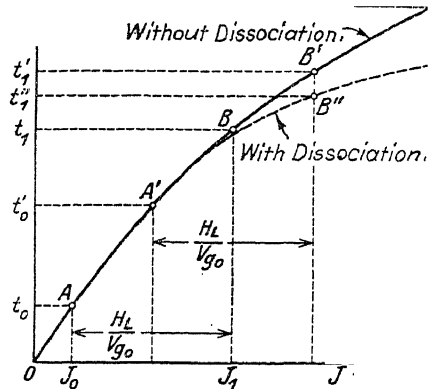


FIG. 26

between the pure gases  $\text{CO}$  and  $\text{H}_2$ , the former values being, on the whole, considerably higher, and equal to  $1875^\circ \text{C}$ . at  $n = 1.6$ . For coke oven gas at  $n = 1.6$  the temperature is only  $1265^\circ \text{C}$ .

In the above, the lower calorific value has been used throughout. If the fuel contains hydrogen or moisture, the steam formed in the products is condensed to water during the cooling process. This condensation commences at the temperature  $t_s$  corresponding to the partial pressure  $p_s$  of the water vapour in the products. The amount of heat required to condense all the water vapour in the products formed from 1 lb. of fuel is about  $600w_m$  C.H.U., where  $w_m$  is the sum of the weights of the combustion water and original moisture contained in 1 lb. of the fuel. The amount of latent heat contained in 1 ft.<sup>3</sup> of products is accordingly  $\frac{600w_m}{V_{g_0}}$ . If now the products with water vapour be cooled down to  $t_0^\circ \text{C}$ .

the quantities of heat given up are—

1. Between  $t$  and  $t_s$ , the heat  $(C_p)'_m t - (C_p)_s'_m t_s$
2. Between  $t_s$  and  $t_0$  the heat  $[1 - v_{\text{H}_2\text{O}}] [(C'_p)_s'_m t_s - (C'_p)_0'_m t_0]$
3. During condensation,  $600w_m/V_{g_0}$ .

The sum of these is equal to  $H_H/V_{g_0}$  where  $H_H$  is the higher calorific value.

The equality gives

$$\begin{aligned} (C'_{ps})_m t - (C'_{ps})_m t_0 &= \frac{H_H - 600w_m}{V_{g_0}} + v_{H_2O} [(C'_{ps})_m t_s - (C'_{ps})_m t_0] \\ &= \frac{H_L}{V_{g_0}} + \Delta \end{aligned} \quad (107)$$

since the lower calorific value  $H_L = H_H - 600w_m$ . The second term on the right can be determined, for a known fractional volume  $v_{H_2O}$  of water vapour in the products, by reading off the quantity in the square brackets between  $t_s$  and  $t_0$  on Chart I. For example, taking the products at

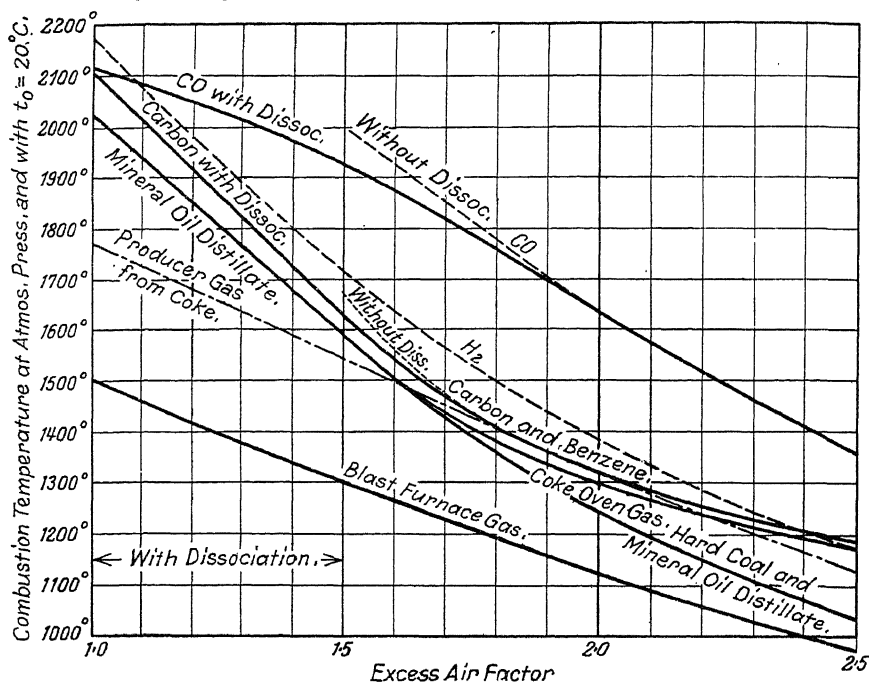


FIG. 27

atmospheric pressure, and assuming the unusually high value  $v_{H_2O} = 0.25$  the partial vapour pressure is  $0.25 \times 14.7 \text{ lb./in.}^2 \text{ abs.} = 3.65 \text{ lb./in.}^2 \text{ abs.}$ , and the corresponding saturation temperature is  $65^\circ \text{ C.}$  If now  $t_0 = 20^\circ \text{ C.}$ , the bracketed quantity is  $1.0 \text{ C.H.U.}$  and  $\Delta = 1.0 \times 0.25 = 0.25 \text{ C.H.U.}$  This is negligible compared with the values  $44.0$  to  $56.0 \text{ C.H.U.s}$  given by the first term on the right of equation (107), so that equations (106) and (107) give practically the same results. If, however, account is to be taken of the second term,  $\frac{H_L}{V_{g_0}}$  has to be increased by  $\Delta$  and the tem-

perature determined with this value as before.

**Volume of the products and thermal concentration at the combustion temperature.** The volume of the products at the combustion temperature

is of basic importance, since the combustion space, in which the products are burned, has to accommodate this volume. The volume  $V_{gt_1}$  at  $t_1^\circ \text{C}$ . and  $B \text{ lb./in.}^2$  abs. is greater than the volume  $V_{g_0}$  at S.T.P. of the same mass, since their relation is expressed by

$$V_{gt_1} = V_{g_0} \left( \frac{273}{273} \right)$$

If  $B = 14.7$ , 
$$V_{gt_1} = V_{g_0} \frac{(273 + t_1)}{273}$$

The heat contained in  $1 \text{ ft.}^3$  of hot products is then

$$Q_{gt} = \frac{H_L}{V_{gt}} = \frac{H_L}{V_{g_0}} \frac{273}{(273 + t_1)}$$

The values of  $Q_{gt}$  have been calculated from this for various fuels and excess air factors by reading the temperatures given in Fig. 27. These  $Q_{gt}$  values are shown to a base of  $n$  values in Fig. 28. With the ordinary

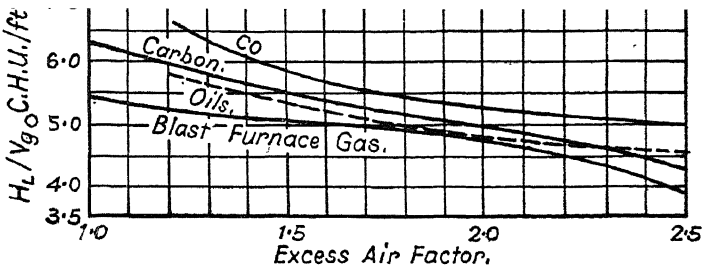


FIG. 28

fuels and  $n = 1$ , the value of  $Q_{gt}$  is about  $6.24 \text{ C.H.U. ft.}^3$  and decrease slightly as  $n$  is increased. Thus, for  $n = 1.5$  the values range from  $5.3$  to  $5.5 \text{ C.H.U./ft.}^3$ , while at  $n = 2$  the range is from  $4.9$  to  $5 \text{ C.H.U./ft.}^3$ . CO shows somewhat higher values throughout, while blast furnace gas shows lower values for  $n$  less than  $1.6$ .

Hence, when the initial temperature is  $20^\circ \text{C}$ ., the heat contained in  $1 \text{ ft.}^3$  of combustion chamber space is not more than  $6.25 \text{ C.H.U.}$  even in an intense fire.

**Combustion chamber volumes and rate of heat supply.** The size of the combustion chamber depends on the volume of products developed in and discharged from it per unit of time. Every fuel requires a certain amount of time  $Z$  in forming the products and in effecting complete combustion. This is particularly noticeable in the case of the combustion of pulverized fuels, and in order to make room for the newly formed products the previously developed products have to leave with a certain mean speed  $c$ .

Now if  $w \text{ lb.}$  of fuel are burned per second, and produce  $V_{gt_1} \text{ ft.}^3$  of products at the temperature  $t_1$ , the volume of products formed per second is  $wV_{gt_1}$ , and this volume has to flow through the mean cross sectional area  $A$  of the combustion chamber per second.

Hence 
$$Ac = wV_{gt_1}$$

If the length of the combustion chamber, measured along the direction of flow, is  $l$ , the time taken for a particle to travel through the chamber is

$$Z = \frac{l}{c}$$

This time must be either equal to, or greater than, the time  $Z_0$  necessary for the complete combustion of every fuel particle, or

$$\frac{l}{c} \geq Z_0 \text{ giving } c \leq \frac{l}{Z_0}$$

With this and the equation given above,

$$Al \geq Z_0 w V_{gt_1}$$

$Al$  is (for a prismatic shaped combustion chamber) the volume of the combustion chamber, which can thus be determined if, in addition to  $w$  and  $V_{gt_1}$ , the combustion time  $Z_0$  is known. The method of finding  $V_{gt_1}$  has been given above.

The heat contained in 1 ft.<sup>3</sup> of products has also been calculated above, and has the value

$$W = \frac{H_L}{V_{gt_1}}$$

which, for any excess air factor, can be read off Fig. 28.

This now gives

$$V = \frac{H_L}{W}$$

and hence

$$Al \geq \frac{H_L}{W}$$

If, in this,  $W$  is taken as 5.6 to 6.2, the value of  $Al$  can be found when  $H_L$ ,  $w$ , and  $Z_0$  are known.

Now, in this expression,  $wH_L$  is the heat developed in the whole combustion space per second, so that the heat developed in 1 ft.<sup>3</sup> of combustion space per second is

$$\frac{wH_L}{V} = Z_0$$

The corresponding quantity per hour is called the "combustion chamber heat load," and is given by

If, in this,  $W$  is taken as 5.6 to 6.1, corresponding to  $n = 1.4$  to 1.1, Fig. 28,

$$L_H = \frac{20000}{\sim} \text{ to } 22000 \text{ C.H.U.}$$

**Values of combustion times  $Z_0$ .** The values of  $Z_0^*$  are influenced by a considerable number of factors, which can only be briefly mentioned

\* See *Z.V.d.I.* (1927), pages 411-418, Allner, "Verbrennungsgeschwindigkeit und Gasgleichgewicht."



here, and vary between a few seconds (for coal dust) and small fractions of a second (for gases). In Fig. 29 the relation between  $Z_0$  and the size of coal dust particles\* is shown. With  $Z_0 = 1$  sec.,  $L_H = 20,000$  to 22,000 C.H.U./ft.<sup>3</sup> hr.

For a given size of particle, however, the combustion chamber heat loads are considerably increased when the air-dust mixture is in a turbulent state. Thus in the A.E.G.† pulverized coal locomotive values of  $L_H$  up to 125,000 were obtained, which corresponds to  $Z_0 = 0.16$  sec.

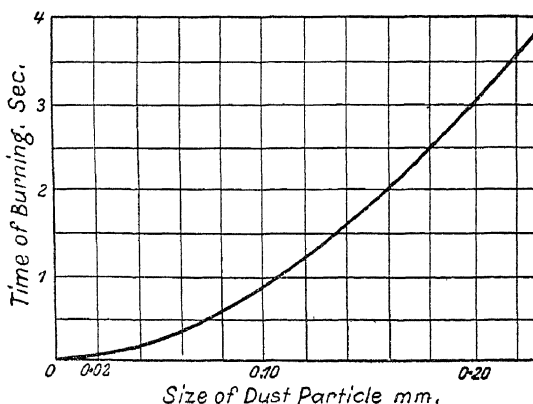


FIG. 29

The combustion chamber heat load is also increased by the absorption of heat at the combustion chamber walls, since the mean temperature of the products is thereby reduced, with a consequent reduction in their volume.

With oil firing, combustion chamber heat loads of about 1,000,000 C.H.U./ft.<sup>3</sup> hr. are obtained, and, with gas firing, about 1,250,000 C.H.U./ft.<sup>3</sup> hr., which corresponds to a value of  $Z_0$  of less than 0.015 sec.

**Heat lost in flue or exhaust gases.** In boilers or internal combustion engines some of the combustion heat of the fuel is always carried away by the flue or exhaust gases. A simple and sufficiently accurate method of determining this heat loss, expressed as a fraction of the calorific value of the fuel, is as follows.

Within the temperature region  $0^\circ$  to  $400^\circ$  C., which covers the present case, the mean molecular specific heat of pure products increases from about 7.4 at  $0^\circ$  C. to 7.7 at  $400^\circ$  C., as shown in Fig. 23, so that for this region,

$$C_{pm} = 7.4 + 0.95 \times t$$

\* *Archiv. der Warmewirtschaft* (1926), page 241, P. Rosin, "Eigenart der Braunkohlenstaubfeuerung." According to page 245 (of this article),  $Z_0$  depends largely on the gas content of the coal. Fig. 29 (above) refers to a coal having 42.8% gas and 13% water content, i.e. 55.8% volatiles, and a maximum size of dust particle of 0.17 mm. A coal containing 20% gas content requires about three times as much time to burn.



## THERMAL PROPERTIES OF GASES

Since  $V_g$  is given by

$$V_g = V_{g_0} \left[ 1 + (n-1) \frac{A_{min}}{V_{g_0}} \right]$$

in which  $V_{g_0}$  is the products volume without excess air,  $A_{min}$  is the theoretical air supply and  $n$  the excess air factor, equation (109) can be written

$$(n-1) \frac{A_{min}}{V} \quad (110)$$

$$\overline{V_{g_0}}$$

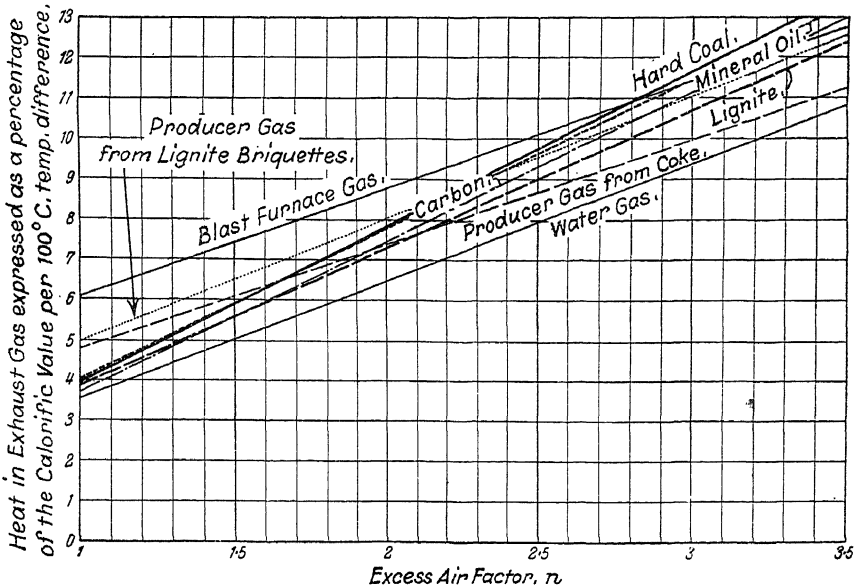


FIG. 31

Now  $\frac{H_L}{V_{g_0}}$ , the total heat per ft.<sup>3</sup> of pure products, is calculable for a fuel of known composition and calorific value. As shown on page 69, its numerical value does not vary greatly for the different types of fuel, and hence still less for the same type (such as the different kinds of coal), so that a mean value can be assumed. The same applies to the ratio  $A_{min}/V_{g_0}$ . In general, therefore, it is sufficiently accurate to determine the waste heat from equation (110) when the type of fuel, from which the products are formed, is known. The only quantities which have to be measured are the exhaust temperature ( $t$ ), the initial air temperature ( $t_0$ ), and the excess air (Fig. 8).

The increase in the waste gas heat above that for pure products, which is represented by the expression in front of the square brackets in equation (110), is proportional to  $n-1$ , so that if the values of  $\frac{s}{t-t_0}$  (i.e. the waste heat loss per °C.) are plotted to a base of  $(n-1)$  or  $n$ ,

a straight line is obtained for each fuel, as shown in Fig. 31. The ordinates of this diagram, however, represent the waste heat losses as a percentage of the calorific values per 100° C. excess temperature of the leaving gases.

If, for example, the temperature of the flue gases leaving a boiler is 320° C., the air temperature 20° C., and the excess air factor  $n = 1.5$ , the waste heat loss per 100° C. temperature difference, from Fig. 31, is 5.9 per cent, so that for 300° C. difference it is  $3 \times 5.9 = 17.7$  per cent.

The values of  $V_{s_0}$  and  $A_{min}$  can also be replaced by Rosin's values for them given on page 69, so that—

1. For solid fuels,

$$\frac{s}{t-t_0} = 0.0212 \left[ \frac{1.422}{100} + \frac{26.403}{H_L} + (n-1) \left( \frac{1.62}{100} + \frac{8}{H_L} \right) \right]$$

or, as a percentage,

$$\frac{100s}{t-t_0} = 0.0212 \left[ 1.422 + \frac{2640.3}{H_L} + (n-1) \left( 1.62 + \frac{800}{H_L} \right) \right] \quad (111)$$

2. For oils,

$$\frac{100s}{t-t_0} = 0.0212 \left[ 1.78 + (n-1) \left( 1.36 + \frac{3200}{H_L} \right) \right] \quad (112)$$

3. For weak gases,

$$\frac{100s}{t-t_0} = 0.0212 \left[ 1.16 + \frac{100}{H_L} + (n-1) 1.4 \right] \quad (113)$$

4. For rich gases,

$$\frac{100s}{t-t_0} = 0.0212 \left[ 1.82 + \frac{25}{H_L} + (n-1) \left( 1.74 - \frac{25}{H_L} \right) \right] \quad (114)$$

Each of these equations is represented, for different values of  $n$ , by a group of radial straight lines when the values  $\frac{100s}{t-t_0}$  are plotted to a base of  $H_L$  values.

**Gas explosions.** If a combustible air-gas mixture, at pressure  $p_0$ , and temperature  $t_0$ , contained in a closed pressure tight space of fixed volume, be ignited, a very rapid combustion results, which causes a rise in temperature from  $t_0$  to  $t_1$  (corresponding to the combustion heat developed), along with a rise in pressure from  $p_0$  to  $p$ . This process is known as a "gas explosion." If, in place of a gas, a liquid or solid fuel (which must be finely and uniformly mixed with the combustion air) is used, the resulting explosions are known as "oil explosions" or "dust explosions" respectively, and are treated in the same way as gas explosions.

So far as the changes in pressure and temperature are concerned, the simplest type of explosion occurs in a vessel of spherical shape with the ignition point placed at the centre. The flame is then propagated in the form of a continuously increasing fire ball from the centre until it reaches the walls. At this instant the combustion is completed and the maximum pressure  $p_1$  attained. The combustion temperature  $t_1$  will only be uniform throughout if, at every point during combustion, thorough mixing of the

products occurs, otherwise the temperature at the core is higher than at the boundary and decreases continuously from the centre outwards. In this section it will be assumed that the temperature  $t_1$  is uniform throughout and that no heat escapes from the gases to the walls until after the combustion is completed. It will also be assumed that sufficient oxygen is present in the original mixture for the complete combustion of the fuel.

The heating of the gases from  $t_0$  to  $t_1$  is effected by the heat liberated during combustion, and the result is the same as would occur with a supply of the same quantity of heat from without in raising the products from the initial temperature  $t_0$  to the final temperature  $t_1$ , at constant volume.

The amount of heat contained in 1 ft.<sup>3</sup> of products (at S.T.P.) is, as shown on page 66,

$$Q_p = \frac{H_L}{V_g}$$

where  $H_L$  is the lower calorific value (see page 71) and  $V_g$  is the volume in ft.<sup>3</sup> at S.T.P. of the products per ft.<sup>3</sup> of gaseous fuel or per lb. of solid or liquid fuel.

Hence, if  $C'_{vm}$  is the mean specific heat per ft.<sup>3</sup> at S.T.P. of the products,

$$\lambda_p = \frac{H_L}{V_g} = C'_{vm}(t_1 - t_0) \quad (115)$$

If  $t_0$  is considerably higher than 0° C., as occurs, for example, in gas or oil engines at the beginning of combustion, it is preferable to take

$$H_L \sim \dots \dots \dots C'_{vm_0} t_0 \quad (116)$$

in which  $C'_{vm_1}$  and  $C'_{vm_0}$  are the mean specific heats between 0° C. and  $t_1$ ° C. and 0° C. and  $t_0$ ° C. respectively. The specific heat used in equation (115) is the mean specific heat between  $t_0$  and  $t_1$ , and, as such, has first to be determined.

If  $C'_{vm}$  were independent of temperature,  $t_1$  could be found simply from

$$t_1 - t_0 = \frac{H_L}{V_g C'_{vm}}$$

or, for the second case, from

$$t_1 = \left( \frac{H_L}{V_g} + C'_{vm_0} t_0 \right) \frac{1}{C'_{vm_1}}$$

Since, however,  $C'_{vm}$  is, for products, actually a function of temperature, these give only approximate values for  $t_1$ . By repeating these approximate calculations, however, which are simplified by the use of Fig. 23, the exact values can be determined.

**Example 2.** The value of  $H_L/V_g$  for most products with  $n = 2$  (i.e. double the theoretical air supply) is about 28 C.H.U./ft.<sup>3</sup> at S.T.P. Find the explosion temperature  $t_1$ , if the initial temperature is  $t_0 = 20^\circ \text{C}$ .

Choosing first  $t_1 - t_0 = 1500^\circ \text{C.}$  gives, from Fig. 23,

$$C_{pm} = 8.0, \text{ so that } C_{vm} = 8 - 1.985 \doteq 6 \text{ and}$$

$$C'_{vm} = \frac{6}{359} = 0.00167$$

$$\frac{28}{0.00167} = 1680^\circ \text{C.}$$

From Fig. 23, the value of  $C_{pm}$  corresponding to  $1680^\circ \text{C.}$  is 8.1, giving

$$C_v \doteq 8.1 - 1.99 = 6.11$$

and

$$C'_{vm} = \frac{6.11}{359} = 0.001702$$

from which

$$\frac{28}{0.001702} = 1646^\circ \text{C.}$$

From this, Fig. 23 gives  $C_{pm} = 8.09$ ,  $C_{vm} = 6.10$ , and  $C'_{vm} = 0.001701$ , so that

$$28$$

By a similar calculation at constant pressure  $t - t_0 = 1300^\circ \text{C.}$  The ratio of the temperature increase at constant volume to that at constant pressure is thus

$$\frac{1648}{1300} = 1.265$$

This ratio is expressed in general by

$$\frac{t_{1v} - t_0}{t_{1p} - t_0} = \frac{C_{pm}}{C_{vm}}$$

in which  $C_{pm}$  and  $C_{vm}$  correspond to the different temperatures  $t_{1p}$  and  $t_{1v}$ , so that their ratio is slightly smaller than the ratio  $\gamma = \frac{C_p}{C_v}$

Along with the temperature rise there is, at the same time, a corresponding pressure rise which can be determined as follows—

The characteristic equation, for the initial state, is

and, for the final state,

$$p_1 v_0 =$$

so that, by division,

$$\frac{p_1}{p_0} =$$

If the gas constants were the same for the unburned mixture and the products, then

$$p_0$$

Actually, however,

$$\frac{R_p}{R_g} = \alpha \begin{matrix} \leq \\ \geq \end{matrix} 1$$

depending on the nature of the fuel and the excess air factor  $n$ .

Hence 
$$\frac{p_1}{p_0} = \alpha \frac{T_1}{T_0} \begin{matrix} \leq \\ \geq \end{matrix} \frac{T_1}{T_0}$$

For gaseous fuels (page 27)

$$\alpha = \frac{V_{g_0} + (n-1) A_{min}}{1 + n A_{min}} = \frac{\text{products volume}}{\text{initial mixture volume}}$$

which gives

for generator gas from coke 
$$\alpha = \frac{0.83 + n}{1 + n} = 0.915 \text{ to } 1$$

for coke oven gas 
$$\alpha = \frac{0.69 + 3.69n}{1 + 3.69n} = 0.935 \text{ to } 1$$

for  $H_2$  and CO 
$$\alpha = \frac{0.5 + 2.38n}{1 + 2.38n} = 0.852 \text{ to } 1$$

and for  $CH_4$  
$$\alpha = 1$$

In the case of solid and liquid fuels the number 1 is absent from the denominator of the expression for  $\alpha$ , so that—

for oils 
$$\alpha = 1 + \frac{0.69}{n} = 1 \text{ to } 1.069$$

and for carbon 
$$\alpha = 1.0$$

With C and  $CH_4$ , therefore,  $\frac{p_1}{p_0}$  is the same as  $\frac{T_1}{T_0}$ , and for the other fuels the difference decreases as the supply of excess air is increased.

If, further, the equation for  $\frac{p_1}{p_0}$  is written in the form

$$\frac{p_1 - p_0}{p_0} T_0$$

or 
$$\frac{p_1}{p_0}$$

it will be recognized that the pressure ratio  $\frac{p_1}{p_0}$ , corresponding to a given temperature rise, can have considerably different values, since it depends not only on  $t_1 - t_0$  but also on the initial temperature  $T_0 = t_0 + 273$ . Thus, with  $t_1 - t_0 = 1700^\circ C$  and  $\alpha = 1$ ,

$$\frac{p_1}{p_0} = 1 + \frac{1700}{T_0}$$

and with 
$$\begin{array}{lll} t_0 = 20 & 200 & 500^\circ C. \\ T_0 = 293 & 473 & 773^\circ C. \text{ abs.} \end{array}$$

$$\frac{p_1}{p_0} = \begin{array}{lll} 6.8 & 4.6 & 2.2 \end{array}$$

The simplest method of determining the combustion temperature is by means of the energy chart.

The internal energy values, found by means of the relation

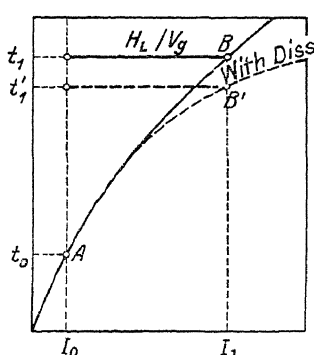


FIG. 32

$I = v_{\text{CO}_2} I_{\text{CO}_2} + v_{\text{H}_2\text{O}} I_{\text{H}_2\text{O}} + v_{(\text{N}_2 + \text{O}_2)} I_{\text{N}_2}$  (using the  $I$  values given in Chart I) are plotted as abscissae against temperatures as ordinates (Fig. 32). The point  $A$  is located at the initial temperature  $t_0$ , and the point  $B$  is such that its abscissa is greater than that of  $A$ , by the amount  $H_L/V_g$ . The ordinate of  $B$  then gives the required temperature  $t_1$ . This follows since

$$\frac{H_L}{V_g} = I_1 - I_0$$

As in the case of combustion at constant pressure, the final state can show an

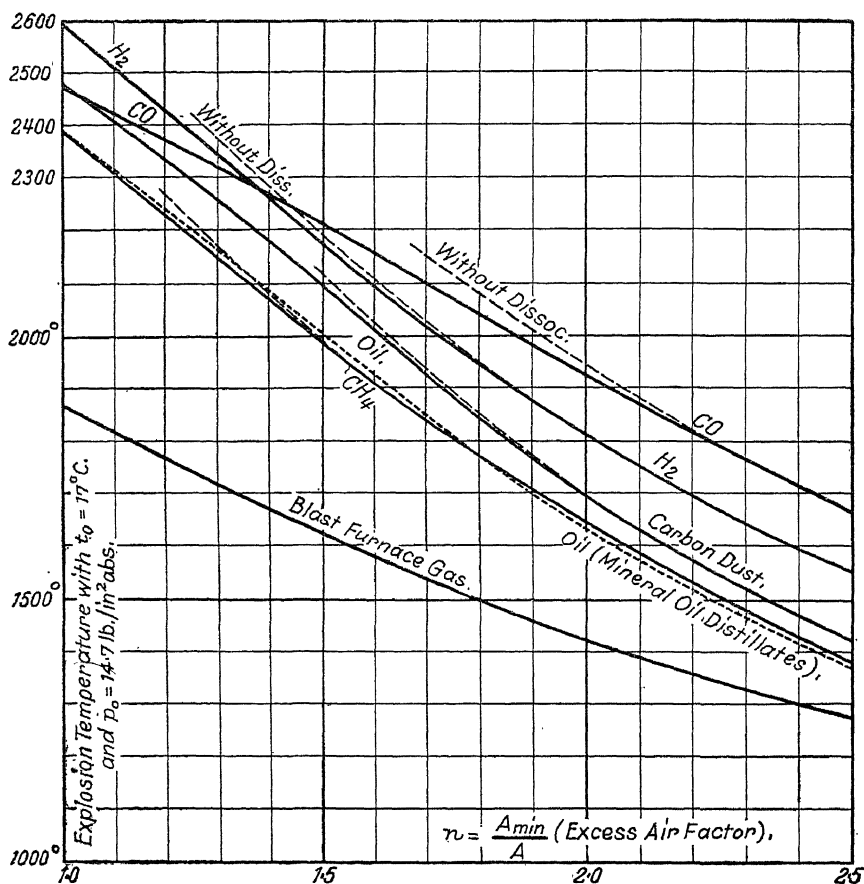


FIG. 33



appreciable degree of dissociation, particularly with rich mixtures or with a high initial temperature  $t_0$ . With dissociation the energy curve deviates from the  $I$  curve, as shown in Fig. 32, so that a reduction in the combustion temperature results (i.e.  $t_1' < t_1$ ).<sup>\*</sup> The location of the curve  $AB'$  depends on the pressure as well as the temperature, and approaches  $AB$  as the pressure is increased.

In the treatment given here for the determination of  $t_1$  and  $p_1$ , the effect of dissociation is not taken into account. In Fig. 33 the combustion temperatures attained by the burning of  $H_2$  at constant volume, and at an initial temperature of  $17^\circ C.$ , are shown to a base of excess air factors  $n$ . These combustion temperatures were calculated in the manner explained above. The value for  $n = 1$ , for which considerable dissociation occurs, was taken from the author's *Neue Tabellen*. The values for the CO-air and methane-air mixtures were similarly determined and plotted in Fig. 33.

In the case of products from ordinary fuels it is preferable to make use of Chart I as follows. A parallel line to  $OD$  (Fig. 34) is drawn through  $A$  (which is at the temperature level  $t_0$ ) and the point  $B$  is located on the curve for the particular products considered, such that the horizontal distance between the parallel line and the curve is equal to  $\frac{H_L}{V_g}$ . The combustion

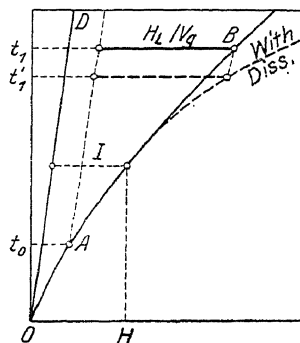


FIG. 34

temperatures obtained on burning carbon, mineral oil distillate, coke oven gas, and blast furnace gas when  $t_0 = 0^\circ C.$  and  $p_0 = 1$  atmosphere were found in this way and plotted in Fig. 33.

From these the pressure ratios  $p_1/p_0$  can also be determined, and are plotted in Fig. 35 to a base of excess air factors. The initial temperatures lie between  $17^\circ$  and  $20^\circ C.$  and the initial pressure is  $p_1 = 1$  atmosphere. If the initial pressure is increased, the explosion pressure is increased in the same ratio. In the dissociation region, the pressure ratios  $p_1/p_0$ † are increased as the initial pressures are increased, since, at these higher pressures, the degree of dissociation is decreased.

See page 340 on the values of explosion temperatures and pressures with high initial temperatures and high initial pressures.

**Pressure and temperature of gaseous mixtures.** Let similar or dissimilar gases, at the pressures  $p_1, p_2, p_3$ , and temperatures  $t_1, t_2, t_3$ , be contained in different vessels having the volumes  $V_1, V_2, V_3$  ft.<sup>3</sup>

The problem considered here is the determination of the pressure and temperature of the gases, when free communication is established between the vessels, so that complete mixing of the gases results in the common volume  $V$ , where

$$V = V_1 + V_2 + V_3$$

The internal energy per lb. of gas is equal to  $c_v t$ . If, during mixing, no external work is done by the gases, the internal energy of the mixture

<sup>\*</sup> See Schüle's *Neue Tabellen und Diagrammen* for a fuller treatment of this case.

† The pressure ratios for  $n = 1$  have been taken from the author's *Neue Tabellen*.

is equal to the sum of the internal energies of the individual gases before mixing. If, therefore, the weights of the gases are  $W_1$ ,  $W_2$ ,  $W_3$ , giving the total weight

$$W = W_1 + W_2 + W_3$$

then 
$$W_1 c_{v_1} t_1 + W_2 c_{v_2} t_2 + W_3 c_{v_3} t_3 = W c_v t$$

from which the mixture temperature is

$$W c_v$$

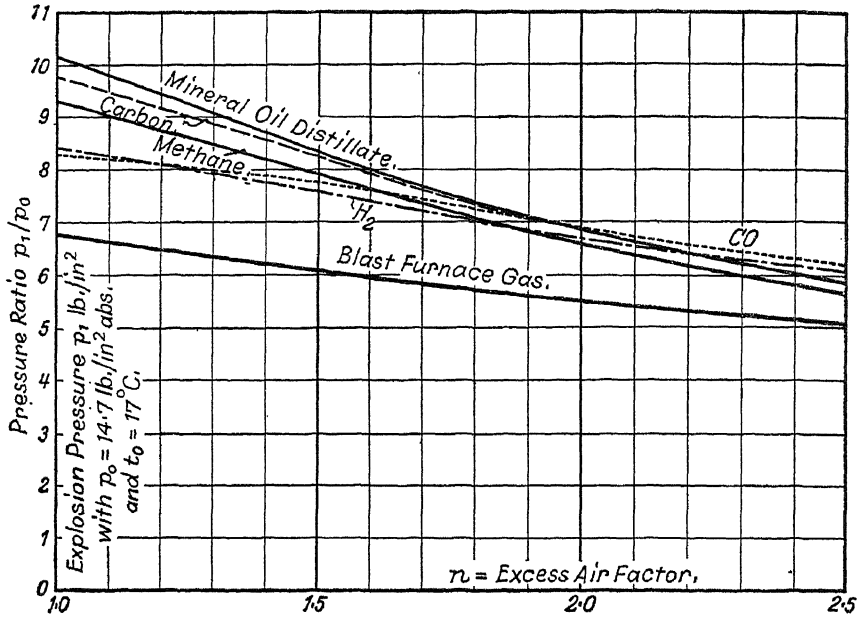


FIG. 35

But

$$W c_v = W_1 c_{v_1} + W_2 c_{v_2} + W_3 c_{v_3}$$

so that

$$t = \frac{W_1 c_{v_1} t_1 + W_2 c_{v_2} t_2 + W_3 c_{v_3} t_3}{W c_v}$$

or, since,

$$T = 273 + t$$

Now, from the characteristic equation of gases,

we have 
$$R_1 = \frac{p_1 v_1}{W_1 T}$$

and similarly for  $R_2$  and  $R_3$ .

$$\text{Hence } T = \frac{p_1 V_1 \frac{c_{v_1}}{R_1} + p_2 V_2 \frac{c_{v_2}}{R_2} + \dots}{\frac{p_1 V_1}{T_1} \frac{c_{v_1}}{R_1} + \frac{p_2 V_2}{T_2} \frac{c_{v_2}}{R_2} + \dots} \quad (117)$$

As shown on page 49,  $mc_v$  has the same value for all diatomic gases, while, as shown on page 12,  $mR$  is also equal for all gases.

Hence, for diatomic gases and their mixtures, the quotients  $\frac{c_{v_1}}{R_1}, \frac{c_{v_2}}{R_2} \dots$  are all equal, and the expression for  $T$  becomes

$$T = \frac{p_1 V_1 + p_2 V_2 + \dots}{\frac{p_1 V_1}{T_1} + \frac{p_2 V_2}{T_2} + \dots} \quad (118)$$

$$\text{With } \frac{p_1 V_1}{T_1} = W_1 R_1 \dots, \text{ etc.}$$

$$\text{then } T = \frac{p_1 V_1 + p_2 V_2 + \dots}{W_1 R_1 + W_2 R_2 + \dots}$$

$$\text{and with } WR = W_1 R_1 + W_2 R_2 + \dots \quad (\text{page 14})$$

$$T = \frac{p_1 V_1 + p_2 V_2 + \dots}{WR}$$

The mixture pressure  $p$  is found from this, since  $T = \frac{pV}{RW}$ ,

$$\text{giving } pV = p_1 V_1 + p_2 V_2 + p_3 V_3 + \dots \quad (119)$$

The sum of the products of pressure and volume before mixing is equal to the product of pressure and volume after mixing.

If the pressures before mixing are equal and the temperatures different, equation (118) gives

$$T = \frac{\frac{V_1}{T_1} + \frac{V_2}{T_2} + \frac{V_3}{T_3} + \dots}{\frac{V_1}{T_1} + \frac{V_2}{T_2} + \frac{V_3}{T_3} + \dots} = T_1 \frac{V_1 + V_2 + V_3 + \dots}{V_1 + V_2 \frac{T_1}{T_2} + V_3 \frac{T_1}{T_3} + \dots} \quad (120)$$

If, on the other hand, the temperatures are equal and the pressures different, then

$$T = T_1$$

i.e. no change in temperature occurs due to mixing.

As shown by equation (119), the pressure of the mixture is independent of the temperature, and hence can be calculated without a knowledge of its value.

If the mixing occurs at constant pressure, instead of at constant volume (in which case the pressure in all the spaces  $V_1, V_2, V_3$ , is equal to  $p_1$ ), the above treatment remains unaltered, except that  $c_v$  is replaced by  $c_p$ . Equations (117), (118), (119), and (120) thus remain the same, and from equation (119) it follows that  $V = V_1 + V_2 + V_3$ , i.e. the volume of the mixture is the sum of the volumes of the individual gases before being mixed, and is irrespective of the gas temperatures before mixing.

**Example 3.** If 0.5 ft.<sup>3</sup> of air at 86 lb./in.<sup>2</sup> abs. and 40° C. are mixed with 2 ft.<sup>3</sup> of air at 14.7 lb./in.<sup>2</sup> abs. and -10° C., find the resulting mixture pressure ( $p$ ) and temperature ( $t$ ).

From equation (119),

$$p(0.5 + 2) = 86 \times 0.5 + 14.7 \times 2 = 72.4$$

$$\therefore p = 29 \text{ lb./in.}^2 \text{ abs.}$$

From equation (117),

$$T = \frac{86 \times 0.5 + 14.7 \times 2}{\frac{86 \times 0.5}{313} + \frac{14.7 \times 2}{263}} = 290^\circ \text{ C. abs.}$$

or

$$t = 290 - 273 = 17^\circ \text{ C.}$$

**Example 4.** In order to use hot products at 1000° C. for a drying process, they are to be mixed with air at 10° C. and reduced to 550° C. Find the volume of air required expressed as a fraction of the products volume.

Denoting the products and air volumes by  $V_1$  and  $V_2$  respectively, we have, from equation (120),

$$(550 + 273) = (1000 + 273) \left\{ \frac{\left(1 + \frac{V_2}{V_1}\right)}{1 + \frac{V_2}{V_1} \frac{(1000 + 273)}{(10 + 273)}} \right\}$$

from which  $\frac{V_2}{V_1} = 0.185$ , i.e. the volume of air required is 18.5% of that of the products.

**Example 5.** The clearance volume of the L.P. cylinder of a compound steam engine is 10 per cent of the stroke volume and contains steam at 10 lb./in.<sup>2</sup> abs. Steam from the receiver at 40 lb./in.<sup>2</sup> abs. mixes with the steam in the clearance volume. Find the pressure drop in the receiver if its volume is 0.75 times the volume of the L.P. cylinder.

The equation for the mixture pressure holds approximately in the case of vapours. If the L.P. cylinder volume is called 1,

$$\text{then } p(0.1 + 0.75) = 10 \times 0.1 + 40 \times 0.75$$

$$\text{or } p = \frac{31}{0.85} = 36.5 \text{ lb./in.}^2 \text{ abs.}$$

The receiver pressure drop is thus  $40 - 36.5 = 3.5 \text{ lb./in.}^2$

**Example 6.** At the beginning of suction, the clearance volume of a gas engine cylinder is filled with hot residual gases left over from the previous working stroke. During suction these hot gases mix with the fresh charge and thus affect the temperature and weight of this charge.

Let the clearance volume be  $V_c \text{ ft.}^3$  and the pressure and temperature of the hot gases contained in it be  $t_r^\circ \text{ C.}$  and  $p_r \text{ lb./in.}^2 \text{ abs.}$  Also let the stroke volume be  $V_s \text{ ft.}^3$  and the temperature

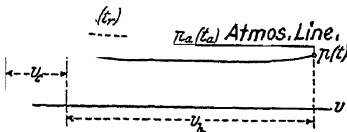


FIG. 36

and pressure of the charge before entering the cylinder be  $t_a$  and  $p_a$ . If now the pressure in the cylinder at the end of suction is  $p$ , the temperature  $t$  has to be found at this position, and also the volume  $V_a$  of gas drawn in at  $p_a$  and  $T_a$ .

For the pressures and volumes, equation (119) holds without reference to the temperatures,

$$\text{hence} \quad p(V_s + V_c) = p_r V_c + p_a V_a$$

That is, during suction, which is effected by a throttling action from the outer pressure  $p_a$  to the inner pressure, the product  $p_a V_a$  remains constant (see page 212).

It follows that

$$V_a = \frac{p}{p_a} (V_s + V_c) - \frac{p_r}{p_a} V_c$$

In the special case of  $p_r = p_a = p$ , we have

$$V_a = V_s$$

i.e. the volume drawn in is equal to the stroke volume, no matter what the temperature of the residuals in the clearance is.

The mixture temperature is

$$T = \frac{p_r V_c + p_a V_a}{\frac{p_r V_c}{T_r} + \frac{p_a V_a}{T_a}} = \frac{p(V_s + V_c)}{\frac{p_r V_c}{T_r} + \frac{p(V_s + V_c) - p_r V_c}{T_a}}$$

If, for example,  $V_c = 0.2 V_s$ ,  $p_r = 14.9 \text{ lb./in.}^2 \text{ abs.}$ ,  $t_r = 500^\circ \text{ C.}$ ,  $p_a = 14.5 \text{ lb./in.}^2 \text{ abs.}$ ,  $t_a = 20^\circ \text{ C.}$ , and  $p = 13.55 \text{ lb./in.}^2 \text{ abs.}$ , then

$$\frac{V_a}{V_s} = \frac{13.55}{14.5} (1 + 0.2) - \frac{14.9}{14.5} \times 0.2 = 0.912 \text{ (volumetric efficiency)}$$

$$\text{and} \quad T = \frac{13.55(1 + 0.2)}{\frac{14.9 \times 0.2}{773} + \frac{14.5 \times 0.912}{293}} = 334^\circ \text{ C. abs., } t = 61^\circ \text{ C.}$$

It should be noted that the volume  $V_a$  is not affected by the temperature of the residuals; the incoming charge expands during mixing by the same amount as the residuals contract. Actually, of course, a certain amount of heat is given to the inflowing air by the hot walls, so that the volumetric efficiency is reduced and the final temperature is higher than that calculated above.

**Exact Method.** In the treatment given above it was assumed that the specific heats, for all the constituents, were independent of temperature and that the molecular specific heats  $mc_v = C_v$  were all equal. Since these assumptions are not, in general, fully justified, particularly with regard to the molecular specific heats, the above equations are only valid within narrow limits. Since the internal energy of a gas at  $t^\circ \text{ C.}$ , reckoned from  $0^\circ \text{ C.}$ , is

$$I = c_{vm} t$$

the sum of the internal energies of the mixture is

$$W_1 c_{vm_1} t_1 + W_2 c_{vm_2} t_2 + \dots = W c_{vm} t$$

in which  $c_{vmt}$ , the mean specific heat of the mixture between  $0^\circ \text{C.}$  and the mixture temperature, is determined from

From this, equation (117) becomes, when  $\frac{c_{v1}}{R} \dots$  is replaced by

$$\overline{mR} = \overline{2780}$$

$$T = \frac{p_1 V_1 C_{vm1t_1} + p_2 V_2 C_{vm2t_2} + \dots}{\frac{p_1 V_1}{T_1} C_{vm1t} + \frac{p_2 V_2}{T_2} C_{vm2t} + \dots}$$

Also, since

$$pV = WR_m T$$

and

$$WR_m = W_1 R_1 + W_2 R_2 + \dots$$

$$= \frac{p_1 V_1}{T_1} + \frac{p_2 V_2}{T_2}$$

we have  $pV = p_1 V_1 \frac{T}{T_1} + p_2 V_2 \frac{T}{T_2} + \dots \dots \dots (120a)$

This equation replaces equation (119), and is only equal to it when the molecular specific heats are equal.

Equation (120a) is valid for all gases which follow the law

$$pV = RT$$

however different their molecular specific heats may be. The mixture temperature  $T$ , however, depends on the molecular specific heats, as shown by equation (119a).

The exact determination of the mixture temperature from equation (119a) is troublesome, since the mean molecular specific heats in the numerator depend on the unknown mixture temperature. The energy ( $I-t$ ) or, in the case of constant pressure, the total heat ( $H-t$ ) chart, offers a convenient graphical method of finding the mixture temperature, since

$$I_m = I_1 + I_2 + I_3 \dots \text{(at constant volume)}$$

and

$$H_m = H_1 + H_2 + H_3 \dots \text{(at constant pressure).}$$

When the volumes  $V_1, V_2, V_3, \dots \text{ft.}^3$  of the different gases, at the pressures  $p_1, p_2, p_3, \dots$  and the temperatures  $t_1, t_2, t_3, \dots$  are known, they have to be reduced to S.T.P. conditions, since Chart I has been constructed for  $1 \text{ ft.}^3$  at S.T.P.

$$\text{This gives } V'_1 = V_1 \frac{273}{T_1} \frac{p_1}{14.7}$$

$$V'_2 = V_2 \frac{273}{T_2} \frac{p_2}{14.7}$$

The ratio of the volume of gas 2 at S.T.P. to that of gas 1 at S.T.P. is thus  $\frac{V'_2}{V'_1}$ . Hence, in order to draw the energy curve for  $\frac{V'_2}{V'_1} \text{ft.}^3$  of gas 2

on the chart, in which the energy curve for 1 ft.<sup>3</sup> of gas 1 is already shown, it is only necessary to multiply the abscissae of this curve by  $V'_1$ ,  
 $\bar{V}'_1$

The energy curve for the mixture of 1 ft.<sup>3</sup> of gas 1 and  $\frac{V'_2}{\bar{V}'_1}$  ft.<sup>3</sup> of gas 2 is then formed by adding the abscissae of the two curves. The energy values  $I_1$  and  $I_2$ , corresponding to  $t_1$  and  $t_2$ , are then marked off on the energy curves of the individual gases, and a point on the mixture

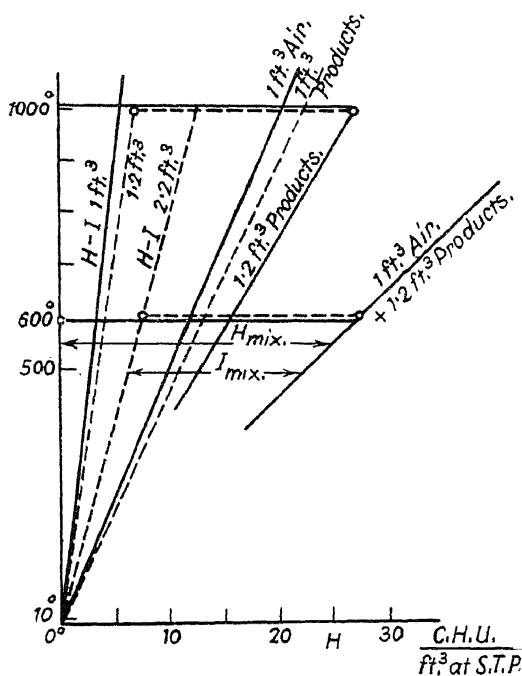


FIG. 37

curve is so located that the internal energy there is equal to  $I_1 + I_2$ . The temperature at this point then gives the required mixture temperature.

**Example 7.** If 5.4 ft.<sup>3</sup> of pure products at 1000° C. are mixed with 1 ft.<sup>3</sup> of air, at 10° C., at constant atmospheric pressure, find the resulting mixture temperature and the volume of the mixture.

The volume of products at S.T.P. per ft.<sup>3</sup> of air at S.T.P. is

$$= \frac{V_2 T_1}{\bar{V}_1 T_2} = \frac{5.4 (273 + 10)}{1 (273 + 1000)} = 1.2 \text{ ft.}^3$$

The abscissae of the pure products curve have thus to be multiplied by 1.2 in order to obtain the  $H$  curve for 1.2 ft.<sup>3</sup> of products (Fig. 37). The abscissae of this curve are added to those of the  $H$  curve for 1 ft.<sup>3</sup> of air, thus giving the  $H$  curve of the mixture. From these the mixture temperature is found to be 600° C., both for mixing at constant pressure

and constant volume. From equation (120a) the mixture volume is 6.77 ft.<sup>3</sup> and is thus greater than  $V_1 + V_2 = 6.4$  ft.<sup>3</sup>

This example is the same as example 4 given above, so far as the quantities and initial temperatures of the gases are concerned. The more exact solution, however, shows a mixture temperature of 600° C. in place of 550° C.

For gaseous mixtures, which do not follow the gas law exactly, due either to high pressures or proximity to the saturated state, the above simple equations (117), (118), (119), and (120) are not valid, but the relations  $H_m = H_1 + H_2 + \dots$  for constant pressure, and  $I_m = I_1 + I_2 + \dots$ , at constant volume, still hold.

### EFFECT OF HEAT ON THE GAS STATE. THE DIFFERENT CHANGES OF STATE

If heat be supplied to a gas contained in a vessel, which is closed by a movable piston, then, in general, the pressure and volume change simultaneously with the temperature, since they depend on the temperature, and on each other, as shown by the characteristic equation  $pV = RT$ . In special cases, one of the three variables may remain constant. Thus the volume  $V$  may remain constant, in which case, if  $T$  is doubled due to a supply of heat,  $p$  also is doubled. On the other hand, if  $p$  remains constant,  $V$  is also doubled when  $T$  is doubled. The heat supplied in the two cases is different and, in general, the supply of heat depends on the particular changes in volume and pressure.

Conversely, any change which may occur in the state of a gas must correspond to a definite supply or removal of heat. Contrary to the case with solids, the temperature change alone of gases is insufficient to determine this quantity of heat, since it depends on the pressure and volume changes also.

Changes of state may be classified as follows—

1. The containing vessel is of constant volume, so that the temperature and pressure alone vary when heat is supplied or withdrawn.

2. The containing vessel is closed by a piston, on which a constant pressure is exerted, so that the gas pressure also remains constant. In this case the temperature and volume both increase when heat is supplied and decrease when heat is removed. Heating under constant atmospheric pressure is a special example of this case.

3. The volume is varied by means of a movable piston in such a way that the temperature remains constant although heat is being supplied. The characteristic law shows that the product  $pV$  is constant in this case, and it is shown later that the volume must increase when heat is supplied, and decrease when heat is removed. This change of state is called "isothermal."

4. The pressure, volume, and temperature may be varied without the supply or removal of heat by mechanically compressing or expanding the gas. This change of state is, strictly speaking, only possible if the walls of the containing vessel are perfect heat insulators, and hence is called an "adiabatic" change of state.

5. The pressure, volume, and temperature all change, when heat is



supplied or withdrawn, according to any arbitrary law. This is the most general case of change of state.

**Change of state at constant volume.** Let a mass of gas at the pressure  $p_1$ , and absolute temperature  $T_1$ , be contained in a vessel of constant volume  $V$ . In order to raise its temperature to  $T_2$  the heat supplied per pound is

$$Q = c_v (T_2 - T_1)$$

If the rise in temperature is small,  $c_v$  may be regarded as constant, i.e. it is independent of  $T_1$  and  $T_2$ , but in heating to the usual temperatures of the products of combustion this assumption is not justified. If  $T_2$  is given,  $Q$  is found from

$$Q = (c_v)_m (T_2 - T_1)$$

where  $(c_v)_m$  is the mean specific heat between  $T_1$  and  $T_2$ . If, however,  $Q$  is given and  $T_2 - T_1$  is required, Chart I can be applied (see Fig. 34).

The weight of gas occupying  $V$  ft.<sup>3</sup> is given by

$$W = \frac{p_1 V}{RT_1} \text{ lb.}$$

The actual heat required is therefore  $WQ$  C.H.U.

Since the total volume is constant, the specific volume is also constant, so that, for the initial state,

$$p_1 v = RT_1$$

and, for the final state,

$$p_2 v = RT_2$$

which gives 
$$\frac{p_2}{p_1} = \frac{T_2}{T_1}$$

i.e. the pressure increases directly as the absolute temperature. The same treatment holds when the gas is cooled. In this case the pressure drops but the formulae remain unchanged. No mechanical work is performed in this change of state.

**Example 8.** If air at 15° C. and 14.7 lb./in.<sup>2</sup> abs. is heated at constant volume until the pressure is 43 lb./in.<sup>2</sup> abs., find the final temperature.

$$\text{Here } \frac{T_2}{(273 + 15)} = \frac{43}{14.7}$$

or 
$$T_2 = 840^\circ \text{ C. abs. and } t_2 = 577^\circ \text{ C.}$$

This example serves to show that, for technical purposes, it is not possible to raise the pressure of gases by external heating (as is done in steam boilers) to more than about 2 atmospheres pressure, owing to the high resulting temperatures.

On the other hand, the internal firing, as adopted in internal combustion engines, is well suited for the production of high pressures required in the performance of mechanical work (see page 340). No harm is caused by the high temperatures in this case, as the duration of maximum temperature is short, and efficient cooling is effected by the walls, the temperature of which is well below the combustion temperature.

**Change of state at constant pressure.** Let a mass of gas at the temperature  $T_1$  be contained in a vessel of volume  $V_1$ , and let its pressure be maintained constant at  $p$  by means of a loaded piston (Fig. 38). Due to a supply of heat from the outside, the gas expands and the increase in volume, as given by Gay-Lussac's Law, is

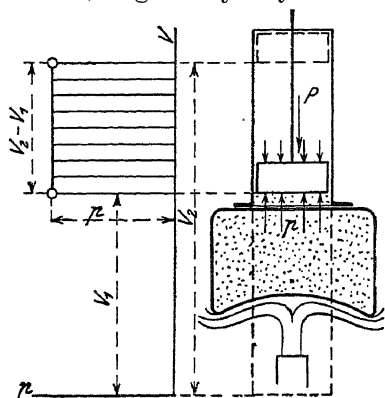


FIG. 38

$$V_2 - V_1 = \quad (\text{see page 2}).$$

If this be divided by the piston area  $A$ , the piston stroke  $S$  is obtained.  $V_0$  is the volume of gas at  $0^\circ \text{C}$ . The proportional increase in volume is

The amount of heat required to raise the temperature by  $t^\circ = T_2 - T_1$  is

$$Q = c_p t$$

where  $c_p$  is the specific heat at constant pressure. The same remarks apply to  $c_p$  as already given on page 91 regarding  $c_v$ . For a given value of  $Q$ ,  $t$  can be found, and when  $t$  is known,  $Q$  is found, when  $c_p$  is variable, from Chart I.

The total heat supply is  $WQ$  C.H.U. where

$$W = \frac{p_1 V_1}{RT_1}$$

**Mechanical work.** In the previous (constant volume) case no mechanical work was performed, but, in this case, mechanical work

$$E_m =$$

is performed, or,

$$\text{since} \quad AS = V_2 - V_1$$

For 1 lb. of gas

$$V_2 = v_2 \text{ and } V_1 = v_1$$

so that

$$E = p(v_2 - v_1)$$

This can also be expressed in terms of the temperatures,

$$\text{since} \quad v_2 = \frac{RT_2}{p_2} \text{ and } v_1 = \frac{RT_1}{p_1}$$

giving

$$E = R(T_2 - T_1)$$

✓ **Mechanical significance of the gas constant  $R$ .**

With  $T_2 - T_1 = 1^\circ \text{C}$ .

$R$  can thus be defined as the absolute work of expansion, in ft. lb., resulting from the heating of 1 lb. of a gas, at constant pressure, through  $1^\circ \text{C}$ . The universal gas constant  $mR = 2780$  is thus the corresponding work per mol., i.e. per  $m$  lb. of the gas.

**Example 9.** Find the quantity of heat transmitted to air flowing through a heater at constant pressure, if 1000 ft.<sup>3</sup> per hour are to be raised from  $-15^\circ \text{C}$ . to  $60^\circ \text{C}$ .

The heat required per lb. of air is

$$c_p(60 + 15) = 0.238 \times 75 = 17.85 \text{ C.H.U.}$$

The density of air at  $60^\circ \text{C}$ . and 14.7 lb./in.<sup>2</sup> abs.

$$= \frac{0.0808 \times 273}{273 + 60} = 0.0662 \text{ lb./ft.}^3$$

so that 1000 ft.<sup>3</sup> weigh  $1000 \times 0.0662 = 66.2$  lb. The heat required per hour is thus  $66.2 \times 17.85 = 1080$  C.H.U.

**Example 10.** Fuel is injected in a cylinder containing air at 500 lb./in.<sup>2</sup> abs. and  $700^\circ \text{C}$ ., and during the subsequent combustion the pressure behind the moving piston remains constant at 500 lb./in.<sup>2</sup> abs. (constant pressure cycle).

Find the temperature  $T_2$  of the gas when the fuel combustion ceases, if the volume behind the piston is then  $2\frac{1}{2}$  times the initial volume. Also find the heat supply per ft.<sup>3</sup> at S.T.P. and the absolute expansion work during combustion.

$$\text{Here,} \quad 273 + 700 \quad \frac{2.5}{1}$$

$$\therefore \quad T_2 = 2430^\circ \text{C. abs. and } t_2 = 2157^\circ \text{C.}$$

The temperature rise is thus  $2157 - 700 = 1457^\circ \text{C}$ ., while the initial temperature is  $700^\circ \text{C}$ . This corresponds, as shown by Chart I, to a heat supply of about 33.8 C.H.U./ft.<sup>3</sup>. The absolute expansion work is

$$E = R(T_2 - T_1) = 96 \times 1457 = 140,000 \text{ ft. lb./lb.}$$

See page 69 regarding the calculation of combustion temperatures.

### CONVERSION OF HEAT TO WORK OR WORK TO HEAT IN A CONSTANT PRESSURE CHANGE OF STATE. MECHANICAL EQUIVALENT OF HEAT. FIRST LAW OF THERMODYNAMICS

If a given mass of gas be heated at constant pressure, the heat supplied to it is greater than that required to heat the same mass of gas at constant volume by  $(c_p - c_v)$  C.H.U. per lb. and per degree centigrade rise in temperature. The increase in sensible heat actually supplied to the gases, is, however, the same, because this depends on the final temperature alone, and not on the method by which the final state is attained. The amount of this increase in sensible heat is  $c_v t$ , since all the heat added at constant volume is used in producing the temperature rise.

Of the heat  $c_p t$  C.H.U. taken up by the gas at constant pressure, therefore, the amount which disappears during expansion is  $(c_p t - c_v t)$  or  $(c_p - c_v)$  per degree rise.

The essential difference between the two changes of state is due solely to the mechanical work delivered by the gas when heated at constant pressure. The expansion and consequent delivery of work is just as much a result of the heat supply as is the temperature rise, whereas at constant volume the heat has only to effect the latter.

It will thus be realized that the heat  $(c_p - c_v)$  C.H.U., which has become latent, is simply required for the performance of the expansion work. As shown on page 92, this amounts to  $R$  ft. lb./lb. ° C., so that 1 ft. lb. of work corresponds to the heat

$$A = \frac{c_p - c_v}{R} \text{ C.H.U.}$$

If the values of  $c_p$ ,  $c_v$ , and  $R$ , as found by direct measurement, are inserted in this equation, it is found that  $A$  is the same for all gases and is independent of the variation of  $c_p$  and  $c_v$  with temperature. Its value

is  $A =$

In order to confirm this from the experimental specific heat values  $c_p$  and  $c_v$ , it should be noted that only  $c_p$ ,  $\gamma = \frac{c_p}{c_v}$ , and  $R$  are given directly, since an exact method, apart from explosion tests, for finding  $c_v$  has not yet been carried out. It is thus convenient to change the expression for  $A$  as follows—

$$\begin{aligned} \text{Since} \quad A &= \frac{c_p \left(1 - \frac{1}{\gamma}\right)}{R} \\ \text{and} \quad R &= \frac{\bar{R}}{\bar{v}_0} v_0 = \frac{\bar{R}}{\bar{v} d_0} \\ A &= \frac{c_p d_0 \left(1 - \frac{1}{\gamma}\right)}{\frac{\bar{R}}{\bar{v}_0}} = \frac{c_p d_0 \left(1 - \frac{1}{\gamma}\right)}{7.75} \end{aligned}$$

where  $d$  is the density at S.T.P., and  $c_p$  and  $\gamma$  are measured at the same temperature.

Thus, for oxygen,\*

$$d_0 = 0.089 \text{ lb./ft.}^3, \quad c_p = 0.2175 \text{ C.H.U./lb.}, \quad \gamma = 1.40$$

$$\text{so that} \quad A = \frac{0.089 \times 0.2175}{7.75} \left(1 - \frac{1}{1.4}\right) \quad 1$$

for hydrogen,

$$d_0 = 0.00561, \quad c_p = 3.40, \quad \gamma = 1.408$$

$$\text{so that} \quad A = \frac{0.00561 \times 3.40 \times 0.408}{7.75 \times 1.408} \quad 1$$

\* From Landolt and Börnstein, *Physik-Chem. Tabellen*. Third Edition (1905).

$$\begin{aligned} \text{and for air, } d_0 &= 0.0808, \quad c_p = 0.240, \quad = 1.40 \\ \text{so that } A &= \frac{0.0808 \times 0.240 \times 0.4}{7.75 \times 1.40} = \frac{1}{1400} \end{aligned}$$

These accurate test values serve to show, therefore, that  $A$  is actually constant and equal to  $\frac{1}{1400}$ .

From the above it follows that the amount of work obtained by the conversion of  $\frac{1}{1400}$  C.H.U. of heat is 1 ft. lb.

If a gas is cooled at constant pressure, so that its temperature decreases by  $t^\circ$  C. (see Fig. 38), the heat taken from it is  $c_p t$ . The amount of sensible heat given up by the gas itself, however, is only  $c_v t$  (for the same reason as explained above when the gas received heat). The excess  $(c_p - c_v)t$ , which is also given up, is obtained from the mechanical work performed on the gas by the external pressure as the volume is decreased. This work is transformed to heat in the gas itself and, as such, is given up during cooling. Hence, when heat is produced from mechanical work, the amount of work required per unit of heat is

$$\frac{R}{c_p - c_v} = \frac{1}{A} = 1400 \text{ ft. lb./C.H.U.}$$

This conversion of heat to work, or work to heat, forms the basis of a perfectly general law, which is independent of the particular properties of gases and is known as the "law of the equivalence of heat and work." It can be stated as—

If, in any way, heat is transformed to mechanical work, or mechanical work to heat, for every unit of heat so transformed the amount of work obtained is 1400 ft. lb., while for every ft. lb. of work transformed to heat the amount of heat generated is  $\frac{1}{1400}$  C.H.U.

This law was first discovered by Mayer in 1842. Its general validity was confirmed by Joule in 1843 by numerous tests. Mayer calculated the mechanical equivalent by means of the values of  $C_p$ ,  $C_v$ , and  $R$ , known at his time in the manner explained above. The value obtained from recently established data for oxygen and nitrogen is, as shown above, 1400, and is the same as the value given by direct tests, in which mechanical work is transformed to heat by friction or turbulence.

Assuming now that this law is valid generally, then for all gaseous substances, to which the characteristic equation  $pV = RT$  applies, we have the important relation

$$c_p - c_v = \frac{R}{1400}$$

or, since

$$R = \frac{2780}{m}$$

$$mc_p - mc_v = C_p - C_v = \frac{2780}{1400} = 1.98$$

i.e. the difference between the molecular specific heats at constant pressure and constant volume is, for all gases and at all temperatures, equal to 1.98.

The number 2780 is called the "universal gas constant" (page 13) and represents the work delivered by 1 mol. of any gas when heated at constant pressure through  $1^\circ\text{C}$ .

### EXPANSION AND COMPRESSION WORK OF GASES AND VAPOURS. ABSOLUTE AND USEFUL WORK

Consider a cylinder fitted with a piston, as shown in Fig. 38, and let it contain  $W$  lb. of gas or vapour. Also let heat be supplied to, or removed from, the gas either internally or externally. Any change in pressure will be transmitted by the piston rod, either from the outside to the gas or from the gas to the outside. In this section the work due to the gas pressures and piston loads will be considered, for any movement of the piston (i.e. for any change in volume of the gas) when the variation of gas pressure with volume is known and equilibrium is maintained between the forces on both sides of the piston.

**Absolute gas work.** The manner in which the gas pressure, acting on the piston, varies with change in volume depends on the heat given to or taken from the gas during this change. Hence, in finding a general expression for the work done by the gas, any arbitrary law connecting the pressures and volumes may be assumed.

For a small movement  $ds$  of the piston the change in pressure is  $dp$ . But  $dp$  is negligible compared with the total pressure  $p$ , so that the work due to the gas pressure is  $apds$ . Now  $ads$  is the change in volume  $dV$  of the gas, due to the movement of the piston of area  $a$ , i.e.  $ads = dV$ .

Hence the work is  $dE = pdV$

or, per lb.,  $dE = p dv$

If the movement of the piston be regarded as a series of steps of length  $ds_1, ds_2, ds_3, \dots$  corresponding to volume increases of  $dV_1, dV_2, dV_3, \dots$  the successive works obtained are  $p_1dV_1, p_2dV_2, p_3dV_3, \dots$ , so that the total work for any travel of the piston is

$$E_w = p_1dV_1$$

(from  $V_1$  to  $V_2$ ) or

$$E_w = \int_{V_1}^{V_2} p dV$$

For  $W = 1$  lb. since  $V = v$

$$E_w = \int_{v_1}^{v_2} p dv$$

$E$  and  $E_w$  are related by the expression

$$E_w = WE$$

since

$V = Wv$  and  $dV = Wdv$  gives

$$E_w = \int_{v_1}^{v_2} p W dv = W \int_{v_1}^{v_2} p dv = WE$$

Work performed by gases is regarded as positive, while work done on gases is regarded as negative. These correspond to the expansion and compression of gases respectively.

**Graphical representation of absolute work.** (Indicator diagram or  $pV$  diagram.) If the absolute pressures  $p$  are plotted to a base of volumes  $V$  (Fig. 39), the element of work  $p dV$  is represented by the small shaded area. The total work delivered by the gas in expanding from  $V''$  to  $V'$ , or the compression work done on the gas between  $V'$  and  $V''$ , is represented by the sum of these elements, i.e. by the area  $ABB'A'$ . For this reason the diagram is called the "work diagram."

The method of locating the pressure axis in practical cases is as follows (Fig. 39). The particular pressure changes are actually given

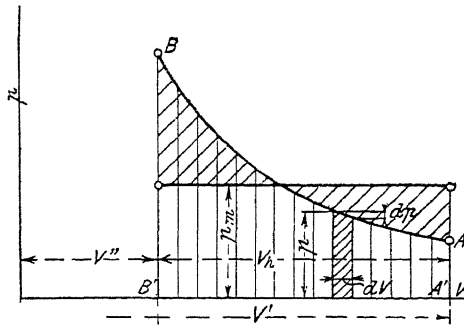


FIG. 39

within the stroke volume, but the corresponding gas volume at any instant is the sum of the volumes swept out by the piston and the volume  $V''$  behind the piston at the inner dead centre position. In the case of gas engines  $V''$  is called the "compression space," and in the case of steam engines the "clearance volume." The pressure axis thus lies at a distance proportional to these volumes from the initial point of the curve.

**Mean effective pressure.** The work delivered by the gas, when the pressure is variable (curve  $BA$ ), may be regarded as delivered by a constant pressure of a certain mean value  $p_m$ , acting during the same volume change.

In the  $pV$  field,  $p_m$  must equal the height of the rectangle, having a base equal to  $B'A'$  and an area equal to  $B'BA A'$ ;  $p_m$  can be found from the diagram by integrating this area with a planimeter and dividing by  $A'B'$  (without reference to the volume scale). The work is then given by

$$E = p_m(V' - V'') \text{ ft. lb.}$$

in which  $p_m$  is in lb./ft.<sup>2</sup> and  $V$  and  $V''$  are in ft.<sup>3</sup>

In practical cases,  $V' - V''$  is usually the stroke volume of a cylinder having a piston area  $A$  and piston stroke  $S$ , so that

In this expression  $p_m$  can be in lb./in.<sup>2</sup> if  $A$  is in in.<sup>2</sup>  $S$ , however, is measured in feet.





In the expansion  $BA$ , however, the work  $ABDC$  has to be performed from without, since the gas pressure  $p$ , being less than the opposing pressure  $p'$ , is not sufficient to maintain equilibrium.

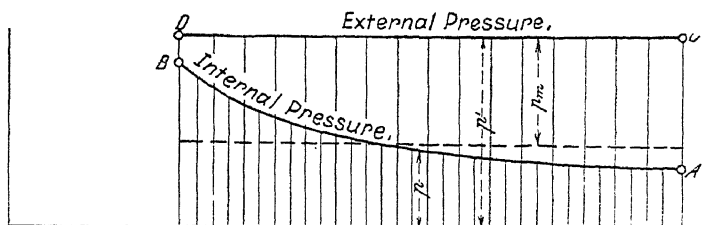


FIG. 42

### ENERGY EQUATION OF GASES. CHANGE IN THE PROPERTIES OF A GAS ACCOMPANYING A CHANGE OF STATE

As shown on page 90, every change of state of a gas, with one exception, is accompanied by a supply or removal of heat; conversely, every heat supply or withdrawal corresponds to a change of state, i.e. of temperature, pressure, and volume.

As already explained, a portion of the heat is used to raise the temperature, and remains as heat in the gas; the remainder, as mechanical work, is used in performing the absolute gas work. For the general case it is not necessary that the pressure of the gas should remain constant (as considered on page 93).

If 1 lb. of the gas is raised in temperature from  $t_1$  to  $t_2$ , the increase in sensible heat (internal energy) is  $c_v(t_2 - t_1)$ , whether or not the volume, or the pressure, remains constant.

This follows, since the internal energy, at any temperature  $t^\circ$ , depends only on the gas state, and not on the path pursued in arriving at this state, from the standard state generally taken at  $0^\circ \text{C}$ . The value of the internal energy above  $0^\circ \text{C}$ . can, therefore, be determined by heating the gas at constant volume from  $0^\circ \text{C}$ . to  $t^\circ \text{C}$ ., whereby all the transmitted heat remains in the gas. Further, for gases which follow the law  $pv = RT$ ,  $c_v$  depends only on the temperature and not on the pressure.

In any change of state, therefore, the amount of heat required solely to raise the temperature of the gas is  $c_v(t_2 - t_1)$  or  $c_v(T_2 - T_1)$ . If, in addition, an amount of absolute work  $E$  is performed by the gas, the extra heat required for this is  $\frac{E}{1400} = EA$ . If now the heat supplied to

the gas is  $Q$ , it follows that

$$Q = c_v(T_2 - T_1) + AE \quad (121)$$

If work is done on the gas, instead of by the gas,  $AE$  is negative.

$E$  is given by the area on the  $pV$  diagram, i.e. the total area below the  $pV$  curve between the ordinates  $p_1$  and  $p_2$  (Fig. 39).

For the initial state we have

and, for the final state,

so that equation (121) can be written

$$Q = \frac{c_v}{R}$$

or, since  $c_p - c_v = AR$  (page 94)

and  $\frac{c_p}{c_v} = \gamma$

$$Q = \frac{A}{\gamma - 1} \quad (122)$$

applicable for 1 lb. of gas.

For  $W$  lb. of gas the equation becomes

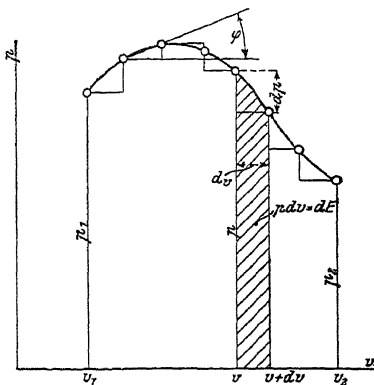


FIG. 43

since

$$E_w = WE \text{ and } Wv_2 = V_2; Wv_1 = V_1$$

The relationships between  $c_p$ ,  $c_v$ ,  $\gamma$ , and  $R$  are used frequently later in the text. The most important are

$$\frac{c_p}{c_v} = \gamma, \quad AR = \gamma - 1, \text{ and } \frac{c_p + c_v}{c_p - c_v} =$$

As an example of the application of equation (122) consider the following case.

If the  $pV$  curve for any change of state be given (as, for example, the expansion or compression curve for a gas engine or air compressor), then, by integrating the area below any two points on the curve, the heat given to or taken from the gas can be determined. In this, of course, the weight of the gas between the two chosen points must remain constant.

**Energy equation applied to an elementary change of state.** When a gas changes its state in any way from the initial state  $p_1, v_1, T_1$  to the final state  $p_2, v_2, T_2$ , the change may be regarded as made up of a series of elementary changes in  $p$ ,  $v$ , and  $T$ , as shown in Fig. 43.

The heat flow  $dQ$  during one of these elementary changes can be expressed in the same way as in the case of a finite change in state, giving

$$dQ = c_v dT + A p dv \quad (123)$$

This is valid, whether or not the specific heat is variable.

✓ **Characteristic equation applied to an elementary change of state.**  
At the commencement of this change, we have

$$pv = RT$$

and, at the end,

$$(p + dp)(v + dv) = R(T + dT)$$

or

$$pv + p dv + v dp + dp dv = RT + R dT$$

Subtraction gives

$$p dv + v dp + dp dv = R dT$$

Dividing by  $dp$  gives

$$p dv/dp + v + dv = R \frac{dT}{dp}$$

On the  $pv$  diagram (Fig. 43),  $dv/dp$  is the slope of the curve at the point considered ( $\cot \phi$ ) and has, therefore, a finite value. Similarly,  $dT/dp$  gives the slope of the pressure-temperature curve, the course of which can be determined from the  $pv$  curve. The last equation contains, therefore, finite amounts, apart from the quantity  $dv$ . This amount disappears, when limiting values are taken, and the expression becomes

$$pdv/dp + v = RdT/dp$$

or

$$p dv + v dp = R dT \quad . \quad . \quad . \quad . \quad . \quad . \quad (124)$$

(Characteristic equation for an elementary change in state.)

Replacing  $R$  by  $pv/T$  and dividing by  $pv$  gives

$$dv + dp = \frac{dT}{T} \quad (125)$$

which is independent of the particular type of gas.

By combining this equation with the energy equation, any one of the quantities  $dp$ ,  $dv$ , or  $dT$  can be eliminated, and the law of variation of  $p$  and  $v$ , or  $p$  and  $T$ , or  $v$  and  $T$ , corresponding to a definite law of heat change  $dQ$ , can be determined, or, conversely, the heat  $dQ$  can be found from a given  $pv$  curve.

Writing equation (124) in the form

$$dT = \frac{1}{R} (pdv + vdp)$$

and inserting this in the energy equation, gives

$$dQ = \frac{c_v}{R} (p dv + v dp) + A p dv$$

Also, since  $\frac{c_v}{R} = \frac{A}{\gamma - 1}$

we have  $\frac{dQ}{\gamma - 1} = \frac{1}{\gamma - 1} v dp = \frac{1}{\gamma - 1} p dv$

or 
$$\frac{dQ}{A dv} = \frac{v}{\gamma - 1} (dv$$

By means of this last equation it is possible to determine whether heat is being supplied to or taken from a gas at any point on a  $pv$  curve (Fig. 44). If, say, with increasing volume ( $dv$  positive) the right-hand side of the equation is positive, then  $dQ$  is also positive, so that heat is being supplied; conversely, when  $dQ$  is negative, heat is being withdrawn.

If, with decreasing volume (compression,  $dv$  negative), the right-hand side is positive, then  $dQ$  is negative, and *vice versa*. It becomes necessary to determine, therefore, whether  $dp/dv + \gamma \frac{p}{v}$  is positive or negative.

In all cases of pressure increase with increasing volume heat is being added (position I). If the pressure and volume are both decreasing, the expression is again positive, but heat is being withdrawn, since  $dv$  is negative (position II).

If the pressure decreases with increasing volume (position III, expansion)  $\frac{dp}{dv} + \gamma \frac{p}{v}$  may either be positive or negative, since  $dp$  is negative

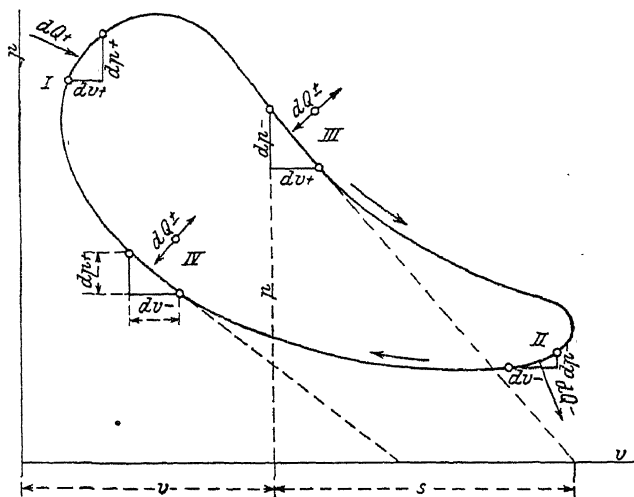


FIG. 44

and  $dv$  is positive. Hence, heat will be added to or taken from the gas if  $\frac{dp}{dv}$  is less or greater than  $\gamma \frac{p}{v}$  respectively. From Fig. 44,

$$\frac{dp}{dv} = -\frac{p}{s}$$

The conditioning equation is thus

$$s >$$

$$v <$$

or

If, therefore, the ratio of the abscissa  $v$  to the subtangent  $s$  is smaller than  $\gamma$ , heat is being added, but, if larger than  $\gamma$ , heat is being withdrawn.

The reverse holds in the case of compression (position IV), i.e. if  $\frac{v}{s} < \gamma$  heat is withdrawn, and if  $\frac{v}{s} > \gamma$  heat is supplied.

Hence, by drawing a tangent at any point, a very simple means is obtained of finding whether heat is being added to, or taken from the gas.

As shown by equation (121) the heat supply during any change of state  $AB$  (Fig. 45) is found by adding the quantity  $c_v(t_2 - t_1)$  to the heat equivalent of the absolute gas work  $AE$ . The quantity  $c_v(t_2 - t_1)$  is represented in mechanical units by the area below the adiabatic expansion curve between the temperatures  $t_2$  and  $t_1$  (see equation 131). Hence, if an adiabatic be drawn through  $B$  and an isothermal through  $A$ , the area below  $BC$  is equal to  $1400 c_v(t_2 - t_1)$ , since the temperatures at  $B$  and  $C$  are  $t_2$  and  $t_1$  respectively. The total shaded area below  $ABC$  represents the heat  $Q$  in mechanical units, i.e.  $1400Q$  ft. lb. The same result is obtained using equation (122).

Hence, when the diagram scales are known, this purely graphical method gives the value of  $Q$ , even if the specific heats and temperatures are not known.

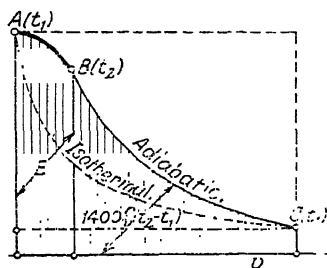


FIG. 45

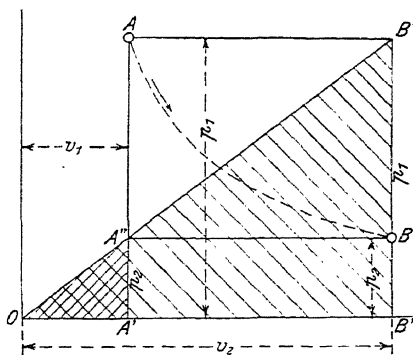


FIG. 46

### CHANGE OF STATE AT CONSTANT TEMPERATURE (Isothermal Change of State)

When the temperature remains constant the pressures are inversely proportional to the volumes, or

$$\frac{p_1}{p_2} = \frac{v_2}{v_1}$$

which gives

$$p_1 v_1 = p_2 v_2$$

or

$$pv = \text{constant}$$

so that the pressure-volume curve is a rectangular hyperbola.

If the pressure and volume in an initial state is represented by the point  $A$  (Fig. 46), the final point  $B$ , corresponding to an increase in volume from  $v_1$  to  $v_2$ , is easily found as follows—

A horizontal and vertical are drawn through  $A$  and a radial line  $OB''$  through  $O$ . Through  $A''$ , where this radial line cuts the vertical through  $A$ , a horizontal is drawn to cut the vertical through  $B''$  at the required point  $B$ .

This follows since  $B'B''/A'A'' = OB'/OA'$

or

which satisfies the condition given above.

This method is used to draw in the complete expansion curve (Fig. 47), or the complete compression curve (Fig. 48), starting in both cases at the point  $A$ . A horizontal and vertical are first drawn through  $A$ . From

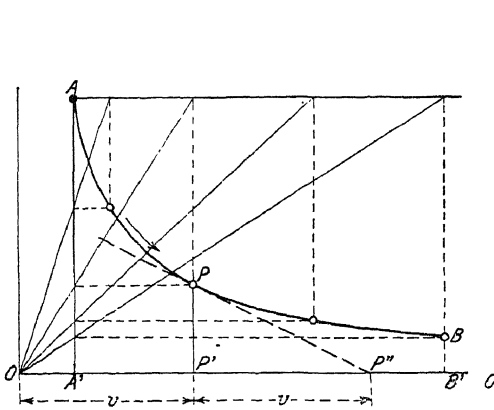


FIG. 47

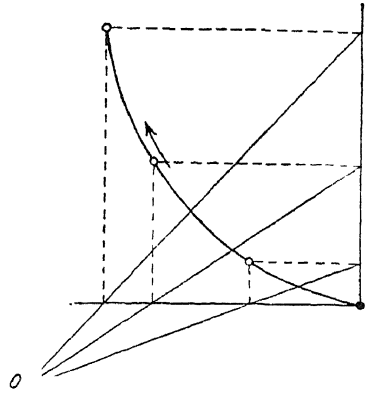


FIG. 48

$O$  any convenient number of radial lines are drawn to cut these at points through which horizontals and verticals are drawn. The points of intersection of these horizontals and verticals lie on the required curve.

The absolute work  $E$ , performed by the gas in expansion, or on the gas in compression, is given by the area  $ABB'A'$  (Fig. 47). From the properties of the rectangular hyperbola this area is

$$E = p_1 v_1 \log_e \frac{p_1}{p_2}$$

or 
$$E = p_1 v_1 \log_e \frac{v_2}{v_1}$$

For the purpose of calculation, it is more convenient to use logarithms to the base 10, which give

$$E = 2.303 p_1 v_1 \log \frac{p_1}{p_2}$$

or 
$$E = 2.303 p_1 v_1 \log \frac{v_2}{v_1}$$

(for 1 lb. of gas).

For the weight  $W$  lb. of gas, having volume  $V_1$  ft.<sup>3</sup>,

$$E_w = 2.303 p_1 V_1 \log \frac{p_1}{p_2}$$

The heat added between  $A$  and  $B$  is given by

$$Q = c_v(T_2 - T_1) + EA$$

but  $T_1 = T_2$ , so that  $c_v(T_2 - T_1) = 0$

and  $Q = EA$

If, therefore, the temperature is to remain constant during expansion, a quantity of heat equal to the work done has to be supplied.

In the case of compression, where  $E$  is negative, we have

$$Q = -AE$$

From this it follows that, if the temperature is not to increase during compression, a quantity of heat equivalent to the absolute compression work has to be removed, i.e. in isothermal compression the whole of the absolute work of compression is converted to heat and carried away by the cooling water.

If the  $pv$  curve is to follow the hyperbola at every point, the heat equivalent of an element of work has to be supplied or removed. From page 100, this gives  $dQ = Apdv$ , and  $dQ$  will be positive for  $+dv$  (expansion) and negative for  $-dv$  (compression). The quantities of heat for the same elementary changes are therefore proportional to the pressures.

In fast running machines, such as air compressors, it is impossible to produce isothermal compression, and the best that can be done is to employ as efficient cooling as possible.

**Example 11.** (See also Boyle's Law, page 1.) Air at  $p_0$  lb./in.<sup>2</sup> abs. is to be compressed isothermally to  $p$  lb./in.<sup>2</sup> abs. Find the absolute work of compression performed on the air. How much heat has to be removed from the air during compression? Express the quantities per ft.<sup>3</sup> of air.

Since  $p$  is in lb./in.<sup>2</sup> abs., we have

$$E = 2.303 \times 144 p V \log \frac{p}{p_0}$$

or, with  $V = 1$  ft.<sup>3</sup>

$$E = 2.303 \times 144 p \log \frac{p}{p_0} \text{ ft. lb./ft.}^3$$

and

$$1400$$

With  $p_0 = 14.7$  lb./in.<sup>2</sup> abs., and  $p$  as follows—

$p =$	$1.5 \times 14.7$	$3 \times 14.7$	$6 \times 14.7$	$9 \times 14.7$	$15 \times 14.7$	lb./in. <sup>2</sup> abs.
we have						
$E =$	1,288	6,990	22,800	41,900	86,000	ft. lb./ft. <sup>3</sup>
and						
$Q =$	0.92	5.0	16.3	30.0	61.5	C.H.U./ft.

### CHANGE OF STATE WITH NO HEAT FLOW EITHER TO OR FROM THE GAS. (Adiabatic Change of State)

In isothermal compression of a gas, heat has to be removed, otherwise the temperature rises, while, in isothermal expansion, heat has to be supplied, otherwise the temperature falls.

In this section, the changes in temperature, pressure, and volume of a gas, and the work done in compression or expansion when no heat is supplied to or taken from the gas, will be considered.

This change of state is of importance in dealing with the expansion and compression of the gases in an internal combustion engine. Even though the metal walls are not perfect heat insulators, and may allow considerable heat flow, the adiabatic change of state is useful in considering the ideal standard of performance, since the work is then delivered entirely by the internal energy of the gas. Again, in the case of high speed engines, the heat flow during expansion is not very serious.

From the energy equation

$$Q = c_v(T_2 - T_1) + AE$$

we have with

$$Q = 0$$

Hence, in the case of expansion,  $T_2 - T_1 < 0$ , i.e.  $T_2 < T_1$ , or the temperature falls. The expansion work is  $\frac{c_v}{A}(T_1 - T_2)$  and is thus equivalent to the heat, expressed in mechanical units, which has been removed from the gas. In the case of compression,  $T_2$  is greater than  $T_1$  (since  $AE$  is negative), therefore  $c_v(T_2 - T_1)$  is negative, and hence the temperature rises. This means that the increase of heat or energy, given to the gas, is equal to the heat equivalent of the work of compression  $AE$ .

In an adiabatic change, therefore, the whole of the absolute work of compression is given to the gas as heat, while, in expansion, the work is obtained entirely from the energy possessed by the gas.

**Expression for the adiabatic work— $pV$  curve.** For the initial state (Fig. 49) we have

$$p_1 v_1 = RT_1$$

and, for the final,

$$p_2 v_2 = RT_2$$

Subtraction gives

so that the work done is

$$\text{or, with} \quad \frac{c_v}{AR} = \frac{1}{\gamma - 1} \quad (\text{page 100})$$

$$E = \frac{1}{\gamma - 1} p_1 v_1$$

Now  $E$  is given by the area below the  $p v$  curve, i.e.  $ABB'A'$ . Hence this curve must satisfy the condition that the area below it, and between the ordinates  $p_1$  and  $p_2$ , must equal the difference between the two rectangles  $p_1 v_1$  and  $p_2 v_2$ , multiplied by the coefficient  $\frac{1}{\gamma - 1}$ . Another property of the  $p v$  curve is established as follows.



The energy equation, for an elementary change of state, is (see page 100)

$$dQ = c_v dT + A p dv$$

so that for

$$dQ = 0$$

$$c_v dT = -A p dv$$

The characteristic equation gives (page 101)

$$p dv + v dp = R dT$$

Eliminating  $dT$ , from both equations, gives

$$-v dp = p dv \left( 1 + \frac{AR}{c_v} \right)$$

but, as shown on page 100,

$$1 + \frac{AR}{c_v} = \gamma$$

so that

$$\frac{dp}{dv} = -\gamma \frac{p}{v} \quad (126)$$

This equation gives the direction of the tangent at any point on the  $pv$  curve having the co-ordinates  $p$  and  $v$ . From Fig. 49,

$$\tan \phi = -dp/dv$$

$$C'D = \frac{CC'}{\tan \phi} \quad \frac{p}{\tan \phi}$$

With  $\tan \phi = \frac{\gamma p}{v}$

we have  $C'D = \frac{v}{\gamma}$

$C'D$  is the projection of the tangent  $CD$  on the abscissa axis (i.e. the subtangent)

and is thus given, for any point on the curve, by the abscissa multiplied by  $\frac{1}{\gamma}$

This geometrical property, as well as the expression for the area given above, is valid generally for all curves in the  $pv$  field which follow the law

$$pv^\gamma = \text{constant} \quad (127)$$

The relation is sometimes called Poisson's Law. It is referred to, in this text, as the "adiabatic equation," while the curve itself is called an "adiabatic."

The value of  $\gamma = \frac{c_p}{c_v}$  is nearly equal for all diatomic gases, and at normal temperatures amounts to 1.4 (see page 52). Hence these gases have the same adiabatics. For other gases, however, the values are

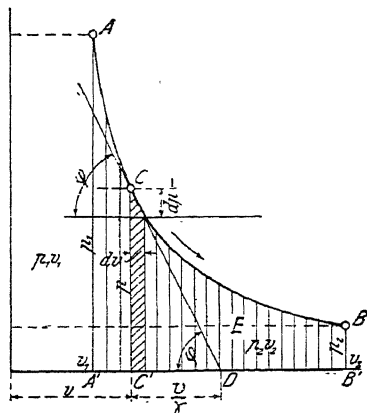


FIG. 49

different; thus for methane  $\gamma = 1.31$ , for ethylene 1.24, and for argon 1.67. At high temperatures the value of  $\gamma$  decreases; for example its value at 1200° C. for air is 1.34 and for pure products 1.28 (Fig. 68).

**Temperature change.** From the characteristic equation at the initial and final states, we have

$$\frac{T_2}{T_1} = \frac{p_2 v_2}{p_1 v_1}$$

and since  $p_1 v_1^\gamma = p_2 v_2^\gamma$

or  $\frac{p_2}{p_1} = \left(\frac{v_1}{v_2}\right)^\gamma$

we have  $\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{\gamma-1}$  . . . . . (128)

Also since  $\frac{v_2}{v_1} = \left(\frac{p_1}{p_2}\right)^{\frac{1}{\gamma}}$

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$$
 . . . . . (129)

For an elementary change of state, we have, when  $dv$  and  $v$  are eliminated from equation (125), by means of the relation  $pv^\gamma = C$ ,

$$\frac{dT}{dp} = \frac{\gamma-1}{\gamma} \frac{T}{p}$$
 . . . . . (130)

**Expressions for the adiabatic work.** This can be given, as already explained, by

$$E = c_v(T_1 - T_2)$$
 . . . . . (131)

or, in terms of the pressures and volumes,

$$E = \frac{1}{\gamma-1} (p_1 v_1 - p_2 v_2)$$
 . . . . . (132)

Again, if  $p_2$  is eliminated by means of the relation,

$$p_2 = p_1 \left(\frac{v_1}{v_2}\right)^\gamma$$

we have  $E = \frac{p_1 v_1}{\gamma-1} \left[1 - \left(\frac{v_1}{v_2}\right)^{\gamma-1}\right]$  . . . . . (133)

and similarly, by eliminating  $v_2$ ,

$$E = \frac{p_1 v_1}{\gamma-1} \left[1 - \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}\right]$$
 . . . . . (134)

These expressions are valid for 1 lb. and are used according to the data available. For the general case, where the weight is  $W$  lb. and the volume  $V$  ft.<sup>3</sup>, the quantity  $v$  is replaced by  $V$ .

Equation (134) can also be expressed as

$$E = \frac{p_1 v_1}{\gamma - 1} \quad (135)$$

In compression processes these expressions are negative, since  $v_2 < v_1$ ,  $p_2 > p_1$ , and  $T_2 > T_1$ , but, in actual calculations, the negative sign need not be retained, so that the work of compression is expressed by, say,

$$E = \frac{p_1 v_1}{\gamma - 1} \left[ \left( \frac{p_2}{p_1} \right)^{\frac{\gamma - 1}{\gamma}} - 1 \right] \quad (136)$$

**Construction of the adiabatics.** Let any point  $A$  on the curve be given (Fig. 50), and let the adiabatic exponent be  $\gamma$ . In order to draw in the curve, points on it may be located by calculation, or graphically, as follows—

**1. By calculation.** This is generally the most rapid and most exact method of drawing the curve. Thus, in Fig. 50, at the point  $A$  let  $p_1$

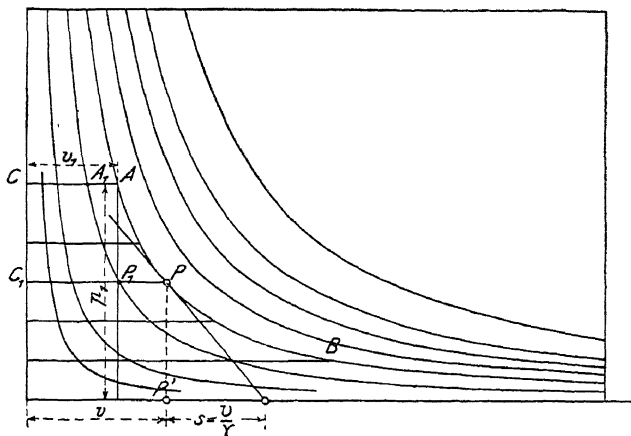


FIG. 50

$= 0.55$  in. and  $v_1 = 0.23$  in., then for  $p = 0.40, 0.30, 0.20$ , and  $0.10$  inches the corresponding abscissae  $v$  are  $0.288, 0.358, 0.480$ , and  $0.79$ , as found

by calculation. Thus, taking the first point,  $v$  is given by  $0.23 \left( \frac{0.55}{0.40} \right)^{\frac{1}{1.4}} = 0.288$ , and so on.

When the curve  $AB$  is drawn, any other curve, such as the one through  $A_1$ , can be drawn from it by making the ratio of the abscissae of the curves constant and equal to  $\frac{CA_1}{CA}$ . Fig. 50 contains a series of adiabatics, between which any other required curve can be easily drawn in the manner just described.

Since, for a given value of  $\gamma$ , the ratio of the volumes  $v/v_0$  is found from the ratio of the pressures, the adiabatic curve for a given initial state  $p_0 v_0$  can be constructed by means of tables such as the following, which are valid for  $\gamma = 1.4$ .

## EXPANSION—

$$\frac{p}{p_0} = 1 \quad 0.9 \quad 0.8 \quad 0.7 \quad 0.6 \quad 0.5 \quad 0.4 \quad 0.3 \quad 0.2 \quad 0.1 \quad 0.05 \quad 0.025$$

$$v = 1 \quad 1.078 \quad 1.173 \quad 1.290 \quad 1.440 \quad 1.640 \quad 1.924 \quad 2.363 \quad 3.157 \quad 5.180 \quad 8.498 \quad 13.942$$

## COMPRESSION—

$$p_0 = 1 \quad 1.5 \quad 2 \quad 3 \quad \quad \quad 9 \quad 10 \quad 20 \quad 40$$

$$v = 1 \quad 0.748 \quad 0.610 \quad 0.457 \quad 0.372 \quad 0.317 \quad 0.279 \quad 0.249 \quad 0.227 \quad 0.209 \quad 0.193 \quad 0.118 \quad 0.0717$$

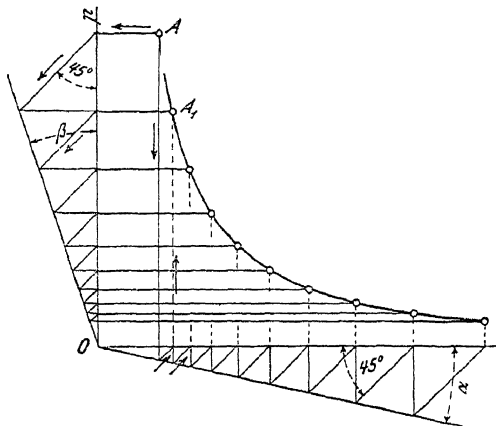


FIG. 51

**2. Brauer's graphical construction** (Fig. 51). From  $O$  draw a line at any angle  $\alpha$  to the horizontal, and a line at the angle  $\beta$  to the pressure axis, such that

$$\tan \beta = (1 + \tan \alpha)^\gamma - 1$$

This is followed by the zig-zag construction shown by the arrows. The parallels must be drawn at  $45^\circ$ . If the angle  $\alpha$  is increased, the points will be further apart.

With  $\tan \alpha = 0.2 = \left(\frac{20}{100}\right)$ , for example, and  $\gamma = 1.4$ , we have

$$= \frac{29.08}{100}.$$

Care must be exercised in this construction, since any small error is carried over to the subsequent points.

**Method of finding the exponent  $\gamma$  from the indicator diagram.** Draw a tangent at the point  $P$  (Fig. 50) where the value of the exponent is required. The subtangent  $s$ , divided by the abscissa of  $P$ , gives the required exponent,

i.e.

See page 116 for an alternative method for finding  $\gamma$ .

In indicator diagrams,  $\gamma$  usually varies at different points, either because the process is not truly adiabatic or because  $\gamma$  itself is variable.

**Example 12.** A mixture of air and gas at 12.8 lb./in.<sup>2</sup> abs. and 50° C. is compressed adiabatically to a fifth of its volume. Find the final pressure and temperature ( $\gamma = 1.38$ ).

$$\text{Here } \frac{p_2}{p} = \left(\frac{v_1}{v_2}\right)^{1.38} \therefore p_2 = 12.8 \times 5^{1.38} = 118 \text{ lb./in.}^2 \text{ abs.}$$

$$\text{and } \frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{\gamma-1} \therefore T_2 = (273 + 50) 5^{0.38} = 595^\circ \text{ C. abs.}$$

$$t_2 = 322^\circ \text{ C.}$$

**Example 13.** Compressed air at 71.6 lb./in.<sup>2</sup> abs. and 40° C. is to be expanded adiabatically so as to give a final pressure of 22 lb./in.<sup>2</sup> abs. Find the proportional increase of volume and final temperature,  $\gamma = 1.4$ .

$$\text{Here } \frac{v_2}{v_1} = \left(\frac{p_1}{p_2}\right)^{\frac{1}{\gamma}}$$

$$\therefore \log \frac{v_2}{v_1} = \frac{1}{\gamma} \log \frac{p_1}{p_2} = \frac{1}{1.4} \log \frac{71.6}{22} = 0.368$$

$$\therefore v_2 = 2.338 v_1$$

$$\text{and } \frac{T_2}{T_1} = \left(\frac{22}{71.6}\right)^{\frac{0.4}{1.4}} = \frac{1}{1.404}$$

$$\therefore T_2 = \frac{273 + 40}{1.404} = 223^\circ \text{ C. abs. or } t_2 = -50^\circ \text{ C.}$$

**Example 14.** A mixture of petrol vapour and air at 100° C. is to be compressed adiabatically to a pressure sufficiently high to produce self ignition. Find this pressure if the ignition temperature is about 430° C. Take  $\gamma = 1.4$ .

$$\text{Here } \frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\therefore \frac{p_2}{p_1} = \left(\frac{273 + 430}{273 + 100}\right)^{\frac{1.4}{0.4}} = 9.2$$

For  $p_1 = 14 \text{ lb./in.}^2 \text{ abs.}$  this gives  $p_2 = 14 \times 9.2 = 129 \text{ lb./in.}^2 \text{ abs.}$

In practice, the compression pressures are not so high as this, since it is necessary to avoid the possibility of pre-ignition. Also at certain parts of the cylinder walls the gas temperature may be higher than that calculated above. If the initial temperature is less than 100° C. a correspondingly higher compression ratio may be used. Benzene permits of considerably higher compression ratios.

**Example 15.** In a Diesel engine the air is compressed to a pressure which gives a temperature above the ignition temperature of the oil used. Find the minimum size of clearance volume expressed as a fraction of the total volume behind the piston at the beginning of compression. Also

find the maximum compression pressure. Take the required final temperature as  $850^{\circ}\text{C}$ . and the initial temperature as  $100^{\circ}\text{C}$ . ( $\gamma = 1.4$ ).

$$\text{Since} \quad \frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{\gamma-1}$$

$$\text{we have} \quad \frac{v_1}{v_2} = \left(\frac{273 + 850}{273 + 100}\right)^{\frac{1}{1.4}} = 15.73$$

$$\text{or} \quad v_2 = 0.0636v_1$$

i.e. 6.36 per cent of the total volume,

$$\text{or} \quad 100 \frac{v_2}{v_1 - v_2} = \frac{100}{14.73} = 6.8 \text{ per cent of the stroke volume.}$$

$$\text{Again, since} \quad \frac{p_2}{p_1} = \frac{v_1 T_2}{v_2 T_1} = 15.73 \times 3.01 = 47.4$$

$$\text{we have, with} \quad p_1 = 12.8 \text{ lb./in.}^2 \text{ abs.,}$$

$$p_2 = 12.8 \times 47.4 = 605 \text{ lb./in.}^2 \text{ abs.}$$

### GENERAL $pV$ CURVES. POLYTROPIC CHANGE OF STATE OR CHANGE OF STATE WITH CONSTANT SPECIFIC HEAT

If a gas be compressed and at the same time gives up heat to the cylinder walls, the total absolute work is always converted to heat. From the general energy equation for compression,

$$Q = c_v(T_2 - T_1) - AE \quad (\text{page 99})$$

$$\text{we have} \quad AE = c_v(T_2 - T_1) - Q$$

When heat is removed,  $Q$  is negative, so that

$$AE = c_v(T_2 - T_1) - (-Q) = c_v(T_2 - T_1) + Q$$

or, the sum of the new supply of heat to the gas  $c_v(T_2 - T_1)$  and the heat carried away  $Q$ , is equal to the heat equivalent of the work done on the gas  $AE$ .

The shape of the  $pV$  curve given by different types of compression merely depends on the proportion of compression work remaining as heat in the gas, and the proportion of heat carried away by radiation or conduction. In isothermal compression, the whole of the compression work is transmitted to the cooling water as heat, while in adiabatic compression it remains in the gas. Both pressure-volume curves are thus different.

With cooling the curve lies below the adiabatic, as shown by  $AB_1$  and  $AB_2$ , in Fig. 52. It is only when the heat withdrawn is equal to the absolute work of compression that the compression curve coincides with the isothermal  $AB_i$ .

If the cooling is sufficient, the temperature may drop continuously, in which case the curve  $AB_4$  is obtained.

If, during compression, heat is supplied, the resulting curve is as shown by  $AB_3$ . A curve such as  $AB_5$  is also possible, as, for example, in

the case of a cooled turbo-blower, where the gas is heated internally by friction and turbulence and at the same time gives up heat to the outside.

In expansion, with heat supply and temperature decrease ( $T_2 < T_1$ ), the whole of the heat supply  $Q$ , together with the internal energy of the gas  $c_v(T_1 - T_2)$  is transformed to mechanical work. This follows directly from the energy equation applied to expansion

$$AE = Q + c_v(T_1 - T_2)$$

Here again, the  $pV$  curves depend on the proportion of work due to the external supply of heat, and the proportion due to the internal energy of the gas. When the whole of the work is developed at the expense of the internal energy, the resulting  $pV$  curve is an adiabatic, while, if the work is obtained from the external heat supply alone, the resulting curve is an isothermal  $AB_i$  (Fig. 53). For a heat supply less

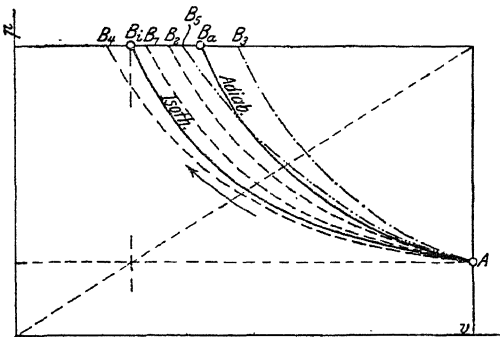


FIG. 52

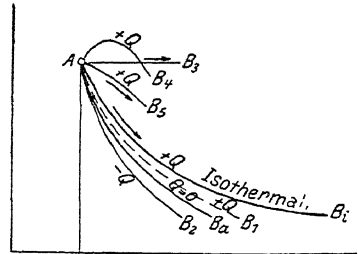


FIG. 53

than that necessary for isothermal expansion, the  $pV$  curve  $AB_1$  lies between the isothermal and the adiabatic.

If sufficient heat be supplied to cause the temperature to rise during expansion, as occurs, for example, in the case of the Diesel engine with expansion at constant pressure  $AB_3$ , or with after burning in the gas engine  $AB_4$  or  $AB_5$ , the  $pV$  line lies above the isothermal  $AB_i$ .

Writing the energy equation in the form

$$AE = Q - c_v(T_2 - T_1)$$

shows that, in addition to performing the work  $AE$ , the heat  $Q$  is used to increase the internal energy of the gas by the amount  $c_v(T_2 - T_1)$ .

If, finally, heat be extracted during expansion, the pressure line  $AB_2$  falls below the adiabatic.

The curves are considerably simplified if, during any change of state, it be assumed that the proportion  $\psi Q$  of the heat supplied, which serves to increase the internal energy of the gas, remains constant throughout the change. The remainder  $(1 - \psi)Q$  is converted to work.

This, then, gives

$$\psi Q = c_v(T_2 - T_1)$$

or

$$Q = \frac{c_v}{\psi} (T_2 - T_1) = c(T_2 - T_1) \quad . \quad . \quad . \quad (137)$$

The constant value  $c = \frac{c_v}{\psi}$  can be called a "specific heat," since it is the heat necessary to raise the temperature by  $1^\circ \text{C}$ . In the special case considered here it is regarded as constant, but for the general case it is variable and depends on the temperature level, since the specific heat varies with temperature.

The shape of the  $pV$  curve is easily determined by combining the energy equation with the characteristic equation.

$$\text{Thus, since} \quad Q = c_v(T_2 - T_1) + AE$$

$$\begin{aligned} \text{we have} \quad AE &= \frac{c_v}{\psi} (T_2 - T_1) - c_v(T_2 - T_1) \\ &= (c - c_v) (T_2 - T_1) \quad . \quad . \quad . \quad . \quad (138) \end{aligned}$$

$$\text{But} \quad T_1 = \frac{n v_1}{R} \quad . \quad . \quad . \quad . \quad \frac{p_2 v_2}{R}$$

$$\text{hence} \quad E =$$

$$\begin{aligned} \text{or, with} \quad AR &= c_p - c_v \\ & \quad . \quad . \quad . \quad . \quad (139) \end{aligned}$$

For this case, therefore, the  $pV$  curve has to satisfy the condition that the work  $E$ , between two ordinates  $p_1$  and  $p_2$ , must equal the difference of the rectangles  $p_1 v_1$  and  $p_2 v_2$ , multiplied by  $\frac{c - c_v}{c_p - c_v}$ . Hence it is a curve of the same type as the adiabatic, i.e. a hyperbola following law

where  $n$  takes the place of  $\gamma$ .

As shown on page 106, the area below a curve of this type is

$$\begin{aligned} \text{or, with} \quad p_1 v_1 &= RT_1 \text{ and } p_2 v_2 = RT_2 \\ E &= \frac{RT_1}{n-1} \left( 1 - \frac{T_2}{T_1} \right) \quad . \end{aligned}$$

In finding  $n$ , we can thus use the relation

$$n-1 = \frac{c_p - c_v}{c_v}$$

$$\text{from which} \quad n = \frac{c_p - c}{c_v - c} \quad . \quad . \quad . \quad . \quad (143)$$

$$\text{and} \quad c = \frac{nc_v - c_p}{n-1} = c_v \left( \frac{n-\gamma}{n-1} \right) \quad . \quad . \quad . \quad . \quad (144)$$



The temperature ratio is found from

$$\frac{T_2}{T_1} = \frac{p_2 v_2}{p_1 v_1}$$

or with

$$p_1 v_1^n = p_2 v_2^n$$

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} = \left(\frac{v_1}{v_2}\right)^{n-1} \quad (145)$$

and the temperature rise is thus

$$T_2 - T_1 = T_1 \left[ \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1 \right] \quad (146)$$

The expression for the work done, as given by equation (141) is

$$E = \frac{p_1 v_1}{n-1} \left[ 1 - \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} \right] = \frac{p_1 v_1}{n-1} \left[ 1 - \left(\frac{v_1}{v_2}\right)^{n-1} \right] \quad (147)$$

The ratio of the heat supplied to the absolute work is, from equations (137) and (138),

$$\frac{Q}{EA} = \frac{c}{c - c_v} = \frac{\gamma - n}{\gamma - 1} \quad (148)$$

**Special cases of polytropic changes.** The previously considered changes of state can be regarded as special cases of polytropic expansion or compression.

Thus, from the equation

$$pv^n = \text{constant}$$

we have,

with	$n = 1$	$pv = \text{const. (isothermal)}$	$c = \pm \infty$
	$n = \gamma$	$pv^\gamma = \text{const. (adiabatic)}$	$c = 0$
	$n = 0$	$p = \text{const. (const. pressure)}$	$c = c_p$
	$n = \pm \infty$	$v = \text{const. (const. volume)}$	$c = c_v$

Figs. (54) and (55) show these cases along with a number of intermediate cases.

In internal combustion engines, air pumps, compressors, etc., the changes of state are neither adiabatic nor isothermal, but in many cases these changes can be represented by a polytropic curve, with sufficient accuracy for tentative calculations.

The value of the index  $n$  is conveniently found by the following logarithmic graphical method.

From  $pv^n = C$

we have  $\log p + n \log v = \log C$

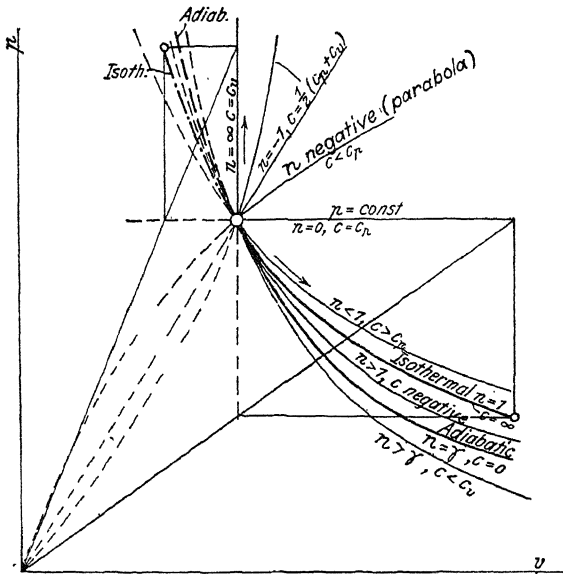
Hence, if the values of  $\log p$  are plotted to a base of  $\log v$  values, the points obtained will lie on a straight line, provided no change occurs in the value of  $n$ . The equation can be written

$$y + nx = C_1$$

so that with  $x = 0, y_0 = C_1$  and with  $y = 0, x_0 = \frac{C_1}{n}$

$$C_1 = y_0 = nx_0 \text{ giving } n = y_0/x_0$$

The points, shown in Fig. 56, were obtained from an expansion line taken from a gas engine indicator diagram (Fig. 58). They do not lie



FIGS. 54 AND 55

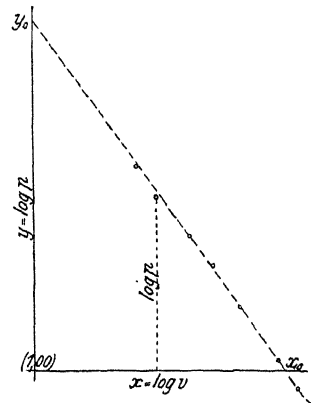


FIG. 56

exactly on a straight line, so that the expansion curve is not a true polytropic. The mean straight line through the points is shown dotted, and gives a mean exponent value of  $n = y_0/x_0 = 1.32$ . Any scale may be used in measuring the points on the  $pV$  field; for example,  $p$  and  $v$  may be measured in inches and used thus in  $\log p$  and  $\log v$ .

### ADIABATIC CHANGES OF STATE WITH LARGE DIFFERENCES IN TEMPERATURE, PRESSURE, AND VOLUME

The adiabatic change of state was considered on page 106 on the assumption that the ratio  $\frac{c_p}{c_v} = \gamma$  and the specific heats  $c_p$  and  $c_v$  were constant. This assumption is justified up to a few hundred degrees centigrade only.

If, however, the temperature changes are of the order of  $1000^\circ \text{C}$ . or more (as in gas or oil engines), the formulae already given are inexact, since  $\gamma$  decreases as  $T$  increases, as shown by the relation

$$\gamma = \gamma_0 - \alpha T$$

For diatomic gases and gas mixtures between 500° and 2000° C. we have, from page 53,

$$\gamma_0 = 1.41 \text{ and } \alpha = \frac{0.5}{10000}$$

and for pure products of combustion (i.e. no excess air)

$$\gamma_0 = 1.36 \text{ and } \alpha = \frac{0.55}{10000}$$

When the changes of state occur at high temperatures with only a few hundred degrees centigrade change in temperature, the value of  $\gamma$  may be regarded as constant, but its value is considerably lower than that at moderate temperatures (see page 60). Thus, with products at

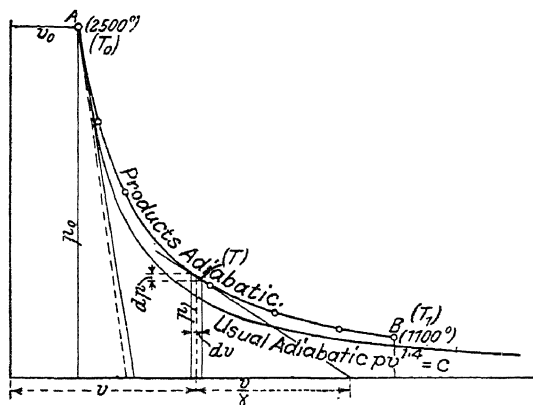


FIG. 57

2000° C., we have  $\gamma = 1.24$  (in place of 1.4) and the adiabatic curve is less steep at this high temperature.

The effect of the variation in the value of  $\gamma$  on the  $pV$  adiabatic curve is shown by the following construction.

For an elementary adiabatic change of state we have (page 107)

$$dp/dv = -\gamma \frac{p}{v}$$

It does not matter here whether  $\gamma$  is variable or constant, since the equation refers to an instantaneous state corresponding to the temperature.

The equation gives the tangent construction explained on page 107. Let  $AB$  (Fig. 57) be an adiabat for a constant value of  $\gamma = 1.4$ . The adiabat for products deviates from this. Thus, at  $A$ , where  $t = 2500^\circ \text{C.}$ ,  $\gamma = 1.24$ , and at  $B$ , where  $t = 1100^\circ \text{C.}$ ,  $\gamma = 1.30$ . The adiabatics for products, at  $A$  and  $B$ , as shown by the tangent construction, are not so steep as with  $\gamma = 1.4$ , so that the whole of the adiabat for products lies above the original curve ( $pV^{1.4} = C$ ) and cannot be drawn accurately from it, owing to the variation in the value of  $\gamma$ .

For the same reason, the equation

$$\frac{T_0}{T} = \left(\frac{v}{v_0}\right)^{\gamma-1}$$

does not give the change in temperature corresponding to a change in volume. In this case, however, the following simple method applies.

From the energy equation

$$dQ = c_v dT + A p dv$$

we have, with  $dQ = 0$

$$\frac{dT}{dv} = -\frac{Ap}{c_v}$$

Eliminating  $p$  by means of  $p = \frac{RT}{v}$

gives  $\frac{dT}{dv} = -\frac{ART}{c_v v}$

and since  $\frac{AR}{c_v} = \gamma - 1$  (page 100)

$$\frac{dT}{dv} = -(\gamma - 1) \frac{T}{v}$$

or, since  $\gamma = \gamma_0 - \alpha T$

$$\frac{dT}{dv} = -(\gamma_0 - 1 - \alpha T) \frac{T}{v}$$

giving  $\frac{dT}{T(\gamma_0 - 1 - \alpha T)} = -\frac{dv}{v}$

Now,  $\frac{1}{T(\gamma_0 - 1 - \alpha T)} = \frac{1}{\gamma_0 - 1} \left( \frac{1}{T} + \frac{\alpha}{\gamma_0 - 1 - \alpha T} \right)$

so that  $\frac{dT}{T} + \frac{\alpha dT}{\gamma_0 - 1 - \alpha T} = -(\gamma_0 - 1) \frac{dv}{v}$

Integrating between the limits  $v_0, T_0$ , and  $v, T$ , gives

$$\log_e \frac{T}{T_0} - \log_e \frac{\gamma_0 - 1 - \alpha T}{\gamma_0 - 1 - \alpha T_0} = -(\gamma_0 - 1) \log_e \frac{v}{v_0}$$

Replacing by common logs and grouping gives

$$\log \left\{ \frac{T}{T_0} \times \frac{(\gamma_0 - 1 - \alpha T_0)}{(\gamma_0 - 1 - \alpha T)} \right\} = (\gamma_0 - 1) \log \frac{v_0}{v}$$

or

$$\frac{T}{T_0} \frac{(\gamma_0 - 1 - \alpha T_0)}{(\gamma_0 - 1 - \alpha T)} = \left(\frac{v_0}{v}\right)^{\gamma_0 - 1}$$

so that 
$$\frac{v_0}{v} = \left\{ \frac{T}{T_0} \frac{(\gamma_0 - 1 - \alpha T_0)}{(\gamma_0 - 1 - \alpha T)} \right\}^{\frac{1}{\gamma_0 - 1}} \quad . \quad . \quad . \quad (149)$$

Conversely, 
$$\frac{T_0}{T} = \left( 1 - \frac{\alpha T_0}{\gamma_0 - 1} \right) \left( \frac{v}{v_0} \right)^{\gamma_0 - 1} + \frac{\alpha T_0}{\gamma_0 - 1} \quad . \quad . \quad (150)$$

Finally, since  $\frac{p_0}{p} = \frac{v T_0}{v_0 T}$

the equation of the  $pv$  curve is

$$\frac{p_0}{p} = \left( 1 - \frac{\alpha T_0}{\gamma_0 - 1} \right) \left( \frac{v}{v_0} \right)^{\gamma_0} + \left( \frac{\alpha T_0}{\gamma_0 - 1} \right) \frac{v}{v_0} \quad . \quad . \quad (151)$$

The absolute gas work follows from the general energy equation for gases (page 121).

$$Q = (C_v)_0'(T - T_0) + AE$$

where  $(C_v)_0'$  is the mean specific heat between  $T_0$  and  $T$ .

With  $Q = 0$  the equation is

$$E = 1400 (C_v)_0'(T_0 - T) \quad . \quad . \quad . \quad (152)$$

or

$$E = 1400 (Q_v)_0'$$

where  $(Q_v)_0'$  is the amount of heat necessary to raise the temperature of the gas from  $T_0$  to  $T$ , at constant volume. Its value can be read from Chart I when  $t$  is calculated.

The simpler equations obtained previously for  $\frac{v}{v_0}$ ,  $\frac{p}{p_0}$ ,  $\frac{T}{T_0}$ , and  $E$  are found directly from those given above, if  $\alpha = 0$ , i.e.  $\gamma = \text{constant}$  and  $(C_v)_0' = c_v = \text{constant}$ . We then have

$$\frac{T_0}{T} = \left( \frac{v}{v_0} \right)^{\gamma - 1}, \quad \frac{p_0}{p} = \left( \frac{v}{v_0} \right)^{\gamma}, \quad \text{and} \quad E = 1400 c_v (T_0 - T)$$

The above equations are used in finding the ideal pressure volume curves, the temperature changes, and absolute work in the case of adiabatic expansion of hot products in a gas or oil engine. From the above it is seen that the expansion line is not likely to coincide with that given by the usual law  $pv^{1.4} = C$ . On account of the high temperatures the exponent will be considerably less than 1.4 and must vary during expansion.

**Example 16.** Air at 100° C. and 12.8 lb./in.<sup>2</sup> abs. is compressed adiabatically in a Diesel engine cylinder, which has a compression ratio of 15.73. Find the values of the pressure and temperature at the end of compression when the variation of the specific heats with temperature is taken account of. Also find the absolute work of compression and its heat equivalent.

From 
$$\frac{T_0}{T_1} = \left( 1 - \frac{\alpha T_0}{\gamma_0 - 1} \right) \left( \frac{v_1}{v_0} \right)^{\gamma_0 - 1} + \frac{\alpha T_0}{\gamma_0 - 1}$$

we have, with  $\gamma_0 = 1.41$  and  $\alpha = \frac{0.5}{10000}$

$$\frac{T_0}{T_1} = \frac{0.9545}{(15.73)^{0.41}} + 0.0455 = 0.354$$

so that  $T_1 = \frac{100 + 273}{0.354} = 1051^\circ \text{C. abs. or } t_1 = 778^\circ \text{C.}$

The compression pressure is

$$p_1 = \frac{p_0 v_0 T_1}{v_1 T_0} = \frac{12.8 \times 15.73}{0.354} = 568 \text{ lb./in.}^2 \text{ abs.}$$

From Chart I the compression work, in heat units, between  $t_0 = 100^\circ \text{C.}$  and  $t_1 = 778^\circ \text{C.}$  is  $(Q_v)_0' = 10.5 \text{ C.H.U./ft.}^3 \text{ at S.T.P., or}$

$$10.5 \times 1400 = 14,700 \text{ ft. lb./ft.}^3 \text{ at S.T.P.}$$

In Example 15, the values, found on the assumption of constant specific heat, were  $t_1 = 850^\circ \text{C.}$  (i.e.  $72^\circ \text{C.}$  higher) and  $p_1 = 605 \text{ lb./in.}^2 \text{ abs.}$  (i.e.  $37 \text{ lb./in.}^2$  higher).

The effect of the variation in specific heat on the temperature and pressure is thus appreciable in this case. Fig. 178 shows the difference between the two  $pv$  curves.

**Example 17.** The effect of the variation in  $c_v$  during compression in gas engines, with normal compression ratios (up to about 8), is slight. The diagram of a gas engine having a compression ratio of 6.84 is shown in Fig. 58. If the initial temperature is assumed to be  $98^\circ \text{C.}$ , the temperature ratio with adiabatic compression is 1/0.4716, which gives  $T_1 = 770^\circ \text{C. abs.}$ , or  $t_1 = 497^\circ \text{C.}$ , and the pressure ratio is 14.5/1. On the assumption of constant specific heat and  $\gamma = 1.4$ , the values are  $t_1 = 509^\circ \text{C.}$  and a pressure ratio of 14.72/1. As shown by the diagram, the final pressures practically coincide.

The calculated compression line, which starts at the point  $a$  and is shown dotted, lies slightly below the diagram line. The adiabatic line with  $\gamma = \text{constant} = 1.4$  coincides almost exactly with the latter, which shows that the cooling effect is negligible. It appears that heat, contained previously in the walls, is transmitted to the gas during compression.

**Example 18.** In the case of the gas engine of Example 17 (Fig. 58) the gases increase in volume from  $A$  to  $B$  in the ratio 3.5:1. Find the ratios of the adiabatic pressure and temperature drops between  $A$  and  $B$ . Show the adiabatic expansion curve in Fig. 58.

The temperature  $T_0$  at  $A$  is calculable from the characteristic equation and the temperature  $T'$  at the beginning of compression. This gives

$$T_0 = \frac{pv}{p'v'}$$

The value of  $\frac{pv}{p'v'}$  is found from the diagram and equals 5.4, so that  $T_0 = 1750^\circ \text{C. abs.}$  (for  $t' = 50^\circ \text{C.}$ ).

Equation (150) gives, for the temperature ratio between  $A$  and  $B$ ,

$$\frac{T_0}{T} = \left(1 - \frac{\alpha T_0}{\gamma_0 - 1}\right) 3.5^{\gamma_0 - 1} + \frac{\alpha T_0}{\gamma_0 - 1}$$

As shown on page 60, the following values apply to the products of combustion of illuminating gas,

$$\gamma_0 = 1.36 \text{ and } \alpha_0 = \frac{0.55}{10000}$$

A more accurate estimate of  $T_0$  (page 8) is then given by

$$T_0 = T' \frac{pv}{p'v'} \frac{R_0}{R_t} \text{ and taking } \frac{R_0}{R_t} = \frac{1}{\alpha} = \frac{1}{0.95}$$

gives 
$$T_0 = \frac{1750 \times 1}{0.95} = 1840^\circ \text{ C. abs. or } t_0 = 1567^\circ \text{ C.}$$

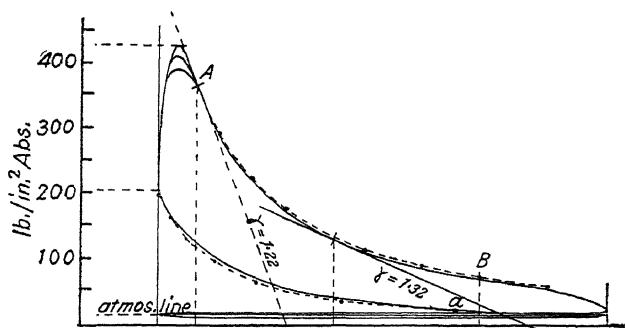


FIG. 58

This now gives

$$\frac{\alpha T_0}{\gamma_0 - 1} = 0.281$$

and

$$\frac{T_0}{T} = (1 - 0.281) 3.5^{0.36} + 0.281 = 1.409$$

Hence,

$$\frac{p}{p_0} = \frac{T v_0}{T_0 v} = \frac{1}{1.409 \times 3.5} = \frac{1}{4.93}$$

As given by the diagram, the pressure ratio between  $B$  and  $A$  is  $\frac{1}{5.1}$ .

The final adiabatic temperature at  $B$  is  $T = 1306^\circ \text{ C. abs.}$ , or  $t = 1033^\circ \text{ C.}$

The adiabatic points have been located in the diagram in the same way. The line thus obtained lies very close to, and above, the diagram line (the highest expansion line in the group of expansion lines shown on the original card). From this it is obvious that the cylinder cooling has only a slight influence on the course of the expansion curve.

If the adiabatic were to follow the previously assumed law  $pv^{1.4} = C$ , the line of the indicator diagram corresponding to the exponent 1.2 to 1.3 would show a large heat flow to the gases, which could only be

accounted for by after-burning throughout the entire expansion. It is now known, however, from the latest specific heat values, that the value of  $\gamma$  for the products is not 1.4, so that this conclusion regarding the heat flow is invalid. After-combustion occurs chiefly in the neighbourhood

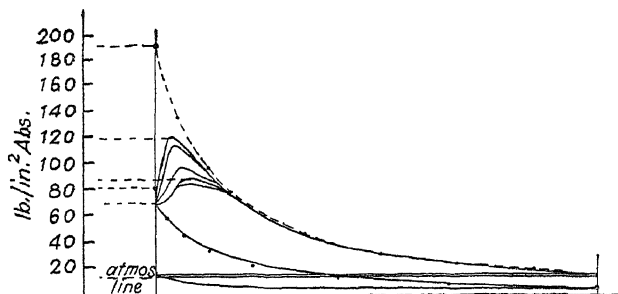


FIG. 59

of  $A$ , where the actual exponent (1.22) is slightly smaller than the adiabatic (1.26). No after-burning occurs beyond this, since the true adiabatic lies above the diagram line. The effect of cooling is thus apparent. The no-load card, shown in Fig. 59, is taken from the same engine and shows similar conditions, although the after-combustion is somewhat prolonged.

The indicator card of a 40 h.p. Diesel engine is shown in Fig. 60. The exponents, found graphically by the tangent method, are about 1.45

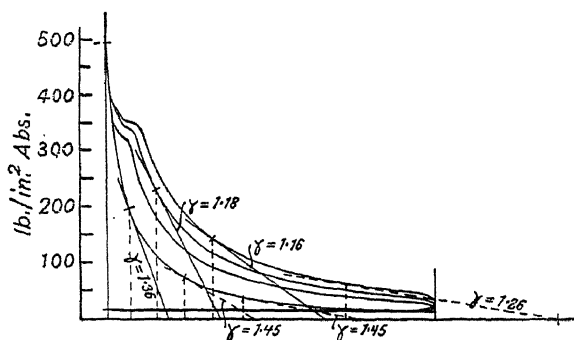


FIG. 60

in the earlier part of the compression curve and, due to the increased temperature (and to cooling), drop to about 1.36 near the end. The exponents for the expansion curve are considerably smaller for all loads, amounting to about 1.26 at the lower part of the curve and varying from 1.16 to 1.18 at the upper part, where the temperatures are in the neighbourhood of 2000° C. These latter values are not quite the same as the adiabatic, due to the cooling effect and incomplete combustion, as well as partial dissociation and continuous re-combining of carbon dioxide during expansion. (See page 345 regarding the abnormal form of the combustion line.)



Münzinger (*Dissert.*, Berlin, 1913), in testing a small (15 h.p.) Diesel engine, found that, during expansion, the exponents continuously increased during expansion at all loads. The values obtained at full load increased from about 1.1 to 1.4, while for light loads they increased from about 1.35 to 1.48. The mean compression value obtained was 1.44.

Regarding tests carried out on nine different Diesel engines, varying from 30 to 150 h.p. per cylinder, see an article in *Z.V.d.I.* (1916), page 276, entitled "Über den Verlauf d. Verbrennung im Dieselmotor," by Weisshaar.

### HEAT DIAGRAM AND ENTROPY OF GASES

The quantities of heat which are given to, or taken from, a gas during any change of state can be represented by an area in the same way as work is represented on the pressure-volume field. To do this, a new magnitude, called entropy, has to be introduced.

The amount of heat contained in 1 lb. of a substance is determined from its temperature and specific capacity (i.e. specific heat) for heat. The determining quantities of the heat, so far as substances are concerned, are thus the temperature and the specific heat. For any small change of state of 1 lb. of a gas the heat given up (or received) is

$$dQ = c dT$$

but, in this,  $c$  is a magnitude which depends both on the kind of gas and the nature of the change of state. The expression becomes perfectly general, however, if in the energy equation  $dQ$  is replaced by  $cdT$ . thus

$$dQ = c_v dT + A p dv$$

so that

From the characteristic equation, we have

$$p dv + v dp = R dT$$

so that

$$c = c_v + AR \cdot \frac{1}{p} \frac{dp}{dv}$$

In this expression,  $c_v$  depends on the particular gas and  $dp/dv$  on the particular shape of the  $pv$  curve (direction of the tangent), so that  $c$  depends on both.

The values of  $c$  could now be plotted to a base of  $T$ , in which case  $dQ = cdT$  would be the area between two adjacent ordinates, and the total heat  $Q$  would be the complete area between the ordinates at the considered temperature limits, exactly as in the case of mechanical work.

This method, however, would only be suitable for changes of state in which the specific heats were constant and finite (i.e.  $c_p = \text{constant}$ ,  $c_v = \text{constant}$ , and  $c = \text{constant}$ ), and would not be applicable to isothermal or nearly isothermal cases. It would be cumbersome and impracticable in a general case and has, therefore, no application to the graphical representation of heat quantities, except perhaps in the determination of  $c_p$  and  $c_v$  from test results.



Replacing  $T$ , in the second term on the right, by  $pv/R$  gives

$$d\phi = c_v \frac{dT}{T} + AR \frac{dv}{v} \quad . \quad . \quad . \quad . \quad . \quad (154)$$

A third form is found from

$$\frac{dv}{v} + \frac{dp}{p} = \frac{dT}{T} \quad (\text{equation 125})$$

and  $c_p - c_v = AR$  (page 94)

These give 
$$d\phi = c_p \frac{dv}{v} + c_v \frac{dp}{p} \quad . \quad . \quad . \quad . \quad . \quad (155)$$

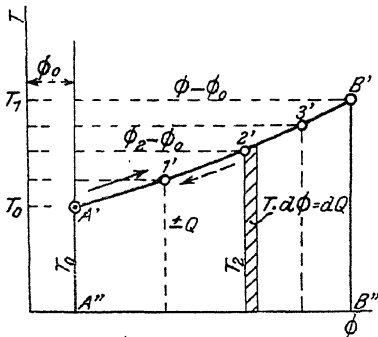


FIG. 61

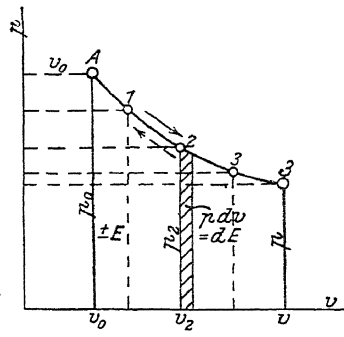


FIG. 62

For any finite change of state between the initial condition  $T_0, v_0$ , and the final condition  $T, v$ , the total change in  $\phi$  is found by integrating equation (154), which gives

$$\phi - \phi_0 = c_v \log_e \frac{T}{T_0} + AR \log_e \frac{v}{v_0} \quad . \quad . \quad . \quad . \quad . \quad (156)$$

Hence, if a gas changes from the state  $T_0, v_0, p_0$ , to the state  $T, v, p$ , in any way, of which there are an infinite number with widely varying values of the addition or extraction of heat, the magnitude  $\phi$  has always the same value, as shown by this equation. The change in the value of  $\phi$  is thus a characteristic of the difference between the initial and final states, which may be denoted by points  $A$  and  $B$ .  $\phi - \phi_0$  is thus entirely independent of the path traversed between  $A$  and  $B$ , so that it is clearly different from, say, the quantity of heat  $Q$  and the work  $E$ , since these quantities depend on the intermediate states of the gas between  $A$  and  $B$ . The quantity  $\phi$  was given the name "entropy" by Clausius, on account of the part played by this quantity when heat is converted to work.

If the absolute temperatures  $T$  are plotted to a base of entropy values  $\phi$ , the resulting diagram is called a "heat diagram" ( $T\phi$  diagram) or "entropy diagram" (Fig. 61). In this diagram the element of heat  $dQ$ , shown shaded, is equal to  $Td\phi$ , and represents the heat absorbed or rejected by the gas during an elementary change of state. The heat supply  $Q$ , during a finite change of state, is thus represented by an area  $A'B'B''A''$  between the initial and final ordinates on the entropy diagram.

When a pressure-volume curve such as  $AB$  (Fig. 62) is given, the entropy equation (156) enables the change of entropy between  $A$  and  $B$ , or between any intermediate points, to be calculated. These values are then plotted in Fig. 61 as abscissae with  $T$  as ordinates, the  $T$  values being also obtained from  $AB$ . The resulting curve  $A'B'$  is then said to be "transferred" from the  $pv$  field to the  $T\phi$  field. The heat supplied between  $A$  and  $B$  is then given by the area  $A'B'B''A''$ . Figs. 61 and 62 are drawn to scale, so that  $AB$  represents a change of state accompanied by a large addition of heat. In this diagram it should be noticed that the abscissa axis passes through the absolute zero temperature point.

The expression given above for the change of entropy can be written, when common logarithms are used, as

$$\phi - \phi_0 = 2.303 \left( c_v \log \frac{T}{T_0} + AR \log \frac{v}{v_0} \right) \quad . \quad . \quad (157)$$

and by a simple transformation, using the characteristic equation and the relations between  $\gamma$ ,  $c_p$ , and  $c_v$ , we have also

$$\phi - \phi_0 = 2.303 \left( c_p \log \frac{T}{T_0} - AR \log \frac{p}{p_0} \right) \quad . \quad . \quad (158)$$

or 
$$\phi - \phi_0 = 2.303 \left( c_p \log \frac{v}{v_0} + c_v \log \frac{p}{p_0} \right) \quad . \quad . \quad (159)$$

Any one of these equations can be applied, accordingly as the  $T$  and  $v$ ,  $T$  and  $p$ , or  $p$  and  $v$  values are known.

Equations (153), (154), and (155) are applicable when the specific heats vary with temperature, but other equations have then to be used in place of equations (157), (158), and (159), depending on the variation of the specific heats with temperature. If, for example, we take the diatomic specific heat equation

$$c_v = c_{v_0} + bt$$

or 
$$c_v = (c_{v_0} - 273b) + bT = a + bT$$

equation (154) becomes

$$d\phi = a \frac{dT}{T} + b dT + AR \frac{dv}{v}$$

and, on integrating,

$$\frac{T}{T_0} + b(T - T_0) + AR \log_e \frac{v}{v_0} \quad . \quad (160)$$

Applying this to 1 lb. of air for which  $c_{v_0} = 0.1806$ ,  $b = 0.000019$ , and  $a = 0.1754$ , gives, for example,

$$\begin{aligned} \phi - \phi_0 = & 0.388 \log \frac{T}{T_0} + 0.000019(T - T_0) \\ & + 0.158 \log \frac{v}{v_0} \quad . \quad . \quad (161) \end{aligned}$$

The validity of the conception of entropy as outlined above extends only to gases which follow the characteristic equation  $pv = RT$ , but the

specific heats may be constant or variable. In the case of these gases, therefore, there exists a magnitude called entropy  $\phi$ , the change of which  $d\phi$  (during an elementary change in state), multiplied by the instantaneous temperature, gives the heat  $dQ$  given to or taken from the gas, or

$$dQ = Td\phi$$

and, for a finite change of state of any type,

$$Q = \int_{T_0}^T Td\phi$$

From this, when the change is isothermal ( $T = \text{constant}$ ),

otherwise, for any finite change of state,

$$\phi - \phi_0 = \int_{T_0}^T \frac{dQ}{T}$$

In an adiabatic change of state,  $dQ = 0$ , so that

$$d\phi = 0 \text{ and } \phi - \phi_0 = 0$$

This change of state, therefore, occurs with constant entropy (i.e. isentropic). Entropy might, therefore, be defined as a magnitude which remains constant during an adiabatic (reversible) change of state.

This conception of entropy and the temperature-entropy diagram explained below can be applied to the processes in compressors, air motors, and internal combustion engines, provided the characteristic equation  $p v = RT$  applies to the working substances in these machines.

The application of the conception of entropy to substances such as liquids, solids, and vapours, which either fail to follow the characteristic equation or follow it only approximately, is somewhat different from the above and involves the application of new principles. It is not possible to establish an expression for the entropy function from the general characteristic equations for these substances, since the equations are known only approximately and within limited regions. Actually, of course, the conception of entropy is applicable to substances of any kind and in any state, in the same way as is the energy conception. But, in order to do this, we require to be familiar with a new universal property of heat, the expression of which is the second law of thermodynamics (page 154). In the case of gases this law is already embodied in the gas law  $p v = RT$ , although this, at first sight, is not apparent.

## PRINCIPAL STATE CHANGES ON THE $T\phi$ DIAGRAM

1. **Isothermal change of state.** ( $T = \text{const.}$ ) If the temperatures, i.e. the ordinates of the entropy line, are constant, the change of state is represented by an horizontal straight line  $A'B'$  (Fig. 63).

The amount of heat added during expansion is equal to the rectangle below  $A'B'$ , or

In the case of compression the same quantity of heat

has to be withdrawn.

The change of entropy between the two states  $A$  and  $B$  is, from equation (156), with  $\log_e \frac{T}{T_0} = 0$ ,

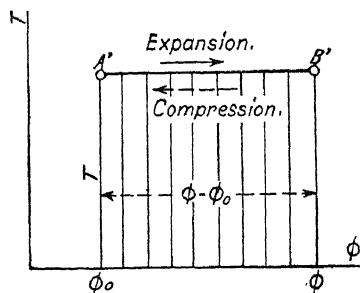


FIG. 63

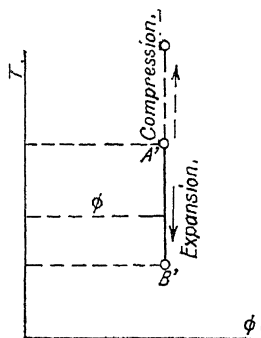


FIG. 64

or, with

$$\phi - \phi_0 = AR \log_e \frac{v}{v_0}$$

$$\frac{v}{v_0} = \frac{p_0}{p}$$

$$\phi - \phi_0 = AR \log_e \frac{p_0}{p}$$

This gives

$$Q = ART \log_e \frac{p_0}{p}$$

or, with

$$pv = p_0 v_0 = RT$$

$$Q = Ap_0 v_0 \log_e \frac{p_0}{p}$$

as already given on pages 104 and 105.

## 2. Adiabatic change of state. (Fig. 64.)

In this case  $dQ = 0$ , and since  $dQ = Td\phi$ ,  $d\phi$  is also  $= 0$  and  $\phi - \phi_0 = 0$ , or the entropy remains constant and the change is therefore said to be isentropic.

This is represented on the  $T\phi$  diagram by a vertical straight line, which corresponds to the adiabatic  $pV$  curve in the work diagram. It serves to indicate the increase or decrease in the temperature, due to compression or expansion

respectively. From the entropy equation for a gas

$$d\phi = c_v \frac{dT}{T} + AR \frac{dv}{v}$$

we have, with  $d\phi = 0$ ,

$$dT = - \frac{ART}{c_v} \frac{dv}{v}$$

so that if  $dv$  is positive (expansion)  $dT$  is negative, and cooling occurs, whereas if  $dv$  is negative,  $dT$  is positive and heating occurs. This result was obtained in a different way on page 106.

If the specific heats are constant the law of expansion of the  $pv$  curve follows from

$$\phi - \phi_0 = 2.303 \left( c_v \log \frac{p}{p_0} + c_p \log \frac{v}{v_0} \right)$$

by putting  $\phi - \phi_0 = 0$ , which gives

$$\log \frac{p}{p_0} = - \frac{c_p}{c_v} \log \frac{v}{v_0}$$

or, since  $\frac{p}{c_v} = \gamma$

$$\frac{p}{p_0} = \left(\frac{v_0}{v}\right)^\gamma$$

i.e.  $pv^\gamma = \text{constant}$  (see page 107)

In the same way, from the two other expressions for entropy, we obtain the relationships already deduced on page 108

$$\frac{T}{T_0} = \left(\frac{p}{p_0}\right)^{\frac{\gamma-1}{\gamma}} \quad \text{and} \quad \frac{T}{T_0} = \left(\frac{v_0}{v}\right)^{\gamma-1}$$

**3. Change of state at constant pressure.** ( $p = \text{const.}$ ) The general equation

$$\phi - \phi_0 = 2.303 c_v \left( \log \frac{p}{p_0} + \gamma \log \frac{v}{v_0} \right)$$

becomes with  $p = p_0$  or  $\log \frac{p}{p_0} = 0$ ,  $z.$

$$\phi - \phi_0 = 2.303 c_v \log \frac{v}{v_0}$$

or, with  $T$

$$\phi - \phi_0 = 2.303 c_p \log \frac{T}{T_0}$$

Hence in the  $T\phi$  field this change of state is represented by a logarithmic curve  $A'B'$  (Fig. 65). The area below  $A'B'$  represents the heat added during the change of state,

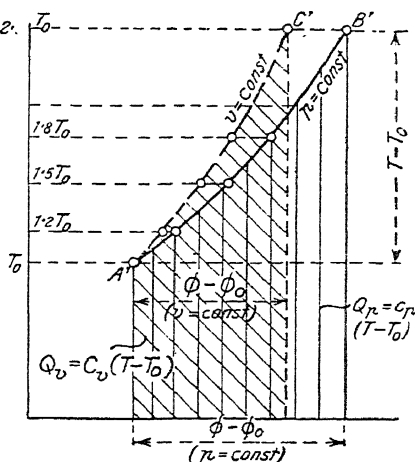


FIG. 65

**4. Change of state at constant volume.** ( $v = \text{const.}$ ) In the same way as in case 3, we have, with  $\log \frac{v}{v_0} = 0$ ,

$$\phi - \phi_0 = 2.303 c_v \log \frac{p}{p_0}$$

or, with  $\frac{p}{p_0} = \frac{T}{T_0}$

$$\phi - \phi_0 = 2.303 c_v \log \frac{T}{T_0}$$

This change of state is again represented by a logarithmic curve  $A'C'$  on the  $T\phi$  chart. The abscissae of the curve  $v = \text{constant}$  are

smaller than those of the curve  $p = \text{constant}$  in the ratio  $\frac{c_v}{c_p} = \frac{1}{\gamma}$  at the same temperature levels  $T$ , when both curves start from the same initial point  $A$ . The curve  $v = \text{constant}$  is thus steeper than the curve  $p = \text{constant}$ .

The heat added  $Q = c_v(T - T_0)$  is represented by the shaded area below  $A'C'$ .

**5. Polytropic change of state.** In this change of state, which also includes the four special cases already considered, the quantities of heat  $Q$  given to, or taken from, the gas are proportional to the temperature changes, or, what amounts to the same thing, they occur with constant specific heat  $c$ . As shown on page 113,

$$Q = c(T_2 - T_1)$$

or, for an elementary change of state,

$$dQ = cdT$$

so that the change in entropy is

$$d\phi = \frac{dQ}{T} = \frac{cdT}{T}$$

From this, as in cases 3 and 4 above, we have

$$\phi - \phi_0 = c \log_e \frac{T}{T_0} = 2.303 \, c \log \frac{T}{T_0}$$

The value of  $c$  is given by (page 114)

$$c = c_v \frac{n - \gamma}{n - 1}$$

where  $n$  is the exponent of the polytropic  $pv$  curve.

The representation of any polytropic change of state which follows the law

$$pv^n = \text{constant}$$

is accordingly a logarithmic line on the  $T\phi$  diagram, like those of constant volume or constant pressure.

The entropy curves, drawn through a definite point, for any polytropic changes are distinguished from one another by the subtangent measured on the entropy scale. This is a maximum  $c = \pm \infty$  for the isothermal change ( $n = 1$ ), so that the logarithmic curve becomes an horizontal straight line. The least value is  $c = 0$  with  $n = \gamma$  (i.e. an adiabatic change), and the logarithmic curve is then a perpendicular line. For the other cases the changes of state are represented by curves having slopes, which depend on the proximity to the isothermal or adiabatic changes of state.

For relatively small changes, to which the law  $pv^n = C$  applies, with values of  $n$  between 1 and 1.4, the logarithmic lines are inclined and almost straight. Their direction is that of the tangent to the curve at the initial point, and the inclination of this tangent to the  $\phi$  axis is equal to  $dT/d\phi$ .



Since  $\frac{d\phi}{dT} = \frac{c}{T}$ , it follows that the projection of the tangent to the  $\phi$  curve on the  $\phi$  axis is equal to the specific heat  $c$ , since the small triangle, with the sides  $dT$  and  $d\phi$ , is similar to the large triangle having the sides  $T$  and the subtangent (Fig. 66). This relationship holds not only for polytropic but also for any other change of state. It is only in the case of the polytropic, however, that the value of the subtangent ( $c$ ) is constant at all points.

### INTERNAL ENERGY, SPECIFIC HEAT, AND ENTROPY OF GASES AT HIGH TEMPERATURES (Based on the Kinetic Theory)

According to the older conception of the kinetic theory of gases, the internal energy of a mass of gas consists of the sum of the kinetic energies of the gas molecules. In the case of the monatomic gases the motion of the individual molecules is purely translatable and consists of zig-zag movements in the three directions of space (three degrees of freedom). In the case of polyatomic gases the rotating motion of the molecules round the three axes of space has to be added to this (two degrees of freedom for diatomics and three degrees for polyatomics). The energy due to the rotation of the atom about its own axis is too small to be considered. The total internal kinetic energy is proportional to the temperature and divides itself uniformly over the different degrees of freedom. For the same temperature it is greater for polyatomic gases than for monatomic gases, due to the additional kinetic energy caused by the rotation of the molecules, and greater for triatomics than diatomics, as is confirmed by test. Taking  $I_t$  as the energy increase due to translatable motion, and  $I_r$  the energy increase due to rotating motion, the theory gives, for the internal energy of 1 mol. of any gas,

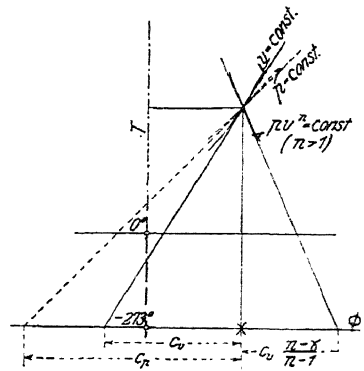


FIG. 66

$$I_t + I_r = R$$

where  $n$  is the number of degrees of freedom and  $R = 1.985$  is the universal gas constant in heat units. If 1 mol. of a gas be raised in temperature by an amount  $dT$  at constant volume, the heat supplied to it from outside is  $dQ = C_v dT$ , and this heat raises the internal energy by an amount  $dI$ , which is used to increase the speed of the molecules. We have, therefore,

$$C_v dT = dI$$

and hence 
$$C_v = \frac{dI}{dT} = n \frac{R}{2}$$

For the monatomic gases, therefore, with  $n = 3$ ,

$$C_v = \frac{3}{2} \times 1.985 = 2.978 \text{ (about 3)}$$

for the diatomic gases, with  $n = 5$ ,

$$C_v = \frac{5}{2} \times 1.985 = 4.96 \text{ (about 5)}$$

and for the triatomic gases, with  $n = 6$ ,

$$C_v = \frac{6}{2} \times 1.985 = 5.955 \text{ (about 6)}$$

From this, the specific heats of all these gases should be independent of temperature and should agree with the values stated. The values obtained by calorimetric test for monatomic gases agree almost exactly with the above, and nearly so in the case of diatomics at normal temperatures. For the triatomic gases  $\text{H}_2\text{O}$  and  $\text{CO}_2$ , the measured specific heats at  $0^\circ \text{C.}$  are about 6.2 and 6.7 respectively, and thus are not greatly different from the theoretical values.

It is known now that the specific heats of diatomic gases increase with temperature, and the increase is still more pronounced in the case of triatomic gases. The older kinetic theory fails to explain this increase. If, however, it is assumed that the atoms in a molecule are in a state of vibration caused by their mutual forces and their masses, the reason for the increase in specific heat with temperature becomes apparent, since the atoms are then capable of absorbing additional energy. This amount of energy is, however, small compared with the translatory and rotating energy at moderate temperatures, and less still at very low temperatures. At high temperatures, however, it becomes of more importance, since its increase is more rapid than would be the case if it were merely proportional to temperature (like the other increments of energy).

For the vibratory energy of a molecular "oscillator" of this type, Planck's quantum theory of light, which was first applied by Einstein to the energy absorbed by solids, gives

$$I_s = R \frac{\beta\nu}{e^{\frac{\beta\nu}{T}} - 1} = E \text{ (C.H.U./mol.)} \quad . \quad . \quad (162)$$

where  $\beta = 4.865 \times 10^{-11}$  is a universal constant,\* and  $\nu$  is the specific number of vibrations per second of the atoms in the molecule.  $\beta\nu$  has the significance of a temperature.

If now this value be added to the internal energy increase, the corresponding increase in specific heat is

$$C_s = \frac{dI_s}{dT} \quad \beta\nu$$

or

$$C_s = 1.985 \frac{\beta\nu}{e^{\frac{\beta\nu}{T}} - 1} = f\left(\frac{\beta\nu}{T}\right) \quad . \quad . \quad (163)$$

\*  $\beta = \frac{h}{k}$  where  $h$  is the Planck quantum element and  $k = \frac{R}{N}$  ( $N$  = number of molecules in 1 mol.).

From this, the total true molecular specific heat at constant volume for diatomics is

$$C_v = 4.96 + f\left(\frac{\beta v}{T}\right) \quad (164)$$

and, since  $C_p - C_v = 1.985$

the corresponding specific heat at constant pressure is

The internal energy per mol. above that at  $0^\circ \text{C.}$  is

$$I = 4.96t + 1.985 \left( \frac{\beta v}{T} - 1 \right) e^{\frac{\beta v}{T}} - 1 \quad (165)$$

and the corresponding total heat per mol. above  $0^\circ \text{C.}$  is

$$H = 6.95t + 1.985 \left( \frac{\beta v}{T} - 1 \right) e^{\frac{\beta v}{T}} - 1 \quad (166)$$

In these, we can take (K. Wohl),

for  $\text{N}_2$   $\beta v = 3340$  (3800)

and for  $\text{O}_2$   $\beta v = 3220$  (2540)

The mean molecular specific heat values between  $0^\circ \text{C.}$  and  $t^\circ \text{C.}$  are given by

$$C_{vm} = \frac{I}{t} \text{ and } C_{pm} = \frac{H}{t}$$

The values of  $I$  and  $H$  per ft.<sup>3</sup> of gas at S.T.P., and the specific heats  $C_v$  and  $C_p$  for the same mass of gas, are found by dividing  $I$  and  $H$ , and  $C_v$  and  $C_p$  by 359 respectively.

Tempera- ture $t^\circ \text{C.}$	Internal Energy above $0^\circ \text{C.}$ , Based on the Kinetic Theory C.H.U./ft. <sup>3</sup>	C.H.U./ft. <sup>3</sup> at S.T.P. From the New Tables	Entropy $\text{N}_2$	$\frac{C_p}{C_v}$	$C_v$
	$\text{O}_2$				
0	0.0	0.0	0.0000	1.400	4.96
100	1.3853	1.410	0.0043	1.400	4.97
200	2.783	2.895	0.0076	1.396	5.00
300	4.206	4.405	0.01023	1.385	5.16
400	5.660	5.997	0.01267	1.374	5.30
500	7.145	7.694	0.01460	1.364	5.46
600	8.711	9.348	0.01635	1.353	5.62
700	10.290	11.088	0.01797	1.344	5.77
800	11.925	12.848	0.01947	1.337	5.90
900	13.553	14.620	0.02103	1.331	6.00
1000	15.282	16.455	0.02246	1.325	6.11
1200	18.757	20.037	0.02496	1.316	6.29
1400	22.233	23.793	0.02702	1.310	6.40
1600	25.846	27.512	0.02864	1.305	6.50
1800	29.503	31.294	0.03026	1.302	6.56
2000	33.166	35.056	0.03170	1.300	6.63
2200	36.835	38.888	0.03301	1.298	6.67
2400	40.635	42.644	0.03426	1.296	6.72

The calculated values of  $I$ , as obtained from equation (165), are given in the table on page 133 for temperatures ranging from  $100^{\circ}\text{C.}$  to  $2400^{\circ}\text{C.}$  For the purpose of comparison the energy values obtained by the latest calorimetric and explosion tests are also given. In Fig. 67 the values for  $\text{N}_2$  have been plotted.

K. Wohl and G. v. Elbe found, for gaseous steam between  $18^{\circ}\text{C.}$ , and  $t^{\circ}\text{C.}$ ,

$$I_{\text{H}_2\text{O}} = 5.96(t - 18) + E(2280) + 2E(5370) \quad . \quad . \quad (167)$$

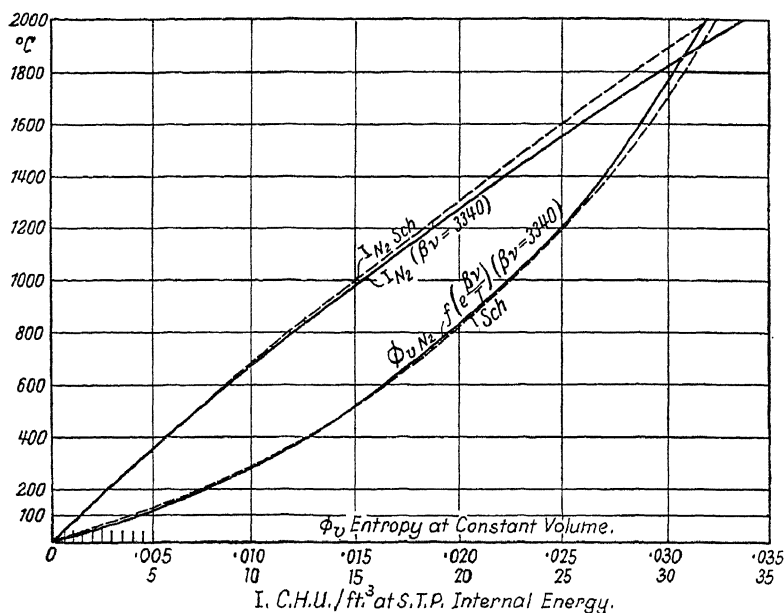


FIG. 67

in which  $E(2280)$  and  $E(5370)$  are the values of the vibration energy (equation 162) for  $\beta v = 2280$  and  $\beta v = 5370$ . This formula shows close agreement with the mean specific heats of steam in the neighbourhood of  $2000^{\circ}\text{C.}$ , as found in explosion tests by Wohl and v. Elbe, and also with the Munich values of the specific heats for low pressures and temperatures up to  $550^{\circ}\text{C.}$  (Fig. 20).

In the case of oxygen the calculated values of  $I$  (as given by the kinetic theory) have not yet been confirmed by test, and these values are considerably higher than those of  $\text{N}_2$ , especially at high temperatures, where the differences range from 3.3 to 8.6 per cent.

The table given below shows the true and mean molecular specific heats  $C_v$  and  $C_{vm}$  (between  $273^{\circ}\text{C. abs.}$  and  $T$ ) for various gases, as determined by K. Wohl,\* and are based on the latest  $\beta v$  values as given here.

\* Communicated by the Phys. Chem. Lab. of Berlin University (29th November, 1929). See also *Zeit. f. Tech. Phys.*, Bd. 10, Nr. 12. W. Nernst and K. Wohl. "Spezifische Wärmen bei hohen Temperaturen."

	HCl			CO			Cl <sub>2</sub>		
	6130	4130	3800	3070	2540	810			
	CO <sub>2</sub>			H <sub>2</sub> O			NH <sub>3</sub>		
	Twice	Once	Once	Once	Twice	Once	Twice	Twice	Once
									6 Times
									3 Times
$\beta\nu$	960	1830	3280	2280	5370	1340	2330	4800	6400
									2000
									4350

TRUE AND MEAN MOLECULAR SPECIFIC HEATS OF DIATOMIC  
AND POLYATOMIC GASES, BY K. WOHL

	<i>T</i> ° C. abs.	273	373	473	573	800	1,200	1,600	2,000	2,400	2,800
True molecular specific heats at constant volume <i>C<sub>v</sub></i> C.H.U./mol.	H <sub>2</sub>	4.82	4.96	4.97	4.98	5.04	5.34	5.72	6.05	6.31	6.54
	HCl	4.97	4.98	5.00	5.01	5.31	5.82	6.20	6.45	6.62	6.73
	N <sub>2</sub>	4.97	4.98	5.05	5.16	5.51	6.05	6.36	6.55	6.66	6.73
	CO	4.97	5.00	5.09	5.23	5.66	6.11	6.41	6.60	6.70	6.76
	O <sub>2</sub>	5.00	5.15	5.38	5.61	6.05	6.47	6.66	6.76	6.81	6.85
	Cl <sub>2</sub>	5.97	6.32	6.53	6.65	6.79	6.88	6.91	6.92	6.93	6.94
	CO <sub>2</sub>	6.63	7.70	8.61	9.24	10.37	11.50	12.02	12.31	12.48	12.60
	H <sub>2</sub> O	5.99	6.11	6.34	6.60	7.28	8.37	9.31	9.99	10.46	10.79
	NH <sub>3</sub>	6.38	7.01	7.65	8.46	9.97	12.2				
	CH <sub>4</sub>	6.38	7.59	9.21	10.48	13.7	17.6				
Mean molecular specific heats at constant volume between 273° C. abs. and <i>T</i> ° C. abs. <i>C<sub>vm</sub></i> C.H.U./mol.	H <sub>2</sub>		4.91	4.93	4.94	4.97	5.06	5.21	5.36	5.52	5.66
	HCl		4.97	4.99	5.00	5.06	5.25	5.49	5.69	5.85	5.98
	N <sub>2</sub>		4.97	4.99	5.03	5.16	5.43	5.67	5.85	6.00	6.11
	CO		4.98	5.00	5.06	5.22	5.52	5.76	5.93	6.07	6.18
	O <sub>2</sub>		5.07	5.17	5.27	5.50	5.85	6.07	6.22	6.31	6.41
	Cl <sub>2</sub>		6.17	6.30	6.40	6.54	6.67	6.74	6.78	6.81	6.83
	CO <sub>2</sub>		7.18	7.67	8.08	8.84	9.77	10.37	10.79	11.10	11.35
	H <sub>2</sub> O		6.05	6.14	6.24	6.53	7.09	7.61	8.10	8.50	8.80
	NH <sub>3</sub>		6.68	7.02	7.39	8.19	9.3				
	CH <sub>4</sub>		6.90	7.71	8.45	10.0	12.7				

**Entropy.** The change of entropy when heat is added at constant volume follows from

$$T d\phi_v = C_v dT$$

or 
$$d\phi_v = \frac{C_v dT}{T}$$

giving, for the diatomic gases with equations (164) and (163),

$$\cdot (168)$$

$$e^T - 1$$

Writing, temporarily,

$$e^{\frac{\beta v}{T}} = x \text{ and } \frac{\beta v}{T} = \log_e x$$

gives, for equation (168),

$$d\phi_v = -R \frac{\log_e x}{(x-1)^2} dx + 4.96 \frac{dT}{T}$$

and hence

$$\phi_v - \phi_{v0} = -R \int_{T_0}^T \frac{\log_e x}{(x-1)^2} dx + 4.96 \int_{T_0}^T \frac{dT}{T}$$

The integration of this gives

$$\phi_v - \phi_{v0} = R \left[ \frac{1}{x-1} \log_e x - \log_e \frac{x}{x-1} \right]_{T_0}^T + 4.96 \log_e \frac{T}{T_0}$$

Inserting the value for  $x$

$$\phi_v - \phi_{v0} = R \left[ \frac{e^{\frac{\beta v}{T}}}{e^{\frac{\beta v}{T}} - 1} \frac{\beta v}{T} - \log_e (e^{\frac{\beta v}{T}} - 1) \right]_{T_0}^T + 4.96 \log_e \frac{T}{T_0} \quad (169)$$

The value per ft.<sup>3</sup> at S.T.P. is found by dividing equation (169) by 359. This gives, along with common logarithms,

$$\begin{aligned} \phi'_v - \phi'_{v0} = 0.0318 \log \frac{T}{T_0} + \left[ 0.00553 \frac{\beta v}{T} \frac{e^{\frac{\beta v}{T}}}{e^{\frac{\beta v}{T}} - 1} \right. \\ \left. - 0.01272 \log (e^{\frac{\beta v}{T}} - 1) \right]_{T_0}^T \quad (170) \end{aligned}$$

The table of values  $\phi' - \phi'_0$  given on page 133 was obtained from this expression with  $T_0 = 273^\circ \text{C. abs.}$  These values, as well as those given by Chart I, are shown in Fig. 67.

## ADIABATIC CHANGE OF STATE FOR DIATOMIC GASES

(Based on the Kinetic Theory Equations)

In the case of an adiabatic change of state in which the gas obeys the law  $p v = RT$ , we have, from page 107,

$$\frac{dp}{dv} = -\gamma \frac{p}{v}$$

where  $\gamma = \frac{c_p}{c_v} = \frac{C_p}{C_v}$

the temperature, and

$$C_p - C_v = 1.985$$

$$\gamma = 1 + \frac{1.985}{C_v}$$

so that, from equation 164,

$$\gamma = 1 + \frac{1.985}{4.96 + f\left(\frac{\beta v}{T}\right)} \quad (171)$$

The values of  $\gamma$  for  $N_2$ , found from this expression, are given in the table on p. 133. They, along with the values given in Schüle's *Neue Tabellen*, have been plotted to a wide scale in Fig. 68.

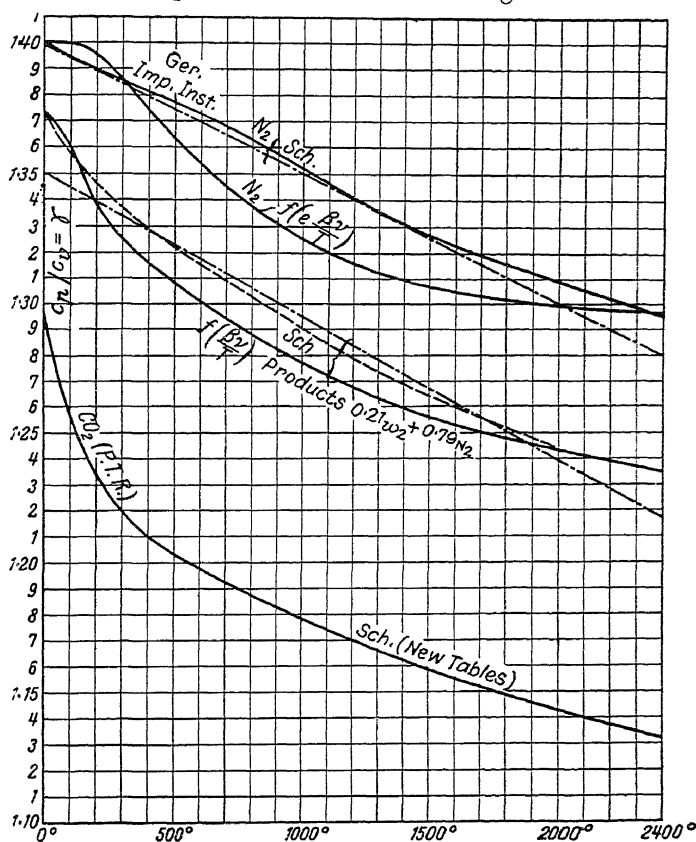


FIG. 68

The equation  $pv^\gamma = \text{constant}$  is thus only valid for the portion of the  $pv$  curve in which  $\gamma$ , corresponding to the mean temperature  $T$ , may be assumed constant.

An exact relationship between  $v$  and  $T$  follows from

$$d\phi = C_v \frac{dT}{T} + \frac{av}{v} \quad (\text{page 125})$$

or, with

$$d\phi = 0$$

$$1.985 \frac{dv}{v} = -C_v \frac{dT}{T}$$

Integration gives, with equations (164) and (163),

$$\log_e v = -2.5 \log_e T - \frac{\beta v}{T} \frac{e^{\frac{\beta v}{T}}}{e^{\frac{\beta v}{T}} - 1} + \log_e (e^{\frac{\beta v}{T}} - 1) + \log_e \text{const.}$$

$$\text{or } v = T^{-2.5} \left( e^{\frac{\beta v}{T}} - 1 \right) \cdot e^{\left\{ -\frac{\beta v}{T} \cdot \frac{e^{\frac{\beta v}{T}}}{e^{\frac{\beta v}{T}} - 1} \right\}} \cdot \text{const.} \quad (172)$$

and since  $\frac{pv}{T} = \text{constant}$

$$p = \frac{T}{v} \text{ constant}$$

$$p = T^{3.5} \cdot \frac{e^{\left\{ \frac{\beta v}{T} \cdot \frac{e^{\frac{\beta v}{T}}}{e^{\frac{\beta v}{T}} - 1} \right\}}}{\left( e^{\frac{\beta v}{T}} - 1 \right)} \cdot \text{const.} \quad (173)$$

With constant specific heat, equations (172) and (173) give the known relations

$$T^{2.5} = \text{constant}$$

and  $p = T^{3.5} \times \text{constant}$

$$\text{or } T \left( \frac{v_0}{v} \right)^{0.4} = \left( \frac{p}{p_0} \right)^{\frac{0.4}{1.4}}$$

The absolute work delivered in the adiabatic expansion between the states  $p_0, v_0, T_0$ , and  $p, v, T$ , is

$$AE = I_0 - I$$

so that, with equation (165),

$$AE = 4.96(T_0 - T) + 1.985 \beta v \frac{\text{C.H.U.}}{\text{mol.}} \cdot \frac{1}{e^{\frac{\beta v}{T}} - 1}$$

This can only be solved, however, if  $T$ , the final adiabatic temperature, is given or calculated from equations (172) or (173), or from the  $T\phi$  diagram, in which case  $AE$  can be read directly on Fig. 67 (for  $N_2$ ). In actual technical problems, however, the values for the products of combustion are more important. For this reason the values of  $\gamma$  for  $CO_2$  have been included in Fig. 68. From these, and the  $\gamma$  values for  $N_2$ , the values of  $\gamma$  for the theoretical products having the analysis  $v_{CO_2} = 0.21$ ;



= 0.79 (formed in burning carbon with air) have been calculated by inserting their values in the general equation

$$\gamma = \frac{1}{\frac{v_{\text{CO}_2}}{\gamma_{\text{CO}_2} - 1} + \frac{v_{\text{N}_2}}{\gamma_{\text{N}_2} - 1}} + 1$$

This relationship follows from  $C_v = v_{\text{CO}_2} C_{v\text{CO}_2} + v_{\text{N}_2} C_{v\text{N}_2}$

and 
$$\gamma = \frac{C_p}{C_v} = 1 + \frac{1.985}{C_v}$$

These  $\gamma$  values have also been plotted in Fig. 68, along with those given by the molecular specific heats found in *Neue Tabellen*, and those given by the linear equation on page 53.

The  $\gamma$  values for products with excess air lie between these lines and the  $\text{N}_2$  lines.

This enables the mean value of  $\gamma$ , over any temperature range, to be determined for the different products. Taking the open scale of the ordinates in Fig. 68 into account, it will be seen that the variation in the value of  $\gamma$ , for moderate ranges of temperature, is not large. For example, the value of  $\gamma$  between  $2000^\circ \text{C.}$  and  $1000^\circ \text{C.}$  (a probable range in the expansion of a gas in a gas engine cylinder) varies from 1.245 to 1.268, so that a mean value of 1.26 can be assumed, thus considerably simplifying the calculations. Use can also be made of the linear equation  $\gamma = \gamma_0 - aT$  over this range, as explained on page 117 *et seq.*

The simplest method, however, is to employ a graphical solution by means of the  $T\phi$  diagram, Chart I. which agrees almost entirely with the  $\text{N}_2$  values up to about  $1400^\circ \text{C.}$ , as shown by Fig. 67.

The above equations do not apply to polyatomic gases, which include therefore the products of combustion. The entropy curves can, however, be drawn for these gases also, when the kinetic energy values are known. The curves are then applied in the same way as those in Chart I.

**Temperature entropy diagram for gases.** In the case of adiabatic changes of state, the calculative treatment is comparatively simple, even when the specific heat is variable (see page 117). Since, however, the entropy remains constant in this change of state, a purely graphical method can be carried out on the entropy diagram as follows (Chart I)—

In Fig. 68*a* the ordinates represent the gas temperatures, while the abscissae give the entropy values. The entropy at  $0^\circ \text{C.}$  of 1 lb. of gas at any arbitrary volume  $V_0$  and a corresponding pressure  $p_0$  is assumed to be zero (point *A*). If, from *A*, the gas be heated at constant pressure, the entropy increases with temperature, as shown by the curve *AB* (see page 129). If heated at constant volume the resulting curve is *AC*. If the gas be compressed adiabatically, before the addition of heat, to a pressure  $12p_0$ , the temperature rises from *A* to  $A_{12}$  such that

$$T_{12} = OA_{12} \text{ which is given by } \frac{T_{12}}{T_0} = \left( \frac{\gamma - 1}{\gamma} \right)^{\frac{\gamma}{\gamma - 1}} = 12$$

(in the case of variable specific heat, see pages 116 and 135).

If now, starting at the point  $A_{12}$ , heat be added at constant pressure, the increase in entropy is shown by the curve  $A_{12}B_{12}$ . In this way the "curves of constant pressure" are drawn in for the pressures  $np_0$  (e.g.  $n = 2, 3, 4, \dots$ ).

Similarly, if the gas be compressed adiabatically to  $\frac{1}{9}V_0$ , giving the line  $AD_9$ , the temperature rises to  $T_9'$ , where

$$\frac{V_0}{\frac{1}{9}V}^{\gamma-1} = 9^{\gamma-1}$$

Heating at this constant volume ( $\frac{1}{9}V_0$ ) gives the curve  $D_9C_9$ . The other "constant volume curves" were drawn in this way, but account was taken of the variation of  $C_v$  and  $\gamma$  with temperature. Other intermediate volumes can, of course, be drawn in, depending on the particular compression ratio and degree of accuracy required.

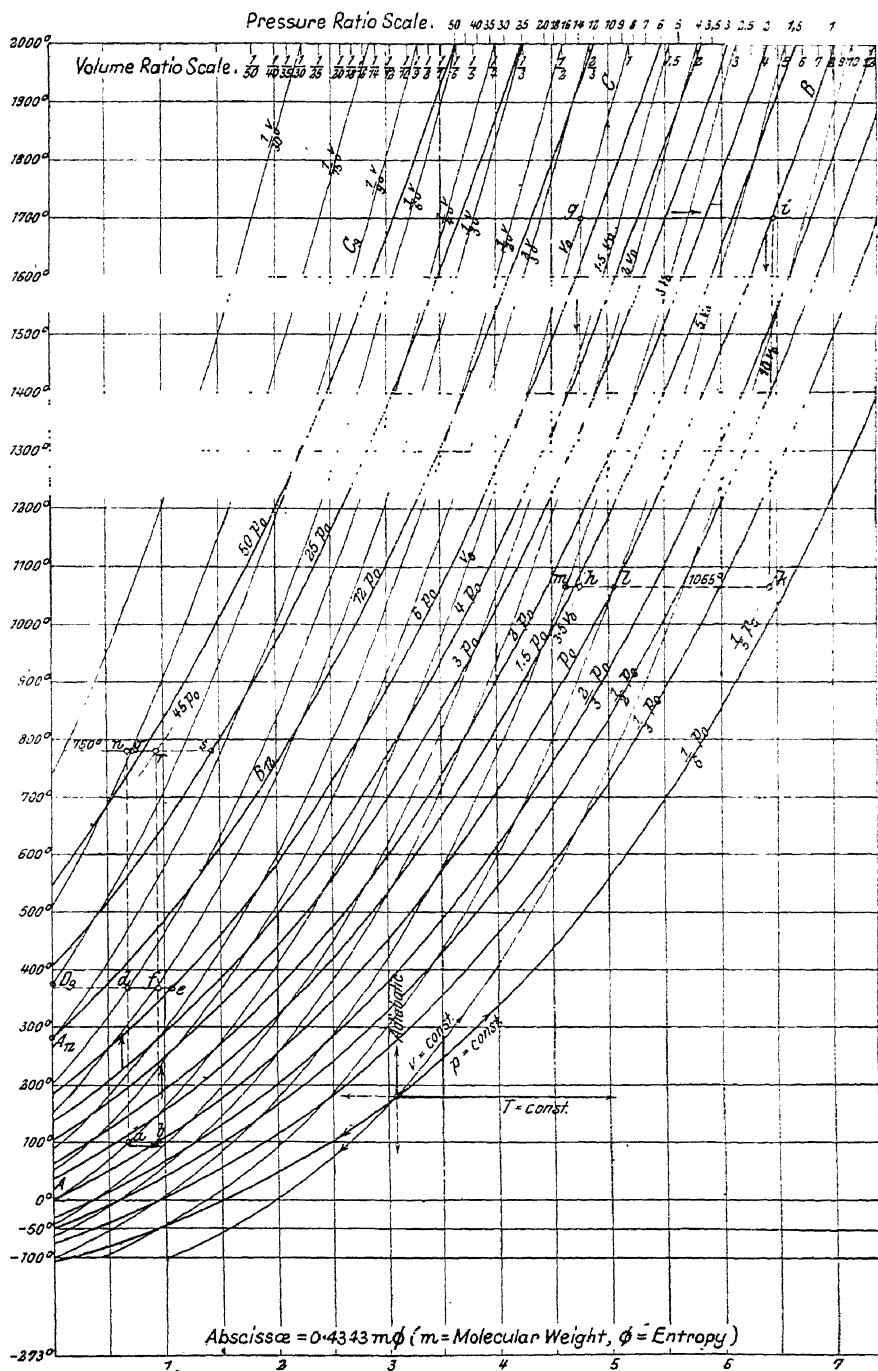
The application of the diagram is as follows. It is required to find, say, the rise in pressure and temperature caused by the adiabatic compression of air at  $100^\circ\text{C}$ . and  $p = 13\text{ lb./in.}^2\text{ abs.}$ , when the volume is reduced to  $\frac{1}{9}$  of the initial volume. The curve  $V_0$  cuts the temperature level of  $100^\circ\text{C}$ . at  $a$ . The vertical line  $ad$ , which represents the adiabatic compression, cuts the constant volume curve  $\frac{1}{9}V_0$  at  $d$ . The temperature at the end of compression is therefore  $370^\circ\text{C}$ . (as shown by  $d$ ). In order to find the pressure rise, a vertical is drawn from  $b$  to the point  $f$ , which is at the same level as  $d$ . This point  $f$  lies on the required pressure line (not shown), so that the pressure is between 6 and 12 times greater than the original pressure. Since the horizontal distances between any two constant pressure curves is the same at all temperature levels, it follows that the distance  $ef$ , when measured to the left of the mark 6 on the pressure scale at the top of the diagram, will give the required compression pressure. This is found to be about 6.9, so that the final pressure is  $6.9 \times 13 = 90\text{ lb./in.}^2\text{ abs.}$

The absolute work of adiabatic compression is equal to the change of internal energy and can be found, per  $\text{ft.}^3$  of gas at S.T.P., from Chart I by taking the difference in internal energies at the temperatures found above.

Chart I has been drawn from the old variable specific heat values for diatomic gases. The new values have been used in Fig. 67 and Fig. 68a.

**Example 19.** Air at  $100^\circ\text{C}$ . is compressed adiabatically to  $\frac{1}{15.73}$  times the original volume. Find the final pressure and temperature (when the specific heats are variable).

Draw the vertical  $an$ ;  $n$  lies to the left of  $q$  by an amount which is measured on the volume scale between the points  $\frac{1}{15}V_0$  and  $\frac{1}{15.73}V_0$ . The required temperature is then given at the point  $n$ , and is equal to  $780^\circ\text{C}$ . In order to find the final pressure, first draw a horizontal from  $a$  to  $b$  (on the curve  $p_0$ ), and then draw a perpendicular from  $b$  to intersect the temperature level  $780^\circ\text{C}$ . at the point  $r$ , which lies on the required pressure curve. As measured on the pressure scale, this pressure is 45 times greater than the initial pressure.





Further  $c_p - c_v = AR$

and  $mc_p - mc_v = mAR = 1.985 \text{ C.H.U.} \quad \dots \quad (179)$

i.e. the difference between the molecular specific heats has the same value for all gases. Since, therefore, the pressure and volume have no effect on  $c_p$  and  $c_v$ , the entropy change between the same absolute temperature limits is constant along any constant volume or constant pressure curve.

The constant pressure curves for different values of  $p$  or constant volume curves for different values of  $v$  are separated on the  $T\phi$  field, since, at constant temperature, the entropy changes when the pressure or volume changes (page 127).

From the energy equation for gases

$$dQ = c_v dT + A p dv$$

we have, at constant temperature,

$$dQ = A p dv$$

i.e. the heat supplied is equal to the heat equivalent of the expansion work, whether or not  $c_v$  (or  $c_p$ ) varies with temperature.

Now, since  $dQ = T d\phi$  or  $d\phi = \frac{dQ}{T}$

the change of entropy in an isothermal expansion or compression is

$$d\phi_T = \frac{A p dv}{T}$$

or, from equation (178),

$$d\phi_T = \frac{AR dv}{v}$$

Integration gives

$$\phi_T - \phi_{T0} = AR \log_e \frac{v}{v_0} \quad \dots \quad (180)$$

or, with equation (178),

$$\phi_T - \phi_{T0} = -AR \log_e \frac{p}{p_0} \quad \dots \quad (181)$$

If, therefore, it is desired to construct the entropy curve for any pressure  $p$  (greater than  $p_0$ , say) from the constant  $p_0$  curve, which passes through  $0^\circ \text{C.}$  when the entropy is zero, it is only necessary to separate these two curves by the same horizontal distance at any temperature level. This horizontal distance is given by equation (181), which shows that the entropy for the pressure  $p$  is less than that for  $p_0$ . All the constant pressure curves are "congruent."

The same applies to the constant volume curves which are separated from one another by the amount given in equation (180).

It is sufficient, therefore, for a given gaseous substance, to plot a

single constant pressure curve ( $p_0$ ) and a single constant volume curve ( $v_0$ ) if a scale of abscissae values be added which gives the displacement of the curves for values of  $\frac{p_0}{p}$  and  $\frac{v}{v_0}$ .

This method is particularly useful if, in the diagram, not only air and diatomic gases, but also  $\text{CO}_2$ ,  $\text{H}_2\text{O}$ , and mixtures of these three gas types, can be represented.

A difficulty arises here, however, in that each gas requires a separate scale, if equal weights are assumed, since each gas has its own constant  $R$ . A common scale can, however, be used for all gases, including  $\text{CO}_2$  and  $\text{H}_2\text{O}$ , if, in place of equal weights, we choose weights which are proportional to the molecular weights  $m$ ,  $m_2$ ,  $m_3$ , . . . of the gases.

If the weights chosen are in mols., the displacement of the lines of constant pressure then becomes, from equation (181),

$$m(\phi_T - \phi_{T_0}) = -mAR \log_e \frac{p}{p_0} \quad . \quad . \quad . \quad . \quad . \quad (182)$$

and, for those at constant volume,

$$m(\phi_T - \phi_{T_0}) = mAR \log_e \frac{v}{v_0}$$

From equations (179) and (182), we have, for the pressure  $p$ ,

$$m(\phi_T - \phi_{T_0})_p = -1.985 \log_e \frac{p}{p_0} \quad . \quad . \quad . \quad . \quad . \quad (183)$$

and, for the volume  $v$ ,

$$m(\phi_T - \phi_{T_0})_v = 1.985 \log_e \frac{v}{v_0} \quad . \quad . \quad . \quad . \quad . \quad (184)$$

The displacements for all gases having the same proportional pressure or volume changes are now equal, so that a single displacement scale suffices.

Advantage can also be taken of the fact that the volume occupied by  $m$  lb. of each particular gas (at the same temperature and pressure) is the same (359 ft.<sup>3</sup> at S.T.P.), so that, if weights corresponding to 1 ft.<sup>3</sup> at S.T.P. of the gases be adopted, a common displacement scale again serves. The weight of this quantity of gas is  $\frac{m}{359}$  lb. (Specific weight at S.T.P.)

The change of entropy at constant pressure for this weight is, from equation (176),

$$\frac{m}{359} \phi_p = \frac{1}{359} \int_{273}^T \frac{mc_p}{T} dT = \phi_p' \quad . \quad . \quad . \quad . \quad . \quad (185)$$

and, at constant volume,

$$\frac{m}{359} \phi_v = \frac{1}{359} \int_{273}^T \frac{mc_v}{T} dT = \phi_v' \quad . \quad . \quad . \quad . \quad . \quad (186)$$

while the displacements, given by dividing equations (183) and (184) by 359, are

$$359 = -\frac{1.985}{359} \times \frac{p}{p_0}$$

and

$$\frac{m(\phi_T - \phi_{T_0})_v}{359} = 0.20405 \log \frac{v}{v_0} \quad (188)$$

The entropy curves for constant pressure given on the left of Chart I, and those of constant volume on the right, have been plotted on this

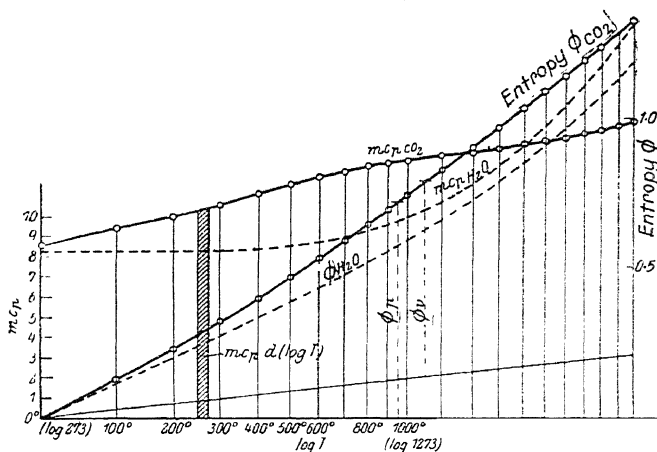


FIG. 69

basis, and are given for diatomic gases (air), carbon dioxide, and steam. For a products mixture, containing these gases, the entropy curves can be formed from

$$\phi = v_{\text{CO}_2} \phi_{\text{CO}_2} + v_{\text{H}_2\text{O}} \phi_{\text{H}_2\text{O}} + (v_{\text{N}_2} + v_{\text{O}_2} + v_{\text{CO}}) \phi_{\text{diat.}}$$

i.e. by multiplying the entropy values by the fractional volume values.

Another method (as in the case of the total heat of products dealt with on pages 57 and 64) is to plot the entropy curve for the mean products (with no excess air), either by using the specific heat of the gas as shown on page 58, or as a curve of pure carbon-air products with  $v_{\text{CO}_2} = 0.21$  and  $v_{\text{N}_2} = 0.79$ . The curves of products containing the fraction  $v_a$  of excess air are then obtained for  $v_a = 0.1, 0.2, 0.3, \dots$ , etc., by dividing the horizontal distances between the curves for air and pure products into ten equal parts and joining the division points.

If an equation showing the dependence of the specific heat on temperature is available, equations (185) and (186) can be integrated (pages 126 and 135). If this is not the case, as, for example, with  $\text{CO}_2$ , or if the equations are too complicated, then, from

$$\frac{dT}{T} = d \log_e T$$

we can write equation (185) in the form

$$\phi_p' = \frac{2.303}{359} \int_{273}^T C_p d(\log T)$$

and find  $\phi_p'$  as the area below the curve obtained by plotting  $C_p$  to a base of  $\log_e T$ , as shown in Fig. 69 for  $\text{CO}_2$ . If, in place of the  $C_p$  curve, the  $H$  or  $I$  curves are given, then, from

$$c_p dT = dH$$

we have 
$$\phi_p = \frac{1}{359} \int_{273}^T \frac{dH}{T}$$

and  $\phi_p$  is therefore given as the area under the curve obtained when  $\frac{1}{T}$  is plotted to a base of  $H$  values.

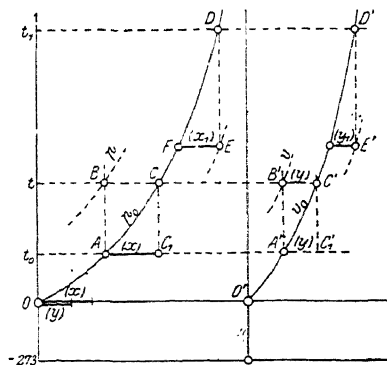


FIG. 70

See Schüle's *Neue Tabellen* regarding the six entropy curves contained in Chart I, and page 64 regarding the total heat curves.

**1. Application of the chart and examples.** In finding the temperature and volume changes, with an adiabatic pressure compression ratio  $x$ , the chart is used as follows—

Let  $A$  be the point at the initial temperature  $t_0$  on the constant pressure curve of the chart (Fig. 70). The final state point  $B$  of compression lies on the vertical through  $A$  and on the constant pressure curve for  $p = xp_0$ . The horizontal displacement of this curve from the reference pressure curve is measured on

the expansion and compression pressure scale, and can be then marked off from  $A$  to give the point  $C_1$ . A vertical through  $C_1$  cuts the reference pressure curve at  $C$ , and thus gives the required temperature  $t$ . The volume ratio is found by marking the point  $A'$  on the constant volume curve at the temperature level  $t_0$ , and the point  $C'$  at the temperature level  $t$ , and measuring the horizontal distance  $y$  between these points. If  $y$  be measured on the volume compression ratio scale with one end on the point marked 1, the other end gives the required volume compression ratio  $v/v_0$ .

**Example 21.** Find the temperature at the end of adiabatic compression for air, pure products, gaseous steam, and carbon dioxide, when the pressure compression ratio is 40.

- With an initial temperature of  $0^\circ \text{C}$ .
- With an initial temperature of  $100^\circ \text{C}$ .

The chart gives (see Fig. 71, left) the following temperatures for

- 493, 440, 384, and  $306^\circ \text{C}$ .
- 760, 675, 610, and  $478^\circ \text{C}$ .



It is thus seen that the differences in the temperature changes between air and the other gases are quite considerable.

The volume ratio shown by the chart (Fig. 71, right) is 1.21 for  $\text{CO}_2$  and 1/15 for air.

**Example 22.** Find the temperatures and pressure compression ratios due to adiabatic compression for the same four gases when the volume compression ratio is 15.73 and the initial temperature is  $100^\circ \text{C}$ .

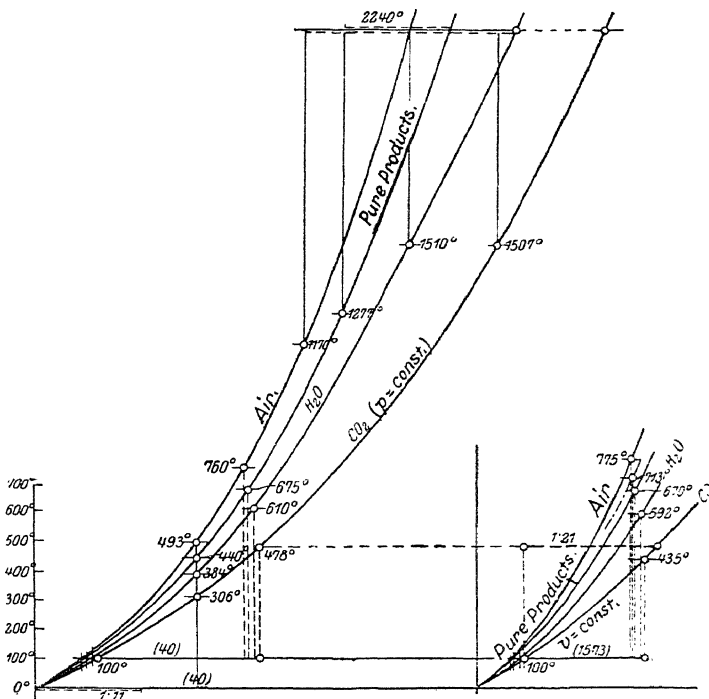


FIG. 71

From the constant volume curves of the diagram (shown on the right of Fig. 71), the final temperatures are—

- 775° C. for air;
- 713° C. for products with 50 per cent excess air;
- 670° C. for pure products;
- 592° C. for  $\text{H}_2\text{O}$ ;
- and 435° C. for  $\text{CO}_2$ .

The pressure compression ratio is 44 for air and 30 for  $\text{CO}_2$ .

**2. Adiabatic expansion.** Mark the points  $D$  and  $D'$  on the constant pressure and constant volume curves at the initial temperature  $t_1$  (Fig. 70).

If, now, the pressure expansion ratio is  $x$ , so that the final pressure is  $p = \frac{p_0}{x_1}$ , the point  $F$  is found on the pressure curve, such that its horizontal distance from  $D$  is equal to that given by the scale between the points

1 and  $\frac{1}{x_1}$ , or 1 and  $x_1$ . This gives the required temperature  $t_2$ . The volume expansion ratio  $y_1$  is found by transferring the distance  $y_1$  from the constant volume curve to the expansion scale. If  $y_1$  be given, the same operation is carried out on the constant volume curve.

**Example 23.** Find the temperature at the end of adiabatic expansion when the pressure expansion ratio is 11 and the initial temperature is  $2240^\circ \text{C}$ .

The diagram (Fig. 71) gives—

- 1170° C. for air;
- 1277° C. for pure products;
- 1510° C. for steam;
- 1507° C. for carbon dioxide.

The differences between steam and carbon dioxide are less, in this case, since the two curves are almost parallel.

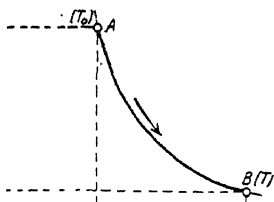


FIG. 72

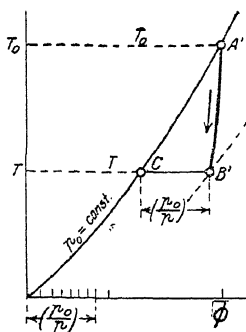


FIG. 73

**3. Transference of points from the  $pv$  field to the  $T\phi$  field.** In Fig. 72 let  $AB$  be the given  $pv$  curve. In order to transfer the curve, the temperature at one point, say  $A$ , must be known. If this is  $T_0$ , the temperature at any other point, such as  $B$ , is given by

$$T = T_0 \cdot p^{\gamma}$$

Mark  $A'$  on the pressure curve  $p_0$ , at the temperature level  $T_0$  on the  $T\phi$  field (Fig. 73). Mark off the distance, given on the expansion scale, corresponding to the ratio  $p_0/p$ , from  $C$  to  $B'$  at the temperature level  $T$ . The point  $B'$  then corresponds to  $B$ . In the same way any intermediate point between  $A$  and  $B$  can be transferred and the curve  $A'B'$  obtained.

## CHAPTER III

### FIRST AND SECOND LAWS OF THERMODYNAMICS

**Heat and mechanical work. First law. The energy law.** The early steam engines served to show that mechanical work could be obtained from heat and the amount was only limited, on the one hand, by the size of engine and steam pressure, and, on the other hand, by the amount of coal burned in the boiler. But the part played by heat, as such, in this production of work, and the numerical relationship between the heat liberated and the work performed remained unexplained for some considerable time after the introduction of the steam engine. A considerable amount of data was, of course, available showing the relation between the amount of coal burned, or steam used, and the work delivered by the engine, and this was of use in measuring the progress made. Improvements were found to be effected, in this way, by the use of higher steam pressures, increased expansion ratios, and compounding. But engineers were not in a position to decide what fraction of the heat given up by the coal was usefully converted to work, so long as the relationship between heat and work remained unknown. Engines could be compared with one another, but the upper limit of the best possible performance remained unknown. The question as to whether other processes than those using steam boilers and engines would show a greater delivery of work for the same fuel consumption had to remain open. Many of the older conclusions regarding the possibility of the conversion of heat to work in air and steam engines were erroneous.

The first successful step in mechanical heat theory was made by the French engineer officer, Sadi Carnot, in 1824. Interested in the fact that steam leaves an engine at a considerably lower temperature than that entering the engine, he established the universally valid principle that heat can only be converted to work if a temperature drop occurs. He also proved that, if perpetual motion be impossible, the greatest amount of work obtainable from a given quantity of heat depends on the temperature limits alone and is independent of the nature of the working substance. From this it is evident that heat causes work to be done and the working substance merely serves as a medium for the transformation.

Carnot compared the capacity of heat in delivering work with that of falling water. The quantity of heat corresponds to the quantity of water, and the temperature drop to the head of water. It is now known that, when heat is converted to work, a certain fraction of the heat supplied disappears as heat. Carnot and his contemporaries assumed that, in spite of its lower temperature, the exhaust steam contained as much heat as the live steam, whereas it is now known that less heat is contained in the exhaust steam owing to the conversion of heat to work in the engine.

This was established by Robert Mayer, a Wurtemberg physician, who first showed (1842) that a quantity of heat is directly comparable with mechanical work and is quite independent of the temperature, and

that it does not correspond to the amount of water in water power, and still less to the product of the quantity of water and the head. In every case in which  $E$  ft. lb. of work are delivered by an expenditure of heat,  $AE$  C.H.U. of heat disappear, and  $A$  is a constant number. Mayer found (in the manner outlined on page 94) a fairly close value to the correct mechanical heat equivalent.\* The value  $\frac{1}{1400}$  was found later in tests

carried out by Joule, and this is generally accepted as the correct value.

The first experimental demonstration of this law applied to a steam engine was carried out by the Alsatian engineer, Gustav Adolph Hirn, who is well known for his work on technical thermodynamics.

The reverse process of obtaining heat by an expenditure of mechanical work (e.g. by friction, impact, action of tools on metals, gas compression, etc.) has long been known. The equivalent law applies here also. If 1400 ft. lb. are converted to heat, the amount obtained is always 1 C.H.U. Almost all tests on the direct determination of the heat equivalent have been carried out in this way, i.e. by measuring, calorimetrically, the heat produced in the complete conversion of mechanical work, or of electrical energy. The same heat equivalent is obtained by other means, which depend on a further application of the mechanical heat theory (e.g. use may be made of measurements of volume, pressure, temperature, and latent heat of steam). Again, processes in which heat is converted to work, as in gas and steam engines, may be applied and lead to the same result, but the method is much more difficult and is less exact, owing to the numerous sources of error.

The application of this first law of heat to other natural processes, like electrical and chemical ones, led Mayer and Helmholtz to the establishment of the principle of conservation of energy, which states that a given quantity of energy cannot be lost nor in any way destroyed. The chief forms of energy are mechanical energy, either as kinetic or pressure energy, heat energy, electrical energy, and chemical energy.

The law goes further, however, and states that energy cannot be created. If energy be obtained in any way it must have come from some existing supply. This law is equal in importance to that which states that perpetual motion is impossible, i.e. a motion which is maintained without an external supply of energy and which delivers useful work or merely overcomes its own frictional work.

**Units of mechanical, thermal, chemical, and electrical energy.** The technical unit of mechanical energy is the "foot-pound" (ft.-lb.), and of thermal energy the "Centigrade heat unit" or "British thermal unit" (C.H.U. or B.Th.U.). On the Continent the heat unit adopted is the Kilocalory (K. Cal.).

As a rule, mechanical energy (such as mechanical or available work and kinetic energy) is measured in mechanical units and heat energy in thermal units. Since, however, the amounts of energy are independent of the type of energy, it frequently happens that the above order is reversed, mechanical energy being expressed in thermal units and quantities of heat in mechanical units.

\* In Great Britain the determination of the mechanical equivalent is generally credited to Joule. (Translator.)

Thus, in steam turbine calculations, it is customary to denote the available work between two given pressures as a "heat drop," expressed in heat units. A heat drop of 160 B.Th.U. means an available energy of  $160 \times 778 = 124,500$  ft. lb. Again, quantities of heat might be expressed in ft. lb. or electrical units, instead of thermal units, in order to obtain uniformity. The specific heat of air would then be  $C_p = 0.24 \times 778 = 186.7$  ft. lb./lb. instead of  $C_p = 0.24$  B.Th.U./lb.

The energy liberated in chemical reactions is generally expressed in heat units.

The amount of work performed when unit force, namely 1 poundal (or 1 dyne), acts through a distance of 1 ft. (or 1 cm.) is called 1 foot poundal (or 1 erg).

But a poundal (or dyne) is the force which gives to the mass of 1 pound (or gram) weight, which is the unit of mass in the absolute system, an acceleration of 1 ft./sec.<sup>2</sup> (or 1 cm./sec.<sup>2</sup>). In the engineering system of units (ft. lb. sec. or m. kg. sec.) this mass is equal to  $\frac{1}{32.2}$  (or  $\frac{1}{1000 \times 9.8}$ ) engineering units of mass, since in this case the unit of force is equal to the unit of weight.

Hence, since

$$\text{force} = \text{mass} \times \text{acceleration}$$

$$1 \text{ poundal} = \frac{1}{32.2} \text{ lb.}$$

$$\text{and} \quad 1 \text{ dyne} = \frac{1}{1000 \times 9.81} \times \frac{1}{100} \text{ kg.} = \frac{1}{9.81 \times 10^5} \text{ kg.} = \frac{1}{981} \text{ gm.}$$

$$\text{Also} \quad 1 \text{ ft. poundal} = \frac{1}{32.2} \text{ ft. lb.}$$

$$\text{and} \quad 1 \text{ erg} = \frac{1}{9.81 \times 10^5} \times \frac{1}{100} = \frac{1}{9.81 \times 10^7} \text{ m. kg.} = \frac{7.38}{10^8} \text{ ft. lb.}$$

Since the erg is an exceedingly small quantity, the unit of electricity is  $10^7$  times greater and is called 1 Joule.

Hence,

$$1 \text{ Joule} = 10^7 \text{ erg} = \frac{1}{9.81} \text{ m. kg.} = 0.102 \text{ m. kg.} = 0.737 \text{ ft. lb.}$$

In thermal units, we have

$$\begin{aligned} 1 \text{ Joule} &= \frac{0.737}{1400} \text{ C.H.U.} = \frac{0.737}{778} \text{ B.Th.U.} = \frac{1}{9.8 \times 427} \text{ K. Cal.} \\ &= \frac{0.527}{1000} \text{ C.H.U.} = \frac{0.948}{1000} \text{ B.Th.U.} = \frac{0.239}{1000} \text{ K. Cal.} \end{aligned}$$

It follows, from these relations, that 1000 Joules of electrical energy are capable of delivering heat amounting to 0.527 C.H.U. or 0.948 B.Th.U. or 0.239 K. Cal.

In considering the performance of engines, it is usual to take into

account the time taken to deliver a quantity of work. The amount of work obtained in 1 sec. is called the power. The power can also be expressed in mechanical or thermal units. The mechanical unit of power is 1 ft. lb./sec. (or 1 m. kg./sec.), but engineers adopt a unit 550 (or 75) times greater, called the horse-power.

$$1 \text{ (British) h.p.} = 550 \text{ ft. lb./sec.} = 33,000 \text{ ft. lb./min.}$$

$$\text{or } 1 \text{ (Continental) h.p.} = 75 \text{ m. kg./sec.} = 32,500 \text{ ft. lb./min.}$$

In thermal units we have

$$1 \text{ h.p.} = \frac{550}{1400} \overset{\text{C.H.}}{\text{B.Th.U.}}/\text{sec.} = 0.392 \overset{\text{C.H.}}{\text{B.Th.U.}}/\text{sec.}$$

$$= 23.6 \text{ C.H.U./min.} = 1415 \text{ C.H.U./hour}$$

$$\text{or } 1 \text{ h.p.} = \frac{550}{778} \text{ B.Th.U./sec.} = 42.4 \text{ B.Th.U./min.}$$

$$= 2545 \text{ B.Th.U./hour.}$$

The unit of power used in electrical measurements is the "watt" and equals 1 Joule/sec., i.e.

$$1 \text{ watt} = 1 \text{ Joule/sec.} = 0.737 \text{ ft. lb./sec.} = 0.102 \text{ m. kg./sec.}$$

1000 watts are called 1 "kilowatt" (kW.).

$$\text{Hence } 1 \text{ kW.} = 1000 \text{ Joules/sec.} = 737 \text{ ft. lb./sec.} = 102 \text{ m. kg./sec.}$$

$$\text{Also } 1 \text{ kW.} = \frac{737}{550} \text{ h.p.} = 1.34 \text{ h.p. or } 1.36 \text{ (Continental) h.p.}$$

The amount of energy delivered in any given time is found by multiplying the power by the time. When the unit chosen is the horse-power, the work delivered in 1 second is called "1 horse-power second," but, as a rule, the "horse-power hour" is used, and

$$1 \text{ h.p. hour} = 550 \times 3600 = 1,980,000 \text{ ft. lb.}$$

or, in thermal units,

$$1980000 \text{ C.H.U.} = 1415 \text{ C.H.U.} = 2545 \text{ B.Th.U.}$$

Hence, if 1 h.p. is converted entirely to heat, for example by a brake fitted to a steam engine, the amount of heat delivered at the brake per hour is 1415 C.H.U. or 2545 B.Th.U.

The British unit of power used in electrical engineering is the B.T.U. (Board of Trade Unit) and equals 1 kilowatt hour.

$$\text{Hence, } 1 \text{ B.T.U.} = 1 \text{ kW. hour} = 3.6 \times 10^6 \text{ Joules}$$

$$\text{and since } 1 \text{ Joule} = \frac{0.527}{1000} \text{ C.H.U.} = \frac{0.948}{1000} \text{ B.Th.U.}$$

$$1 \text{ kW. hour} = \frac{3.6 \times 10^6 \times 0.527}{1000} = 1900 \text{ C.H.U.}$$

$$\text{or } 1 \text{ kW. hour} = \frac{3.6 \times 10^6 \times 0.948}{1000} = 3414 \text{ B.Th.U.}$$

If, therefore, 1 kilowatt hour be completely converted to heat, for example, by means of an electrical resistance, the heat developed is 1900 C.H.U. or 3414 B.Th.U. The maximum amount of steam obtainable from 1 kw. of electrical energy is  $\frac{3414}{H}$ , where  $H$  is the total heat of the steam in B.Th.U. reckoned from the water temperature. Thus, with  $H = 1140$  B.Th.U. (steam at 15 lb./in.<sup>2</sup> abs.) the weight of steam generated is  $\frac{3414}{1140} \doteq 3lb.$

**Efficiency of heat engines.** Steam engines, steam turbines, and internal combustion engines are essentially heat engines, since their capacity for performing work is due to the heat supplied to them, either in the steam or the hot gases. A portion of this heat is converted to mechanical work in the engine by means of the working substance (steam or gases), while the remainder is rejected, as heat, in the exhaust steam or in the exhaust gases and jacket water. It is obvious that the first portion should be made as large as possible, so that the cost of fuel for a given power may be as low as possible, while for the same size of engine more work will be performed.

A gas engine, in which the combustion is poor, may conceivably fail to deliver the anticipated output in spite of an excessive gas consumption. An engine which converts heat to work efficiently rejects a smaller quantity of heat than a less efficient engine when the loads are the same.

It was only after the discovery of the mechanical equivalent of heat that it was possible to determine what fraction of a given heat supply could be usefully converted to work, and compare this fraction with the heat equivalent of the actual work done.

The term "thermal efficiency" means the ratio of the useful work (net load) of the engine to the heat used in driving the engine in the same time, and is found from the fuel consumption  $C$  per horse-power as follows—

If the calorific value of the fuel is  $H$   $\frac{C.H.}{B.Th.U.}$ , lb. or per ft.<sup>3</sup>, the amount of heat used per h.p. hour is  $CH$   $\frac{B.Th.U.}{h.p.}$ . If this were completely converted to work, the amount obtained would be  $1400CH$  ft. lb. Actually the work delivered is  $33,000 \times 60$  ft. lb., so that the thermal efficiency is

$$\eta = \frac{60 \times 33000}{1400CH} = \frac{1415}{CH}$$

If the heat quantities are measured in B.Th.U., the expression is

$$\eta = \frac{60 \times 33000}{778CH} = \frac{2545}{CH}$$

The heat required per h.p. hour, i.e. the product  $W = CH$ , is also used as a measure of the heat usefully converted, and may be called the "economy." The smallest imaginable value of this would be  $W = 2545$ . In the two following examples it is 23,200 B.Th.U./h.p. hour, and 10,279 B.Th.U./h.p. hour respectively. For the latest steam locomotive the figure is about 12,000 B.Th.U., for high pressure steam plants about 9200 B.Th.U., and for internal combustion engines about 8000 B.Th.U.

If the load is given in kilowatts and the consumption as  $C$  lb. (or ft.<sup>3</sup>) per kW. hour,

then 
$$\eta = \frac{3414}{CH}$$

so that  $W = CH = \frac{3414}{\eta}$  B.Th.U./kW. hour

The values of  $\eta$  and  $W$  enable comparisons to be made between steam prime movers and internal combustion engines for any state of the steam and for any fuel.

**Example 1.** A steam locomotive uses 2.0 lb. of coal per h.p. hour, having a calorific value of 12,600 B.Th.U./lb.

Find the fraction of the heat energy contained in the coal which is usefully converted to work.

$$\eta = \frac{2545}{2 \times 12600} = 0.10 \text{ or } 10 \text{ per cent.}$$

**Example 2.** A gas engine uses 18 ft.<sup>3</sup> of gas per h.p. hour.  $CV$  of gas 571 B.Th.U./ft.<sup>3</sup>. Find  $\eta$ .

$$\eta = \frac{2545}{18 \times 571} = 0.248 \text{ or } 24.8 \text{ per cent.}$$

**Example 3.** A steam engine uses 22 lb. of steam per h.p. hour. If the heat given to the steam in the boiler is 1160 B.Th.U./lb. find the fraction of heat transformed to mechanical work.

Here 
$$\eta = \frac{2545}{22 \times 1160} = 0.10 \text{ or } 10 \text{ per cent.}$$

## SECOND LAW OF THERMODYNAMICS

**Carnot cycle.** (With gas as the working substance.) A fundamental difference exists between the transformation of mechanical work to heat (as, for example, with friction between solids or liquids) and the reverse process, in which heat is transformed to mechanical work.

A given quantity of mechanical work, delivered, say, by a water turbine, a falling weight, or a moving mass, can be entirely transformed to heat by means of friction, and, as such, can be measured calorimetrically. When care is taken to avoid any loss of heat, it is then found that the mechanical energy  $E$  is equivalent to the heat generated or  $Q = E$

In the reverse process, however, such as occurs in a steam or gas engine, neither the indicated nor the brake work is equivalent to the heat  $Q$ , supplied to the cylinder. Experience shows that only a fraction (5 to 20 per cent in steam engines and up to 35 per cent in gas engines) of the heat is transformed to work, however much care is taken to effect as complete a transformation as possible.

Hence, in order to measure the equivalent factor  $\frac{1}{1400}$  by means of a known heat supply to, and work delivered by, a heat engine, it is necessary



to find the unconverted heat carried away partly by the exhaust steam (or exhaust gases and cooling water) and partly by direct conduction and radiation. If these amount to  $Q_2$ ,

then  $Q_1 = AE + Q_2$

so that  $A$  can be determined. It is obvious that considerable difficulties will be encountered in trying to find  $A$  in this way.

The reason for this incomplete conversion of heat to work can be explained, and the amount determined, as follows—

The first law merely states that when work is obtained from heat in any way the ratio of this work  $E$  to the converted fraction  $Q$  of the total heat  $Q_1$ , supplied to the working substance, is constant and independent of the manner in which the work is obtained. This ratio is  $\frac{E}{Q} = 1400$ . The law says nothing, however, about the amount of this

converted or convertible fraction  $Q$ . It is quite wrong to assume that the greatest work obtainable is  $1400 Q_1$ . The law also fails to give any information regarding the possibility of being able to transform a larger fraction than  $Q$  to work, by altering the operating conditions. Thus, when gas and steam engines are compared, a considerably greater portion of the heat supplied is converted to work in the gas engine, but this result could not have been anticipated from the first law alone.

It will thus be seen that the first law in itself is incomplete in dealing with such thermodynamic problems, and it becomes necessary to introduce a second independent law of far-reaching importance.

The basis of this law was really established by Carnot, who recognized the necessity of establishing a temperature difference in order to obtain mechanical work from heat. In spite of the enormous amount of heat contained in substances, due to radiation from the sun, practically none of it is available, since these substances are all at approximately the same temperature. Experience shows that combustion processes create the highest possible temperature differences, and these are accordingly adopted in steam boiler firing and internal combustion engines. Some of the high temperature heat, developed in this way, can be transformed to work, which is delivered by the engine.

Carnot, however, was unable to establish a relation between the available work in a given quantity of heat and the temperature drop, since a knowledge of the equivalent law is required for this. After the discovery of the latter, Clausius and Lord Kelvin established the second law of thermodynamics.

Clausius showed that, while, from Carnot's principle, a temperature drop must accompany a delivery of work from heat, part of the heat, equivalent to the work done, always disappears. It does not necessarily follow, of course, that work is obtained whenever a temperature drop occurs. Thus, no work is done, under ordinary conditions, by radiation or conduction. But if the heat is forced to deliver work in a special arrangement, such as an engine, with an accompanying temperature drop, the equivalent fraction of heat which is converted to work always disappears as heat.

When work is delivered by a heat engine, the heat changes from a high to a low temperature. Thus, the exhaust steam from a steam

engine is always colder than the supply steam. The reverse occurs in refrigerating machines, in which substances are cooled below the temperature of the surroundings. In order to do this, it is necessary to withdraw heat from these substances and deliver it to the warmer surroundings. As the substances are more and more cooled, these quantities of heat must be raised from a continuously decreasing temperature level to that of the surroundings. If water is to be changed to ice, after its temperature has been reduced to the freezing point, the latent heat of fusion has to be taken from it and raised to the temperature of the surroundings before it can be transferred to the surroundings. This "heat transfer" is just as impossible without the application of mechanical work as is the gaining of mechanical work from heat without an

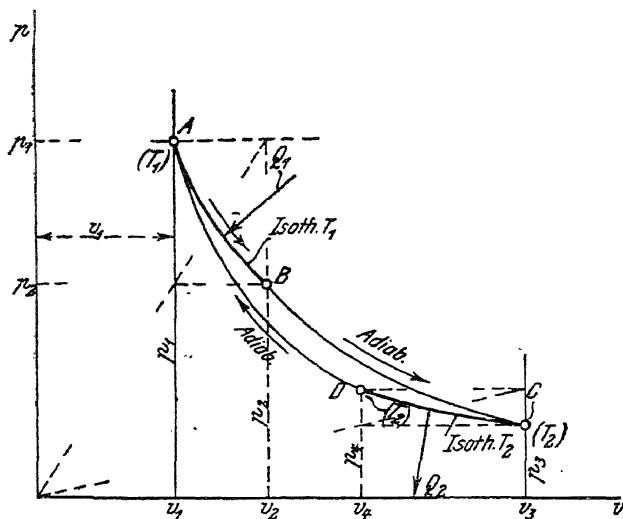


FIG. 74

equivalent conversion of heat. In the same way it is also impossible for heat by itself, i.e. without the application of mechanical work, to increase its temperature above that of the surroundings.

**Conditions which limit the amount of mechanical work obtained from heat.** In order to determine how much work  $E$  can be obtained, under the most favourable conditions, from a quantity of heat  $Q$  when a temperature drop occurs with a fixed upper limit  $T_1$  and a fixed lower limit  $T_2$ , consider the following process in which any gas may be used as the working substance.

Let the pressure  $p_1$  and temperature  $T_1$  be given for 1 lb. of the gas, point  $A$  (Fig. 74). This mass of gas is now allowed to expand isothermally to  $B$  in a cylinder, having walls which are heat conductors, and performs external work. For this operation, a quantity of heat  $Q_1$ , calculable as shown on page 103, must be supplied. This heat may be considered as flowing from a relatively large source of heat contained in a fluid surrounding the cylinder, so that no appreciable drop in temperature occurs in the fluid due to the delivery of heat. From  $B$  onwards the gas expands

without any heat being given to, or taken from, the gas (i.e. adiabatically) and continues to deliver work till the temperature drops to the lower limit  $T_2$ . During this stage the walls have to be regarded as being non-conductors of heat.

In order to return the gas to its original state, from the state  $C$ , so that the cycle may be closed and the same gas used repeatedly (cyclic process), the following processes are introduced.

The gas is first compressed at the constant lower temperature  $T_2$ , so that a quantity of heat  $Q_2$ , calculable as shown on page 103, is removed from it. The cooling substance can be regarded as a fluid which surrounds the cylinder (the walls of which are now heat conductors), and the amount of this fluid is so large that no appreciable rise above  $T_2$  occurs. Finally, the gas is compressed adiabatically in order to bring it back to its original state at  $A$  on the diagram. This is arranged by choosing the point  $D$ , at the end of isothermal compression, such that it lies on the adiabatic through  $A$ . The cycle which the gas has described is merely a means of obtaining the work  $E$ , which is represented by the closed area  $ABCD$ . Since no change has occurred between the initial and final state of the gas, the work must have come from the heat which disappeared in the operation of the cycle. Since  $Q_1$  units of heat were given to, and  $Q_2$  units taken from the gas, it follows that  $Q_1 - Q_2$  units have disappeared. This heat has been transformed to the equivalent work  $E$ , and hence, from the first law,

$$AE = Q_1 - Q_2 = Q_1 \left( 1 - \frac{Q_2}{Q_1} \right)$$

The heat supplied to the gas, during the isothermal expansion  $AB$  (Fig. 74), is, from page 104,

$$Q_1 = ART_1 \log_e \frac{v_2}{v_1}$$

and, similarly, the heat removed during isothermal compression  $CD$ , is

$$Q_2 = ART_2 \log_e \frac{v_3}{v_4}$$

The following relations apply to the adiabatics  $BC$  and  $AD$ ,

$$\frac{T_1}{T_2} = \left( \frac{v_2}{v_3} \right)^\gamma \quad \text{and} \quad \frac{T_1}{T_2} = \left( \frac{v_1}{v_4} \right)^\gamma$$

hence 
$$\frac{v_2}{v_3} = \frac{v_1}{v_4} \quad \text{or} \quad \frac{v_2}{v_1} = \frac{v_3}{v_4}$$

Now 
$$\frac{Q_2}{Q_1} = \frac{T_2 \log_e \frac{v_3}{v_4}}{T_1 \log_e \frac{v_2}{v_1}} = \frac{T_2}{T_1}$$

so that 
$$AE = Q_1 \left( 1 - \frac{T_2}{T_1} \right)$$

and the thermal efficiency is

$$\eta = \frac{AL}{Q_1} = 1 - \frac{T_2}{T_1}$$

The fraction  $\eta$  of the heat supplied  $Q_1$  is thus converted to work in the Carnot cycle, and is dependent only on the absolute temperatures between which the process occurs.

Writing the expression for the efficiency in the form

$$\eta = \frac{T_1 - T_2}{T_1}$$

it will be seen that  $\eta$  depends not only on the temperature drop, but also on the absolute value of the higher temperature.

The whole of the heat supplied  $Q_1$  would only be converted to work if  $\frac{T_2}{T_1} = 0$ , i.e. if  $T_2 = 0$ . The withdrawal of heat would thus occur at  $-273^\circ \text{C.}$ , which is impossible, since cooling substances do not exist at this temperature.

The lowest temperatures of the substances used in removing heat from engines are those of the usual cooling water and range from  $5^\circ$  to  $20^\circ \text{C.}$  Assuming an average value of about  $10^\circ \text{C.}$  gives

$$T_2 = 10 + 273 = 283^\circ \text{C. abs.}$$

With a lower temperature of  $293^\circ \text{C.}$ , and the following upper temperature values, we have—

1200	1000	800	600	400	200	100
1473	1273	1073	873	673	473	373
0.81	0.77	0.73	0.66	0.56	0.38	0.21

If the heat, whose transformation to work is desired, is at the same temperature as its surroundings, i.e. if  $T_1 = T_2$ , then none of it can be transformed to work by means of the Carnot cycle. The heat of the surroundings is thus of no use for the production of mechanical work.

This does not mean that it is impossible for these quantities of heat to be transformed to work. As shown on page 99, this transformation is possible. If, for example, a store of compressed air is available and is allowed to expand isothermally in a cylinder, by taking up heat from the surroundings, then all this heat is transformed to work. In this, however, the compressed air is reduced to the pressure of the atmosphere, so that, in addition to heat, a supply of compressed air is required. This compressed air must, however, have been previously raised in pressure, for which an expenditure of work at least equal to, and in actual cases greater than, the delivered work is required.

As opposed to this process, the working substance in the Carnot, and every other closed cycle, is found to be in the same state finally as it was initially. After the air has been compressed the cycle can be repeated as often as desired, so that the initial work of compression becomes negligibly small compared with the work delivered. This initial compression work could also be re-delivered in an air motor after any number of repetitions of the cyclic process. Hence, no compressed air is required

in the cyclic process; all that is necessary is the heat supply  $Q_1$ . The fraction  $(1 - \eta) Q_1 = Q_2$  of the heat supply drops to the lower temperature  $T_2$  and is carried away by the cooling water at this temperature. Hence the law remains as before. Heat is only capable of delivering work if it is at a temperature above that of the surroundings.

**Carnot cycle with gases having variable specific heat.** It is of basic importance to determine whether or not the efficiency of the Carnot cycle alters when, in place of an ideal gas, an actual gas with its varying properties, or a superheated or saturated vapour, is used as the working substance. Actual gases possess specific heats, which vary with the temperature even though they obey the ideal gas law  $pv = RT$  almost exactly. Hence the upper adiabatic curve in the Carnot cycle has a different exponent  $\gamma$  from that of the lower curve, as shown on page 60, and both exponents are not quite constant. The pressure diagram, therefore, is different in form and the treatment given above does not apply in this case.

The treatment is simplified by considering the  $T\phi$  diagram of the Carnot cycle. Since the entropy conception applies to gases even if their specific heats  $C_p$  and  $C_v$  vary with temperature, it follows that the  $T\phi$  diagram for both variable and constant specific heat is of the same form (Fig. 75). The isothermal  $AB$  of the pressure diagram in Fig. 74 is represented by the horizontal line  $A'B'$  in Fig. 75, the adiabatic  $BC$  by the vertical line  $B'C'$ , the isothermal  $CD$  by the horizontal line  $C'D'$ , and the adiabatic by the perpendicular line  $D'A'$  (see page 127). The rectangle  $A'B'\phi'\phi''$  represents the heat supplied  $Q_1$ , while the rectangle under  $C'D'$  gives the heat discharged  $Q_2$ , so that  $A'B'C'D' = Q_1 - Q_2$  is the heat converted to work, and corresponds to the area  $ABCD$  in the pressure diagram. No matter what shape the adiabatics have in the  $pv$  field, they remain vertical on the  $T\phi$  field, so that  $A'B'C'D'$  retains its rectangular form and size, and the efficiency is given by

$$\eta = \frac{Q_1 - Q_2}{Q_1} = \frac{A'B'C'D'}{A'B'\phi'\phi''}$$

Hence the efficiency of the Carnot cycle remains unaltered whether or not the specific heats of the working gases are constant, so long as the gas law  $pv = RT$  is followed.

**Reverse Carnot process.** (Warming or refrigerating process.) It is of interest to discover whether this efficiency is also given by any other working substance which does not obey the gas law, in which case it will be a property of the heat alone. This can only be done, as first shown by Clausius, by making use of a new law, the basis of which will be made clear by considering the reversed Carnot cycle. Starting at  $A$  in Fig. 74, air is expanded adiabatically to  $D$ , so that the temperature drops from  $T_1$  to  $T_2$ . The air is then allowed to expand isothermally at  $T_2$  to  $C$ ,

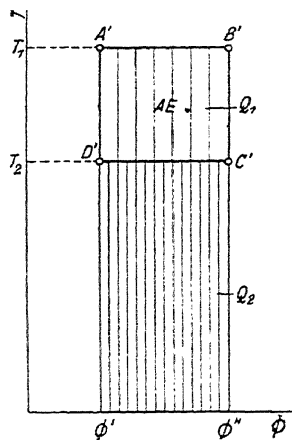


FIG. 75

where the volume is  $v_3$ . In this isothermal expansion the same quantity of heat  $Q_2$  has to be added to the air as was previously taken from it in the motor process. From  $C$  to  $B$  the air is compressed adiabatically so that its temperature rises from  $T_2$  to  $T_1$ . Finally, the air is compressed at the constant temperature  $T_1$  from  $B$  to  $A$ , where it is again at the initial state and the cycle is completed. In the last isothermal compression the same quantity of heat  $Q_1$  has to be withdrawn from the air as was supplied to it in the motor process. The result of this process is that a quantity of work, equal to the area  $ABCD$ , has been supplied from the outside, a quantity of heat  $Q_2$  has been withdrawn from the cold source at the lower temperature  $T_2$  and a quantity of heat  $Q_1$  has been delivered to the hot source at the upper temperature  $T_1$ . Hence the hot source has been heated at the expense of the cold source. The same relations hold between the quantities of work and heat as in the motor process, so that, applying the law of the conservation of energy, we have

$$Q_1 = Q_2 + AE$$

or the heat  $Q_1$  given to the upper heat container is greater than the heat  $Q_2$  taken from the lower heat container by the heat equivalent of the work done  $E$ .

Since, as before,  $\frac{Q_1}{Q_2} = \frac{T_1}{T_2}$  and therefore  $AE = Q_2 \frac{T_1 - T_2}{T_2}$ , it will be recognized that, in order to deliver the heat  $Q_2$  from the cold to the hot source, i.e. from the lower temperature  $T_2$  to the higher temperature  $T_1$ , a quantity of work  $E$  is required, which is proportional to the temperature difference  $T_1 - T_2$ . If, for example, at  $T_2 = 300^\circ \text{C. abs.}$  ( $27^\circ \text{C.}$ )  $T_1 - T_2 = 15^\circ \text{C.}$ , the work necessary would be

$$AE = \frac{15}{300} Q_2 = \frac{1}{20} Q_2$$

while for  $T_1 - T_2 = 1200$

$$AE = \frac{1200}{300} Q_2 = 4Q_2$$

Conversely, the quantity of heat which can be raised from  $T_1$  to  $T_2$ , with a given quantity of work  $AE$ , is

$$Q_2 = AE \frac{T_2}{T_1 - T_2}$$

so that, in the two examples above,

$$Q_2 = 20AE$$

and

$$Q_2 = \frac{1}{4} AE$$

The conveyed heat quantities can, accordingly, be a large multiple or only a fraction of the heat equivalent of the work done, depending on the amount of the required temperature rise.

The total increase in heat  $Q_1$  of the hot source is greater than  $Q_2$  by the amount  $AE$ , so that, in the two cases,

$$Q_1 = 21AE$$

and

$$Q_1 = \frac{5}{4} AE$$

The heat delivered to the hot source amounts, therefore, in the first case almost entirely to the heat given up by the source (20 out of 21 parts), while, in the latter case, most of the heat is due to the mechanical work (4 out of 5 parts). Hence, in the latter case, not much more heat is obtained with the reversed Carnot process than would be obtained by direct braking, whereas in the former case 21 times as much would be obtained. Hence, when mechanical work is available (e.g. water power or electrical energy), considerable heating effects can be obtained by means of the Carnot cycle with a relatively small expenditure of mechanical or electrical energy, provided the temperature rise is moderate (from 50 to 100° C., say).

This method of heating was first proposed by Lord Kelvin, and can be described as "reversible heating," since it is effected, as opposed to heating by mechanical friction or electrical resistance, by means of a reversible process, such as the Carnot.\*

In its practical application, the reverse process has been chiefly used for refrigeration. Thus, taking the normal air temperature of  $t_1 = 15^\circ \text{C}$ . or  $T_1 = 288^\circ \text{C}$ . abs., the lower temperature produced by adiabatic expansion ( $AD$  in Fig. 74) is conditioned by the chosen pressure ratio  $p_1/p_4$ . By using brine, which freezes only at low temperatures, as the cold source, a sufficient repetition of the cycle enables the brine to be reduced to, say,  $-20^\circ \text{C}$ . The brine can then be maintained at this temperature in spite of the natural flow of heat into it from outside simply by continuing to remove the quantity of heat  $Q_2$  by means of the reverse process.  $Q_2$  is called the "refrigerating effect," and the process itself "refrigeration."

**The second law.** Regarding the heat contained in our surroundings, the result of the above is expressed by the following law. Heat, at normal temperatures, cannot by itself be raised to a higher temperature without an expenditure of mechanical work, and, similarly, can only be taken from substances at a lower temperature than that of the surroundings by means of mechanical work, which thus enables this heat to be transferred to the surroundings. Clausius recognized that this law applies generally and is independent of the properties of the working gases. This law, along with the previously established law, which states that the heat contained in our surroundings can deliver no mechanical work in cyclic processes, using gas as the working substance, forms the basis of the second law of thermodynamics. In its most general sense, the law can be expressed as follows. It is impossible for a prime mover, using the heat of its surroundings alone, without any other driving agent, to deliver work.

A prime mover of this type would be described as a perpetual motion of the second class, since it would continuously deliver work by converting the enormous stores of heat in the surroundings without, however, violating the law of the conservation of energy, as is the case in the usual conception of perpetual motion. The shortest way of expressing the second law is thus as follows. As shown by natural laws, perpetual motion of the second class is impossible.

\* See *Zeitsch. f. d. ges Kälteindustrie* (1919), E. Altenkirch. "Die Erhöhung der Wirtschaftlichkeit von Heizungsanlage durch den Einbau von Kältemaschinen." (Increased economy of heating plants by the introduction of refrigerators.) The principle has been applied lately in the "heat pump."

**Carnot cycle using any working substance.** By means of the second law the effect on the efficiency of the Carnot cycle when substances other than gases are used can now be determined. If, for instance, saturated steam or ammonia vapour be used, the diagram shown on the  $pv$  field (Fig. 76) is totally different from that shown in Fig. 74, since the isothermals are now horizontal lines. Starting, for example, with the liquid at the boiling point, the pressure at I corresponds to the temperature  $T_1$  of the heat source and IV represents the volume increase during evaporation. At III the vapour is usually wet and the line III II represents the decrease in volume due to condensation. Fig. 76 can be regarded as the Carnot diagram for an engine using saturated vapour. The same

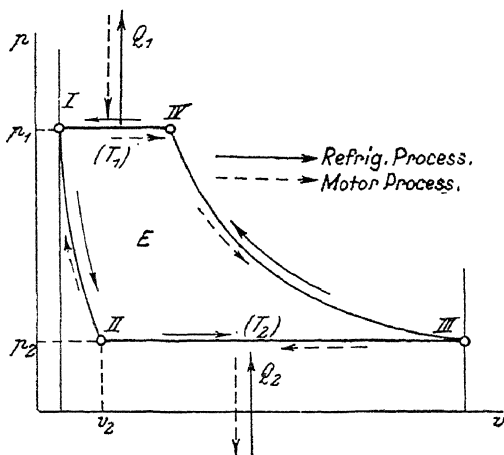


FIG. 76

diagram, in a reverse sense, represents a refrigerating process using a saturated vapour such as ammonia.

The quantities of heat  $Q_1$  and  $Q_2$  given to, and taken from, the working substance from I to IV and III to II are the evaporation and condensation heats respectively. The shape of the two adiabatics IV, III and II, I is, however, unknown at present, which means that the area of the diagram is also unknown, so that the useful work and efficiency of the process cannot be determined.

It can, however, be shown in another way that, for the same temperature limits, the efficiency of the Carnot cycle using vapours is the same as that using gases. Thus, consider a refrigerating process using gas, and let it be driven by a motor process using vapour. Let both processes work between the same temperature limits and let the work delivered by the motor be  $E$ . Due to the refrigerator, a certain quantity of heat  $Q_2'$  would be transferred from the cold to the hot source. If now  $Q_2'$  is greater than the heat  $Q_2$  given to the cold source by the gas operating in the motor process, the net result of the whole process would be that a quantity of heat  $Q_2' - Q_2$  would be transferred from the cold to the hot source without an expenditure of work, since  $+E - E = 0$ . This, however, would be contrary to the second law. Again, by means of the heat  $Q_2' - Q_2$  at the temperature  $T_2$ , mechanical work could be done by



making use of the temperature drop  $T_1 - T_2$ , and in this way the heat of the cold source, i.e. of the surroundings, could be converted to work. This, again, is contrary to the second law, so that  $Q_2'$  cannot be greater than  $Q_2$ . If now  $Q_2'$  is less than  $Q_2$  it would only be necessary to perform the motor process with the gas and the refrigerating process with the vapour in order to prove in the same way that  $Q_2$  cannot be greater than  $Q_2'$ . Hence,  $Q_2'$  must equal  $Q_2$ . Now, from the first law, we have, for the motor process using vapour,

$$Q_1 - Q_2 = AE$$

and, for the refrigerating process using gas,

$$Q_1' - Q_2' = AE$$

hence,  $Q_1 - Q_2 = Q_1' - Q_2'$

and, since  $Q_2' = Q_2$  it follows that  $Q_1'$

and also  $\frac{Q_1 - Q_2}{Q_1} = \frac{Q_1' - Q_2'}{Q_1'} = \eta$

i.e. the value of  $\eta$  is the same in both processes, so that, as with gases,

$$\eta = \frac{T_1 - T_2}{T_1}$$

The thermal efficiency of a Carnot cycle is accordingly independent of the nature of the working substance and depends on the temperatures of the hot and cold sources alone.

If steam is used and the heat supplied  $Q_1$  is equal to the latent heat  $L$  per lb. of water at the saturation temperature ( $T_1$ ) so that from I to IV in Fig. 76 the substance changes from water to dry saturated steam, the work obtained in the Carnot process is

$$AE = L \frac{T_1 - T_2}{T_1} = \eta L$$

Thus with  $t_2 = 60^\circ \text{ F.}$ , and

$t_1$	=	212	302.5	401.2	477	
$L$	=	971.4	908.7	821.9	740.1	(Steam tables)
$\eta$	=	0.226	0.318	0.396	0.445	
$AE$	=	220	289	326	330	B.Th.U./lb.

The actual operations in a steam engine are not the same as the Carnot cycle, chiefly owing to the difficulty of compressing the very wet condensed steam along II I, but the Carnot process is useful in showing the largest amount of available work which can be obtained from saturated steam.

The Carnot process is not applicable to superheated steam so long as the superheating is effected in the usual way, i.e. at constant pressure and increasing temperature, and is, therefore, not generally used as a standard of comparison under these conditions.

### ABSOLUTE TEMPERATURE

Before the second law was established nothing was known regarding the lower limit of temperature, and this appeared to be just as inconceivable as an upper limit of temperature. The conception of an "absolute" temperature, which is rather more complete than that found convenient in dealing with calculations in the Gay-Lussac Law, is only reasonable, however, if a lower limit of temperature exists which is independent of the substance of which the thermometer is made and of the temperature measurements. This degree of cold limits the lower attainable value, not only from a practical, but also from a theoretical point of view.

The Carnot-Clausius principle, as expressed in the representation of the Carnot cycle, enables this lower limit of temperature to be conceived, and by combining this principle with the energy law the absolute zero point can be determined.

When efforts are made to convert a given quantity of heat  $Q_1$ , supplied by a large heat source at  $t_1$ , to mechanical work by means of the Carnot cycle, the difficulty (which cannot be overcome by using any other cycle) immediately arises that (when the usual temperature limits are employed) only a fraction of  $Q_1$  is available for mechanical work. Thus, if  $t_2$  is the cooling water temperature, the available work is

$$\left( \frac{273}{273 + t_1} \right) Q_1$$

In practice, therefore,  $t_1$  has to be made as high as possible.

If, however, it were assumed that cooling substances were available, at any temperature below  $0^\circ \text{C.}$ , the heat in the surroundings could be made to perform work with any desired efficiency.

Thus, with  $t_1 = 100^\circ \text{C.}$ , and

$t_2 =$	$0^\circ \text{C.}$	$-100^\circ \text{C.}$	$-200^\circ \text{C.}$	$-273^\circ \text{C.}$	$-300^\circ \text{C.}$
$\eta =$	0.268	0.536	0.805	1.0	1.07

Hence, with a cooling temperature of  $-273^\circ \text{C.}$ , the whole of the heat supplied in the process could be converted to useful work. This would also be the case with any higher or lower initial temperature  $t_1$ , since,

$$\text{with } t_2 = -273$$

$$\eta Q_1 = \frac{t_1}{t_1 + 273}$$

for every value of  $t_1$ .

If, now, a lower temperature than  $-273$  exists, it is possible to conceive that a cooling agent could be procured at this temperature. If we assume this to be  $-300^\circ \text{C.}$ , the results above show that the efficiency would then be 1.07, and for any temperature  $t_2$  less than  $-273^\circ \text{C.}$  the efficiency is more than unity. It would thus be possible to obtain more work from the heat  $Q_1$  than corresponds to the mechanical heat equivalent. This, however, would be contrary to the first law and to the principle of the conservation of energy. The lowest attainable temperature is, therefore,  $-273^\circ \text{C.}$ , and this corresponds to the absolute zero point on the Centigrade scale.

Direct temperature measurements, carried out by the application of special arrangements (expansion of liquefied gases), have shown that this temperature can be approached within a few degrees, but the measured temperature never falls below it. Thus when liquid helium, which has a liquid temperature of  $-268.5^{\circ}\text{C}$ . at atmospheric pressure, is cooled by expansion it is only 2 or 3 degrees above the absolute zero point.

The measurement of these very low temperatures is effected by means of gas thermometers (by measuring the change in pressure due to cooling a mass of gas when the volume remains constant). It is necessary that the gases should be at a very low pressure at these temperatures, in order to prevent them from assuming the vapour or liquid state. According to J. Dewar, it is possible to measure with accuracy the following low temperatures with gas thermometers, containing the different gases shown—

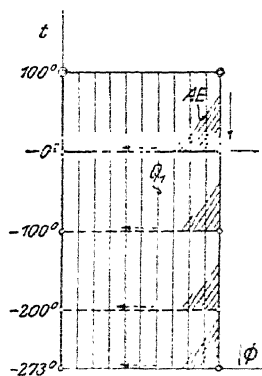


FIG. 76A

Gas Used	Initial Pressure of Gas in mm. Hg at $0^{\circ}\text{C}$ .	Lower Measurable Temperature	
		$T^{\circ}\text{C. abs.}$	$t^{\circ}\text{C.}$
$\text{CO}_2$ . . . . .	$\frac{1}{2}$ 273	171.4	$-101.6$
$\text{O}_2$ . . . . .	$\frac{1}{3}$ 273	69.2	$-203.8$
$\text{H}_2$ . . . . .	$\frac{1}{2}$ 273	11.72	$-261.28$
$\text{H}_2$ . . . . .	$\frac{1}{10}$ 273	10.40	$-262.60$
Helium . . . . .	$3.8 \times$ 273	2.95	$-270.05$
Helium . . . . .	$\frac{1}{2} \times$ 273	1.90	$-271.10$
Helium . . . . .	$\frac{1}{4} \times$ 273	1.71	$-271.29$

By the evaporation or sublimation of the following liquids or solids at very low pressures, the low temperatures shown alongside are attained (as given by Dewar).

	$p$ (mm. Hg)	$T^{\circ}\text{C. abs.}$	$t^{\circ}\text{C.}$
Ice ( $\text{H}_2\text{O}$ )	0.001	203	$-70$
$\text{CO}_2$	0.63	132	$-141$
$\text{O}_2$ .	0.19	51	$-222$
$\text{N}_2$ .	0.70	37	$-236$
$\text{H}_2$ .	0.59	9	$-264$
Helium	0.26	1.8	$-271.2$
	—	0.4	$-272.6$

These temperatures amount, as shown by Dewar, to about a third of the absolute critical temperatures of the corresponding substance.  $X$  is a hypothetical substance which is as “volatile” compared with helium as helium is compared with hydrogen.

**Temperatures given by the gas thermometer and the absolute temperature scale.** Strictly speaking, gases do not obey the characteristic equation  $p_v = RT$  completely, and the deviations from this increase as the gas approaches the critical region or the saturated state. The equation

$$\frac{p_1}{p_2} = \frac{T_1}{T_2}$$

used in gas thermometry applies only to an ideal gas, so that it is only with such a gas that a truly accurate temperature measurement can be made. With the different gases used in gas thermometry, a given unknown temperature change shows thermometer readings (i.e. pressure ratios  $p_1/p_2$ ), which are not quite the same, so that if the temperature is calculated from these readings, different values are obtained. At normal or high temperatures the deviations are slight, but at low temperatures they become more serious, unless the pressures are made very low. According to Henning,\* for example, the readings of a nitrogen thermometer are lower than those of a hydrogen thermometer by the amounts shown below when the pressure is between 620 and 780 mm. Hg.

At	445° C.	- 99° C.	- 183° C.	- 193·6° C.
About	0·11°	0·00°	0·19°	0·26°

Neither hydrogen nor helium behave as perfect gases, both deviating considerably from Boyle's Law, even at low pressures.

Temperature scales between 0° and 100° C., as given by means of a nitrogen or hydrogen thermometer deviate, therefore, from one another and from the absolute scale obtained by using a perfect gas. The problem of temperature measurement on the absolute scale, which involves considerable practical and theoretical difficulty, is of more importance to the physicist than to the engineer.†

From the latest researches‡ it appears to be established that the scale of the hydrogen thermometer agrees sufficiently closely with the ideal gas scale between 0° C. and 450° C. The deviation at 450° C. is only about 0·01° C. (Constant volume hydrogen thermometer with an initial pressure of 620 mm. Hg.)

The following boiling and melting or freezing points of pure liquids and solids serve for the purpose of calibrating thermometers—

Boiling point of—

Oxygen at 760 mm. Hg	. . . . .	$t_s = -183·0° \text{ C.}$
Oxygen at the pressure $p$ in the neighbourhood of 760 mm.	. . . . .	$t_s = -183·0 + 0·01258(p - 760) - 0·0000079(p - 760)^2° \text{ C.}$
Carbon dioxide at 760 mm. Hg	. . . . .	$t_s = -78·5° \text{ C.}$
and at $p$ as above	. . . . .	$t_s = -78·5 + 0·01595(p - 760) - 0·000011(p - 760)^2° \text{ C.}$

Freezing points of—

*Ethyl ether	. . . . .	$t_f = -123·6° \text{ C.}$
*Carbon disulphide	. . . . .	$t_f = -112·0° \text{ C.}$
*Chloroform	. . . . .	$t_f = -63·7° \text{ C.}$
*Chlorobenzene	. . . . .	$t_f = -45·5° \text{ C.}$
Mercury	. . . . .	$t_f = -38·89° \text{ C.}$
Water	. . . . .	$t_f = 0° \text{ C.}$

Boiling point of—

Water at 760 mm. Hg	. . . . .	$t_b = 100° \text{ C.}$
Naphthalene	. . . . .	$t_b = 217·96 + 0·058(p - 760)° \text{ C.}$
Benzene phenolate	. . . . .	$t_b = 305·9 + 0·063(p - 760)° \text{ C.}$
Sulphur	. . . . .	$t_b = 444·55 + 0·0908(p - 760) - 0·000047(p - 760)^2° \text{ C.}$

\* See *Tätigkeit der Phys. Techn. Reichsanst.* für 1914.

† For a full discussion on scientific thermometry, see F. Henning's *Grundlagen, Methoden und Ergebnisse der Temperaturmessung* (1915).

‡ *Tätigkeit der Phys. Techn. Reichsanst.* (1911).

Melting and freezing points of—

Tin .	= 231.84° C.
Cadmium .	= 320.9° C.
Zinc ‡ .	= 419.4° C.
Antimony .	= 630.0° C.
Silver .	= 960.5° C.
Gold .	= 1063° C.
Copper .	= 1083° C.
Palladium .	= 1557° C.
Platinum .	= 1764° C.

With the exception of those\* marked with a star the above have been taken from the *Bekanntmachung der Physik. Techn. Reichsanst. vom 16 Dez. über die Prüfung von Thermometern*. The platinum resistance thermometer can be used to interpolate between 0° C. and the boiling point of sulphur. For higher temperatures the platinum and 10 per cent platinum rhodium thermocouple may be used, while for low temperatures a platinum resistance thermometer corrected by means of the hydrogen thermometer may be used.

**Entropy of any substance.** It has been shown on page 125 that, for gases which obey the law  $p v = RT$ , a magnitude of state exists called entropy ( $\phi$ ), the change in which, multiplied by the absolute temperature, gives the heat supplied. In the particular case of an isothermal change of state,

$$Q = T(\phi - \phi_0)$$

or 
$$\phi - \phi_0 = \frac{Q}{T}$$

and, for an adiabatic change of state,

$$\phi - \phi_0 = 0$$

Since the Carnot process consists of isothermal and adiabatic changes of state, and the relations deduced from this cycle are valid for any type of working substance (page 162), it follows that this cycle enables us to determine whether the entropy function also applies to substances other than gases.

In the Carnot cycle we have, in general,

$$\frac{Q_1}{Q_2} = \frac{T_1}{T_2}$$

or 
$$\frac{Q_1}{T_1} = \frac{Q_2}{T_2}$$

in which  $Q_1$  and  $Q_2$  are the heats supplied to, and taken from, the substance along the upper and lower isothermals respectively. Substituting the letters  $\phi_1$  and  $\phi_2$  (without assuming that these represent entropy values) for the quotients  $\frac{Q_1}{T_1}$  and  $\frac{Q_2}{T_2}$ , and plotting these values, as abscissae, at the temperatures  $T_1$  and  $T_2$  (i.e. on the isothermals), the points  $B'$  and  $C'$  are obtained (Fig. 78). The line joining these two points along with the horizontals  $B'A'$  and  $C'D'$  give the rectangle  $A'B'C'D'$ . The area

\* From Henning, *Annal. Physik* (1914).

lying below  $A'B'$  to the abscissae axis is equal to  $\phi_1 T_1$  and is thus equal to  $Q_1$ , while the area below  $C'D'$  is  $\phi_2 T_2 = Q_2$ . Hence the rectangle  $A'B'C'D'$  is equal to  $Q_2 - Q_1$ , i.e. the work obtained from the process in heat units. Now the shape, shown in Fig. 78, remains the same (whatever shape the  $pv$  diagram may have) in the Carnot process for all substances, and if  $Q_1$  and  $Q_2$  remain the same, the magnitudes are also equal for all substances.

Consider a second Carnot cycle, using the same working substance, at the same initial state but operating with a smaller heat supply  $Q_1'$ . The resulting  $pv$  diagram is given by  $AB_1C_1D$  in Fig. 77. If the values  $\frac{Q_1'}{T_1} = \phi_1'$  and  $\frac{Q_2'}{T_2} = \phi_2'$  are plotted in Fig. 78, the points  $B_1'$  and  $C_1'$  are obtained, giving the rectangle  $A'B_1'C_1'D'$ . Any number of such processes can be imagined with different heat supplies, giving, when

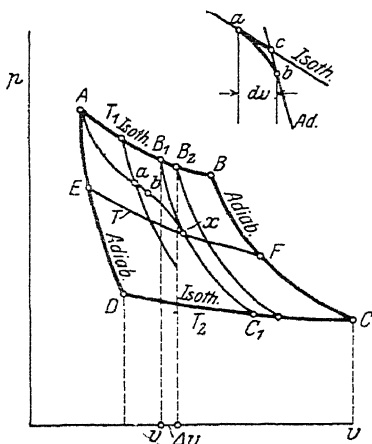


FIG. 77

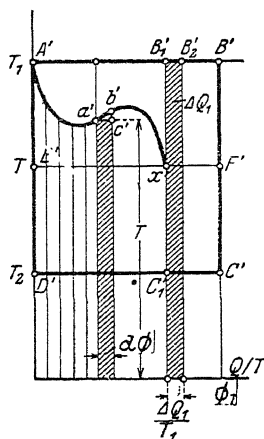


FIG. 78

plotted in Fig. 78, any number of points like  $B_1'$  lying on  $A'B'$  and corresponding to definite points on the isothermal  $AB$ . The abscissae  $\frac{Q_1'}{T_1}$  of these points depend on the heat supplied  $Q_1'$  between  $A$  and the point  $B_1$  and the temperature of the isothermal.

If, again, the process be carried out with the lower temperature raised from  $T_2$  to  $T$  so that  $Q_2'$  is greater than  $Q_2$ , the  $pv$  diagram is now shown by  $ABFE$  and when transferred to Fig. 78 gives the rectangle  $A'B'F'E'$ , where  $F$  corresponds to  $F'$ . The point  $F'$  lies vertically below  $B'$  and possesses the same value  $\frac{Q_1}{T_1}$  as  $B'$  does, since  $F$  is the final point of the lower isothermal  $EF$ , for which the value  $\frac{Q_2'}{T}$  is the same as the value  $\frac{Q_1}{T_1}$  of the upper isothermal  $AB$ . In the same way all other points on the adiabatic  $BC$  give corresponding points on the vertical line  $B'C'$ . Hence  $B'C'$  represents the adiabatic curve  $BC$ . Every such point on

$B'C'$  has the same abscissa  $\frac{Q}{T} = \frac{Q_1}{T_1}$  which is given by the point  $B$  at the end of the upper isothermal or the initial point of the adiabatic. Hence the value  $\frac{Q}{T}$  remains constant along the adiabatic, so that for any substance, operating on the Carnot cycle, there exists a function, the value of which remains constant if the substance is expanded adiabatically (while delivering work). Also, for any point  $x$  in the  $pv$  field (Fig. 77) there corresponds a definite value  $\frac{Q}{T}$ , which is found by carrying out the Carnot cycle  $AB_1xE$ , and which is equal to  $\frac{Q_1'}{T_1}$  (the value found from the upper isothermal). Hence, by tracing the quite arbitrary path  $Aax$  from  $A$  to  $x$  the same value is always obtained for  $\frac{Q}{T}$ , since this depends on the position of  $B_1$  alone. By plotting the values of  $\frac{Q}{T}$  in Fig. 78 corresponding to any points such as  $a$  on  $Ax$ , a definite line  $A'a'x'$  is obtained, so that any state curve on the  $pv$  field has a corresponding curve in the  $T\phi$  field, which can be located if the heats  $Q$ , supplied during an isothermal change of state, are known. From this it follows that any change which occurs in  $\phi$ , accompanying a change of state, depends on these states alone and not on the path traversed during the change. In the particular case in which the temperature remains constant but the pressures and volumes vary, the difference in  $\phi$  is given directly by considering the quantity of heat  $\Delta Q_T$  supplied during the change. Thus, as shown in Fig. 78, when the difference in the value of  $\frac{Q_1}{T}$  is taken for the distances  $AB_2$  and  $AB_1$  in Fig. 77, we have

$$\Delta\phi_T = \frac{\Delta Q_T}{T}$$

or

$$\Delta Q_T = T \Delta\phi_T$$

i.e. the heat added, during an elementary isothermal change in state of any substance, is equal to the product of the absolute temperature and the change in the function  $\phi$  and is thus given by the area below  $B_1'B_2'$  in Fig. 78.

This can be extended to include any change of state, as is the case with gases, since in any elementary change of state, such as the expansion  $ab$  (Fig. 77), the heat supplied is the same as in the isothermal expansion  $ac$ , where  $c$  lies on the adiabatic through  $b$ . This follows, since the path  $acb$  in Fig. 77 is represented by  $a'c'b'$  in Fig. 78 and the heat  $dQ_T$  supplied in the isothermal expansion  $ac$  is equal to the heat  $dQ$  supplied along  $ab$  and is given by the area below  $a'c'$ .

In the limit the small triangle is negligible compared with the strip, and we then have  $dQ = Td\phi$ . . . . . (189)

In a finite change of state such as  $Aax$  the heat supplied is the sum of all these strips and equals the area below  $A'a'x'$ , or

$$Q = \int_A^x Td\phi \quad (190)$$

In this way it is shown that, for any substance which changes its state in any way, the heat supplied can again be represented by an area on the  $T\phi$  diagram. Hence the function  $\phi$  possesses all the properties of the entropy function of gases and can, therefore, be regarded itself as entropy.

Equation (189) or (190) is the simplest mathematical expression of the second law, as applied to any substance experiencing a reversible change of state.

For further discussions on the entropy function see page 187 and 204.

## INTERNAL ENERGY OF SUBSTANCES

All substances contain a certain amount of internal energy ( $I$ ) irrespective of their state. The absolute amount\* of this energy in a given weight cannot, however, be determined at a given state (fixed by  $p$ ,  $v$ , and  $T$ ), e.g. in 1 lb. of air at  $100^\circ\text{C}$ . and 75 lb./in.<sup>2</sup> abs., but can only be expressed as the excess above that at some arbitrarily chosen standard state, such as  $0^\circ\text{C}$ . and 14.7 lb./in.<sup>2</sup> abs. (as is usual in the case of gases), or the state of liquid at  $0^\circ\text{C}$ . (as is usual with vapours and liquids).

There is no advantage, at least in technical work, in choosing the absolute zero point of temperature as the standard temperature, since the condition of substances at that point is not fully known and it is quite convenient to regard substances at temperatures below  $0^\circ\text{C}$ . as containing a negative quantity of energy.

If the substance happens to be in motion, so that it possesses kinetic energy, this is not included in its internal energy, since this is regarded more as external work, as shown in the chapter on "The Flow of Gases."

In the following the term "internal energy" of a substance will be regarded as the sum of the quantities of heat and mechanical work (in heat units) which a definite weight (such as 1 lb.) of the substance can deliver externally (or can receive from outside) in being brought from its existing state to the standard state.

**Energy as a magnitude of state.** If the above definition of energy is to be consistent, it must give the same value of  $I$ , irrespective of the nature of the changes of state during the reduction to the standard state.

Thus, a gas at the state  $A$  (Fig. 79), defined by  $p$ ,  $v$ , and  $T$ , can be reduced to the standard temperature  $0^\circ\text{C}$ . at  $B$  by adiabatic expansion. If, now, the gas be compressed isothermally from  $B$  to  $A_0$  it can be reduced to the standard pressure  $p_0$ . During the expansion an amount of work

\* Regarding this, Planck states, in *Phys. Zeit.* (1912), page 168 (Modern Thermodynamic Theories): "According to the modern Lorentz-Einstein principle of relativity, the absolute amount of energy of a body at rest, if the external pressure is neglected, is equal to the product of its mass into the square of the propagating velocity of light in a vacuum; an extremely large number, which, up till the present, however, has not proved of any practical significance."

It is also known that, in the breaking down of radium, very considerable quantities of energy are liberated from within the atom, regarding the existence of which nothing was known till lately. This energy is capable of performing work, but only in so far as the temperature of the body exceeds that of the surroundings. The heat which can be obtained from 1 lb. of the latest, specially prepared, pure radium is 132.3 C.H.U., provided the whole of the radiation is transferred to heat.

The amount of radiant heat obtainable from the sun per minute on 1 ft.<sup>2</sup> at right angles to the rays, amounts to from 3.7 to 4.1 C.H.U.



$E_1$  is delivered, while during compression the work  $E_2$  has to be done on the gas and a quantity of heat  $Q$  has to be withdrawn. The total change of energy is thus

$$I = +AE_1 - AE_2 + Q$$

But, for gases, it is shown on page 105 that

$$Q = AE_2$$

so that

$$I = AE,$$

The energy of a gas is thus the same as the absolute work delivered by it, in an adiabatic expansion, from its initial temperature to the standard temperature. If the initial temperature is the same as the standard temperature, the internal energy is zero, however high the pressure may be. If a gas, at any temperature, be expanded or compressed isothermally, its internal energy suffers no change. From page 106 we have the following expressions for  $I$ .

$$I = \frac{A}{\gamma - 1} (pv - p_0 v_0)$$

$$\text{or } I = c_p(T - T_0) = c_p(t - t_0)$$

and if  $t_0 = 0$

$$I = c, t$$

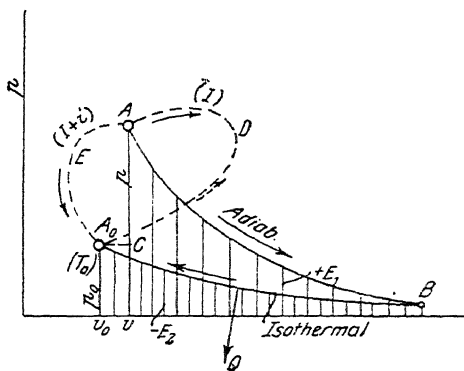


FIG. 79

The internal energy thus depends on temperature alone,

and the same result would be obtained along any other path such as  $AEA_0$  or  $ADA_0$ .

This agreement is a result, however, of the assumptions made in establishing the energy equation for gases (page 99), namely, that  $c_v$  is independent of pressure and temperature, and that no internal heat is absorbed by the gas apart from that required in raising its temperature. Both assumptions are only valid within certain limits for gases and not at all for vapours.

In spite of this, however, the energy given up by any substance in being reduced to the standard state, is again independent of the path followed.

Thus, imagine the substance, which has been brought to the normal state  $A_0$  along any path  $ADA_0$ , to be returned to the old state  $A$  along the same path. The same energy  $I$  will be returned as was previously liberated, since the change of state occurs with no losses, i.e. it is reversible at all points. This condition is easily fulfilled in isothermal and adiabatic processes.

It might, however, be considered that along some other path such as  $AEA_0$  a greater quantity of energy  $I + i$  could be liberated than before. By allowing the working substance to suffer this change of state from  $A$  to  $A_0$  and then returning it along the path  $A_0DA$  to  $A$ , the substance would again be in exactly the same state, so far as the pressure,

volume, and temperature are concerned, but an excess of energy would be liberated along  $AEA_0$  given by

This could be repeated any number of times and as much energy could be liberated as desired without any corresponding expenditure. The same applies to the heat absorbed  $I - i$  when the process is reversed.

This creation of energy is, of course, impossible, and the process amounts to a perpetual motion of the first class. Hence it follows that  $i = 0$  and the energy  $I$  is independent of the path chosen in determining it, and depends only on the instantaneous state of the substance. Internal energy, therefore, like pressure, volume, and temperature, serves to define the state of a body.

**Nature of the energy contained in substances.** The energy stored in gases exists solely as sensible heat (apart from a negligibly small amount in other forms). In the case of saturated vapours, however, the sensible heat is a relatively small fraction of the total energy contained in the vapour. This becomes apparent from the consideration of an evaporative process (page 358). The heat required for evaporation becomes latent in the vapour itself. It is required to separate the molecules and only reappears as heat when the vapour is reduced to a liquid. It is stored in the vapour as "internal potential energy."

This potential energy is also present in gases, but, owing to the relatively large distance apart of the molecules, it is small compared with the heat energy which is used to produce the kinetic energy of the molecules.

Under certain conditions, excessive heating can effect a breakdown in the molecules of compound gases (i.e. dissociation).\*

For example, carbon dioxide can be split up into carbon monoxide and oxygen. Experiments show, in this case again, that the heat becomes latent, so that the "splitting" of the molecules requires an expenditure of energy.

It is necessary to regard the internal energy  $I$  of a body, therefore, as the sum of its sensible and internal potential energy. (The external energy, which a substance possesses in virtue of its motion and weight, is not included in  $I$ .)

## CHANGE OF STATE OF A GAS OR VAPOUR

### FIRST LAW

Let the pressure and volume of any gas or vapour change as shown by the curve  $ACB$  in Fig. 80. Let the heat supplied be  $Q$ .

Due to the change in state, the energy of the substance increases by an amount  $I_2 - I_1$ , and the absolute work of expansion  $E$  is represented by the area  $ACBDE$ . The heat supplied must suffice for the change of energy (which may be negative) and the work done.

\* According to modern chemical theories, a degree of dissociation always exists (although it may be very small) in dissociable gases at every temperature and pressure. The term "dissociation temperature" usually means the temperature at which an appreciable degree of dissociation is present. See Schüle's *Tech. Thl* Bd. II, and *Neue Tabellen*.

$$\text{Hence, } Q = I_2 - I_1 + AE \quad . \quad (191)$$

and, for an elementary change of state,

$$dQ = dI + AdE$$

$$\text{or } dQ = dI + Apdv \quad (192)$$

The following special cases of this equation will now be considered—

1. For a change of state at constant volume  $dv = 0$ , so that  $dQ = dI$ , or the heat added or removed is equal to the change in internal energy.

2. With  $d\phi = 0$ , i.e. an adiabatic change,

$$dI = -Apdv$$

or the absolute work is equal to the decrease in internal energy.

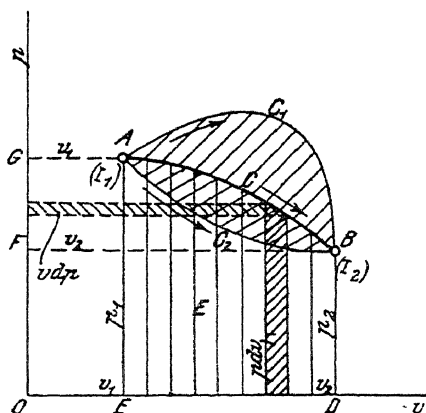


FIG. 80

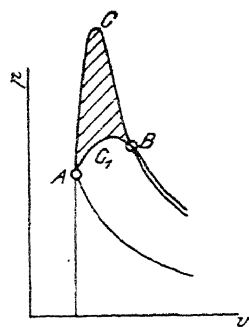


FIG. 81

3. If  $I_2 = I_1$  or  $I_2 - I_1 = 0$ , the change of state occurs at constant internal energy, in which case

$$Q = AE$$

or

$$dQ = Apdv$$

i.e. the heat supplied during expansion is equal to the heat equivalent of the absolute work delivered during expansion. For gases, whose internal energy depends only on the temperature, this change of state coincides with the isothermal.

4. If the change on the  $pv$  field is as shown by  $AC_1B$  (Fig. 80) or  $AC_2B$  instead of  $ACB$ , no change occurs in  $I_2 - I_1$ , since  $I_2$  and  $I_1$  are determined solely by the limiting states  $A$  and  $B$ . The mechanical works are, however, different, being greater along  $AC_1B$  and less along  $AC_2B$ . This means that, in the above equation, the heat quantities  $Q_1$  and  $Q_2$ , supplied *via*  $C_1$  and  $C_2$ , are greater and less respectively than that supplied *via*  $C$ . The difference is given by the heat equivalent of the areas shown shaded.

If, for example, the explosion line of a gas engine, in one case, is as shown by  $ACB$  in Fig. 81, and in another case as shown by  $AC_1B$ , the heat, given to the walls, is less in the latter case than in the first by the

heat equivalent of the shaded area. This may be caused either by a reduced supply of fuel gas or by retarded combustion (in which case the continuing line of  $AC_1B$  on the  $pv$  field lies above that of  $ACB$ ) or by incomplete combustion. It may also be caused by a combination of these conditions.

5. The heat  $Q$  added to a gas in any change of state such as  $ACB$  in Fig. 80 can also be represented by an area on the  $pv$  field (Fig. 82). Thus, instead of allowing the gas to expand from  $A$  to  $B$  directly, first expand it isothermally from  $A$  to  $C$  (Fig. 82) and then return it along the adiabatic

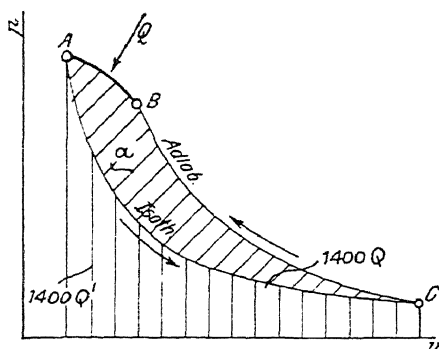


FIG. 82

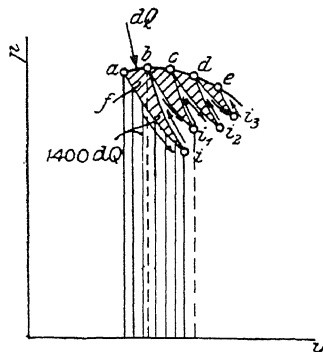


FIG. 83

to  $B$ . In this case the heat  $Q'$  added is smaller than the heat  $Q$  added in the direct expansion, and the difference is equal to the heat equivalent of the obliquely shaded area  $a$  ( $ACB$ ) or

$$Q' = Q - \Delta a$$

and

$$Q = Q' + \Delta a$$

Heat is neither supplied nor rejected along the adiabatic  $BC$ , so that  $Q'$  is the heat supplied along the isothermal  $AC$ . For gases this heat is equal to the absolute work and is thus given by the area below  $AC$  in heat units.  $Q$  is therefore the complete shaded area below  $ABC$  down to the abscissa axis.

In this way, quantities of heat, supplied or rejected during certain periods, such as combustion, expansion, or compression in a gas engine cylinder, can be determined simply by measuring areas on the  $pv$  diagram. In the case of vapours, the isothermal shown in Fig. 82 is replaced by a curve for which  $\bar{I} = \text{constant}$ . (See case 3.)

6. Every finite change of state can be represented by a successive series of elementary changes of state  $ab, bc, cd, \dots$  (Fig. 83). The heat supplied along  $ab$  is obtained, in the case of gases, by measuring the area under  $abi$  (see 5), where  $ai$  is an isothermal and  $bi$  an adiabatic. By drawing the points  $a, b, c, d, \dots$  closer and closer together, the small three-cornered areas  $abi, bci, \dots$  become small compared with the strips lying below them. The sum of these small areas is, in the limiting case, negligible compared with the sum of the strips lying below the isothermals.

Hence, so far as heat quantities are concerned, any finite change of state can be replaced by a succession of elementary alternate isothermals

and adiabatics, or the continuous curve  $abcde$  can be replaced by the zig-zag lines  $ai, bi_1, ci_2$ .

This also applies to any kind of vapour if, in place of the isothermals, a curve of constant internal energy is substituted (Fig. 84), which passes through  $a$  and meets the adiabetic through  $b$  at  $i'$ . The heat supplied along  $ab$  is greater than that supplied along the isothermal  $ai$  by the area  $abi$  (see 5), while the heat supplied along this isothermal ( $ai$ ) is equal to the area below  $aii'$ , as shown in case 6 (Fig. 83). In the limiting case the small triangle becomes negligible compared with the heat added along the isothermal. For saturated vapours the  $I$  curve lies below the horizontal isothermals (Fig. 85).

7. Any kind of state changes, accompanied by heat supplies, can be substituted in place of the small isothermals  $ai, bi_1, \dots$  in Fig. 83. Hence

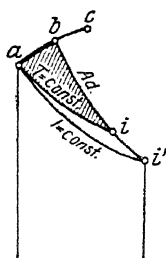


FIG. 84

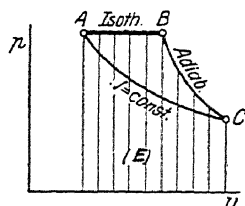


FIG. 85

a continuous change of state with heat supply can be replaced by any consecutive series of elementary changes of state, with heat supply, alternating with adiabatics. The case in which the changes of state are isothermals and adiabatics is, however, of special significance in connection with the elementary Carnot cycle (see page 185).

**Second form of the first law. Total heat at constant pressure.** In any change of state  $AB$ , Fig. 80, the absolute work  $ABDE = E$  can also be expressed by the areas  $GABF = E'$ ,  $FBDO = p_2v_2$ , and  $GAEO =$

$$\text{Thus} \quad = E' + p_2v_2 - p_1v_1$$

$$E' \text{ is equal to } - \int_{p_1}^{p_2} v \, dp.$$

The negative sign is required since, for decreasing pressures (i.e.  $dp$  negative), the product  $v \, dp$  is negative, whereas  $E'$  has to be positive.

The first law

$$\text{then becomes} \quad Q = I_2 - I_1 + Ap_2v_2 - A \quad AE'$$

$$\text{or} \quad Q = (I_2 + Ap_2v_2) - (I_1 + p_1v_1) - A \int_{p_1}^{p_2} v \, dp$$

$I_1$  and  $I_2$  depend only on the states  $A$  and  $B$ , i.e. on  $p_1, v_1, T_1$ , and  $p_2, v_2, T_2$ , so that the same applies to the sums  $(I_1 + Ap_1v_1)$  and  $(I_2 + Ap_2v_2)$ . These quantities are independent of the nature of the change between  $A$  and  $B$  and depend merely on the instantaneous state.

By writing

$$I_1 = I_1 + A p_1 v_1$$

and

$$= I_2 - A p_2 v_2$$

we have

$$Q = H_2 - H_1 - A \int_{p_1}^{p_2} v dp \quad . \quad (193)$$

For an elementary change of state this becomes

$$dQ = dH - A v dp \quad . \quad . \quad . \quad . \quad . \quad (194)$$

The quantity  $H$  is called the "total heat." For a change of state at constant pressure ( $p_2 = p_1 = p$ )  $v dp = 0$  and  $\int_{p_1}^{p_2} v dp = 0$ , so that the area  $E'$  is reduced to a straight line  $p_1$ . We then have

$$Q = H_2 - H_1$$

The difference between the values of  $H$ , in two different states having the same pressure, is, therefore, equal to the quantity of heat required to change the substance from the one state to the other, while the pressure remains constant.

**Evaluation of  $H$  for gases and vapours.** For gases, at the initial state, we have

$$\begin{aligned} H_1 &= I_1 + A p_1 v_1 \\ &= c_v t_1 + A p_1 v_1 \end{aligned}$$

and since  $p_1 v_1 = RT_1$

$$H_1 = c_v t_1 + A R T_1$$

Similarly, at the final state,

$$H_2 = c_v t_2 + A R T_2$$

hence  $H_2 - H_1 = c_v(t_2 - t_1) + A R(T_2 - T_1)$

or, since,  $t_2 - t_1 = T_2 - T_1$

$$H_2 - H_1 = (c_v + A R) (T_2 - T_1)$$

But, since  $c_v + A R = c_p$ ,

$$H_2 - H_1 = c_p(T_2 - T_1) \quad . \quad . \quad . \quad . \quad . \quad (195)$$

Again, since,  $p_1 v_1 = RT_1$ ,  $p_2 v_2 = RT_2$  and  $\frac{c_p}{A R} = \frac{\gamma}{\gamma - 1}$ ,

we have  $H_2 - H_1 = A \frac{\gamma}{\gamma - 1} (p_2 v_2 - p_1 v_1) \quad . \quad . \quad . \quad . \quad . \quad (196)$

In an adiabatic change of state  $Q = 0$ , so the first law becomes

$$0 = H_2 - H_1 + A E'$$

or

$$E' = -\frac{H_1}{\gamma} \quad (\text{for all substances}).$$

This quantity  $E'$  is of importance in engine processes and in problems on flow through nozzles. The method of determining the heat drop for steam turbines from the  $H\phi$  Mollier diagram depends on this equation.

For further discussion on the quantitative determination of total heat and its change in adiabatic process, see pages 64 and 455. In this section only the graphical representation on the  $pv$  field is considered. If  $AB$  (Fig. 86) is an adiabatic, the area below it is equal to the change in internal energy, while the area between  $AB$  and the ordinate axis gives the change of total heat. For gases the latter is (in heat units)  $c_p(t_2 - t_1)$  and the former  $c_v(t_1 - t_2)$ .

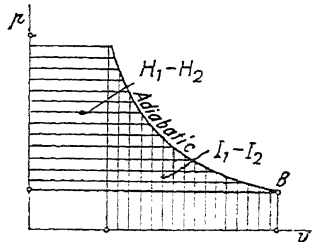


FIG. 86

**Available work in constant pressure and constant volume processes.** The available work in  $W$  lb. of gas at the pressure  $p_1$  and temperature  $T_1$  means the useful work obtained from the gas when it expands adiabatically to the external pressure  $p_2$ . Two main cases can be considered here.

1. The working substance is supplied by a large container in which the pressure remains constant at  $p_1$ .

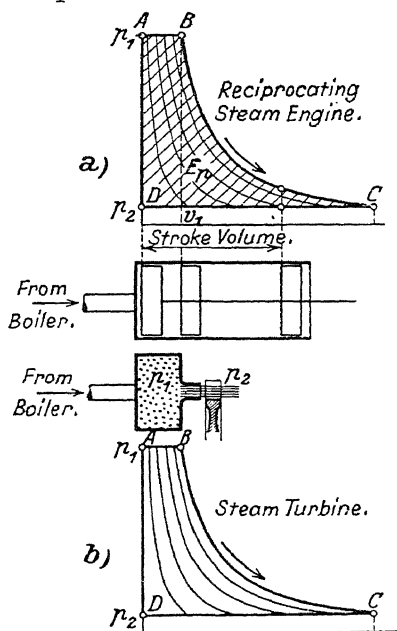


FIG. 87

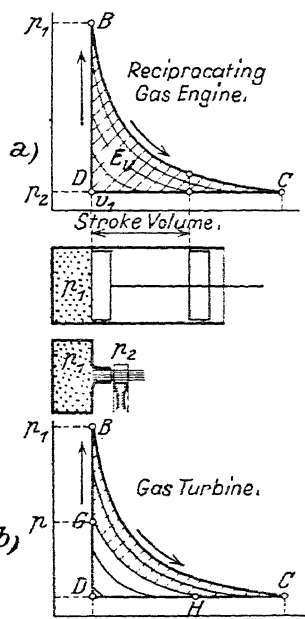


FIG. 88

2. The pressure  $p_1$  of the working substance is first established in a working chamber of constant volume.

Examples of the first case are the compressed air motor, the reciprocating steam engine, and the steam turbine (Fig. 87, *a* and *b*). The second case is represented by the gas engine operating on the Otto cycle, and by the explosion gas turbine (Figs. 88, *a* and *b*).

The available work, in the first case, is given by the area  $ABCD$  on the  $pv$  field. Air motors and steam engines generally deliver a part only of this work, because of incomplete expansion. The steam turbine, however, is capable of complete expansion and the whole of the weight  $W$  experiences the complete pressure drop.

In Fig. 88 the available work is given by the area  $BCD$ , since no work is done at constant pressure (i.e. there is no cut-off). In the case of the gas engine this work is reduced, since the expansion is incomplete. In the gas turbine, however, the whole mass of gas expands to the lower pressure. If, in the latter case, the explosion chamber is emptied until the pressure drops to  $p > p_2$ , then, up to this point the work delivered is represented by the fraction  $BCHG$  of the total available work, where

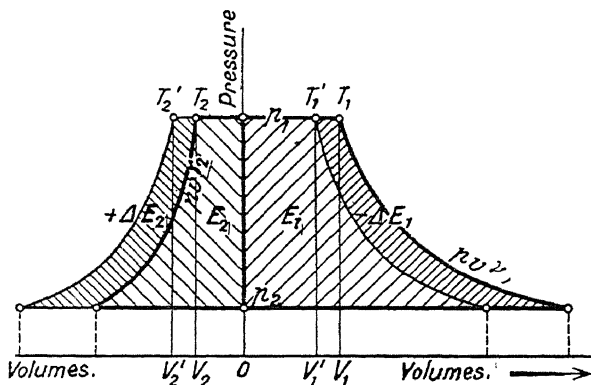


FIG. 89

$GH$  is the adiabatic through  $G$ . Each small pressure drop  $\Delta p$  in the explosion chamber corresponds to a strip between two adiabatics on the  $pv$  field.

In the first case (constant pressure) the available work, as shown above, is

$$H_1 - H_2 = AE_p$$

and in the second case (constant volume) it is

**Maintenance of available work during heat exchange at constant pressure.** Two gases, weighing  $W_1$  and  $W_2$  lb., having the volumes  $V_1$  and  $V_2$  ft.<sup>3</sup> temperatures  $t_1$  and  $t_2$  °C. with  $t_1 > t_2$ , and at the same pressure  $p$  possess a total available work, which is given by the sum of their available works  $E_1 + E_2$  (Fig. 89).

If now, before delivering work, the colder gas II takes up part of the heat from gas I, we have to determine here the new sum of the available works.

Let the warmer gas I be cooled from  $T_1$  to  $T'_1$  so that it gives up the heat  $W_1 c_{p1} (T_1 - T'_1)$ . The colder gas is raised in temperature from  $T_2$  to  $T'_2$ , so that it absorbs the heat  $W_2 c_{p2} (T'_2 - T_2)$ . These heats are equal or



$$\text{which gives } \frac{W_1}{W_2} = \frac{c_{p2} (T'_2 - T_2)}{c_{p1} (T_1 - T'_1)} \quad . \quad . \quad . \quad . \quad . \quad . \quad (197)$$

From the characteristic equations,

$$p_1 V_1 = W_1 R_1 T_1$$

$$p_1 V_2 = W_2 R_2 T'_2$$

$$\text{hence } \frac{W_1}{W_2} = \frac{V_1 R_2 T_2}{V_2 R_1 T_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad (198)$$

Equating equations (197) and (198) gives

$$\frac{c_{p2} (T'_2 - T_2)}{c_{p1} (T_1 - T'_1)} = \frac{V_1 R_2 T_2}{V_2 R_1 T_1}$$

$$\text{and since } \frac{T_2}{T'_2} = \frac{V_2}{V'_2}; \quad \frac{T'_1}{T_1} = \frac{V'_1}{V_1} \text{ and } \frac{R_2}{R_1} = \frac{m_1}{m_2}$$

$$\text{then } \frac{V'_2 - V_2}{V_1 - V'_1} = \frac{m_1 c_{p1}}{m_2 c_{p2}} = \frac{C_{p1}}{C_{p2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (199)$$

$V'_2 - V_2$  is the increase in volume of gas II due to its being heated, while  $V_1 - V'_1$  is the reduction in volume of gas I due to its being cooled. These volume changes are inversely proportional to the molecular specific heats, so that if these are equal, the changes in volume are equal, and the total gas volume remains the same. If the molecular specific heats are different, as, for example, if gas II is cold air at  $20^\circ \text{C.}$  and gas I is hot air, or hot products, at  $1200^\circ \text{C.}$ , giving  $C_{p2} < C_{p1}$ , the increase in volume of the cold gas is greater than the decrease in volume of the hot gas, so that the sum of the two volumes is increased by the heat interchange.

The original available work  $E_1$  of gas I is now reduced by the amount  $\Delta E_1$ , while the original work  $E_2$  of the hot gas is increased by the amount  $\Delta E_2$  (Fig. 89).

For the same gas, the available works before and after the heat exchange are proportional to the initial volumes,

$$\text{i.e. } \frac{E_1}{E_1 - \Delta E_1} = \frac{V_1}{V'_1}$$

$$\text{and } \frac{E_2}{E_2 + \Delta E_2} = \frac{V_2}{V'_2}$$

from which it follows that

$$\begin{aligned} \frac{\Delta E_1}{\Delta E_2} &= \frac{E_1 V_2}{E_2 V_1} \left( \frac{V_1 - V'_1}{V'_2 - V_2} \right) \\ &= \frac{E_1}{E_2} \frac{V_2}{V_1} \frac{C_{p2}}{C_{p1}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (200) \end{aligned}$$

Now, 
$$E_1 = p_1 V_1 \frac{\gamma_1}{\gamma_1 - 1} \left[ 1 - \left( \frac{p_2}{p_1} \right)^{\frac{\gamma_1 - 1}{\gamma_1}} \right]$$

and 
$$E_2 = p_1 V_2 \frac{\gamma_2}{\gamma_2 - 1} \left[ 1 - \left( \frac{p_2}{p_1} \right)^{\frac{\gamma_2 - 1}{\gamma_2}} \right]$$

which, along with equation (200), gives

$$\frac{\Delta E_1}{\Delta E_2} = \frac{1 - \left( \frac{p_2}{p_1} \right)^{\frac{\gamma_1 - 1}{\gamma_1}}}{1 - \left( \frac{p_2}{p_1} \right)^{\frac{\gamma_2 - 1}{\gamma_2}}}$$

Hence, if both gases have the same adiabatic exponents  $\gamma_1 = \gamma_2 = \gamma$ , then  $\Delta E_1 = \Delta E_2$

and the total available work remains unaltered.

If, on the other hand, the exponents are different, e.g. for air at 20° C.  $\gamma_2 = 1.40$ , and for hot products at 1200° C.  $\gamma_1 = 1.27$ , then for

$$\frac{p_2}{p_1} = \frac{1}{3}, \frac{1}{6}, \frac{1}{12}$$

$$\frac{\Delta E_2}{\Delta E_1} = 1.292, 1.263, 1.239$$

Hence, the available work of the cold gases increases by a greater amount than that of the hot gases decreases, and the total available work increases, due to the heat exchange, by the amount  $\Delta E_2 - \Delta E_1$ , giving the ratio of available work after and before the exchange as

$$\eta = 1 + \frac{\Delta E_2 - \Delta E_1}{E_1 + E_2}$$

In the example given above  $\eta$  is greater than 1, and this always occurs when the adiabatic exponent of the hot gases is smaller than that of the cold, and hence always in heat exchange between the same gases and also between air and hot products.

The above relations apply also, with fair accuracy, to heat exchange, using hot products containing steam, since superheated steam follows the adiabatic law  $pv^{1.3} = c$ , and also obeys the characteristic equation, approximately. It should be noted, however, that it has been assumed that  $c_p$  is independent of temperature, although this only applies within a limited range of temperature. A more exact treatment can be carried out by the heat entropy chart for gases or steam, since, in this way, the values of  $E_1$ ,  $E'_1$ ,  $E_2$ , and  $E'_2$  are directly determined.

For an important application of the above see page 608.

**Cyclic processes.** The conversion of heat to work in cylinders of steam and internal combustion engines is effected by the pressure of the



Heat is now supplied to the air so that, at the inner dead centre position, the initial pressure  $p_0$  is again attained.

Let the heat supplied, from the outside, between I and II be  $Q_1$ , and the heat rejected along III  $c d$  IV be  $Q_2$ . The useful work  $E$  is represented by the closed area of the diagram.

A working process of this type is called a "cyclic process," because the working substance, after suffering a series of pressure, volume, and temperature changes, is in the same state finally as it was initially. If the process is repeated  $n$  times the total heat supplied is  $nQ_1$ , the heat rejected is  $nQ_2$ , and the useful work is  $nE$ .

The process of an actual engine is different from the above in that the pressure drop along III  $c d$  IV is not caused by removing heat from a constant weight of the charge, but by removing some of the gas or vapour from the cylinder. Again, in gas or oil engines, the heat is supplied internally instead of externally, and in steam engines the portion I II of the diagram represents admission of live steam to the cylinder. Consider, for example, the diagram of the Carnot cycle using steam (Fig. 76), which can be regarded as the indicator diagram of a steam engine. In the theoretical cycle the line I IV then corresponds to the heating of the water (evaporation), while in the actual engine it represents the admission to the cylinder of a previously evaporated quantity of steam. Line III II represents the condensation of the steam in the cyclic process, whereas in the actual engine it represents the exhausting of the steam from the cylinder. So far as the delivery of work is concerned, it is immaterial which method of supply and withdrawal of heat is considered. The nature of the heat conversion, however, is much more easily followed if the mass and chemical properties of the working substance are regarded as constant. It is only necessary to ensure, then, that the actual process of the engine is replaced by a cyclic process which gives the same pressure diagram. (See Schüle's *Tech. Ther.*, Bd. II, Art. 47.)

The heat supplied to the working substance during a cycle is  $Q_1$ , and the heat withdrawn  $Q_2$ . At the end, therefore, if  $Q_2 < Q_1$  the excess heat  $Q_1 - Q_2$  might be contained in the gas, or if  $Q_2 > Q_1$ , the heat lost,  $Q_2 - Q_1$ , might be taken from the gas. Neither of these cases is possible, since the final state of the working substance is the same as the initial state. We would have, therefore (as Carnot assumed),  $Q_2 = Q_1$ .

But the useful work  $E$  would then be obtained without an expenditure of heat, and this contradicts the first law. Hence it follows that  $Q_2$  must be less than  $Q_1$ .

The heat  $Q = Q_1 - Q_2$  has accordingly disappeared during the process, i.e. it has been transformed to work, and since the heat equivalent of the work done is  $AE$ , we have

$$AE = Q_1 - Q_2$$

Of the heat supplied, therefore, between I and II, at the high temperatures, only the portion  $Q = AE = Q_1 - Q_2$  is transformed to mechanical work.

The unconverted portion,  $Q_2 = Q_1 - AE$ , of the supplied heat has to be removed from the working substance at a reduced temperature and must be regarded as a loss.

The quotient

is called the thermal efficiency of the cyclic process, and gives the fraction of the supplied heat converted to work.

In the cyclic processes, the adiabatics, in which heat is neither supplied nor rejected, might conceivably be absent, in which case the points II and III, as well as IV and I, would coincide, and the heat withdrawal would merge with the heat supply. This case, however, would be less general. In actual engines there are always two main periods, one in which heat is supplied and one in which heat is rejected (as in Fig. 91), while the periods II to III and IV to I can, in the ideal case, be regarded as adiabatics.

The cyclic process is merely a changing of states of the working substance, with a return to the same initial state. If, in a complete cycle, mechanical work is done, a motor process results. The first law then becomes, with  $I_1 = I_2$ ,

$$Q = AE$$

The whole heat  $Q$ , which is the algebraic sum of all the heat supplied and rejected in the process, is thus equal to the closed area of the diagram, which can have any general form like Fig. 91. If alternate heat supply and removal occurs, then  $Q = Q_1 - Q_2 + Q_3 - Q_4 \dots = AE$ , while in the case of continuous changes in the heat supply and removal,

$$\int dQ = AE$$

This is the simplest mathematical expression of the first law applied to any cyclic process.

**Reversed cyclic processes (Refrigeration).** The transfer of heat from low temperature to high temperature substances, as required in refrigeration, can be represented by cyclic processes, which operate in the reverse direction to motor cyclic processes.

For the purpose of simplicity and of comparison with practice, the process (Fig. 76) in which ammonia is the working substance will be considered.

At the beginning of the cycle (point I, Fig. 76) let the cylinder contain 1 lb. of very wet ammonia at the higher pressure  $p_1$ . Due to the outward movement of the piston, the vapour first expands adiabatically so that the pressure drops to  $p_2$  and the volume increases to  $v_2$ . At the same time a temperature drop, corresponding to the pressure drop, occurs, and some of the liquid evaporates (see page 379). From II onwards the pressure remains constant at  $p_2$  in spite of the volume increase caused by the movement of the piston. This is only possible if sufficient heat  $Q_2$  be supplied between II and III to maintain the evaporation of the ammonia. During this period no change occurs in the temperature.

On the return stroke of the piston the vapour is first compressed adiabatically to the initial pressure  $p$ , as shown by III IV. This causes the temperature to rise to the initial value, provided the vapour is not

superheated. From IV onwards the pressure remains constant, in spite of the reduction in volume due to the movement of the piston. This is only possible if the heat given up is equal to that required to condense the vapour during this reduction in volume. Let  $Q_1$  be the complete heat withdrawn between IV and I. At I the pressure, volume, and temperature have been returned to their initial values, and the cyclic process is completed.

The total work  $E$  performed on the ammonia is given by the closed area of the diagram. The process, similar to the motor process, consists of a heat supply (II III), a heat withdrawal (IV I), and two adiabatics. While, however, in the motor process, heat is supplied at the high pressure and temperature, and withdrawn at the low pressure and temperature, the reverse now takes place, and in place of the delivery of useful work the process demands a supply of useful work equal to the area of the diagram (Fig. 76). By reversing the arrows of the refrigerating process diagram, the diagram of a vapour engine is obtained.

In spite of the supply of heat  $Q_2$ , and the removal of the (greater) quantity of heat  $Q_1$ , there is, as in the case of the motor, neither an excess nor a deficiency of heat in the ammonia after completing a cycle, since the final state is the same as the initial. The excess heat withdrawn,  $Q_1 - Q_2$ , must be accounted for, therefore, by the work  $E$  done on the vapour. Hence

$$AE = Q_1 - Q_2$$

or

$$Q_1 = AE + Q_2$$

The result of the cyclic process, therefore, merely means that the quantity of heat  $Q_2$  taken up by the ammonia at the lower temperature is withdrawn from it at the higher temperature. Along with  $Q_2$  the heat equivalent of the work done  $AE = Q$  is withdrawn.

Hence, in order to raise the quantity of heat  $Q_2$  at the lower temperature  $T_2$  to the higher temperature  $T_1$ , the work  $E$  has to be performed. The heat equivalent of this work is equal to the difference between the heat carried away by the cooling water and the heat (refrigerating effect) taken up by the ammonia.

This process would not be possible in a single cylinder, as it is necessary to have separate cylinders for compression (III IV) and expansion (I II). A separate condenser and an evaporator are also required.

The actual process of vapour compression refrigerators differs from that of Fig. 76 in that the adiabatic expansion (I II) is replaced by throttling.

If the process shown in Fig. 91 is reversed, a refrigerating process of a more general nature than that just described is obtained.

Both the refrigerating and motor processes are also represented in Fig. 92. The machine is either a motor or refrigerator, depending on whether the positive or negative areas are greater.

The refrigerating process can also be applied in raising a given quantity of heat, at the temperature of the surroundings, to some higher temperature.

**Elementary cyclic processes.** If the indicator diagram of a cyclic process (Fig. 93) be split up, by a series of closely adjacent adiabatic curves, into small strips, each of these strips, bounded by two adiabatics

and by two elements of the state curve, is called an elementary cyclic process. The nature of these is the same as that of the finite process described on page 180.

If it is imagined that the working substance goes through a series of these elementary processes, one after the other, the total mechanical work delivered is the same as in the actual process. Again, the sum of the heat quantities supplied  $dQ_1$ , i.e.  $+Q_1$ , and the sum of the heat quantities removed  $dQ_2$ , i.e.  $-Q_2$ , are the same as in the actual case (see

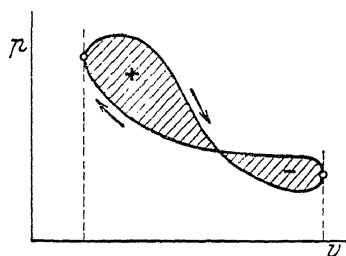


FIG. 92

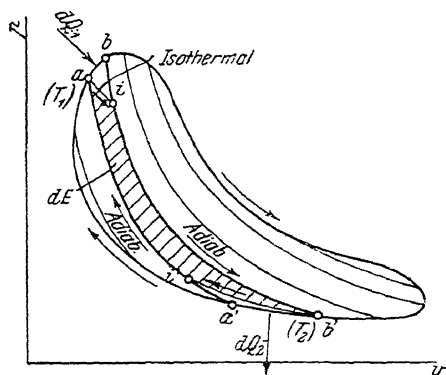


FIG. 93

page 174, 6). Hence the actual case may be considered as being made up of the sum of the elementary processes.

**Heat conversion in the elementary process.** Let the heat supplied along  $ab$  be  $dQ$ , the heat rejected along  $a'b'$  be  $dQ_2$ , and the work  $abb'a'$  of the elementary process be  $dE$ , then

$$AdE = dQ_1 - dQ_2$$

If  $ai$  is an isothermal through  $a$ , the heat supplied along  $ai$  is, as shown on page 174, only very slightly different from the actual amount  $dQ_1$  supplied along  $ab$ , and in the limiting case these quantities become equal.

Similarly, if  $b'i'$  is an isothermal through  $b'$  the heat withdrawn along  $b'i'$  is the same as that along  $b'a'$ . We have now obtained a new elementary process, to which the same heat quantities  $dQ_1$  and  $dQ_2$  apply as in the actual case, and in which the work area differs from  $dE$  by a negligibly small amount.

This proposed process, however, is a Carnot cycle, so that, as shown on page 163,

$$AdE = d\epsilon$$

where  $T_1$  and  $T_2$  are the absolute temperatures along  $ai$  and  $b'i'$  respectively.

We also have

$$\overline{dQ_2}$$

In the case of the actual elementary process we have, from page 183,

so that, here again,

$$T_2$$

The efficiency of any arbitrarily chosen elementary adiabatic process is thus equal to that of the Carnot process between the same temperature limits.

In finite cyclic processes, therefore, occurring with heat supply at varying temperature, the fractions of the heat supplied at the highest temperature will be most advantageously employed.

The following relation holds between the elementary quantities of heat  $dQ_1$  and  $dQ_2$  and the temperature limits  $T_1$  and  $T_2$

$$\frac{dQ_1}{T_1} = \frac{dQ_2}{T_2}$$

While, therefore,  $dQ_1$  and  $dQ_2$  are always different, with  $dQ_1 > dQ_2$  the above quotients are equal. Writing, therefore, in a finite cyclic process such as Fig. 93, this relationship for all the elementary processes and summing gives

$$\int_I^{II} \frac{dQ_1}{T_1} = \int_{III}^{IV} \frac{dQ_2}{T_2}$$

Hence the sum of all the quotients  $dQ/T$  decreases during the heat withdrawal by the amount that it increases during the heat supply.

If equation (204) be written in the form

$$\frac{dQ_1}{T_1} - \int_{III} \frac{dQ_2}{T_2} = 0$$

and be regarded as the algebraic sum of all the quotients  $dQ/T$  in the complete cycle, in which the elements of heat supply are regarded as positive and those of heat removal as negative, we can write

$$\int_I^I \frac{dQ}{T} = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (205)$$

Writing  $d\phi = dQ/T$ , and calling  $\phi$ , as in the case of gases on page 125, entropy, equation (205) can be expressed as follows—

The total change of entropy of the working substance in a reversible cycle is zero.

This law, or the corresponding equation (205), is the simplest expression of the second law in its application to cycles using any working substance.

In the case of a cyclic process in which each change of state is an isothermal equation (205) becomes

$$= 0$$

or

$$Q = 0$$



Since the first law gives  $\int dQ = AE$ , we also have

$$AE = 0$$

Hence, in a purely isothermal cyclic process, the sum of the heat quantities and the total useful work are both zero.

Such cyclic processes cannot be carried out with physical changes of state alone, but consist of physical and chemical changes of state. An important application of this case for saturated vapours is given in Schüle's *Technische Thermodynamik*, Bd. II, Sect. 15.

## ENTROPY AS A MAGNITUDE OF STATE

**Statement of the second law applied to any changes of state.** From the result deduced in the previous section, the further deduction can now be established that the entropy of any substance is a magnitude of state. This property of entropy has already been established, in the case of gases, from their characteristic equation, and in the case of any substance from the second law and the Carnot cycle.

Let the entropy at any initial state point  $p_0, v_0, T_0$  (point A, Fig. 94), be  $\phi_0$ . Due to a change of state such as  $ACB$ , resulting from a heat supply, let the entropy increase by  $\int_A^B \frac{dQ}{T}$ . This value

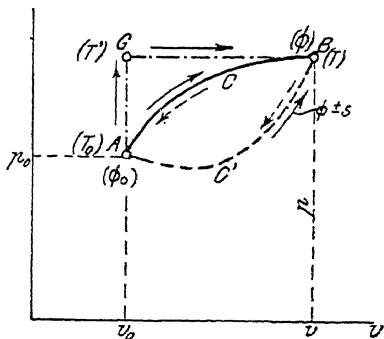


FIG. 94

can be determined if the heat quantities

supplied, and the temperatures along the state line, are known. Let  $\phi$  be the final value of the entropy (at B), so that  $\phi - \phi_0$  is the increase in entropy along  $AB$ .

If the substance be returned to its initial state along the same path  $BCA$ , the entropy decreases again to  $\phi_0$ , for at each point the temperatures are the same as on the forward path, and the same quantities have to be removed as were previously supplied. Hence the sum of all the quantities  $dQ/T$  must be the same on the return path.

It might, however, be argued that, if the substance changed its state along  $AC'B$ , instead of along  $ACB$ , the increase in entropy would be different and equal to  $\phi - \phi_0 \pm s$ . At first sight this seems reasonable, since the heat quantities supplied along  $AC'B$  are now different, so that the quotients  $dQ/T$  are also different. On the return path  $BC'A$  the entropy would decrease by the same amount.

If now the substance passes from A to B through C, and returns through C', it completes a reversible cyclic process. As shown on page 186, this means that the total change of entropy is zero. If, however, as assumed above, the entropy increase on the outward path were  $\phi - \phi_0$ , and on the return path the decrease were  $\phi - \phi_0 \pm s$ , then, in the complete cycle, the entropy would change by the amount  $\pm s$ . This contradicts the second law, so that  $\pm s$  must be zero. The difference in the

entropy values between  $A$  and  $B$  is thus purely determined by the positions of  $A$  and  $B$ , i.e. by  $p_0$ ,  $v_0$ ,  $T_0$  and  $p$ ,  $v$ ,  $T$ . The entropy increase is always the same, whatever path is followed between  $A$  and  $B$ .

Entropy shares this property in common with the internal energy and total heat of substances, in addition to the magnitude of state, pressure, volume, and temperature. The entropy, internal energy, and total heat have thus quite definite values in a given state and can be used, in a reverse sense, to indicate the state of a substance.

This is analogous to the available work possessed by a body in virtue of its weight. A body of weight  $W$  always delivers the same potential work  $Wh$ , whether it drops vertically or along an inclined straight, or curved, path through the vertical height  $h$ . As opposed to this, however, the work  $E$ , performed by compressed gases or vapours, depends not only on the initial and final values of  $p$  and  $v$ , but also, to a large extent, on the path pursued in the change of state.  $E$  does not indicate the state, nor does the heat  $Q$  supplied between two states, since  $Q$  can have any value.

This is expressed mathematically by saying that the expression

$$. \quad (206)$$

is a complete differential. This means that the sum

$$\int_{T_0} \frac{dQ}{T} = \phi - \quad (207)$$

between two fixed limits, within which  $Q$  and  $T$  vary, is independent of these intermediate values (and of  $p$  and  $v$ ), which can be of widely varying amounts.

The same applies to the function, expressed in the form of a general integral, by equation (206),

$$\text{i.e.} \quad = \int \frac{dQ}{T} \quad \text{constant} \quad . \quad (208)$$

$$\text{The equation } dQ = Td\phi \quad . \quad (209)$$

is called the "equation of the second law" (for reversible changes of state). It, along with the equivalent equations (206), (207), and (208), applies to all substances.

A change of entropy always occurs when heat is supplied to or withdrawn from a substance, as is shown by writing  $dQ/T$  for the change of entropy. Hence it can be stated that sensible heat is neither supplied to, nor taken from, a substance when its entropy remains constant. This is similar to the statement that mechanical work performed on, or by, a gas or vapour, is only possible if accompanied by a change in volume. The heat taken up, or rejected by, any substance in any small reversible change of state is, by equation (209), the product of the absolute temperature and the small change in entropy of the substance.

It must not be assumed unconditionally, however, that, if no heat is supplied to, or taken from, a substance, its entropy cannot change. This assumption is valid only for reversible changes of state. In irreversible changes of state the entropy always increases (see page 208).

**Evaluation of the entropy function for gases and vapours.** Entropy changes accompanying state changes can always be determined when the relations between the heat quantities, supplied or withdrawn, and the temperatures are known.

(a) GASES. Let a gas be heated, first at constant volume, from S.T.P. conditions so that the pressure rises from  $p_0$  to  $p$  (line  $AG$ , Fig. 94), and the temperature rises from  $T_0$  to  $T'$ . The gas is then heated at constant pressure till the volume increases to  $v$  and the temperature to  $T$ .

In the first step the heat supply, for a rise  $dT$  is  $dQ = c_v dT$ , hence the elementary increase in entropy is

$$d\phi = \frac{dQ}{T} = c_v \frac{dT}{T}$$

The sum of these quantities between  $A$  and  $G$  is

$$\phi_G - \phi_A = c_v \log_e \frac{T'}{T_0}$$

or, since  $\frac{T'}{T_0} = \frac{p}{p_0}$

$$\phi_G - \phi_A = c_v \log_e \frac{p}{p_0}$$

This is given, either by the integral

$$\int_{T_0}^T \frac{dT}{T} = \log_e \frac{T}{T_0}$$

or by the area of a rectangular hyperbola in which  $\frac{1}{T}$  is plotted to a base of  $T$ .

For the second step  $GB$  we have

$$dQ = c_p dT$$

so that, as before,

$$\phi_B - \phi_G = c_p \log_e \frac{T}{T'}$$

or, since  $\frac{T}{T'} = \frac{v}{v_0}$

$$\phi_B - \phi_G = c_p \log_e \frac{v}{v_0}$$

The total change in entropy between the arbitrarily chosen states  $A$  and  $B$  is, therefore,

$$\phi_B - \phi_A = c_v \log_e \frac{p}{p_0} + c_p \log_e \frac{v}{v_0}$$

The general form, from equation (208), is

$$\phi = c_v \log_e p + c_p \log_e v + \text{constant}$$

The same result would be obtained along any other path. See also pages 123, 127, and 131.

(b) VAPOURS. See pages 373 and 430.

As shown above, it is only the change in, and not the absolute value of, entropy which is determined. The zero point of entropy can be at any arbitrarily assumed state.

From a new heat theorem, however, established by Nernst,\* on the condition of solid and liquid homogeneous substances at absolute zero temperature and its neighbourhood, Planck† has shown that at  $T = 0$  the entropy of every chemically homogeneous solid or liquid is zero. This is of special importance in the thermodynamics of chemical processes.

### ENTROPY DIAGRAM OF CYCLIC PROCESSES THERMAL EFFICIENCY

**Process of maximum heat conversion.** By plotting the absolute temperature  $T$ , of points on the diagram of Fig. 91, to a base of entropy values  $\phi$ , a closed diagram shown in Fig. 95 is obtained. The curve I II of heat supply in Fig. 91 corresponds to the curve I' II' in Fig. 95, and the curve III IV of heat withdrawal to the curve III' IV'. The adiabatic curves of Fig. 91 are represented by vertical lines, since the entropy changes of state are constant (see page 128).

The area below I' II' (Fig. 95) is the heat supplied  $Q_1$ , and that below III' IV' the heat withdrawn  $Q_2$ . Hence the closed area of the entropy diagram I' II' III' IV' is equal to the difference  $Q_1 - Q_2$ . This, however, as shown on page 182, is the heat converted to work, of which the mechanical equivalent  $E$  is given by the corresponding indicator diagram (Fig. 91).

The area of the closed entropy diagram of the cycle thus represents, in heat units, the same work as the area of the  $pv$  diagram. This applies to the complete cycle, however, and not to the separate state changes.

The unconverted or rejected heat  $Q_2$  depends on two factors, as shown by the entropy diagram. The first of these is the value of the absolute temperature at which the heat withdrawal occurs (i.e. the mean height  $T'_m$  of the area  $Q_2$ ), and the second is the increase in entropy during the heat supply (i.e. the width of the area  $Q_2$ ).

If, now, the greatest possible amount of the heat  $Q_1$  supplied in the cycle is to be converted to work,  $Q_2$  should be as small as possible. This occurs when the temperatures, during the withdrawal, are as small as possible, but these temperatures are limited by that of the atmosphere or of the cooling water supply. So far as the conversion to work is concerned, the most suitable heat withdrawal occurs when this temperature is maintained constant. The lower limit of the entropy diagram is then an horizontal line at the height  $T_2$  above the abscissa axes, while in the  $pv$  field it is an isothermal, i.e. a straight line for vapours and a rectangular hyperbola for gases, such as  $AD$  in Figs. 98 and 100.

For a given supply of heat  $Q_1$ , i.e. a given area below I' II', the width of the area depends on its mean height  $T_m$  on the entropy diagram. Hence, as the mean temperature is increased, the entropy decreases, so that the loss, due to the heat removed  $Q_2$ , is reduced.

\* W. Nernst, *Theoret. Chem.*, 6 Auf (1909), page 699. First published in the *Nachr. d. Gesellsch. d. Wissensch. zu Göttingen. Math.-physik. Kl.* (1906), Heft. 1. See also Schüles, *Tech. Ther.*, Vol. II.

† M. Planck, *Vorlesungen über Thermodynamik*, 3 Auf. (1911), page 268.

For a given supply of heat  $Q_1$  with a fixed mean temperature  $T_m$ , it is obvious that the nature of the state change, during this supply of heat, has no effect on the heat usefully converted to work. The temperature can increase, decrease, or remain stationary throughout the change. (Curves  $I' II'$ ,  $gh$  and  $ik$ .)

Hence there is no "best" change of state during the heat supply. It is just as suitable at a high as at a low initial temperature, provided

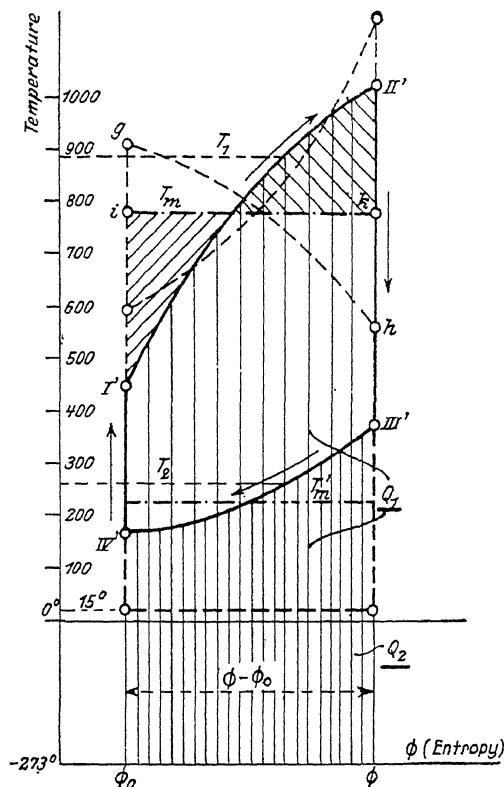


FIG. 95

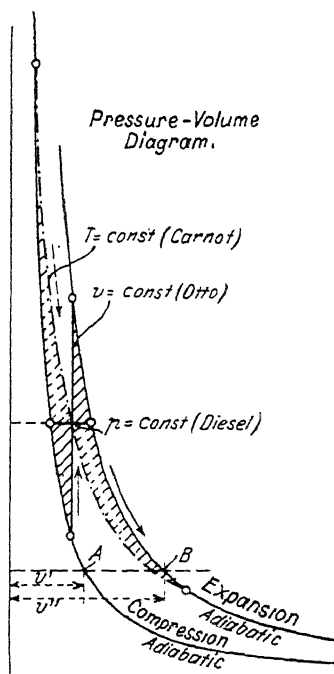


FIG. 96

the mean temperature on the entropy field is the same, or, what amounts to the same thing, provided the entropy increase is the same.

In the particular case of the Carnot cycle, for instance, with its isothermal heat supply, there is no advantage over other cycles in which heat is supplied with increasing temperature. (Gas and oil engines.)

An isothermal heat removal is, however, the most advantageous on account of the fixed lower available temperature of the large cooling water quantities.

On the  $pV$  field the two adiabatics, separated by a definite amount, correspond to the adiabatics  $I' IV'$  and  $II' III'$  on the entropy diagram.

Where the specific heat is constant, the ratio  $\frac{v''}{v'}$  (Fig. 96), determined by this distance, is the same at all pressure levels. Where the specific

heat is variable, however,  $\frac{v''}{v'}$  is, to a certain extent, variable. In the first case the ratio is determined as follows—

The increase in entropy  $\phi - \phi_0$  during the heat supply is given by

$$Q_1 = (\phi - \phi_0) T_m$$

or 
$$\phi - \phi_0 = \frac{Q_1}{T_m}$$

As shown by the entropy diagram, the increase of entropy, between any two points on the upper and lower adiabatics, is constant. In the case of gases, this increase is given in terms of the ratio  $\frac{v''}{v'}$ , if a path of constant pressure be followed between the lower and higher adiabatics, corresponding to the straight line  $AB$  of the  $pV$  diagram (Fig. 96). This gives (page 128),

$$\phi - \phi_0 = 2.303 c_p \log \frac{v''}{v'}$$

hence 
$$\frac{Q_1}{T_m} = 2.303 c_p \log \frac{v''}{v'}$$

or 
$$\log \frac{v''}{v'} = \frac{Q_1}{2.303 c_p T_m}$$

Thus, if  $Q_1 = 450$  C.H.U./lb. (overload),  $c_p = 0.3$ , and  $T_m = 1200$

we have 
$$\log \frac{v''}{v'} = \frac{450}{2.303 \times 0.3 \times 1473} = 0.442$$

and 
$$\frac{v''}{v'} = 2.77$$

The expression is not so simple when  $c_p$  is variable, in which case it is better to employ the entropy diagram. The entropy increase  $\frac{Q_1}{T_m}$  is set off to the right of the given pressure curve and the change in volume determined from the constant volume curves.

The value of  $v''/v'$  for the gas engine indicator diagram shown in Fig. 58 is 2.4 at the compression pressure level and 2.3 at the release pressure level.

The upper part of the Diesel engine overload diagram (Fig. 60) gives  $\frac{v''}{v'} = 1.93$  and the lower part 2.5.

As  $\frac{v''}{v'}$  is reduced, the mean temperature, during the heat supply, increases, so that the unavoidable loss  $Q_2$  is reduced.

All areas, representing useful work on the entropy diagram, between two given adiabatics, will be equal, if they have the same mean temperature  $T_m$  during the heat supply and a common lower temperature. The line of heat supply can thus be of any form.

Hence the corresponding indicator diagrams, on the  $pv$  field, are also equal. Three cases are shown in Fig. 96, viz. isothermal heat supply (Carnot), constant pressure heat supply (Diesel), and constant volume heat supply (Otto). These heat supplies are all equal. It will be observed that the isothermal case requires a considerably higher maximum pressure than the other two cases, and shows no compensating advantage. For a given thermal efficiency the constant pressure Diesel process shows the lowest maximum pressure. Diesel engines which do not operate at a constant pressure combustion, but with a pressure increase above that of compression (as in solid injection engines) demand larger maximum pressures for the same efficiency.

The highest compression is required by the isothermal process and the lowest by the Otto.

Engines of equal power, which operate on these three cycles with equal thermal efficiency, have to have the same stroke volume if the heat supplies are to be equal.\* The clearance volumes, however, are different, the smallest being that of the isothermal process and the largest that of the Otto.

Since, in a process having any arbitrarily chosen upper and lower limits,  $Q_1 = T_m(\phi - \phi_0)$  and  $Q_2 = T'_m(\phi - \phi_0)$ , it follows that the thermal efficiency is

$$\eta = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{T'_m}{T_m}$$

Thus, if  $T_m = 1200 + 273$  and  $T'_m = 100 + 273$   
 then  $\eta = \frac{373}{927} = 0.402$

while, with  $T'_m = 350 + 273$   $\eta = 0.58$

The absolute value of the upper temperature  $T_m$  depends on the heat  $Q_1$  supplied to 1 lb. of the charge (i.e. the calorific value of the charge) and on the adiabatic pre-compression. It is due to the latter that the high temperatures necessary for a reasonable conversion of heat to work are rendered possible. Theoretically, any temperature could be attained by means of compression, but a practical limit is imposed on the compression pressure by the strength of the cylinder, by the difficulties of lubrication, and, in the case of the compression of explosive mixtures, by the pre-ignition temperature.

If the upper temperature exceeds  $2000^\circ \text{C}$ ., partial dissociation occurs (for  $\text{CO}_2$  it occurs at  $1700^\circ \text{C}$ .). This causes a part of the combustion heat to be absorbed in splitting up the gas molecules, so that this heat is not available for raising the temperature. Hence the conditions at high temperatures are different from those considered above.

If it were possible, by separate compression of the air and fuel, to attain so high a temperature, that, on uniting the two gases, no combustion occurred, owing to the maintenance of complete dissociation, and this were followed by a continuous chemical combining during expansion, the highest possible thermal efficiency would be obtained.

\*It is assumed here that exhaust occurs in the usual way, that the full stroke is utilized during suction, and that the same fuel is used.





Fig. 97 shows the entropy diagram, to scale, with initial and final compression temperatures of  $20^{\circ}\text{C}$ . and  $600^{\circ}\text{C}$ . respectively and a final combustion temperature of  $1300^{\circ}\text{C}$ . (The corresponding indicator diagram shown in Fig. 98 is shown for considerably lower pressures and temperatures in order to avoid a confusing diagram.)  $B'C'$ , in Fig. 97, shows the line of heat supply,  $C'D'$  the adiabatic expansion,  $D'A'$  the isothermal compression, during which heat is removed and the pressure raised to the initial value, and  $A'B'$  the adiabatic compression, with which the process commences.

The area below  $B'C'$  gives the heat supplied  $Q_1$ , the area below  $A'D'$  the unavoidable heat loss  $Q_2$ , and the area  $A'B'C'D'$  the heat  $Q_2 - Q_1$

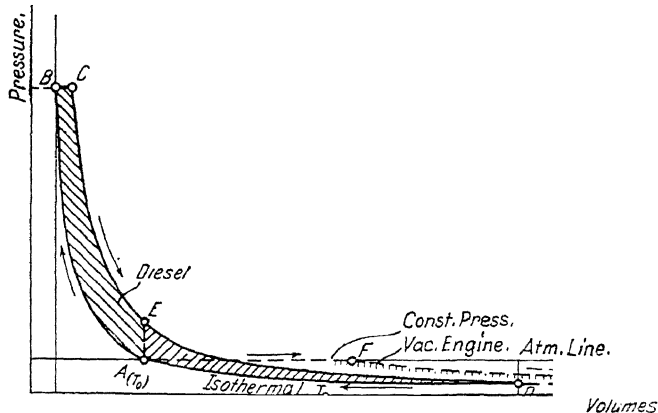


FIG. 98

which is converted to work. With the figures given above,  $Q_2$  is about 24 per cent of  $Q_1$ , so that the thermal efficiency is 76 per cent.

In the Diesel cycle (or constant pressure cycle) the portions  $EAD$  on the  $pV$  field and  $E'A'D'$  on the  $T\phi$  field are omitted.  $E'A'$  and  $EA$  represent the constant volume exhaust period. The heat loss is thus increased, so that the thermal efficiency is reduced to 62 per cent. As regards the actual process, see page 343.

If combustion occurs at constant atmospheric pressure, as in furnaces, instead of at  $850\text{ lb./in.}^2$  abs., as shown in Fig. 97, the  $T\phi$  curve is now represented by  $A'F'$ . The area below  $A'F'$  is the same as that below  $B'C'$ , since the same heat is supplied in the two cases. The unavoidable heat loss, however, given by the area below  $A'D''$ , is now about twice as large as before. Hence combustion in furnaces, with moderate initial temperatures, is not so advantageous, as regards the possibility of heat conversion to work, as combustion in constant pressure engines with initial compression of the charge.

For a condensing steam engine or turbine operating on the Carnot cycle with the same heat supply and supply steam at  $225\text{ lb./in.}^2$ , the unavoidable loss is still greater and is given by the area below  $A'J$  ( $\eta = 0.38$ ). The ideal cycle of the actual steam engine shows a still higher loss (area below  $A'J_1$ ,  $\eta = 0.34$ ). In these steam cycles a condenser temperature of  $20^{\circ}\text{C}$ . has been assumed.

If the heat generated by combustion at atmospheric pressure were to be used directly in a gas engine (instead of being used to generate steam), the resulting diagram would be  $A'F'D''A'$ , for which the efficiency is  $\eta = 0.46$ . This is represented by the cycle  $AFD''DA$  on the  $pV$  field (Fig. 98) and lies entirely below the atmospheric line.

**Available work from heat generated at constant volume.** The  $T\phi$  diagram of the cycle process showing the greatest conversion of heat to

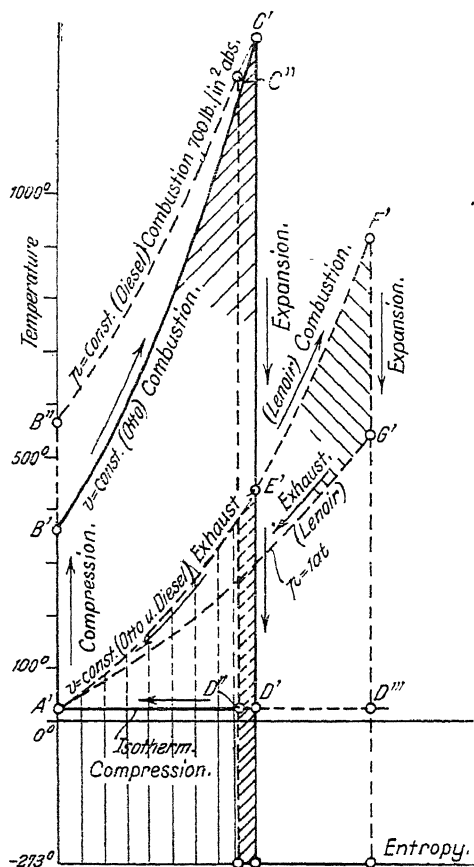


FIG. 99

work in this case (Figs. 99 and 100) is similar to that at constant pressure, except that the constant volume line  $B'C'$  now takes the place of the constant pressure line. In the indicator diagram (Fig. 100) it is shown by the vertical  $BC$ . The other lines are the same as before.

With assumed temperatures of  $360^\circ \text{C.}$  at the end of compression and  $1300^\circ \text{C.}$  at the end of combustion, the unavoidable loss is 29 per cent of the heat supplied, so that the efficiency is  $\eta = 0.71$ .

In the gas engine (Otto) cycle, the areas  $EDA$  on the  $pV$  field and  $E'D'A'$  on the  $T\phi$  field are absent. This gives  $\eta = 0.46$ .

The efficiency of the ideal constant pressure cycle, with the same maximum pressure, is easily found by drawing the curve of this pressure through  $C'$  and locating  $C''$  so that the area below  $B''C''$  is equal to the area below  $B'C'$ . The heat lost is then less than before by the area below  $D'D''$  or, in the case of the Diesel with constant volume exhaust, by the shaded area below  $E'$ , which is about double the area below  $D'D''$ . Hence, for the same maximum pressure, the Diesel cycle is slightly more efficient than the Otto cycle.

The absolute value of the efficiency depends primarily on the amount of compression (point  $B'$ ). With no compression the heat supply takes

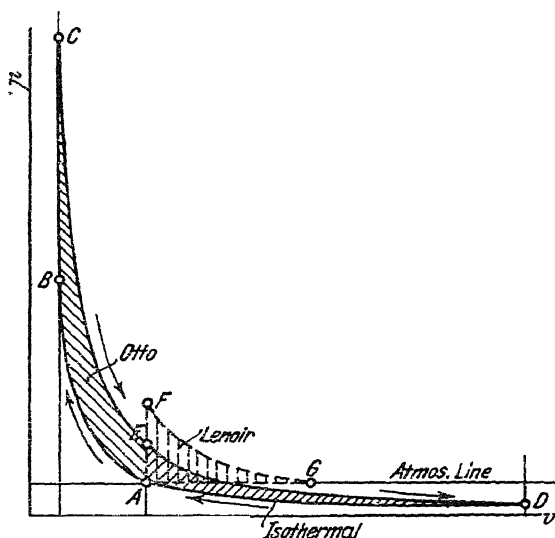


FIG. 100

place along  $A'F'$ . The heat rejected, given by the area below  $A'D''$ , has then its greatest value, giving a resulting least efficiency.

By drawing a line of constant pressure  $A'G'$  through the initial point  $A'$ , it will be seen that, due to the adiabatic expansion  $C'D'$  or  $F'D''$ , the pressure drops below the initial pressure, i.e. below atmospheric pressure (see page 142). This sub-atmospheric pressure is shown directly in Fig. 100.

If, in the constant volume process with no compression, expansion proceeds to atmospheric pressure (i.e. to  $G$  on the  $pV$  field and  $G'$  on the  $T\phi$  field), and is followed by exhaust at the same pressure ( $G'A'$ ), the ideal cycle of the earliest gas engine—the Lenoir—is obtained. It corresponds to the areas  $AFGA$  and  $A'F'G'A'$ . The useful work is only 19 per cent of the heat supplied.

In the actual Lenoir engine the expansion was incomplete, but in the Langen and Otto atmospheric engines it was carried below atmospheric pressure, which partly accounts for the considerably reduced gas consumption in these engines.

In constant volume and constant pressure engines, as now built, no

work is delivered below atmospheric pressure. As seen from the  $T\phi$  chart, this means a considerable loss of work, but the loss is reduced as the pressures and temperatures are increased, i.e. as the compression pressures are increased,

The above entropy curves have been drawn on the assumption of constant specific heat. When exact values are required, use should be made of the entropy charts I and II, in which the variation of specific heat has been taken into account.

**Explosion turbine** (Holzwarth). The cycle is shown in Fig. 101. A combustible mixture of air and gas is drawn into a compressor at atmo-

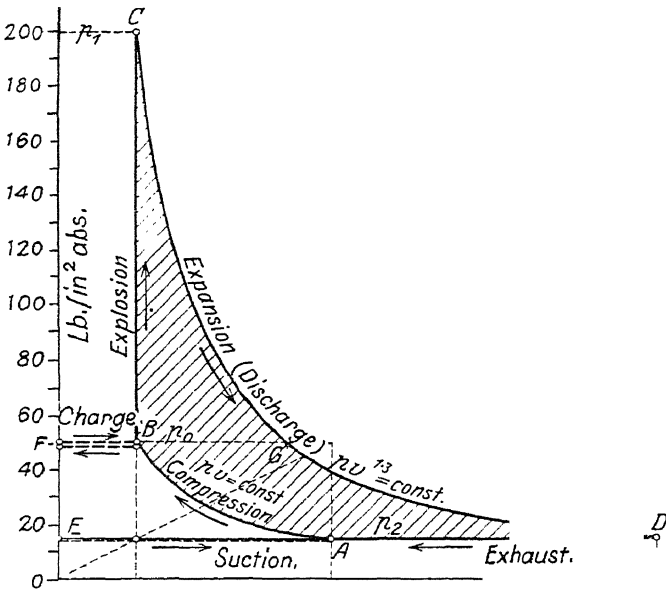


FIG. 101

spheric pressure ( $EA$ ) and compressed isothermally ( $AB$ ). This compressed mixture then fills ( $FB$ ) a combustion chamber, which has been previously emptied to atmospheric pressure. At  $B$  the mixture is ignited and the pressure rises from  $p_0$  to  $p_1$  ( $BC$ ). The discharge valves are then opened, allowing the hot gases to expand adiabatically, through de Laval nozzles, to the back pressure  $p_2$ . The kinetic energy thus given to the gases is transferred to a turbine wheel. The gases then exhaust to the atmosphere (after having given up some of their heat in a waste heat boiler). The ideal available work of the cycle, neglecting the heat given to the boiler, is given by the area  $ABCD$  (Fig. 101). The process is shown, on the entropy field, by the closed cycle  $A'B'C'D'$  (Fig. 102), which also represents the ideal available work. The area below  $B'C'$  gives the heat supplied, and the ratio of the work area to this area gives the ideal thermal efficiency  $\eta_{th}$ . In Fig. 102 the reciprocating Otto cycle for the same heat supply has also been drawn. Its area is given by  $A'B'C''D''A'$  and is considerably smaller than that of the gas turbine. The ideal

efficiency of the Holzwarth cycle is thus greater than that of the Otto cycle for the same amount of compression and the same weight of charge.

A diagram is also shown in Fig. 102 for a richer mixture, i.e. for the same weight of charge having more gas and less air (explosion temperature  $1450^{\circ}\text{C}.$ ). The ideal work  $A'B'C'D'$  is considerably greater, although the compression work  $B'A'H N$  remains the same.

**The Holzwarth-Schüle cycle.** In practice, it is difficult to keep the heat and mechanical losses of the gas turbine within such limits that the thermal efficiency is reasonably high. Hence, as in the steam turbine, it is necessary to introduce multistage expansion. The useful available work is then divided into two main parts (Fig. 101), the high pressure work  $BCG$ , which may be delivered either by a single or a two-row wheel, and the low pressure work  $FGDE$ , which is delivered by a low pressure turbine

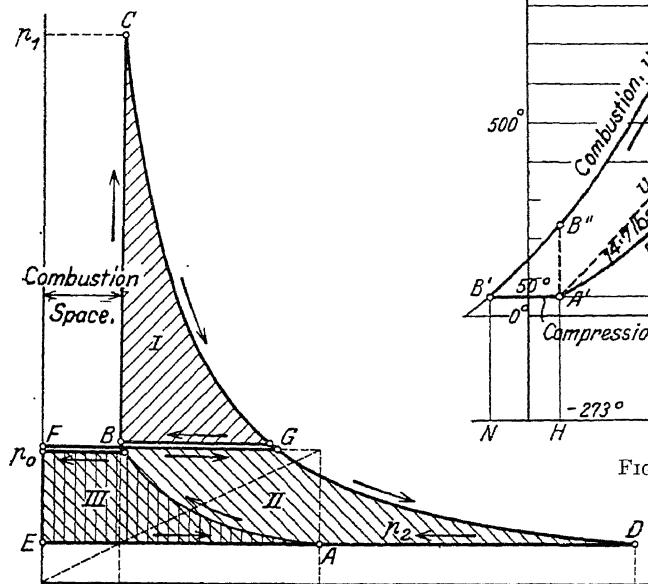


FIG. 103

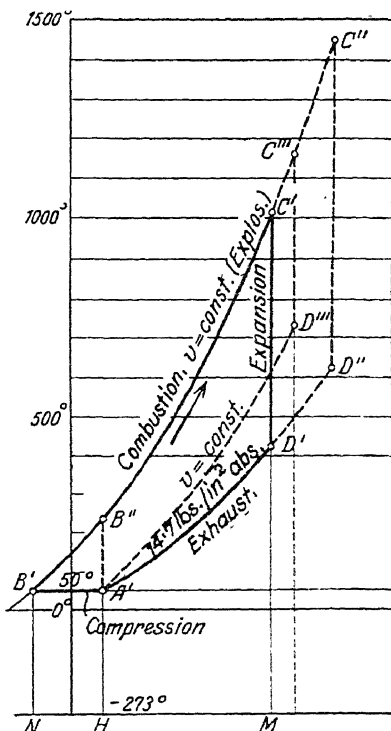


FIG. 102

fed from a receiver in which the pressure  $p_0$  is maintained constant by the exhaust gases from the high pressure stage. Hence the L.P. turbine is similar to a steam turbine in that the supply pressure remains constant. The L.P. turbine is further improved by having a steam superheater fitted in the receiver, so that the temperatures in the turbine are reduced without any loss in the total available work (see pages 178 and 608). The cycle of this combined explosion and constant pressure turbine is shown in Fig. 103.

### IRREVERSIBLE PROCESSES

**Fundamental conditions of reversibility.** In the treatment of state changes of substances, considered in the previous sections, it was assumed that such changes as the isothermal or adiabatic could proceed in either direction, so that, if an expansion were immediately followed by a compression back to the original volume, exactly the same states would be passed through. These state changes are said to be "reversible."

In this, two conditions which are never quite realized in practice have to be fulfilled. The first is that complete equilibrium exists within the gas during the entire course of the change. This equilibrium is such that all portions of a substance which is changing its state must have the same pressure and temperature. These quantities must also satisfy the characteristic equation

$$pV = WRT$$

which itself is based on the assumption of complete equilibrium.

In the case of a gas or vapour expanding so slowly in a cylinder that the movement of the piston is negligibly slow compared with the propagating velocity of the gas pressure (which is equal to the acoustic velocity) this condition is almost completely realized. Even with low piston speeds, however, a disturbance of the equilibrium can occur. Thus, when a diaphragm having a small orifice is fitted in the cylinder, any movement of the piston establishes different pressures and temperatures in the separated spaces of the cylinder, since the gas has to flow through the orifice. This flow requires a pressure difference between the two spaces and turbulence results in the space next the piston. If the orifice is made very small, throttling occurs, causing a still greater deviation from the state of equilibrium. It is obvious that state changes of this nature, even if unaccompanied by heat supply or removal (i.e. adiabatic changes), do not follow the law  $pv' = \text{constant}$ , and are not reversible. Further, the work done on the piston is always less than that given by the law of expansion  $pv' = C$ , due to friction and turbulence.

In the flow of gases, therefore, the simple state equations only apply when the flow is frictionless and unaccompanied by turbulence. If these conditions are fulfilled, a compressed gas flowing at high velocity may, by means of its own kinetic energy and suitably designed nozzles, be returned to the same state as that at which the flow commenced.

Actually, these conditions are never realized, so that no flow is completely reversible and the state changes cannot be expressed by the adiabatic law  $pv' = C$ , even when all heat flow to, or from, the gas is prevented. The kinetic energy of the jet is never quite so high as that corresponding to the work given by this equation, and the gas cannot be brought to the original pressure by means of its own velocity. Hence, since the condition of complete equilibrium cannot be attained in practice, it follows that completely reversible state changes never occur.

When air flows from one heat insulated vessel into another, the process is again adiabatic, but is not such that the adiabatic equilibrium equation  $pv' = \text{constant}$  is satisfied. The whole of the flow energy is reconverted to heat by friction, turbulence, and impact in the second vessel as conditions are settling down. This heat is just sufficient to return the air, which was cooled during the flow, to its original temperature.

In the final state, therefore, the internal energy is still the same, but the pressure is reduced. This pressure is higher, however, than the final pressure found from the adiabatic law of expansion, and since, during expansion, no external work has been done, it follows that no work is now available to compress the gas back to the initial pressure. This process is, therefore, also irreversible, since it occurs with complete disturbance of internal equilibrium.

The second condition is that there must be no temperature difference between the substance which is changing its state and the walls surrounding the substance. Hence, in changes of state accompanied by a heat supply or rejection, the temperature difference, necessary for the heat transfer, between the substance and the hot or cold body must be indefinitely small. Also, in adiabatic changes, either the wall temperature must follow the gas temperature exactly or no heat flow must occur between the gas and wall in spite of a temperature difference. These conditions are never attained in practice, since in all heat transmission a finite temperature difference is required, and, conversely, where temperature differences exist, heat flow necessarily results. A purely adiabatic state change, which satisfies the equation  $pv^\gamma = \text{constant}$ , is thus never attained. During expansion a certain amount of heat always flows from the surroundings to the gas and, during compression, heat flows from the gas to the surroundings, so that the state lines of alternate expansions and compressions do not coincide.

The simplest example of a state change accompanied by heat supply is that of the isothermal. In this case the whole of the compression work is transformed to heat, which must be entirely transferred to the cooling water. In the case of expansion the whole of the work done must come from the equivalent heat supplied by the hot source to the gas.

Consider now that during compression  $AB$ , Fig. 104, the gas temperature at every instant is equal to the cooling water temperature. This is conceivable with very slow compression, large cooling surfaces, and a large supply of cooling water. During the succeeding expansion  $BA$  the heat given to the cooling water in the previous compression could again be supplied to the gas under the same conditions. The expansion line thus coincides with the compression line, and the work required for compression can be returned in expansion. The change of state is completely reversible.

If, however, during the isothermal compression, the gas temperature is slightly higher than that of the cooling water (as always occurs in high speed machines), no appreciable change occurs in the indicator diagram and the heat carried away is about the same, but this heat takes up the temperature of the cooling water. In the succeeding expansion  $BA$

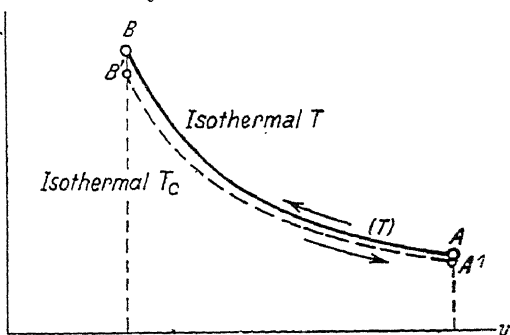


FIG. 104

this heat would have to be returned to the gas if the process is to be reversible in every respect. This, however, is impossible, since the gas temperature is now higher than that of the water. An isothermal expansion is, of course, possible, but only at a lower temperature  $T_c$  than that of compression. Hence the process is not reversible, and the work regained below  $B'A'$ , between the same volume limits, is less than the original compression work. When the gas and cooling water are considered as a whole, nothing has been lost of the heat given up during compression, but this heat has suffered a drop in temperature, so that the available work is reduced.

The generation of steam in boilers is an important example of irreversible state change. Even if all the heat generated in the furnace could be transmitted to the steam, there would still be a considerable reduction

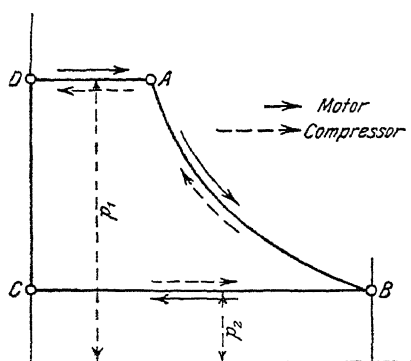


FIG. 105

in available energy, due to the large temperature difference between the furnace and the steam (see page 195 and Fig. 97). The evaporation is reversible as shown on page 358, but the temperature drop in the furnace is irreversible. There is no means, apart from a supply of fresh energy, of again increasing this temperature.

Summing up, it can be stated that all actual state changes are either partially or completely irreversible, since they always occur with some disturbance of equilibrium and with some finite difference in temperature.

Completely reversible state changes must be regarded as ideal changes from which all actual changes deviate more or less.

Again, all irreversible state changes are always accompanied by a loss of available work.

In the case of prime movers, the heat lost by radiation and conduction, as well as by friction at the moving parts, have to be added to those mentioned above.

Consider, for example, the adiabatic reversible expansion of air in an air motor (Fig. 105). Let the motor (having no clearance volume) be supplied with compressed air from a large container I ( $DA$ ), after which it expands adiabatically to the back pressure  $p_2$  ( $AB$ ). It is then removed at  $p_2$  ( $BC$ ) to a large container II, where air at the same state is stored. From this container air is now drawn into an air compressor cylinder ( $CB$ ), compressed adiabatically to  $p_1$  ( $AB$ ), and re-delivered to the container I. If no heat losses occur by conduction or radiation, and there is no conversion of mechanical energy to heat by turbulence, friction, and impact, the motor would be capable of driving the air compressor without extra power. The air would perform a cycle in which its final state would be the same as the initial state. Such a process, however, is impossible, since there are always losses. Due to these losses, the weight of air which can be compressed is less than that required to operate the air motor. Complete reversibility is thus an ideal state



change not attainable, either in individual changes of state or in complete processes.

**Important examples of irreversible state changes and their relation to the second law.** The most important irreversible state changes are as follows—

1. Heat transfer from hot to cold bodies by conduction or radiation, unaccompanied by the performance of useful work by the heat.

As shown by the second law, the heat  $Q$  could only be re-transferred by an expenditure of mechanical work given by

$$E = \frac{Q}{A} \frac{T_1 - T_2}{T_2}$$

The process is thus irreversible, since this work is not supplied in the heat transfer.

2. Expansion of gases, vapours, or liquids from a vessel, in which the pressure is  $p_0$ , to another vessel, in which the pressure is  $p$ , without an accompanying delivery of external work.

When a gas flows into an empty vessel, or into a vessel containing the same gas at the same temperature, no change occurs in the temperature. Hence, to return the gas to the original state, work equal to that given by isothermal compression  $pV \log_e \frac{p_0}{p}$  is at least required, and this must come from the outside, since no work was delivered in the original flow.

3. The normal case of gas mixing occurs without a delivery of work, while the separating of a mixture of gases demands an expenditure of work. Hence the usual type of mixing (by diffusion and turbulence) is an irreversible process.

4. The flow of gases, vapours, and liquids along pipes and through valves, diaphragms, and nozzles is irreversible, since the internal heat generated by friction and turbulence cannot be completely re-converted to the kinetic energy of flow. Throttling processes are also included in this group.

5. The generation of heat from mechanical work by friction, impact, and turbulence.

Of the heat  $Q$  generated, only the fraction  $\frac{T_1 - T_2}{T_1} Q$  is capable of being regained as mechanical work. In this,  $T_1$  is the temperature of this heat and  $T_2$  is the temperature of the surroundings. Hence the process is irreversible. (See page 161 regarding the reversible generation of heat by means of mechanical work.)

6. The usual types of combustion processes in which chemical energy is liberated as heat. The ideal available work from this heat depends on the temperature drop (see the ideal gas engine process, page 338), and is, therefore, never complete and is never sufficiently large to effect the chemical separation of the products of combustion in the exhaust.

The above six processes can all occur in nature by themselves, i.e. without the aid of external energy. In certain cases they may actually deliver work. All that is necessary to start them is some sort of releasing action, after which no work is required. Thus in 1 the process may be started by removing an insulated separator; in 2, 3, and 4 by opening a

valve; in 5 by releasing the stop of a weight, and in 6 by local heating (ignition). When these processes are reversed, however, mechanical energy is required, and in every case this energy is greater than the corresponding energy liberated in the original process. The reverse process, therefore, can never be completed by itself in nature.

These processes serve to show that irreversibility is closely related to the second law of thermodynamics, since the criterion of reversibility depends, in all cases, on the completeness of the reconversion of heat to work. The statement that in nature a reversible change of state never occurs, is merely another expression of the second law, and, since these self-acting processes are irreversible, the second law indicates how such natural processes tend to become run down.

The mathematical expression for this is contained in the next section.

### Entropy and energy changes accompanying irreversible state changes.

It must be noted at the outset that, when a substance proceeds from a definite initial state  $A$  to a definite final state  $B$ , the change in the entropy is the same whether the path followed is reversible or irreversible, since, as shown on pages 124 and 187, entropy is a magnitude which depends only on the state. The entropy change in irreversible processes cannot, of course, be determined from the intermediate states, since these are generally indefinite and not in a condition of equilibrium. It is necessary to consider the final state, in which equilibrium has again been established. This then enables the difference in entropy between the equilibrium states  $A$  and  $B$  to be determined by choosing any convenient reversible path between  $A$  and  $B$ .

Consider, for example, a quantity of air at the state  $p_1$  and  $T_1$ , and let it flow from a vessel of volume  $V_1$  ft.<sup>3</sup> into an empty vessel of volume  $V_2$  ft.<sup>3</sup> Both vessels are heat insulated. The final temperature, pressure, and entropy, when the flow ceases and conditions have settled down, are found as follows—

The internal energy remains unchanged, since neither work nor heat have been delivered by the air to the surroundings. Hence the temperature remains constant, so that

$$V_2)$$

$$\text{or} \quad p_2 = \frac{p_1}{\frac{V_1}{V_2} + 1}$$

as in reversible isothermal expansion (although the state change occurs adiabatically).

From the general entropy equation for gases (page 125), we have

$$\begin{aligned} \phi_2 - \phi_1 &= c_v \log_e \frac{p_2}{p_1} + c_p \log_e \frac{p_1}{p_2} = (c_p - c_v) \log_e \frac{p_1}{p_2} \\ &= (c_p - c_v) \log_e \frac{V_1 + V_2}{V_1} \end{aligned}$$

This quantity is always positive and has the same value as that given by an isothermal reversible expansion. In all cases of expansion of this type the entropy thus increases.

The determination of the entropy changes in cases 3 and 6 (page 203) is more involved, since a reversible path cannot be directly substituted.

In any change of state, except the adiabatic, two substances are always involved, namely, the working substance and the hot or cold body, which delivers or receives heat during the state change. Now, since in all cases of heat reception or rejection, the substance involved suffers a change in entropy, the hot or cold body, along with the working substance, must suffer a change in entropy. As a rule, however, it is the changes in entropy of the working substance alone which are considered, and these are quite independent of the corresponding changes in the hot or cold body. A distinction has generally to be made, as shown on page 200, between processes which are irreversible, on account of a temperature gradient, and those which are irreversible, on account of a lack of equilibrium.

**Changes of state with a temperature gradient between the hot or cold source and the working substance.** The entropy diagram can, as in the case of a corresponding reversible state change, be applied directly here to the change of state of the working substance, since the change in entropy of this working substance depends only on the heat supplied to, or given up by, it, and on its temperature,

$$\text{i.e.} \quad d\phi = \frac{dQ}{T}$$

Hence, as before, the entropy diagram represents the quantity of heat by the area below the entropy curve,

$$\text{i.e.} \quad dQ = Td\phi$$

The relation of this to the simultaneous changes in the entropy of the hot or cold source will be readily understood by considering the case of isothermal compression of a gas, with a temperature gradient, as dealt with on page 201 (Fig. 104). It was shown there that the entropy of the gas decreased, during compression, by the amount  $\frac{Q}{T}$ . At the same time the entropy of the cooling water, which received the heat  $Q$  at the lower temperature  $T_c$ , increased by the amount  $\frac{Q}{T_c}$ . The gas and cooling water together thus experienced the entropy increase of  $+\Delta\phi = \frac{Q}{T_c} - \frac{Q}{T}$ . If  $T_c = T$ , i.e. if there is no temperature gradient,  $\Delta\phi = 0$ . The above holds also for isothermal expansion. Thus, if the hot source has a temperature  $T_c > T$ , the entropy of the gas increases by the amount  $\frac{Q}{T}$ , due to the heat supply, and that of the hot source decreases by  $\frac{Q}{T_c}$ , due to the heat removed. Hence the entropy as a whole, for both bodies, increases by the amount  $+\Delta\phi = \frac{Q}{T} - \frac{Q}{T_c}$ .

The same applies to any change of state with varying temperature, since the change can be regarded as made up of elementary alternate isothermals and adiabatics.



$\bar{v}' = \int p' dv$ . Hence equation (212) must not be written as

$$Q = I - I_1 + A \int p dv$$

because then it would appear that the values of  $p$  in the integral and in the expression for  $I$  were identical. Thus, for gases,

$$I - I_1 = \frac{1}{\gamma - 1} (pv - p_1v_1)$$

but it is wrong to state that

$$Q = \frac{1}{\gamma - 1} (pv - p_1v_1) + A \int p dv$$

To correct this, we must write,

$$Q = \frac{1}{\gamma - 1} (p'v - p_1v_1) + A \int p dv$$

It is thus preferable to leave the equation in the form shown by equations (210) and (212) and to insert the work  $E$  corresponding to particular cases.

For the same heat supply the value  $I_2'$ , in a reversible process, is found as follows. For this process, the heat supplied is

$$Q = I_2' - I_1 + AE_0 \quad (214)$$

where  $I_2'$  is the final value of the internal energy and  $E_0$  is the work done. As shown on page 201,  $E_0$  is always greater than the work  $E$  in the irreversible process. (For this reason the work done in a reversible process is also called the maximum work. See Schüle's *Tech. Ther.*, II, Sect. 27.)

By combining equations (210) and (214), we have

$$I_2' - I_1 = E + AE_0 \quad (215)$$

and, since  $E_0$  is greater than  $E$ , we also have

$$I_2' > I_2 \quad (216)$$

The internal energy in the final state is, therefore, greater after the irreversible process than after the reversible process.

As in the case of internal energy, the entropy  $\phi$  can only be determined under equilibrium conditions, either at any intermediate point or finally. Its value can be found by changing reversibly, at constant volume, from the final state of the reversible process, to the final equilibrium state of the irreversible process. If  $Q'$  is the heat added in this change, the second law gives

$$Td\phi' = dQ' = dI + A p dv$$

but, since,  $v = \text{constant}$  or  $dv = 0$

$$Td\phi' = dQ' = dI$$

and hence  $Q' = \int Td\phi' = I_2 -$



In the particular case of adiabatic flow, the change of entropy is

since  $dQ = 0$ . As  $dQ_v$  is always positive, it follows that  $d\phi$  must also be positive. That is, in all irreversible adiabatic processes, either of compression or expansion, the entropy can only increase, not decrease. When adiabatic expansion occurs with friction, a curve, such as  $AB$  in Fig. 106, represents the state change and not  $AB'$ . In the same way, irreversible adiabatic compression is represented by a curve such as  $CD$  and not by  $CD'$ . Compression along  $CD''$  and expansion along  $AB''$  means a heat removal in all cases. If these processes are also accompanied by friction they become irreversible.

A process, such as  $AB$ , can also be accompanied by heat supply or rejection, and still remain irreversible.

The same applies to such irreversible adiabatic processes as throttling or emptying and filling of vessels, where the final state alone, and not the intermediate state, has to be considered.

If now the actual irreversible state change, under equilibrium conditions, is represented by a curve such as  $AB$  (Fig. 155) on the  $pV$  field, the corresponding curve on the entropy field  $A_1B_1$  (Fig. 156) is located by means of the general entropy equation, since entropy is a state function.

As opposed to a reversible state change, however, the area below  $A_1B_1$  does not give the heat supplied or rejected, but represents—

- (a) the heat  $Q_v$  developed internally by friction in an adiabatic change of state; or
- (b) the algebraic sum of the heat  $\pm Q$  supplied or rejected externally and the frictional heat  $Q_v$  developed internally in any change of state.

Hence the area  $A_1B_1E_1F_1$  (Fig. 156), or the area below  $AB$  (Fig. 106) is equal to  $Q_v \pm Q$  or  $\frac{F}{T} + Q$ , where  $E_v$  is the frictional work in mechanical units.

Thus, if  $Q$  is known by measurement,  $E_v$  can be found from the diagram. On the other hand, and as opposed to reversible changes, the heat  $Q$ , supplied or rejected externally, cannot be found from the entropy diagram when  $E_v$  is not known.

See pages 307 to 316 for the application to steam turbines and injectors.

The second law, when applied to an irreversible change of state with internal friction, does not give the change of entropy as

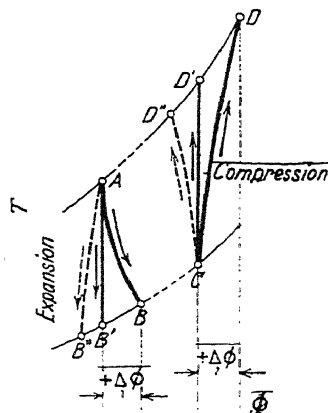


FIG. 106

In this case we must write

$$d\phi = \frac{dQ + dQ_v}{T}$$

or, in general,  $dQ < T d\phi$  as already given above.

In applying the entropy diagram to processes in a reciprocating engine, it is of basic importance to notice that the diagram only gives the heat supplied or rejected correctly if no disturbance of equilibrium, causing a generation of heat, occurs in the working substance during the change in state considered. The heat ( $AE$ ) corresponding to this disturbance is included in the entropy diagram and cannot be separated from the other heats.

**Change in state caused by throttling.** If the cross sectional area, at one point in a pipe of uniform or varying diameter, is considerably reduced, it is known that, when gases or vapours flow along the pipe, the

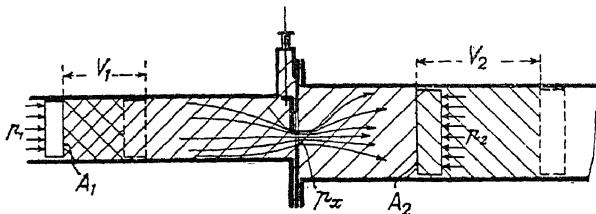


FIG. 107

pressure beyond the reduced section is less than that before it. The pressure reduction carried out in this way is known as "throttling." The amount of throttling depends on several factors, including the nature and state of the substance flowing, the relative reduction in the cross-sectional area, the velocity of flow, and the density of the substance flowing. Valves and cocks fitted in the pipe line cause throttling, particularly when only partially opened. The throttle or butterfly valve is used in this way to control the supply of steam to engines and turbines. Throttling is also used in pressure control systems and to measure the weight of steam or gas flowing in pipes.

In this section only the change in temperature and specific volume caused by throttling between given pressure limits will be considered, and not the probable pressure reduction due to given conditions of throttling.

The problem is simplified by imagining two pistons to be fitted in the pipe line, one before the narrowing and one after, as shown in Fig. 107. The change of state of the gas then occurs in the space between these two pistons. A driving pressure  $A_1 p_1$  is exerted by the left-hand piston on the gas, while the gas itself exerts the total pressure  $A_2 p_2$  on the right-hand piston. If the velocity of the L.H. piston is  $V_1$  ft./sec., the work done by it, on the gas per second, is  $A_1 p_1 V_1 = p_1 V_1'$ , where  $V_1'$  is the volume flowing per second. In addition to this pressure work, the volume  $V_1'$  (having the weight  $W$ , say) carries with it the internal energy  $WI$ , and the kinetic energy  $\frac{1}{2} W V_1^2$ . Hence the total energy in the form of mech-



anical work, sensible and latent heat, and kinetic energy before throttling

$$\text{is } p_1 V'_1 + \frac{W I_1}{A} + \frac{W V_1^2}{2g} \text{ (ft. lb.)}$$

The work done after throttling on the right-hand piston is  $p_2 A_2 V'_2 = p_2 V'_2$ . This work, along with the internal energy  $\frac{W I_2}{A}$  and the kinetic energy  $\frac{W V_2^2}{2g}$ , is carried away by the gas, or the total energy carried away is

$$p_2 V'_2 + \frac{W I_2}{A} + \frac{W V_2^2}{2g}$$

Since, during throttling, energy is neither supplied nor rejected externally, either as heat or mechanical work, these expressions are equal,

$$\text{i.e. } p_1 V'_1 + \frac{W I_1}{A} + \frac{W V_1^2}{2g} = p_2 V'_2 + \frac{W I_2}{A} + \frac{W V_2^2}{2g}$$

Dividing by  $\frac{W}{A}$  and noting that  $\frac{V'_1}{W} = \frac{1}{\rho_1}$  and  $\frac{V'_2}{W} = \frac{1}{\rho_2}$ , where  $\rho_1$  and  $\rho_2$  are the densities before and after throttling, we have

$$I_1 + \frac{A p_1}{\rho_1} + \frac{A V_1^2}{2g} = I_2 + \frac{A p_2}{\rho_2} + \frac{A V_2^2}{2g} \quad (221)$$

The quantities  $I_1 + \frac{A p_1}{\rho_1}$  and  $I_2 + \frac{A p_2}{\rho_2}$  represent the total heats  $H_1$  and  $H_2$ , which depend only on the initial and final states. (See pages 175 and 455.)

This now gives

$$H_1 + A \frac{V_1^2}{2g} = H_2 + A \frac{V_2^2}{2g}$$

$$\text{or } H_1 - H_2 = A \left( \frac{V_2^2}{2g} - \frac{V_1^2}{2g} \right) \quad (222)$$

or the reduction in total heat is equal to the increase in kinetic energy. In an elementary state change, therefore,

$$dH = - A d \frac{V^2}{2g}$$

In many cases the change in kinetic energy is negligible,

$$\text{so that } H_1 = H_2 \quad (223)$$

or  $H = \text{constant}$

This applies particularly to the case of throttling in a pipe of uniform diameter. Throttling is, then, an irreversible change of state with constant total heat. This applies not only to gases, but also to wet, dry, and superheated vapours.





This gives

$$\frac{A V_1^2}{c_p 2g} = 1 - \frac{A V_1^2}{A V_1^2} \quad (228)$$

which is a linear equation in  $T_1$  or  $t_1$ . Equation (227) differs from this equation only in the denominator. This is always greater than 1, hence  $t_1 - t_2$ , as given by equation (228), is somewhat smaller than that given by (227). The difference is small for moderate velocities.

Temperature increases occur when a gas discharges from a pipe into a large vessel, because the whole of the efflux energy is converted to heat. With  $V_2 = 0$ , we have

$$t_1 - t = \frac{A V_1^2}{c_p 2g}$$

which makes  $t_2 > t_1$ .  $V_1$  must, however, be large if  $t_2 - t_1$  is to be appreciable. If, for example,  $V_1 = 650$  ft./sec., we have, for air,

$$t_2 - t_1 = 19.9^\circ \text{ C.}$$

## CHAPTER IV

### FLOW OF GASES

**General relations of state in liquid and gaseous flow.** The internal state of any flowing substance is defined by the pressure, volume, or specific weight and temperature in the same way as for a stationary substance. The relations between these three magnitudes are independent of the velocity.

The simplest example is that of parallel flow. Here all particles move parallel to the pipe axis and have equal speeds (Fig. 108, I). The pipe itself is straight and of uniform diameter. Flow of this type, in which the speed remains uniform, is said to be stationary.

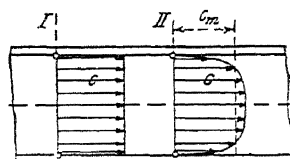


FIG. 108

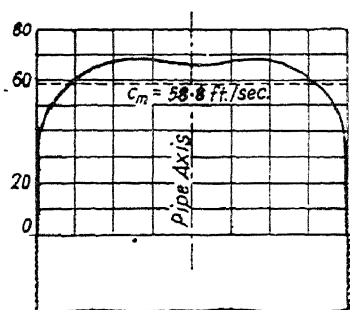


FIG. 109

In actual cases the speeds of the particles at different points in a cross section are not equal, due to the influence of the walls. The velocity near the walls is always less than that at the centre, while just at the wall itself the speed is zero, because a thin film of liquid clings to the wall. The speed variation, therefore, is similar to that shown in Fig. 108 at II. The volume flowing per second in case I is  $V = AC$ , while in case II it is the sum of the quantities  $C \times dA$  taken over the whole area. Thus, if it is required to measure the quantity of air flowing through large pipes or channels by means of the speed of flow, it is necessary to measure these speeds at several points in the cross-sectional area.

The velocity  $C_m$ , which gives the same volume flowing as the actual velocities, is called the mean velocity. This gives

$$V = C_m A$$

Fig. 109 shows the distribution of velocity across a pipe diameter, as determined by very careful measurements made by Jakob in a mild steel pipe of  $15\frac{3}{4}$  in. diameter (400 mm.).

It has been found that parallel flow in pipes and channels, in the sense that all particles move in straight and parallel paths, occurs only at low speeds or in narrow pipes. In the case of the usual speeds and diameters, the flow is turbulent. In many practical cases the flow just at entrance to the pipe is turbulent, and along the pipe itself disturbances are caused by valves, bends, branches, and roughness of the wall surface. The nature

of this continuous turbulent flow is illustrated by the smoke issuing from a chimney.

In the case of turbulent flow, the mean velocity is again found by dividing the volume  $V'$ , flowing per second, by the cross sectional area  $A$  of the pipe, i.e.

$$C_m = \frac{V'}{A} = \frac{Wv}{A} = \frac{W}{\rho A}$$

where  $v$  is the specific volume and  $\rho$  is the density at the section  $A$ .

If the flow occurs in a pipe of variable section, the mean speed changes with these variations. Under steady conditions, the weight flowing per second at every section is the same.

$$\text{Hence } A_1 c_1 \rho_1 = A_2 c_2 \rho_2 = A_3 c_3 \rho_3 \text{ (Equation of continuity) } . \quad (229)$$

If  $\rho$  is constant, as for liquids,

$$\text{or } \frac{c_2}{c_1} = \frac{A_1}{A_2}$$

The mean speeds are then inversely proportional to the cross-sectional areas. This applies also to gases and vapours with considerable accuracy, when the pressure changes are small.

The paths (or stream lines) of the individual particles in a case like that shown in Fig. 110 are not straight. Hence the velocity directions are different at different points in a cross section, but this is only of importance in very rapid changes of the cross section.

**Pressure changes as dependent on speeds.** The absolute pressure of a flowing fluid is that pressure which is measured by an absolute pressure gauge at a point in the stream. An example of this is afforded by a barometer carried in a balloon which is driven by an air stream.

The gauge pressure or vacuum in a stream, as in the case of a fluid at rest, is the difference between the absolute pressure of the liquid and the barometric pressure. The absolute pressure, thus measured, is called the "static pressure" of the fluid, and it is this pressure which has to be used in the characteristic equation of the flowing substance.

In channels and pipes this pressure is the same as that exerted by the gases on the walls. The static pressure can therefore be determined by boring a hole in the pipe wall and connecting a pipe from this point to a manometer.

Exact measurements have shown, however, that this method gives slightly inaccurate results, due to a disturbance at the sharp inside edge of the manometer pipe, but, if this is slightly rounded, the errors are reduced.

The static pressure at every section in a pipe of varying cross section (Fig. 110) is different. The particles increase in velocity between the sections  $A_1$  and  $A_2$ , and the work necessary for this is supplied by the static pressure, which consequently decreases.

An element, of mass  $m$ , bordered by two illimitably close cross sections exerts a total pressure of  $ap$  on the first section and a pressure of  $a(p + dp)$  on the second (downstream) section, so that the driving pressure acting

in the direction of flow is  $-a \, dp$ . If  $dV$  is the increase in velocity between the two sections, the acceleration is  $\frac{dV}{dt}$ , and hence  $m \frac{dV}{dt}$  is the accelerating pressure. This gives

$$-a \, dp = m \frac{dV}{dt}$$

or, since  $m = \frac{\rho \, ds}{g}$

$$dp = -\frac{\rho \, ds}{g \, dt}$$

and, since  $\frac{ds}{dt} = V$

$$(230)$$

A decrease in speed, therefore, corresponds to an increase in pressure, so that in flowing through a reduced area the pressure generally decreases, since, at least for liquids, the reduced area corresponds to an increase in velocity. In the case of vapours and gases, however, the reverse may occur, due to the dependence of the specific volume on the pressure. Thus, in the nozzles of a steam turbine, an increased area may correspond to a reduced pressure. This, however, occurs only when the speed of the jet exceeds the acoustic velocity, which ranges from 1000 to 1300 ft./sec.

Equation (230) can be expressed in another form. Thus, with

$$V \, dV = \frac{a^2}{\gamma} \frac{dp}{p}$$

and  $\rho = \frac{1}{v}$

the equation becomes

$$(231)$$

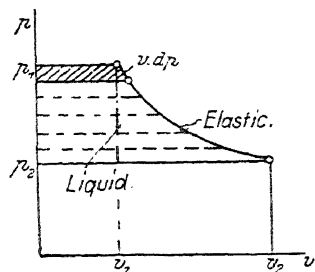


FIG. 111

Now  $\frac{V^2}{2g}$  is the kinetic energy per pound of the fluid, so that  $\frac{V^2}{2g}$  is the increase in kinetic energy corresponding to the useful work  $-v \, dp$  performed by 1 lb. of the gas, due to its pressure reduction. The equation could, of course, be established directly owing to the equality of these two quantities. On the  $p \, v$  field,  $v \, dp$  is represented by the shaded area in Fig. 111. The curve on the right of this area applies to elastic fluids but becomes a perpendicular for liquids.

In order to apply this relationship, which is valid for an elementary change in pressure and velocity, to a finite change between given limits, we have to consider the change as made up of a succession of such elementary changes. By summing (integrating) these we have

$$\frac{V^2}{2g} = \frac{v_2}{g} - \frac{v_1}{g} \quad (232)$$

where the expression on the left represents the complete shaded area on the  $pv$  field and gives the useful work between  $p_1$  and  $p_2$ .

**Flow with small pressure changes.** If, in any type of flow, the pressure changes are small, so that the volume changes are also small, the work area is approximately a rectangle (Fig. 112) having the same value as for liquids, namely,  $v(p_1 - p_2)$ .

It is of importance to decide how great the pressure drop may be, in order that the error may not be appreciable in applying this to gases and vapours.

The  $pv$  curve  $BC$ , shown in Fig. 112, is an adiabetic, since it is assumed that no heat flow occurs. The law of expansion is, therefore,  $pv^\gamma = C$ , with  $\gamma$  ranging from 1.1 to 1.4. In small changes the curve and the tangent coincide approximately, and the tangent is easily drawn as

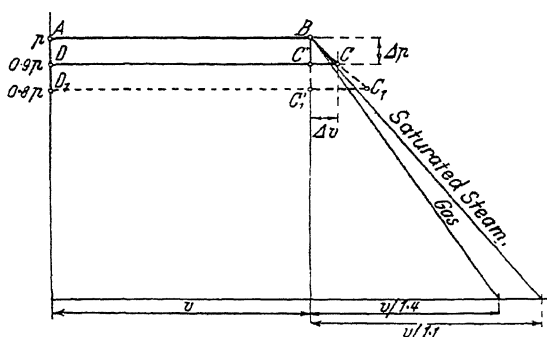


FIG. 112

explained on page 107. The error due to taking the area  $ABC'D$  in place of  $ABCD$  is given by the triangle  $BCC'$ . The ratio of the area of this triangle to the area of the rectangle  $ABC'D$  is given by  $\frac{CC'}{2C'D} = \frac{\Delta p}{2v}$  since both these areas have the same height.

As shown on page 107, however,

$$\frac{\Delta p}{\Delta v} = -\gamma \frac{p}{v}$$

so that, leaving out the negative sign, the proportional error is

$$\frac{\Delta v}{2v} = \frac{\Delta p}{2\gamma p}$$

The greatest error occurs with  $\gamma = 1$  and is then  $\frac{\Delta p}{2p}$ . If, for example, this error is not to exceed  $\frac{1}{2}$  per cent, the pressure drop  $\Delta p$  must not exceed 1 per cent of the pressure. Thus, if  $p = 1$  atm. = 14.7 lb./in.<sup>2</sup> abs. the change in pressure must not be more than 0.147 lb./in.<sup>2</sup>

On the assumption made above, equation (232) becomes

$$v(p_1 - p_2) = \frac{V_2^2}{2g} - \frac{V_1^2}{2g}$$

since the work diagram is now a rectangle, having the area  $v(p_1 - p_2)$ .



Replacing  $v$  by  $\frac{1}{\rho}$  gives

$$(233)$$

The reduction in pressure energy is thus equal to the gain in kinetic energy.

By writing the equation in the form

$$\frac{p_1}{\rho} + \frac{V_1^2}{2g} = \frac{p_2}{\rho} + \frac{V_2^2}{2g}$$

it is seen that the sum of the pressure and kinetic energies is constant for flow of this type, i.e.

$$\frac{p}{\rho} + \frac{V^2}{2g} = \text{constant}$$

$$\text{or} \quad p + \frac{\rho V^2}{2g} = \text{constant} \quad (234)$$

Since a pressure difference is involved in the equation

$$p_1 - p_2 = \frac{\rho}{2g} (V_2^2 - V_1^2)$$

$p_1$  and  $p_2$  may be expressed either as gauge or absolute pressures, but they should be given in lb./ft.<sup>2</sup> This means that where the pressures are in lb./in.<sup>2</sup>, as is usual, the quantity on the left has to be multiplied by 144. The fundamental equation, in place of (234), is then

$$p + \frac{\rho V^2}{2g} = \text{constant} \quad (235)$$

$$\text{The value } \frac{\rho V^2}{2g} = h_d \quad (236)$$

is called the "dynamic pressure" of the flowing fluid, and the sum of the static ( $h_s$ ) and dynamic ( $h_d$ ) pressures is called the total pressure, i.e.

$$h_s + h_d = H \quad (237)$$

If, at some point in the cross-sectional area the velocity  $V$  is reduced to zero by means of an obstruction, the static pressure at this point

risks to  $H$ , i.e. it increases by the amount  $\frac{\rho V^2}{2g}$ .

Before the obstruction, we have

and at the obstruction,

$$h_s' + 0 = H$$

so that the static pressure, at the obstruction, is

$$h'_s = h_s + \frac{\rho V^2}{2g} = H$$

A suitably fitted manometer, with the opening facing the stream (Fig. 113), shows in this way the total pressure, provided the mouth is so shaped that it forms an obstruction which satisfies the condition mentioned above. Instruments of this type are known as Pitot tubes.

These have been tested in various ways and show that the total pressure is given very accurately, even when a simple smooth tube is merely inserted in the stream.

**Measurement of air velocities with a Pitot tube.** If, at a particular section of a pipe, the static pressure  $h_s$  be measured, along with the total pressure  $H$ , as given by the Pitot tube, the air velocity is found from the equation

$$\frac{\rho V^2}{2g} = H - h_s$$

as

$$V = \sqrt{\frac{2g(H - h_s)}{\rho}} \text{ ft./sec.}$$

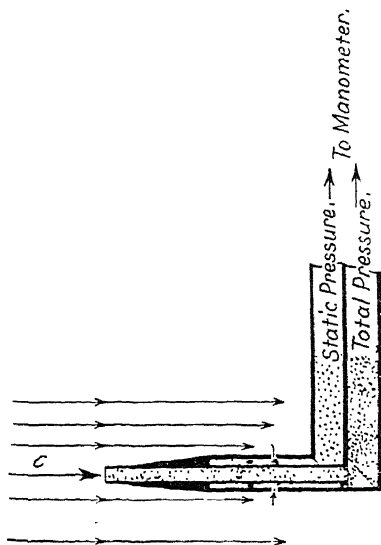


FIG. 113

The Pitot tube can be arranged to give both  $H$  and the static pressure  $h_s$  (Fig. 113). For this purpose, holes are bored in the outer tube behind the conical portion. The value  $H - h_s$  then enables the air velocity to be calculated.

**Temperature changes accompanying velocity changes.** Where the static pressure suffers a change, due to a change in velocity, it follows, from the properties of the gas, that the temperature also changes. On the assumption that no heat flow occurs during these changes, we have, for a small state change (page 107),

$$\frac{\Delta t}{\Delta p} = \frac{Av}{c_p} = \frac{A}{\rho c_p}$$

so that

$$\Delta t = \frac{1}{1400 \rho c_p} \Delta p$$

**Example I.** Air at a static pressure of 1.18 in. of water and at a temperature of 15° C. is moving at a speed of 33 ft./sec. in a pipe 6 in. diameter. If the pipe is gradually reduced to 4 in. diameter, find, neglecting friction, the velocity, pressure, and temperature at the reduced section.

The velocity is

$$V_2 = 33 \left(\frac{4}{6}\right)^2 = 74.3 \text{ ft./sec.}; 1 \text{ in. water} = \frac{62.5}{1728} = 0.036 \text{ lb./in.}^2$$

and the static pressure at the reduced section is given by

$$144 \times 0.036 \times 1.18 + \frac{33^2}{2g} \times \rho = h_s + \frac{74.3}{2g} \rho$$

$$h_s = 144 \times 0.036 \times 1.18 - \frac{\rho}{2g} (74.3^2 - 33^2)$$

Taking  $\rho = 0.0780 \text{ lb./ft.}^3$

$$h_s = 6.12 - 5.36 = 0.76 \text{ lb./ft.}^2 = 0.147'' \text{ H}_2\text{O}$$

The temperature drop is

$$\Delta t = \frac{5.36}{1400 \times 0.0780 \times 0.237} = 0.207 \text{ C}^\circ.$$

**Example 2.** A Pitot tube, fitted in an air pipe, shows a total gauge pressure of  $3.937'' \text{ H}_2\text{O}$ , while the static gauge pressure is  $0.787'' \text{ H}_2\text{O}$ . Find the air velocity.

The dynamic pressure is  $(3.937 - 0.787) 0.036 \times 144 = 16.33 \text{ lb./ft.}^2$ ,

hence 
$$v = \sqrt{\frac{2 \times 32.2 \times 16.33}{\rho}} = \frac{32.4}{\sqrt{\rho}}$$

If the air in the pipe is at atmospheric pressure with  $\rho = 0.078 \text{ lb./ft.}^3$ ,

then 
$$v = \frac{32.4}{\sqrt{0.078}} = 116 \text{ ft./sec.}$$

If, however, the pressure is very low and equals, say,  $\frac{1}{10}$  of an atmosphere, i.e.  $1.47 \text{ lb./in.}^2 \text{ abs.}$ , then  $\rho = 0.0078 \text{ lb./ft.}^3$  and, for the same Pitot tube readings,

$$V = 367 \text{ ft./sec.}$$

If the pipe is used for town gas, for which  $\rho = 0.0312 \text{ lb./ft.}^3$ ,

then 
$$V = 184 \text{ ft./sec.}$$

**Example 3.** A ventilating fan supplies air to a pressure pipe at a speed of  $115 \text{ ft./sec.}$  By how much can the static pressure of the air be raised if, by gradually enlarging the pipe, the speed is reduced to  $40 \text{ ft./sec.}$ ?

From equation (235),

$$h_{s1} + \frac{115^2}{2g} \rho = h_{s2} + \frac{40^2}{2g} \rho$$

we have 
$$h_{s2} - h_{s1} = \frac{\rho}{2g} (115^2 - 40^2)$$

$$= \frac{0.078}{2 \times 32.2} (115^2 - 40^2) = 14.1 \text{ lb./ft.}^2$$

$$= 2.72'' \text{ H}_2\text{O}$$

This result is based on the assumption of non-turbulent flow. Actually the pressure increase will be about 70 per cent of this.





with the increase in velocity. Later, however, the changes in volume are large compared with the changes in velocity. Hence, when the pressure drop is considerable, the areas first decrease and then increase, giving a minimum section, called the "throat" section.

These conditions are shown in Fig. 114, which has been drawn to scale for air at an initial state of 147 lb./in.<sup>2</sup> abs., and 100° C. (point *A*). *AB* is the adiabatic *p**v* curve. An initial velocity of  $V_0 = 460$  ft./sec., giving a kinetic energy of 3,300 ft. lb./lb. (*DC*), has been chosen. The energy drops, corresponding to the curve *AB*, have been plotted, as abscissae, against the pressures as ordinates, and give the curve *CC*<sub>1</sub>. The line *CG* represents the initial area *A*<sub>0</sub>, and subsequent areas are given by the curve marked "Cross sectional area *A*."

From equation (246), we have

The diagram shows how the areas first decrease (with decreasing pressure) until a minimum area *a* is reached. Thereafter the areas increase. The circular pipe shown alongside also depicts these area changes. The area becomes a minimum when the pressure is about 0.57 *p*<sub>0</sub>. In the case of liquids the areas would continuously decrease, with decreasing pressures, whereas with elastic fluids they commence to increase after a certain "critical" pressure is passed.

**Case 2. Compression flow. (Conversion from velocity to pressure.)**  
The fundamental equation (231)

$$-v \, dp = d \left( \frac{V^2}{2g} \right),$$

applies in this case also. If the pressure is to increase (*dp* +),  $d \left( \frac{V^2}{2g} \right)$  must be negative, i.e. the kinetic energy, and with it the velocity, must decrease.

If the pressure rises from *p*<sub>0</sub> to *p*, while retardation in the flow occurs, then

$$- \int_{p_0}^p v \, dp = \frac{V^2}{2g} - \frac{V_0^2}{2g}$$

or 
$$\int_{p_0}^p v \, dp = \frac{V_0^2}{2g} - \frac{V^2}{2g}$$

The integral is represented by the area *E*<sub>*x*</sub> in Fig. 115, so that

$$E_x = \frac{V_0^2}{2g} - \frac{V^2}{2g} \quad . \quad . \quad . \quad . \quad . \quad . \quad (247)$$

If the final velocity is zero, i.e. *V*<sub>0</sub> = 0, the whole of the kinetic energy due to the initial velocity has been transformed to the work *E*<sub>*x*</sub>, and the value of this initial velocity is given by

$$\frac{V_x^2}{2g} = E_x \quad . \quad . \quad . \quad . \quad . \quad . \quad (248)$$

## FLOW OF GASES

We have, therefore,

$$\frac{v_x}{2g} = \frac{v_0}{2g} - \frac{v}{2g} \quad \dots \quad (249)$$

or in heat units, as in case I,

$$h_x = h_0 - h \quad \dots \quad (250)$$

where  $h_0$  is the original kinetic energy in heat units,  $h$  the kinetic energy left finally, and  $h_x$  the work of compression due to the conversion of kinetic energy.

The final pressure  $p$ , due to compression, is obtained by calculation or graphically from the compression work  $E_x$  or  $h_x$ . The analytical expression for the work done  $E$  is employed in the calculative method.

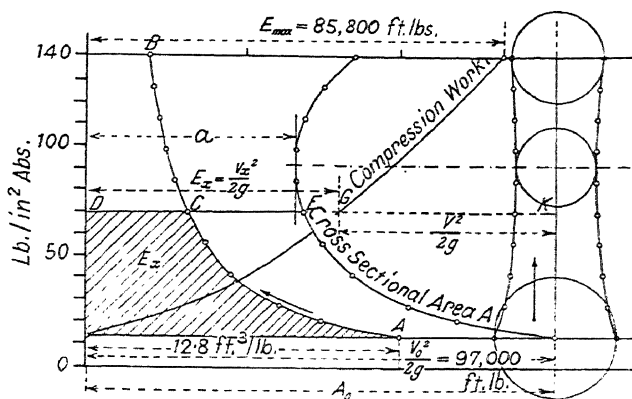


FIG. 115

Starting with the adiabatic equation

$$pv^\gamma = \text{constant}$$

we have, for gases and vapours (pages 108 and 328),

$$\text{from which} \quad E \propto \frac{v^2}{\gamma-1} \quad (251)$$

where  $v^2$

is determined from the given initial and final velocities.

Equation (251) now becomes

$$p_0 \quad (252)$$

**Example 4.** Air at 14.2 lb./in.<sup>2</sup> abs. and 0° C. flowing at a speed of 2500 ft./sec. is to be compressed by reducing the speed to 850 ft./sec. Find the rise in pressure and temperature. Take  $\rho = 0.078$  lb./ft.<sup>3</sup>, i.e.  $v_0 = 12.82$  ft.<sup>3</sup>/lb.

From equation (252)

$$p_0 = \frac{1.4 \times 2 \times 32.2 \times 14.2 \times 144 \times 12.82}{0.4 (2500^2 - 850^2)} = 1.934^{3.5} = 10.03$$

and 
$$\frac{T}{T_0} = \left(\frac{p}{p_0}\right)^{\frac{1}{\gamma}} = (10.03)^{\frac{0.4}{1.4}} = 1.934$$

so that  $T = 1.934 \times 273 = 528^\circ \text{C. abs. or } 255^\circ \text{C.}$

Alternatively, the adiabatic compression work is

$$\text{ft. lb. or } \frac{1}{1400} \left( \frac{V_0^2}{2g} - \frac{V^2}{2g} \right)$$

and is equal to  $1400 c_p t$ , where  $t$  is the rise in temperature (see page 327).

This gives  $h_x = c_p t$  or  $t = \frac{h_x}{c_p}$  and

$$\frac{p}{p_0} =$$

In this example,

$$h_x = \frac{1}{1400} \left( \frac{2500^2 - 850^2}{2 \times 32.2} \right) = 61.3 \text{ C.H.U.}$$

$$\therefore t = \frac{61.3}{0.24} = 255^\circ \text{C. as above.}$$

**Variation in cross-sectional areas.** As in Case I, the equation of continuity gives

$$A = A_0 \frac{V_0}{V} \frac{v}{v_0}$$

Hence, to find if  $A$  is greater or less than  $A_0$ , we have to find if  $\frac{V_0}{V} \frac{v}{v_0}$  is greater or less than unity.

With a continuous increase in pressure, in the direction of flow, the velocity decreases, or  $\frac{V_0}{V}$  is greater than unity. On the other hand, the volume decreases with increasing pressure, or  $\frac{v}{v_0}$  is less than unity.

Fig. 115 has been drawn to show these relationships for air at 14.2 lb./in.<sup>2</sup> abs. and 0° C., having a velocity of  $V_0 = 2500$  ft./sec., which corresponds to a kinetic energy of 97,000 ft. lb./lb.



Retardation occurs continuously throughout the flow, so that the pressure increases and the volume decreases, as shown by the adiabatic  $AB$ . The corresponding compression works  $E_x$  have been plotted as abscissae against compression pressures as ordinates. Thus, the line  $DG$  represents the compression work between  $p_0$  and  $p$ , and is equal to the shaded area. If the initial kinetic energy is  $\frac{V_0^2}{2g}$ , the distance

$$GK = \frac{V_0^2}{2g} - \frac{V_x^2}{2g}$$

is the kinetic energy still existing at the pressure  $p$ . The cross-sectional areas  $A$  are also plotted against the pressures as ordinates. It will be seen that, with increasing pressures, the areas first decrease until a minimum value  $a$  is reached, and then increase. At a compression pressure of 142 lb./in.<sup>2</sup> abs. (i.e. 10 times the original pressure) the area is about 0.57  $A_0$ . The final kinetic energy is 11,200 ft. lb./lb., which gives the final velocity as 850 ft./sec.

The variation in the cross-sectional areas is thus similar to the case of accelerated flow, although the process itself is reversed. It follows, therefore, that the two processes can be carried out in the same pipe.

From this example it would appear that the cross-sectional areas must first decrease in the direction of flow when gaseous substances are to be compressed, and this is opposite to what occurs in the compression of liquids. This, however, only happens to be the case owing to the special conditions of the example, as will be apparent from the following.

**General conditions governing the changes in the cross-sectional area for accelerated and retarded adiabatic flow.** The equation of continuity demands, in general, that

$$AV\rho = A_1V_1\rho_1$$

Considering two closely adjacent sections, we have

$$V_1 = V + dV, \quad A_1 = A + dA, \quad \text{and} \quad \rho_1 = \rho + d\rho$$

so that  $AV\rho = (A + dA)(V + dV)(\rho + d\rho)$

Multiplying out and neglecting infinitesimals of the second order, gives

$$A\rho dV + V\rho dA + AVd\rho = 0$$

$$\text{or} \quad \frac{dA}{A} + \frac{dV}{V} + \frac{d\rho}{\rho} = 0 \quad (253)$$

For adiabatic changes, equation (126), page 107, applies,

$$\text{i.e.} \quad \frac{dv}{v} = -\frac{dp}{\gamma p}$$

$$\text{or with} \quad v = \frac{1}{\rho} \quad \text{and} \quad dv = -\frac{d\rho}{\rho^2}$$

$$dp = \frac{\gamma p^2 dv}{\rho}$$

$$\text{or} \quad \frac{d\rho}{\rho} = \frac{1}{\gamma} \frac{dp}{p}$$



**Retarded flow.** In this case  $dp$  is always positive. Hence, if  $dA$  is to be positive, i.e. if the pipe is to increase in area, then

$$\frac{gv}{V^2} > \frac{1}{\gamma p}$$

or  $V < \sqrt{g\gamma p v}$

On the other hand, if the pipe decreases in area,  $dA$  is negative, so that

$$\frac{gv}{V^2} < \frac{1}{\gamma p}$$

and  $V > \sqrt{g\gamma p v}$

Compressive flow, therefore, which commences with a velocity less than the acoustic, must take place in a divergent pipe, as is the case with liquids.

When a compressive flow commences with a velocity above, and finishes with a velocity below, the acoustic velocity (as in Fig. 115), the pipe must first converge and then diverge. A compressive flow, in which the velocities everywhere are above the acoustic, can only occur in a convergent pipe.

Instead of starting with a given type of flow, the shape of the pipes may be given, in which case the pressure changes are found as follows—

Equation (254) gives

$$dp = \frac{dA}{A} \cdot \frac{gv}{V^2} \cdot A(g\gamma p v - V^2) \quad (255)$$

In a converging pipe ( $dA$  negative), the pressure decreases and the velocity increases when the denominator is positive, i.e. when the speed of flow is below the acoustic velocity. In the same portion of pipe, however, the pressure increases when the denominator is negative, i.e. when the velocity of flow is above the acoustic.

In a divergent pipe ( $dA +$ ) the pressure decreases and the velocity increases for velocities above the acoustic velocity, while, for the same portion of pipe, when the velocities are below the acoustic velocity, compression occurs.

In a pipe which first converges and then diverges, the following can occur. If the flow commences at a speed below the acoustic, the velocity continues to increase and the pressure decrease until the throat is reached. If, up to this point, the acoustic velocity has not been reached, compression occurs in the second portion of the pipe (diverging portion). If the acoustic velocity is reached the further flow can continue as an expansion with velocities above the acoustic; it is, however, also possible that, from the throat outwards, compression may occur. The back pressure at the end of the pipe decides which of the two cases will occur.

If the flow commences with a speed above the acoustic, compression occurs. If now the velocity remains higher than the acoustic beyond the throat, acceleration occurs in the divergent portion and the pressure

decreases. If, on the other hand, the acoustic velocity be attained at the throat, from there onwards compression may occur, although it is also possible for expansion to occur, depending again on the back pressure.

In the above it has been assumed that the pressure in the space into which the pipe discharges is the same as the pressure at the final section of the pipe, as given by the initial pressure and shape of the pipe. If this is not arranged, considerable alterations may occur in the conditions, since the influence of the back pressure, whether above or below the correct value, extends some distance within the pipe. Again, in the actual process, the changes from one section to another must be gradual.

**Velocity and pressure at the throat section.** A convergent divergent pipe represents the most general case, since it involves wide limits of pressure and velocity, both for accelerated and retarded flow. The conditions at the throat have an important influence on the flow beyond this section, as is illustrated above. At the throat, where the changes in area are gradual,  $dA = 0$  and the general equation

$$gv$$

can satisfy this condition in two ways. First by making  $dp = 0$ , i.e. by making the pressure constant in the neighbourhood of the throat. This, however, does not agree with the cases shown in Figs. 114 and 115, where the pressure changes at this section are not sensibly different from those at any other section. The case in which  $dp = 0$  at the throat will only occur when the process in the diverging portion is opposite to that in the converging portion, so that, at the throat, the previously falling pressure commences to rise, or *vice versa*.

We can, however, have  $dA = 0$ , whether  $dp \lesseqgtr 0$ , i.e. whether the pressure is increasing or decreasing, if

$$\frac{gv}{V^2} - \frac{1}{\gamma p} = 0$$

or  $V^2 = g\gamma p v$

In the normal cases, shown in Figs. 114 and 115, the velocity of the throat is thus the acoustic velocity, which the gas would have when its state is the same as that given by the throat conditions. This applies to both compressive and expansive flow.

The above result can be deduced in another way, as follows. Dividing both sides of the fundamental equation by  $dx$ , i.e. an element of the length of the pipe in the direction of flow, we have

$$\frac{1}{A} \frac{dA}{dx} = \left( \frac{gv}{V^2} - \frac{1}{\gamma p} \right) \frac{dp}{dx}$$

or  $\frac{dp}{dx} = \frac{\frac{dA}{A}}{\frac{gv}{V^2} - \frac{1}{\gamma p}}$

At the throat,  $\frac{dA}{dx}$ , which gives the rate of change in the cross-sectional area along the pipe, is zero. If now, in spite of this,  $\frac{dp}{dx}$  is not zero, i.e. if the pressure changes are as shown in Figs. 114 and 115, the only possible value of the denominator on the right is zero. This, then, gives the indeterminate value  $\frac{dp}{dx} = \frac{0}{0}$ , which can be above or below zero.

Denoting the values of  $V$ ,  $p$ , and  $v$  at the throat by  $V_t$ ,  $p_t$ , and  $v_t$ , we have

$$V_t^2 = g\gamma p_t v_t \quad (256)$$

Also, if the initial state in the pipe is defined by  $V_0$ ,  $p_0$ ,  $v_0$ , then, for the case of expansion,

$$V_t^2 - V_0^2 = 2gE_t$$

and, of compression,

$$V_0^2 - V_t^2 = 2gE_t$$

where  $E_t$  is the conversion to work.

For both cases we can thus write

$$V_t^2 - V_0^2 = \pm 2gE_t$$

and, inserting the value of  $V_t$  given by equation (256), we have

$$g\gamma p_t v_t - V_0^2 = \pm 2gE_t \quad (257)$$

We also have

$$p_t v_t^{\gamma'} = p_0 v_0^{\gamma'}$$

$$\text{and} \quad E_t = \frac{\gamma}{\gamma-1} p_0 v_0 \left[ 1 - \left( \frac{p_t}{p_0} \right)^{\frac{\gamma'-1}{\gamma'}} \right] \text{ for expansion}$$

$$\text{or} \quad E_t = \frac{\gamma}{\gamma-1} p_0 v_0 \left[ \left( \frac{p_t}{p_0} \right)^{\frac{\gamma'-1}{\gamma'}} - 1 \right] \text{ for compression}$$

and hence, for both cases,

$$\text{since} \quad p_t v_t = p_0 v_0 \left( \frac{p_t}{p_0} \right)^{\frac{\gamma'-1}{\gamma'}}$$

we have, from equation (257),

$$g\gamma p_0 v_0 \left( \frac{p_t}{p_0} \right)^{\frac{\gamma'-1}{\gamma'}} - V_0^2 = 2g \frac{\gamma}{\gamma-1} p_0 v_0 \left[ 1 - \left( \frac{p_t}{p_0} \right)^{\frac{\gamma'-1}{\gamma'}} \right]$$

which gives

$$\left( \frac{p_t}{p_0} \right)^{\frac{\gamma'-1}{\gamma'}} = \frac{\gamma-1}{\gamma+1} \frac{V_0^2}{g\gamma p_0 v_0} + \frac{2}{\gamma+1} \quad (258)$$

This equation shows that the pressure at the throat depends on the initial state  $p_0$ ,  $v_0$ , on the initial velocity  $V_0$ , and on the kind of substance flowing ( $\gamma$ ).

With  $V_{a0}$  as the acoustic velocity in the initial state, we have

$$g\gamma p_0 v_0 = V_{a0}^2$$

so that 
$$\left(\frac{p_t}{p_0}\right)^{\frac{\gamma-1}{\gamma}} = \frac{\gamma-1}{\gamma+1} \frac{V_0^2}{V_{a0}^2} + \frac{2}{\gamma+1} \quad (259)$$

Again, since

$$V_t = \sqrt{g\gamma p_t v_t}$$

or, with the value of  $p_t v_t$  given above,

$$V_t = \sqrt{g\gamma p_0 v_0 \left(\frac{p_t}{p_0}\right)^{\frac{\gamma-1}{\gamma}}}$$

and substituting the value of  $\left(\frac{p_t}{p_0}\right)^{\frac{\gamma-1}{\gamma}}$  given by equation (259), we have

$$V_t = \frac{V_0^2}{V_{a0}^2} \quad (260)$$

**Accelerated flow.** When the initial velocity is small compared with the acoustic velocity at the initial state (as frequently occurs in accelerated

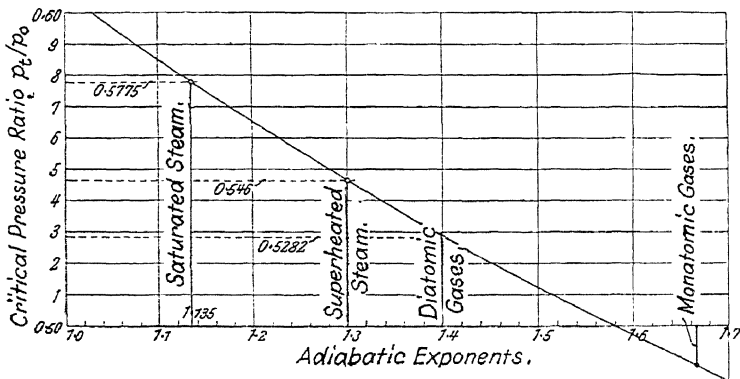


FIG. 116

flow), the first member on the right of equation (259) is small compared with the second, so that

$$\frac{p_t}{p_0} \approx \frac{2}{\gamma+1} \left(\frac{p_t}{p_0}\right)^{\frac{\gamma-1}{\gamma}} \quad (\text{critical pressure ratio})$$

This is exact when  $V_0 = 0$ , i.e. when flow commences from rest. In this case, as also for moderate initial velocities, the pressure at the throat depends only on the kind of gas or vapour ( $\gamma$ ) and not on the

initial state. Fig. 116 shows the values of  $\frac{p_t}{p_0}$  corresponding to the values of  $\gamma$  met with in practice (1 to 1.67). The changes in  $\frac{p_t}{p_0}$  are relatively small for different substances and range from 0.5 to 0.6. Hence the pressure at the throat cannot drop below half the initial pressure.

If, however, the ratio of the initial velocity  $V_0$  to the acoustic velocity  $V_{a0}$  is not negligible,  $\frac{p_t}{p_0}$  is larger, i.e. the reduction in pressure is less. In the extreme case, where  $V_0 = V_{a0}$ ,  $\frac{p_t}{p_0} = 1$ , as easily shown from equation (259), so that the throat is at the inlet, i.e. the pipe is entirely divergent (as shown previously). In convergent pipes, with the throat at the outlet section, the pressure at this section cannot drop below  $p_t$  and the velocity cannot exceed the acoustic. This is of considerable importance in the application to nozzle design. (See page 246.)

With  $V_0 = 0$ , we have

$$V_t = \sqrt{2g \frac{\gamma}{\gamma + 1} p_0 v_0}$$

which also represents the greatest possible discharge velocity from a convergent pipe.

**Compressive flow.** For a convergent divergent pipe the inlet velocity must be greater than the acoustic. Equation (258) gives

$$\left(\frac{p_t}{p_0}\right)^{\frac{\gamma-1}{\gamma}} = \frac{\gamma-1}{\gamma+1} \frac{V_0^2}{V_{a0}^2} + \frac{2}{\gamma+1}$$

where  $V_0 > V_{a0}$ . In the limiting case  $V_0 = V_{a0}$  and  $\frac{p_t}{p_0} = 1$ , i.e. a divergent pipe must be used.

Fig. 117 shows the amounts of compression obtained at the throat for different values of the ratio  $\frac{V_0}{V_{a0}}$  and for  $\gamma = 1.4$  (diatomics at normal temperatures), as well as  $\gamma = 1.135$  (initially saturated steam) and  $\gamma = 1.3$  (superheated steam).

For values of  $\frac{V_0}{V_{a0}}$  up to 2 the curves practically coincide. For higher ratios, however, the pressure rise for steam is considerably greater than for gases. The line for steam applies only so long as no superheating occurs due to the compression. In the case of initially dry steam the curve for superheated steam should be applied. If the inlet velocity is more than double the acoustic velocity the pressure rise up to the throat is very rapid and so produces a considerable amount of compression. The values of  $\frac{p_t}{p_0}$  fail to agree entirely with the corresponding values in expansive processes and are very much higher.

The velocity at the throat is given by equation (260),

$$\text{i.e.} \quad V_t = \sqrt{g \gamma p_0 v_0 \left( \frac{\gamma-1}{\gamma+1} \frac{V_0^2}{V_{a0}^2} + \frac{2}{\gamma+1} \right)}$$

This velocity is always greater than in the expansion nozzle (assuming the same  $p_0 v_0$ ), since  $V_0 > V_{a0}$  and can have widely different values, depending on the ratio  $\frac{V_0}{V_{a0}}$ , whereas in the expansion nozzle (assuming small inlet velocities)  $V_t$  has a definite value, which depends on the initial state alone.

The ratio of the inlet area  $a_0$  to the throat area  $a$  follows from

$$\frac{a_0}{a} = \frac{V_t v_0}{V_0 v_t} = \frac{V_t}{V_0} \left( \frac{p_t}{p_0} \right)^{\frac{1}{\gamma}}$$

which, with equations (259) and (260), gives

$$\frac{a_0}{a} = \left[ \frac{\gamma - 1}{\gamma + 1} \frac{V_0^2}{V_{a0}^2} + \frac{2}{\gamma + 1} \right]^{\frac{\gamma + 1}{2(\gamma - 1)}} \frac{V_{a0}}{V_0} \quad (261)$$

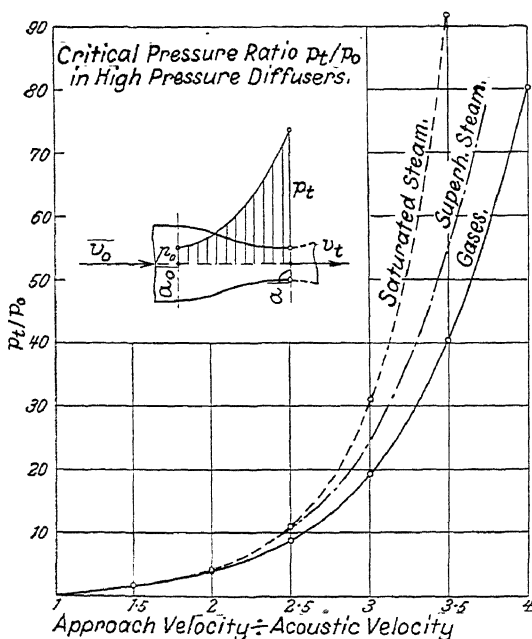


FIG. 117

From this it follows that a nozzle which is used for compression, and which has a given throat-inlet ratio, can be correct only for a single value of the inlet velocity  $V_0$ , since the acoustic velocity changes but slightly with changes in the inlet state of the flowing substance. An alternative method of expressing this is to state that a given nozzle can only be correct for different initial values of  $p_0$ ,  $v_0$ , and  $T_0$  provided it has the correct value of  $V_0$ .



The ratios of the areas, i.e.  $\frac{a_0}{a}$  are found from equation (261). and are given below for  $\gamma = 1.4$  and for the values of  $\frac{V_0}{V_{a0}}$  shown.

$\frac{V_0}{V_{a0}} =$	1.1	1.25	1.5	1.75	2	2.5	3
$\frac{V_0 a_0}{V_{a0} a} =$	1.109	1.309	1.763	2.426	3.375	6.592	12.70
$\frac{a_0}{a} =$	1.008	1.047	1.173	1.627	1.687	2.637	4.233

while for  $\gamma = 1.3$ , i.e. for superheated steam,

$\frac{V_0 a_0}{V_{a0} a} =$	1.110	1.314	1.791	2.50	3.55	7.39	15.50
$\frac{a_0}{a} =$	1.009	1.051	1.194	1.429	1.775	2.956	5.167

**Mass flow through convergent divergent nozzles.** In the normal process (i.e. expansion only or compression only throughout), the velocity at the throat of area  $a$  is, from equation (256),

$$= \sqrt{g \gamma p_t v_t}$$

and the mass flow per second through this section (and hence through every other section) is

$$W = \frac{a \dot{V}_t}{v_t}$$

or 
$$W = a \sqrt{g \gamma \frac{p_t}{v_t}}$$

Now 
$$v_t = v_0 \left( \frac{p_0}{p_t} \right)^{\frac{1}{\gamma}}$$

so that 
$$\frac{p_t}{v_t} = \frac{p_t}{p_0} \left( \frac{p_t}{p_0} \right)^{\frac{1}{\gamma}} = \frac{p_0}{v_0} \left( \frac{p_t}{p_0} \right)^{\frac{\gamma+1}{\gamma}}$$

and from equation (258)

$$\frac{p_t}{p_0} = \left( \frac{\gamma-1}{\gamma+1} \frac{V_0^2}{g \gamma p_0 v_0} + \frac{2}{\gamma+1} \right)^{\frac{\gamma}{\gamma-1}}$$

hence 
$$W = a \sqrt{g \gamma \frac{p_0}{v_0} \left( \frac{\gamma-1}{\gamma+1} \frac{V_0^2}{g \gamma p_0 v_0} + \frac{2}{\gamma+1} \right)^{\frac{\gamma+1}{\gamma-1}}} \quad (262)$$

The mass flow through nozzles of this type is thus dependent on the throat, the inlet state, the inlet velocity, and the kind of substance flowing. The back pressure at the final section does not appear in (262), so that, for all back pressures for which the normal

operation is possible, the mass flowing per second depends merely on the initial state. It is only necessary to ensure that the outlet area of the nozzle is such that the pressure there is the same as the outlet pressure.

If the inlet velocity is zero, i.e.  $V_0 = 0$ , equation (262) gives

$$W = a \quad (263)$$

This is also the greatest possible flow through a convergent nozzle. (See equation (276), page 243.)

In compression nozzles, since  $V_0$  is greater than  $V_{a0}$ , equation (262) must be used. For the same initial pressure  $p_0$ , the mass flowing can be considerably greater than that given in expansion when the value of  $\frac{V_0^2}{g\gamma p_0 v_0}$  or  $\frac{V_0}{V_{a0}}$  is large. Also the flow is considerably influenced by the approach velocity, whereas in expansion the effect of this is negligible.

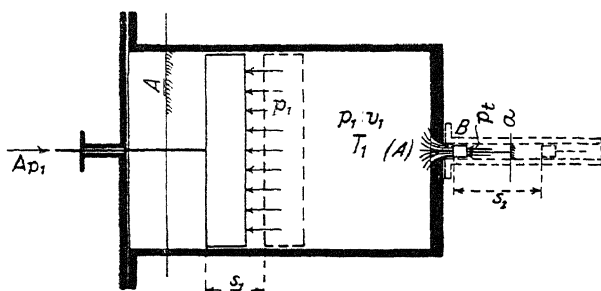


FIG. 118

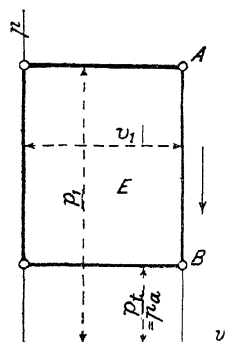


FIG. 119

**Discharge of gases and vapours from nozzles.** The discharge of gases and vapours, i.e. of elastic substances, is different from that of liquids in that, during discharge, not only the pressure but also the specific volume varies, due to pressure changes.

The discharge velocity of water, under constant pressure supply, is found as follows. (Figs. 118 and 119.)

Consider a piston of area  $A$  ft.<sup>2</sup> which exerts a pressure of  $p$  lb./ft.<sup>2</sup> on the water in the cylinder. When 1 lb. of water flows through a nozzle fitted in the cylinder end, the piston moves through a distance  $s_1$  such that  $As_1$  is the specific volume of the water,

$$\text{i.e.} \quad v_1 = 0.016 \text{ ft.}^3$$

$$\text{hence} \quad s_1 = \frac{v_1}{A}$$

The work performed by the piston rod is  $E = Ap_1s_1 = p_1v_1$  ft. lb. In the same way the jet performs the work  $p_tv_t$  ft. lb. where  $p_t$  is the pressure and  $v_t$  the specific volume of the liquid at exit. In the case of liquids, however, the jet pressure is the same as the pressure  $p_a$  of the space into which the jet discharges. This now gives  $p_t = p_a$ . Also  $v_t = v_1 = v_a$ .

In the case of gases and vapours, however,  $p_t$  is not necessarily the same as  $p_a$ , as sometimes  $p_t$  is greater than  $p_a$ , and  $v_t$  is always greater than  $v_1$ .

The excess work done on the water, per pound, is

$$E = p_1 v_1 - p_t v_t$$

and serves to produce the kinetic energy of the water at the jet.

If the jet area is small compared with the area  $A$  of the cylinder, the kinetic energy of the water in the cylinder may be neglected (approach velocity). Hence, for liquids, the discharge velocity is

$$V = \sqrt{2gE}$$

$$\text{or} \quad V = \sqrt{(p_1 v_1 - p_t v_t)}$$

$$\text{or} \quad V = \sqrt{v_1(p_1 - p_a)}$$

$$\frac{V(p_1 - p_a)}{\rho_1} = 8.025 \sqrt{\frac{p_1 - p_a}{\rho_1}}$$

If  $p_1 - p_a = H$  is the pressure difference in lb. ft.<sup>2</sup> and  $\rho_1$  is the density in lb./ft.<sup>3</sup>,  $\rho_1 H = p_1 - p_a$ , or, since  $\rho_1 = \frac{1}{v_1}$ ,  $H = \frac{p_1 - p_a}{\rho_1}$ , so that

$$V = \sqrt{2gH}$$

which is the usual expression for the discharge velocity of liquids.

**Small pressure differences.** The above expressions apply to gases also, when the pressure differences are so small that no serious error occurs, due to neglecting the changes in specific volume, i.e. on the assumption that  $v_a = v_t = v_1$ . For values of  $\frac{p_a}{p_1}$  below 0.9, however, the formulae deduced later must be applied, since the simple equations deduced here are then invalid.

Small pressure differences are usually expressed in inches of water. In the equations above  $p_1$  and  $p_a$  are in lb. ft.<sup>2</sup>, so that if  $h$  is the pressure difference in inches of water, the equation for the velocity is

$$V = \sqrt{\frac{2g \times 0.036 \times 144h}{\rho_1}} = 18.27 \sqrt{\frac{h}{\rho_1}} \quad (264)$$

with  $\rho_1$  in lb./ft.<sup>3</sup>

The volume discharged in ft.<sup>3</sup>/sec. from a nozzle having a cross-sectional area  $A$  ft.<sup>2</sup> is

$$V' = AV = 18.27A \sqrt{\frac{h}{\rho_1}} \quad (265)$$

and the weight discharged in lb./sec. is

$$W = AV\rho_1 = 18.27A \quad (266)$$

**Example 5.** Air at atmospheric pressure (14.7 lb./in.<sup>2</sup> abs. with  $\rho = 0.0808$  lb./ft.<sup>3</sup>) and atmospheric temperature flows through a nozzle with

the pressure differences 0.04, 0.4, 4, and 40 in. of water. Find the corresponding outlet velocities.

Equation (264) gives

$$V = \frac{18.27}{\sqrt{0.0808}} \sqrt{h} = \sqrt{h}$$

so that the velocities are  $V = 12.88, 40.7, 128.8$ , and  $407$  ft./sec. respectively.

**Example 6.** If, in Example 5, the area of the nozzle is  $0.155$  in.<sup>2</sup>, find the volumes and weights discharged per second.

Equation (265) gives

$$V' = \frac{18.27 \times 0.155}{144 \sqrt{0.0808}} \sqrt{h} = 0.0695 \sqrt{h} \text{ ft.}^3/\text{sec.}$$

and equation (266) gives  $W = 0.0808 \times 0.0695 \sqrt{h} = 0.00562 \sqrt{h}$ . These give for

$h$	=	0.04	0.4	4	40	in. of water
$V'$	=	0.0139	0.044	0.139	0.44	ft. <sup>3</sup> /sec.
and $W$	=	0.00112	0.00356	0.0112	0.0356	lb./sec.

**Example 7.** In a pipe, used for conveying illuminating gas, the gauge pressure is  $1.18$  H<sub>2</sub>O and the temperature  $12^\circ$  C. The barometric pressure is  $29.5$  Hg. Find the velocity and volume discharged in 10 min. from a circular orifice  $0.04$  in. in diameter.

The density of the gas at  $0^\circ$  C. and  $30$  Hg is  $0.03$  lb./ft.<sup>3</sup>, hence at  $29.5$  Hg and  $12^\circ$  C. it is

$$\rho_1 = 0.03 \times \frac{29.5}{30} \times \frac{273}{(273 + 12)} = 0.0282 \text{ lb./ft.}^3$$

Equation (264) gives the velocity as

$$V = 18.27 \sqrt{\frac{1.18}{0.0282}} = 118 \text{ ft./sec.}$$

The area of the orifice is  $\frac{0.785 \times 0.04^2}{144}$  ft.<sup>2</sup>

so that the volume discharged per second is

$$V' = AV = \frac{0.785 \times 0.04^2 \times 118}{144}$$

and, in 10 min.,

$$V' = \frac{0.785 \times 0.04^2 \times 118 \times 600}{144}$$

If the orifice is sharp edged the jet area is smaller than the area of the orifice and the amount flowing is about  $\frac{2}{3}$  of the value found above. (See page 253.)

**Discharge with large pressure differences.** If the pressures are plotted

to a base of volumes (Fig. 119) the states of the liquid before and after discharge are represented by the points  $A$  and  $B$ , where  $B$  lies vertically below  $A$ . The work  $E$  is represented by the rectangle  $(p_1 - p_a)v$ .

In the case of gases and vapours, however, a volume increase occurs, corresponding to the pressure drop from  $p_1$  to  $p_a$ . In Fig. 120 a portion of the gas at  $A$ , in traversing the path  $AB$ , experiences a volume increase, due to the pressure decrease, and the velocity increases from  $V = 0$  at  $A$  to  $V_t$  at  $B$ .  $v_t$  is greater than  $v_1$  so that, on the  $pv$  field (Fig. 121),  $B$  does not lie on the vertical through  $A$ , but to the right of this, by an amount equal to the increase in volume between  $p_1$  and  $p_t$ .

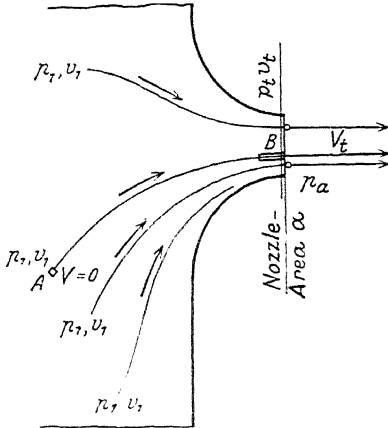


FIG. 120

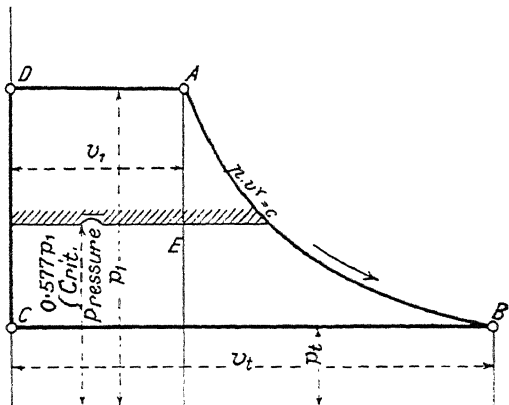


FIG. 121

The position of the points at which the pressure reductions commence in the chamber and the course of the pressure changes through the nozzle have no effect on the discharge. All the gas particles experience the same pressure and volume change in their passage from the vessel to the plane of the orifice. The curve  $AB$  in Fig. 121 merely serves to express the instantaneous relationships between the pressure and volume, i.e. it serves to show the change in state in the same sense as previously considered. The fact that 1 lb. of gas or vapour is accelerated in flowing through the nozzle, in addition to suffering a change of state, has no effect on the change of state itself.

As in the case of expansion in a cylinder, a continuous increase in volume occurs here from  $v_1$  to  $v_t$ . This expansion is adiabatic and isentropic if no cooling or heating occurs at the nozzle and the flow is frictionless. As before, the law of expansion is given by  $pv^\gamma = \text{constant}$ , where  $\gamma = 1.4$  for gases at moderate temperatures,\* 1.3 for superheated steam, 1.135 for dry saturated steam, and  $1.035 + 0.1q$  for wet steam, having the dryness fraction  $q$ .

In the case of liquids, the work required to accelerate the jet is  $p_1 v_1 - p_t v_t = (p_1 - p_a) v_a$  but with elastic fluids extra work is available, due to expansion, and is supplied by the internal energy of the substance. This extra work is represented by the area  $E$ , below the state curve  $AB$

\* See pages 53, 60, and 137 for hot gases and products.

down to the abscissae axis. The total accelerating work, per pound of gas or vapour, is thus

$$E = p_1 v_1 - p_t v_t + E_e$$

This expression is represented in Fig. 121 by the area  $ABCD$ .

$$\text{As before, } E = \frac{1}{2g} \frac{v_t^2}{v_t}$$

$$\text{so that } V_t = \bar{E} \text{ (discharge velocity) .} \quad (267)$$

In this form the expression corresponds to that for liquids, namely,

$$V = \sqrt{2gH}$$

The pressure head  $H$  is replaced by the work value  $E$ , or, if expressed in heat units, by the heat drop  $AE$ . It should be noted that  $H$ , as given above, represents a quantity of work and not merely a length or height.

The mass flow per second follows from the velocity  $V_t$  thus—

The volume flowing through the orifice area  $a$  per second is  $V' = V_t a$ .

Since the specific volume is  $v_t$  and the density  $\frac{1}{v_t}$ , the weight flowing per second is, therefore,

$$W = \frac{V'}{v_t} = \frac{a V_t}{v_t}$$

The work area  $ABCD$ , Fig. 121, is, from page 327,

$$E = \frac{\gamma}{\gamma-1} (p_1 v_1 - p_t v_t)$$

which, since  $p_1 v_1^{\gamma'} = p_t v_t^{\gamma'}$  can be written as

$$E = \frac{\gamma}{\gamma-1} p_1 v_1 \left[ 1 - \left( \frac{p_t}{p_1} \right)^{\frac{\gamma-1}{\gamma}} \right]$$

and hence the discharge velocity at the orifice is

$$V_t = \sqrt{2g \frac{\gamma}{\gamma-1} p_1 v_1 \left[ 1 - \left( \frac{p_t}{p_1} \right)^{\frac{\gamma-1}{\gamma}} \right]} \quad (268)$$

The weight discharged per second is, then, with  $v_t = v_1 \left( \frac{p_1}{p_t} \right)^{\frac{1}{\gamma}}$

$$W = a \sqrt{2g \frac{\gamma}{\gamma-1} \frac{p_1}{v_1} \left[ \left( \frac{p_t}{p_1} \right)^{\frac{2}{\gamma}} - \left( \frac{p_t}{p_1} \right)^{\frac{\gamma+1}{\gamma}} \right]} \quad (269)$$

**Back pressures above the critical.** If the ratio  $\frac{p_1}{p_a}$ , i.e. the ratio of the absolute initial pressure in the chamber to the absolute external pressure, be varied, it is found, as shown later, that down to a definite limit, the value of the pressure at discharge from the nozzle is equal to the external pressure, i.e.  $p_t = p_a$ .

Hence, within this region, equations (268) and (269) give

$$V_t = \sqrt{2g \frac{\gamma}{\gamma-1} p_1 v_1 \left[ 1 - \left( \frac{p_a}{p_1} \right)^{\frac{\gamma-1}{\gamma}} \right]} \quad . \quad . \quad . \quad (270)$$

and 
$$W = a \sqrt{2g \frac{\gamma}{\gamma-1} \frac{p_1}{v_1} \left[ \left( \frac{p_a}{p_1} \right)^{\frac{2}{\gamma}} - \left( \frac{p_a}{p_1} \right)^{\frac{\gamma+1}{\gamma}} \right]} \quad . \quad . \quad . \quad (271)$$

When  $p_1 = p_a$  both  $V$  and  $W$  are zero. For values of  $\frac{p_a}{p_1}$  which are not much less than unity, equations (270) and (271) become the same as equations (264) and (266).

Writing 
$$\psi = \sqrt{\frac{\gamma}{\gamma-1} \left[ \left( \frac{p_a}{p_1} \right)^{\frac{2}{\gamma}} - \left( \frac{p_a}{p_1} \right)^{\frac{\gamma+1}{\gamma}} \right]} \quad . \quad . \quad . \quad (272)$$

gives, for equation (271),

$$\frac{W}{a} = \psi \sqrt{2g \frac{p_1}{v_1}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (273)$$

or 
$$= 8.025 \psi \sqrt{\frac{p_1}{v_1}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (274)$$

where  $a$  is the area in ft.<sup>2</sup> and  $p_1$  is in lb. ft.<sup>2</sup> abs.

With a constant internal pressure  $p_1$  and a varying external pressure  $p_a$ , equation (274) shows that  $W$  depends only on  $\psi$ , i.e. on the value of the ratio  $\frac{p_a}{p_1}$ , equation (272). For the same pressure ratio, but different values of  $p_1$  and  $p_a$ ,  $W$  is proportional to  $\sqrt{\frac{p_1}{v_1}}$ . The value of this quantity, for gases, follows from

$$p_1 \quad \frac{p_1^2}{RT_1}$$

giving  $p_1$

i.e. with a fixed initial temperature and fixed pressure ratio the weight discharged per second is proportional to the pressure.

For a given constant initial state the discharge velocity  $V$  depends only on the pressure ratio and increases as the back pressure is decreased. When the pressure ratio is constant,  $V$  is proportional to  $\sqrt{p_1 v_1}$ , which, for dry saturated steam, depends but slightly on the pressure, and, for gases, only on the temperature. Hence, for the same initial temperature,  $V$  depends only on the pressure ratio and, therefore, is entirely independent of the absolute pressure in the case of gases, and almost so in the case of saturated vapours.

If the values of  $\psi$  are plotted to a base of the pressure ratios  $\frac{p_a}{p_1}$ , the values of which range from 0 to 1 (Fig. 122), it will be observed that

$\psi$  (and therefore the mass flow) does not increase continuously as the back pressure is decreased, but attains a maximum value for a definite value of the pressure ratio (depending on  $\gamma$ ) and thereafter decreases. With an absolute vacuum in the outer space ( $\frac{p_a}{p_1} = 0$ ) the value of  $W$ , as thus

given, is zero, as is the case when  $\frac{p_a}{p_1} = 1$ . This, however, is contrary

what actually occurs, for gases or vapours do flow into a space in which the back pressure is almost or entirely zero. Hence the equations

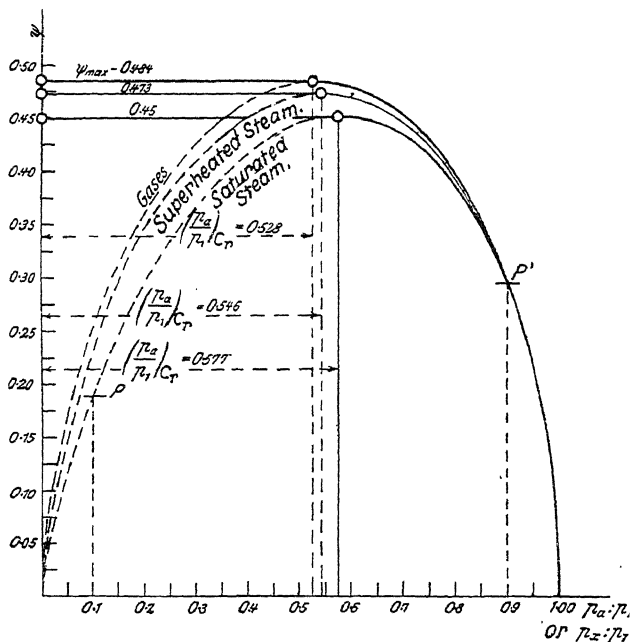


FIG. 122

do not apply in this region. It is impossible for the flow to decrease when the back pressure decreases. Tests show that, below a certain pressure ratio, the rate of flow remains constant at the maximum value however small the back pressure may be. The dotted portions of the curves in Fig. 122, therefore, fail to give the mass discharged,\* and should be replaced by straight horizontal lines, continued from the highest points of the curves. Tests have further shown that the equations, established above, give the correct mass flow from zero up to the maximum values.

**Critical pressure ratio.** The pressure ratio  $\frac{p_a}{p_1}$ , at which the mass flow first becomes a maximum, is called the "critical pressure ratio." As shown in Fig. 122, its value for gases (with  $\gamma = 1.4$ ) is  $\left(\frac{p_a}{p_1}\right)_{cr} = 0.528$ , for superheated steam 0.546, and for dry saturated steam 0.577.

\* For their true significance, see page 248.



The critical value can be determined from the maximum value of

$$\left(\frac{p_a}{p_1}\right)^{\frac{2}{\gamma}} - \left(\frac{p_a}{p_1}\right)^{\frac{\gamma+1}{\gamma}}$$

which, for a fixed internal pressure, determines the rate of flow (equation (271)).

Differentiating this with respect to  $\frac{p_a}{p_1}$ , and equating to zero,

$$\text{gives} \quad \frac{2}{\gamma} \left(\frac{p_a}{p_1}\right)^{\frac{2}{\gamma}-1} - \frac{\gamma+1}{\gamma} \left(\frac{p_a}{p_1}\right)^{\frac{1}{\gamma}} = 0$$

$$\text{from which} \quad \left(\frac{p_a}{p_1}\right) = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} \quad (275)$$

as in Fig. 116.

The maximum rate of flow, expressed in the form of equation (273), is then

$$\begin{aligned} \left(\frac{W}{a}\right)_{max} &= \psi_{max} \sqrt{2g \frac{p_1}{v_1}} \\ \text{and} \quad \psi_{max} &= \sqrt{\left(\frac{2}{\gamma+1}\right)^{\frac{2}{\gamma-1}} - \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}} \sqrt{\frac{\gamma}{\gamma-1}} \\ &= \left(\frac{2}{\gamma+1}\right)^{\frac{1}{\gamma-1}} \sqrt{\frac{\gamma}{\gamma+1}} \\ \text{so that} \quad \left(\frac{W}{a}\right)_{max} &= \left(\frac{2}{\gamma+1}\right)^{\frac{1}{\gamma-1}} \sqrt{\frac{p_1}{v_1} \frac{2g\gamma}{\gamma+1}} \quad (276) \end{aligned}$$

as in equation (263).

For gases with  $\gamma = 1.4$ ,

$$W_{max} = 3.89a \sqrt{\frac{p_1}{v_1}}$$

and, for superheated steam,

$$W_{max} = 3.80a \sqrt{\frac{p_1}{v_1}}$$

where  $a$  is in ft.<sup>2</sup> and  $p_1$  in lb./ft.<sup>2</sup> abs.

**Back pressures below the critical.** The fact that the rate of mass flow remains constant, when the back pressure is below a certain value, or the inlet pressure is above a certain value, means also, as mentioned above, that the pressure just at the nozzle outlet is different from the back pressure. This outlet pressure remains constant and is equal to the critical pressure, whatever low value the back pressure may have. The

gas or vapour flows in lines parallel to  $AB$  (Fig. 123) only so long as  $p_t$  remains equal to  $p_a$ . If  $p_t$  is greater than  $p_a$  the gas deviates from this direction, as shown by  $AC$ , and the amount of this deviation (i.e. the angle  $\delta$ ) depends on the difference  $p_t - p_a$ . It is not until the jet has passed the exit section that the further expansion from  $p_t$  to  $p_a$  occurs, as also established experimentally by Stodola.

The nozzle exit pressure is given by

$$p_t = \left( \frac{2}{\gamma + 1} \right)^{\frac{\gamma}{\gamma - 1}} p_1$$

which gives, for saturated steam,

$$p_t = 0.577 p_1$$

Now, since, under these conditions,  $p_t$  remains constant, it follows that the available work, between  $p_1$  and  $p_t$ , used in accelerating the gas, is also constant, and with it the discharge velocity  $V_t$  and the mass flow.

The velocity, therefore, cannot exceed the maximum velocity given by

$$V_t = \sqrt{2g \frac{\gamma}{\gamma - 1} p_1 v_1 \left[ 1 - \left( \frac{p_t}{p_1} \right)^{\frac{\gamma - 1}{\gamma}} \right]}$$

which, with  $\frac{p_t}{p_1} = \left( \frac{2}{\gamma + 1} \right)^{\frac{\gamma}{\gamma - 1}}$  . . . . . (277)

gives  $(V_t)_{max} = \sqrt{2g \left( \frac{\gamma}{\gamma + 1} \right) p_1 v_1}$  . . . . . (278)

Replacing  $p_1$  and  $v_1$  by  $p_t$  and  $v_t$ , as given by  $p_1 v_1^\gamma = p_t v_t^\gamma$ , and substituting the value for  $\frac{p_t}{p_1}$  as given by equation (277), we have

$$(V_t)_{max} =$$

This expression represents the acoustic velocity at the state existing in the nozzle exit plane. Hence, in the normal type of convergent nozzle, the discharge velocity does not exceed the acoustic velocity. This is of importance in steam turbine nozzle design. In nozzles of prismatic or convergent shape, having the smallest sectional area at exit, the exit velocity cannot exceed the acoustic velocity, however low the back pressure may be.\* The pressure at the exit section cannot be less than  $0.528 p_1$  for air,  $0.577 p_1$  for saturated steam, and  $0.546 p_1$  for superheated steam and hot gases. With high pressure differences the work which can be transformed to kinetic energy is only that available between  $p_1$  and  $0.528 p_1$  to  $0.577 p_1$ .

In Fig. 121 this is represented by the area above the shaded horizontal line. The transformation of the remainder, lying below this line, is

\* In the free jet beyond the nozzle, and in nozzles having an oblique exit section, the steam can assume a velocity above the acoustic, but only up to certain limits. (See page 270.)

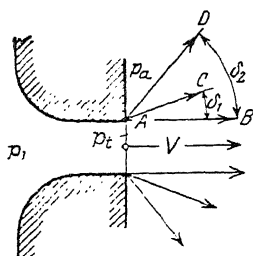


FIG. 123



where  $v_1 = \frac{155(273 + t_1)}{p_1} - 0.0525$  ( $p_1$  in lb./ft.<sup>2</sup> abs. and  $t_1$  in ° C.)

For saturated steam, with  $\gamma = 1.135$ , we have

$$V_{max} = 5.84 \sqrt{p_1 v_1} \quad (284)$$

Between 40 and 170 lb. in.<sup>2</sup> abs. (5760 and 24,500 lb./ft.<sup>2</sup> abs.),  $\sqrt{p_1 v_1}$  ranges from 246 to 257, so that the steam velocity, issuing from convergent nozzles, cannot exceed 1500 ft./sec.

The weight discharged per ft.<sup>2</sup> with high pressure ratios (i.e. above

$$\frac{1}{0.577} = \frac{1.735}{1} \left) \text{ is} \right. \\ \left. \left( \frac{W}{a} \right)_{max} = 3.62 \sqrt{p_1} \quad (285)$$

**Influence of the approach velocity.** The above equations apply when the discharge occurs from a vessel, in which the contents may be regarded as being at rest. If, however, the flowing substance approaches the nozzle with an appreciable velocity  $V_0$ , the discharge velocity is higher than those given above. An example of this is shown in the passage of steam through reaction blades. Here the steam enters with considerable velocity and is accelerated in passing through the blades, owing to the further pressure drop occurring there. In multistage turbines, again, the guide blades receive the steam from the moving blades of the previous stage, with the absolute leaving velocity of this stage.

The kinetic energy  $\frac{V^2}{2g}$  of the discharged jet is equal to the sum of the original kinetic energy  $\frac{V_0^2}{2g}$  and  $E$  the equivalent of the heat drop, which is liberated by the expansion from the inlet to the outlet pressure.

This gives

$$\frac{V^2}{2g} = \frac{V_0^2}{2g} + E$$

so that  $V = \sqrt{V_0^2 + 2gE}$

$E$  is determined, as before, from the heat drop.

### EXPANSION NOZZLES (de LAVAL NOZZLES)

As shown on page 242, the discharge velocity of a gas or vapour, from a convergent or parallel nozzle (upper part of Fig. 124), of any cross-sectional shape, cannot exceed the acoustic velocity, and the available work for producing the discharge velocity is limited by the initial pressure  $p_1$  and the pressure at the exit section, i.e.  $p_t = 0.577p_1$  (for saturated steam). This is a serious disadvantage in impulse turbines, which operate with convergent nozzles discharging steam to a wheel chamber, in which the pressure is equal to the exhaust pressure, since only the kinetic energy contained in the jet is convertible to mechanical work in a wheel of this type. The remaining steam energy, i.e. the portion represented by the area below the hatched line in Fig. 121, is entirely

lost. The loss is increased as the initial pressure is increased, or the exhaust pressure is decreased.

This loss, which renders the operation of turbines of this type uneconomical, can, however, be avoided as first shown by de Laval of Stockholm.

If the nozzle converges down to the minimum section and thereafter diverges (Fig. 124), it is found that the pressures in the diverging portion drop below  $0.577p_1$ . The work available for the production of the velocity is thus increased, the amount of this increase depending more or less on the amount of the divergence. In this way the discharge velocity is higher than the acoustic velocity at the throat, and the diverging portion can be so designed that the steam expands down to the back pressure,

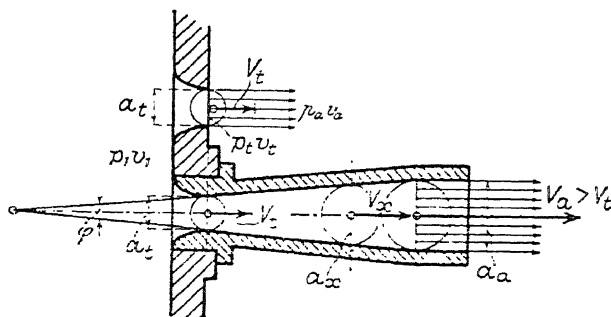


FIG. 124

i.e. to atmospheric pressure in non-condensing turbines, to the exhaust pressure in condensing turbines or to the stage pressure in multistage turbines.

In order to carry this out in practice, it is necessary that the cone angle of the diverging portion of the nozzle should not exceed about  $10^\circ$ , as otherwise the jet tends to leave the wall. The length of the nozzle thus depends on the amount of expansion.

In this type of nozzle the frictional effect is considerably greater than in the convergent type, due to the higher velocities (up to 3500 ft./sec.) and larger wall surfaces. The frictional effect will, however, be neglected in this preliminary treatment.

On this assumption the change of state occurs adiabatically. The pressure  $p_x$  and specific volume  $v_x$  at any section are then found from

$$p_x v_x^\gamma = p_t v_t^\gamma \quad (\text{or} = p_1 v_1^\gamma)$$

The same mass flow per second occurs through every cross section ( $a_x$ ). Since the pressure at the throat ( $a_t$ ) is

$$p_t = p_1$$

the mass flow is given by equation (273),

$$\text{i.e.} \quad W = 0.025 a_t \psi_{\max} \sqrt{\frac{p_1}{v_1}} = b a_t \sqrt{\frac{p_1}{v_1}} \quad (286)$$

( $a_t$  in ft.<sup>2</sup> and  $p_1$  in lb./ft.<sup>2</sup> abs.)



shows that the ratio of the highest ordinate  $\psi_{max}$  to that of any other ordinate  $\psi_x$ , corresponding to the pressure ratio  $\frac{p_x}{p_1}$ , is the same as the ratio of the nozzle area (at the section where the pressure is  $p_x$ ) to the throat area.

It should be remembered, however, that expansion also occurs before the throat is reached, and from inlet to the throat the pressure drops from  $p_1$  to  $p_t$ , i.e. in the ratio  $\frac{p_t}{p_1}$ . This region is represented by the full curves in Fig. 122, since, for the pressure ratios between 1 and  $\frac{p_t}{p_1}$ , the same equations apply as in the diverging portion.

Since the ratio of the maximum ordinates ( $\psi_{max}$ ) of the curves to the ordinates  $\psi$  on the dotted or full lines, at any arbitrarily chosen points such as  $P'$  or  $P$ , is the same as the ratio of the nozzle areas, at these points, to the throat areas, it follows that the ordinates of the  $\psi$  curve will represent the reciprocal values of the cross-sectional areas if the maximum ordinate is made equal to the throat area.

It also follows that, with a continuous pressure drop accompanied by a velocity increase, the nozzle areas must first decrease and then increase. In the first portion the steam expands from  $p_1$  to  $p_t = 0.577p_1$  (for saturated steam), and the velocity becomes equal to the acoustic velocity under the throat conditions, i.e. at  $p_t$  and  $v_t$ . In the second portion the pressure drops still further, depending on the ratio of the exit area to the throat area. The velocity beyond the throat is thus higher than the acoustic velocity.

In the upper part of Fig. 125 the ratios  $\frac{a_x}{a_t}$  have been plotted to a base of pressure ratios  $\frac{p_1}{p_x}$  for saturated steam, superheated steam, and gases. With a given pressure drop, the largest increase in area occurs with saturated steam and the smallest with gases. The portion on the right of the diagram was drawn for expansion ratios between 25 and 100 for which a smaller scale was necessary.

In the lower portion of Fig. 125 the diameters of a nozzle of circular cross section have been plotted to the same pressure ratio base. The resulting nozzle is conoidal in shape. The pressures have been plotted below the nozzle and give, for example, a value  $p_1$  against the pressure ratio 1, and the value  $\frac{1}{17}p_1$  against the pressure ratio of 17. This type of diagram can also be used to determine the pressure changes in a given nozzle. It is only necessary to determine the ratios  $a_x/a_t$  at various sections of the given (e.g. conical) nozzle and from these determine the pressure ratios by means of the upper curve in Fig. 125.

When the exit area  $a_a$  is such that the pressure just at outlet is equal to the back pressure, the discharge velocity is given by

$$V_a =$$

where  $E$  is the available energy between  $p_1$  and  $p_a$ .

This gives

$$V_a = \sqrt{2g \frac{\gamma}{\gamma-1} p_1 v_1 \left[ 1 - \left( \frac{p_a}{p_1} \right)^{\frac{\gamma-1}{\gamma}} \right]} \quad (292)$$

Values of  $\frac{v}{V_a}$  have also been plotted in Fig. 125 for saturated steam.

When the pressure ratio is about 11 the velocity is double the acoustic velocity. For a pressure expansion ratio of 12 the discharge velocity from the nozzle is  $2.03 V_t$  (i.e. about 3060 ft./sec. if  $p_1 = 176.4$  lb./in.<sup>2</sup> abs.) and for a ratio of 100 it is  $2.6 V_t = 3920$  ft./sec.

If superheated steam is supplied to a nozzle and, at a particular section of the nozzle, changes to the saturated state, equations (289), (290), and (292) no longer apply, owing to the change in the adiabatic exponent from 1.3 to 1.135. If the change occurs before the throat, equation (286) and the expressions for the throat pressure are also inapplicable. In these cases it is better to apply graphical solutions by means of the entropy diagrams (see pages 455 and 465), although general expressions can also be deduced.

**Effect of increasing the back pressure.** The ratio  $a_a/a_t$  of the divergent portion of a nozzle determines the ratio of the initial to the final pressure for a given gas or vapour. Nozzles are, therefore, only correctly designed provided the back pressure is the same as the exit pressure, as given by this ratio. If the back pressure is increased above this, it has been shown by Stodola's tests that the expansion of the substance is different from that in the normal expansion in that its pressure drops below the back pressure. At some point  $X$  (Fig. 125), however, the pressure commences to rise again and continues to rise until it becomes equal to the back pressure. Impact occurs at  $X$ , causing a loss in the jet energy. The position of  $X$  depends on the back pressure, and as this is increased  $X$  recedes from the nozzle end.

**Steam consumption of nozzles with varying initial pressures.** The rate of flow from a given nozzle having a throat area  $a_t$  is

$$W = ba_t \sqrt{\frac{p_1}{v_1}}, \quad ba_t \quad p_1$$

Now in the case of dry saturated steam, the product  $p_1 v_1$  varies but slightly with the pressure, as can easily be verified by reference to steam tables. In the case of superheated steam, the equation  $p_1 v_1 = RT_1 - Cp_1$  applies, and the term  $Cp_1$  is small compared with  $RT_1$ . For the same steam temperatures, therefore,  $p_1 v_1$  depends but slightly on the pressure. Hence it follows that the rate of flow is proportional to the initial pressure, both for saturated and superheated steam. Thus, at 147 lb./in.<sup>2</sup> abs. (10 atm.), the rate of flow is  $\frac{1.0}{0.8} = 1.25$  times that at 118 lb./in.<sup>2</sup> abs. (8 atm.), and twice as great as that at 73.5 lb./in.<sup>2</sup> abs. (5 atm.). The back pressure does not affect this, so long as it is not high enough to influence the conditions as far back as the throat or does not exceed the critical pressure.

On the other hand, by changing the amount of superheat and keeping the pressure constant, the rate of flow decreases as the temperature rises, being approximately inversely proportional to the square root of the absolute temperature.

If steam is only moderately superheated, so that it becomes saturated before the throat is reached, this rule does not apply.

**Expansion in non-divergent nozzles or channels having an oblique exit section.** The exit sections of the nozzles or channels of a steam



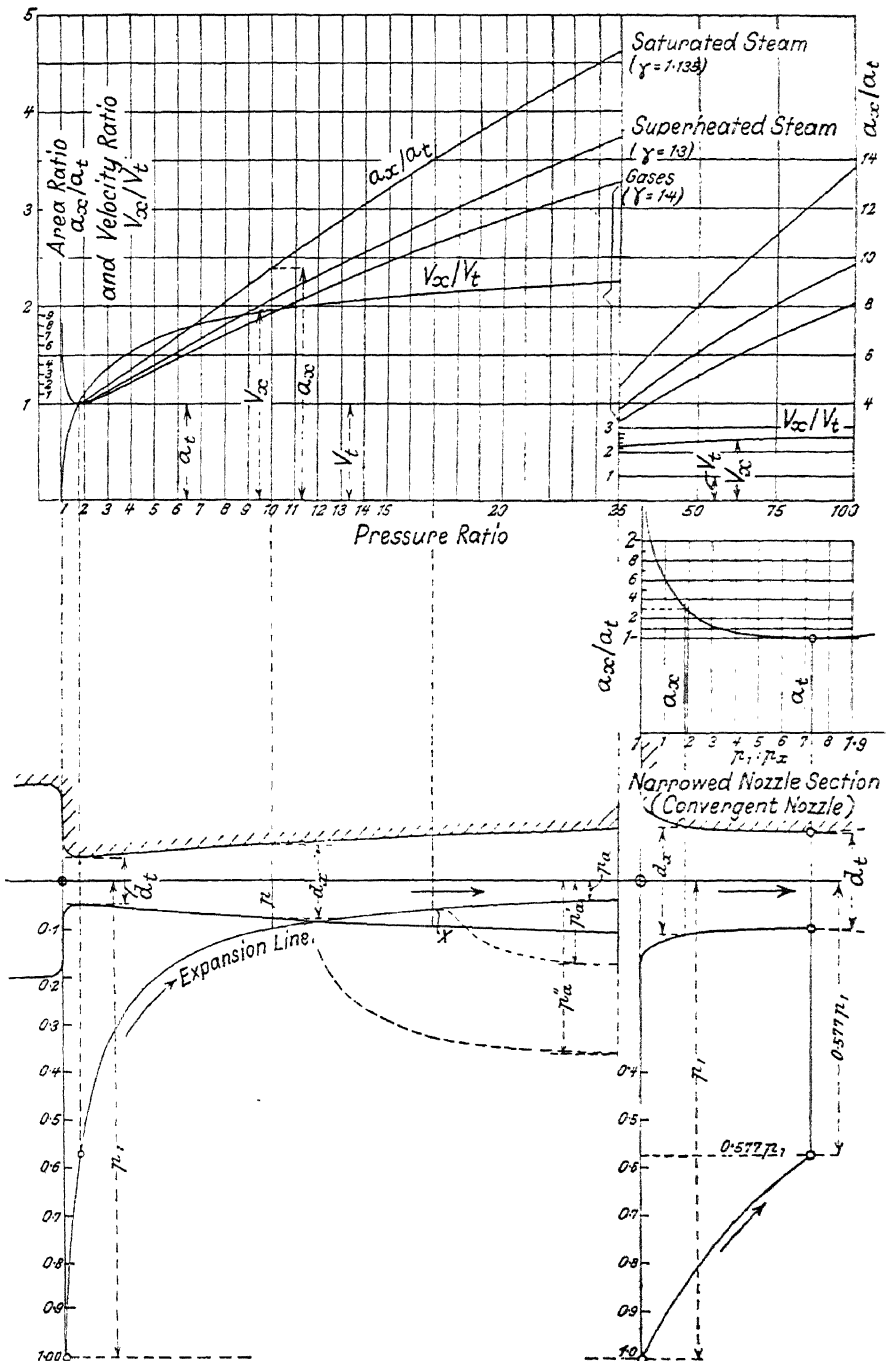


FIG. 125

turbine are not perpendicular to the direction of the jet at discharge but inclined to it, as in Fig. 126. Now, in the case of convergent nozzles, so long as the pressure in the wheel chamber is not below the critical value, the jet is discharged in the direction  $BE$ , i.e. at the angle  $\alpha$  and with a velocity  $V < V_a$ . If, however,  $\frac{p_a}{p_1}$  is less than 0.545, the velocity of

the jet at the narrowest section  $AC = a_t$  cannot exceed the acoustic velocity  $V_a$  however great  $p_1$  may be compared with  $p_a$  and the pressure at  $AC$  cannot exceed the critical pressure, i.e.  $0.545p_1$ . Beyond  $AC$ , however, the steam tends to expand from  $p_t$  to  $p_a$ . The area required

for this is obtained by a deflection of the jet at the angle  $\alpha' > \alpha$ . This deflection commences in  $ACB$ , and the jet thereafter becomes a parallel stream of area  $a_a$ . Equation (291), which applied to de Laval nozzles, can be used here

in finding the ratio  $\frac{a_a}{a_t}$  from the pressure ratio  $\frac{p_a}{p_1}$ . From Fig. 126, we have  $\sin \alpha' = \frac{a_a}{a_t} \sin \alpha$  since

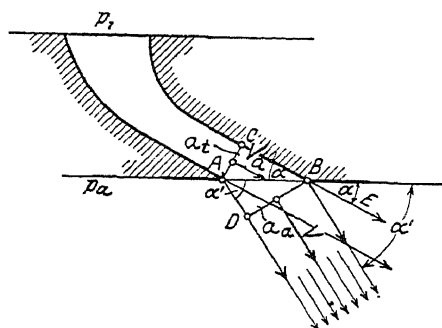


FIG. 126

$$\frac{t}{r} = \sin \alpha \text{ and } \frac{a_a}{a_t} = \sin \alpha'$$

This jet deflection has been confirmed by observation.

It follows that non-divergent nozzles, with oblique exit sections, are capable of operating with an increased pressure drop, so that the discharge velocity is increased, accompanied, however, by a deflection of the jet, and this deflection increases as the jet velocity  $V$  exceeds the acoustic velocity. Expansion also occurs in the oblique portion of convergent-divergent nozzles when they operate with pressures above the normal pressures.

A theoretical treatment of this will be found on page 270.

**Actual rate of flow and discharge velocities.** The actual rate of flow and the discharge velocity depend not only on the area of the throat but also on the shape of the nozzle. In this, the shape of the cross section (i.e. circular, square, rectangular, etc.) is of less importance than the shape of the axial section.

All the tests carried out on air or steam show agreement with the equations (264) to (285) provided the nozzle is well rounded at inlet, as shown in Figs. (120), (123), and (124).

In all cases the substance requires a certain amount of time and a corresponding distance to acquire the discharge velocity from a state of rest. If the nozzle is sharp edged, as shown in Fig. 127, the formation of the jet commences within the vessel. Tests with liquids show visibly that the direction of flow of the outer surface of the jet, at the plane of the orifice, is not parallel to the jet axis. In the case of a circular orifice, the free jet is conoidal in shape and only becomes cylindrical at some distance beyond the orifice. The maximum jet velocity  $V$  occurs at the narrowest jet area  $A' < A$ , as shown by the equation of continuity, and

the flow per second is, therefore, not  $A V \rho$  but  $A' V \rho$ . The ratio  $\frac{A'}{A} = C_c$  is called the "coefficient of contraction," and gives

$$W = C_c A V \rho$$

If a nozzle be designed so that its inner surface has the same shape as the free jet (see lower part of jet in Fig. 127), and also, so that all the stream lines at the throat are parallel, the flow through it will be approximately the same as that given by calculation. This has been confirmed by tests, which also show that the perimeter is of less account than the rounding of the nozzle at inlet. Even a small amount of rounding has a considerable effect.

When the nozzle, however, is well rounded, the actual mass discharged is still below the calculated. This is due to the discharge velocity  $V$  being less than the ideal  $V_0$ , as found from the pressure ratio and the properties of the fluid. A part of this velocity is destroyed by friction in the nozzle, so that

$$V = C_v V_0$$

where  $C_v$  is the "coefficient of velocity."

Hence, for a convergent nozzle,

$$W = C_c C_v A V_0 \rho$$

The product  $C_c C_v = C_d$  gives

$$W = C_d A V_0 \rho$$

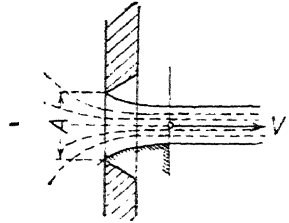


FIG. 127

For the general case, however, the relation  $C_d = C_c C_v$  does not apply. Thus, in the case of a sharp-edged entrance to a pipe, it can be assumed that the contraction within the pipe is similar to that of the free opening shown in Fig. 127. The discharge velocity  $V$ , however, is determined at some distance in the pipe beyond this section, by taking account of the impact loss due to the expansion from the narrowest section to the pipe section. Due to this impact loss, some of the kinetic energy is converted to heat. Hence  $C_c$  and  $C_v$  are not related in the above simple manner, since they correspond to different sections.

In all nozzles, therefore, which are not sharp-edged orifices of the type shown in Fig. 127, the rate of flow is written as

$$W = C_d W_0 \quad (293)$$

where  $W_0$  is the theoretical rate of flow, neglecting friction and contraction.  $C_d$  is called the "coefficient of discharge."

Further, 
$$V = C_v V_0 \quad (294)$$

where  $V$  is the actual discharge velocity.  $C_d$  and  $C_v$  are the only values which can be found by test directly.

In place of the coefficient  $C_v$ , which is of importance in turbine design, another quantity will be introduced here. The value  $C_v$  enables the loss in energy per pound to be determined without reference to the coefficient of discharge  $C_d$ . The energy of the jet, with no velocity loss, is the same

as the available work  $E_0 = \frac{V_0^2}{2g}$ , but the actual jet energy is  $\frac{V^2}{2g}$ , so that the loss is

$$W_v = \frac{V_0^2}{2g} - \frac{V^2}{2g} = \frac{V^2}{2g}$$

or  $E_v = E_0 (1 - C_v^2)$

Denoting the ratio  $\frac{E_v}{E_0}$  by  $\zeta$

gives  $\zeta = 1 - C_v^2$  . . . . . (295)

$\zeta$  is called the "coefficient of resistance."

For gases and vapours a further relationship has to be established between  $C_d$  (of equation (293)) and  $C_v$  or  $\zeta$  (of equations (294) and (295)). Due to the frictional work the jet is heated so that the specific volume  $v'$  at the throat is greater than that given by the usual equation with adiabatic expansion. As will be shown later, this increase in volume is the same as would be obtained by heating the gas, after adiabatic expansion at the exit pressure, if the heat supplied is  $A\zeta E_0$ .

If now we write  $\frac{v'}{v} = 1 + \beta$ , the rate of flow,

with  $W = \frac{a_a V'}{v'}$

is  $W = \frac{a_a C_v V_0}{v(1 + \beta)} = \frac{C_v}{1 + \beta} \frac{a_a V_0}{v}$

but  $\frac{a_a V_0}{v} = W_0$

so that  $W = \frac{C_v}{1 + \beta} W_0$

from which  $\frac{C_v}{1 + \beta} = C_d$  . . . . . (296)

The value to be given to  $a_a$  is that corresponding to the final velocity.

In equation (296)  $\beta$  is still dependent on  $C_v$  but the relationship is somewhat involved. Thus, in the case of gases, we have, due to the heating effect caused by friction,

$$A\zeta E_0 = c_p(T' - T) = c_p T \left( \frac{T'}{T} - 1 \right)$$

or  $\frac{T'}{T} = \frac{A\zeta E_0}{c_p T} + 1$

in which  $T'$  is the actual, and  $T$  the adiabatic, final temperature. From Gay-Lussac's Law, this is also the proportional volume increase, so that

$$= 1 + \frac{A\zeta E_0}{c_p T} \quad . \quad (297)$$

and, if  $AE_0$  is replaced by the heat drop  $h_0$ ,

$$C_a = \frac{\sqrt{1-\zeta}}{1 + \frac{\zeta h_0}{m}} \quad (298)$$

since  $C_v = \sqrt{1-\zeta}$  from equation (295).

In this way the value of  $\zeta$  can be found when  $C_a$  is known from test results.

(See page 309 on the representation of this on the entropy diagram and the difference between the real frictional work and the loss in energy, which, however, has no effect on the above treatment.)

**Experimental verification of the discharge equations.** As previously mentioned, tests for this purpose can only be carried out with short well-rounded nozzles. Such tests, with up-to-date equipment and due regard

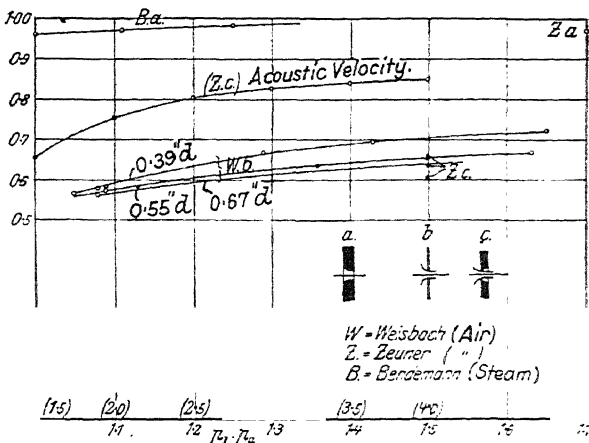


FIG. 128

to the various sources of error, have been carried out by Zeuner\* on air, by Gutermuth and Blaess† on dry saturated steam, and by Bendemann,‡ as well as Loschge,§ on superheated steam. They all agree in showing that the measured amounts discharged differ by only a few per cent from those given by calculation.

In Zeuner's earlier tests, in which compressed air was discharged from a vessel, he found that, for pressures more than double the exit pressure (thus giving the acoustic velocity) the agreement with equation (276) was such that the discrepancies could be accounted for by the errors of observation. He thus confirmed St. Venant's and Wantzel's Law of constant rate of flow for pressures above the critical.

In later and more exact tests by Zeuner (flow of atmospheric air into

\* Zeuner, *Technische Thermodynamik*, 1st Edn. (1887), page 250. 2nd Edn. (new tests) (1900), page 256. As carried out in the Dresden Tech. High School.

† *Forsch. Arb.* No. 19 (1904).

‡ *Forsch. Arb.* No. 37, in which appear remarks on other tests.

§ *Z.V.d.I.* (1913), page 60.

an evacuated vessel, through nozzles of 5, 11, and 15 mm. diameter) the rates of flow were found to be slightly less than the calculated, and gave  $\zeta = 0.066, 0.068$ , and  $0.044$  each at the acoustic velocity. These correspond to  $C_v \doteq 0.97$ , which is about the same as  $C_d$ .

Weisbach had found earlier that the value of  $C_d$ , for smaller pressure ratios, was also  $0.97$  when the nozzles were well rounded and short. His results, along with Zeuner's, as shown in Fig. 128, give an horizontal straight line throughout. For low pressure ratios the values seem to be slightly less.

Gutermuth found, in the case of steam flowing through a well-rounded convergent nozzle, "complete agreement with the calculated values," both for small and high pressure ratios, and for initial steam pressures ranging from  $15$  to  $135$  lb./in.<sup>2</sup> abs.

Bendemann, whose results confirmed all the earlier tests, found, in the case of moderately superheated steam and for all pressure ratios, that the values of  $W$ , as found by test, were from  $3$  to  $4$  per cent smaller than the calculated values, so that  $C_d = 0.96 - 0.97$ . For saturated steam, however, the actual discharges, with pressure ratios above  $1.3$ , are slightly higher than the calculated values, so that apparently  $C_d$  is greater than unity. Again, saturated steam and superheated steam showed the same discharge, for the same pressure ratio, which is also contrary to the theory already given.

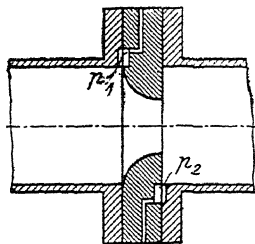


FIG. 129

Loschge, however, confirmed these results. For superheated steam at about  $260^\circ \text{C}$ . and between  $75$  and  $105$  lb./in.<sup>2</sup> abs., he found  $C_d = 0.97$ , as also for pressure ratios below and above the critical up to about  $0.85$ . In

the case of saturated steam, however, with super-critical pressure ratios, the actual discharge was about  $3$  per cent greater than the calculated. This apparent contradiction is now explained by the theory of supersaturation, in which the steam continues to behave as if it were superheated, and so becomes undercooled.

The velocity coefficient for nozzles, as used in the above tests, is about the same as  $C_d$ . The difference is found from

giving  $C_v$  slightly higher than  $C_d$ . By taking account of the difference we have

$$C_v = 0.975.$$

**Measuring nozzles.** The volume (or weight) of a gas or vapour flowing in a pipe can be measured by inserting a suitable measuring nozzle in the pipe (Fig. 129). By means of a differential manometer the pressure drop in the stream between inlet to, and exit from, the nozzle is measured, while the pressure at inlet is measured by means of a normal gauge (or the pressures  $p_1$  and  $p_2$  alone are measured). The mass discharged per second is then given by

$$W = C_d W_0$$

## FLOW OF GASES

where, for a small pressure drop (page 237),

$$W_0 = AV\rho =$$

so that  $W = C_d A$

and  $V = C_d A$

The value of  $\rho$  is that corresponding to the outlet conditions, and  $V$  is the volume of gas discharged per second. Since, however, this equation is only valid for small pressure differences and, in addition, the approach velocity is neglected, it is better, as pointed out by Jakob and Fritz,\*

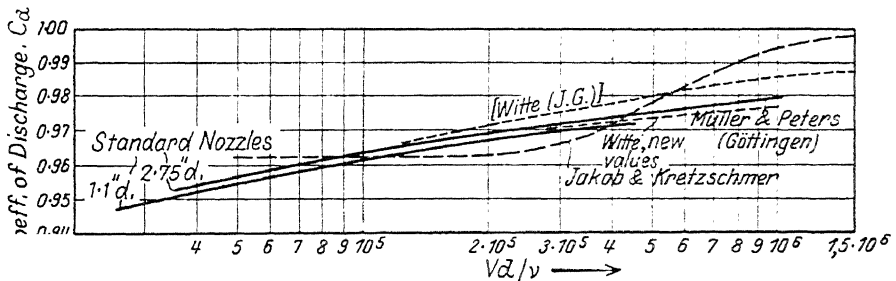


FIG. 130

to take the mean value  $\rho_2$  where  $\rho_1$  and  $\rho_2$  are the densities before and after the nozzle.

Using German† standard nozzles, Jakob and Erk‡ obtained the following values—

Nozzle Diameter Inches	Material	Pipe Diameter Inches	$C_d$ Nozzle at Pipe End	$C_d$ Nozzle in Pipe
7.88	C.I. Machined	19.7	0.955	0.9645
5.50	C.I. Machined	13.8	0.957	0.963
2.75	C.I. Nickelled and polished	6.87	0.965	0.955
3.13	Wood. Painted and varnished	6.87	0.944	0.946
3.13	Wood. Painted and varnished	Very large	0.972	—

The mean value of  $C_d$  from these is 0.96.

Very complete tests have recently been carried out on air, steam, and water by Jakob and Kretzschmer,§ by Witte,|| and by H. Müller

\* *Z.V.d.I.* (1928), page 116. "Application of simplified formulae to nozzles and channels."

† See *Regeln für Leistungsversuche an Ventilatoren und Compressoren*. (Rules for output tests of fans and compressors.)

‡ *Z.V.d.I.* (1924), page 581, and *Forsch. Arb.* No. 267.

§ *Forsch. Arb.* No. 311. "Coefficients of discharge of standard nozzles." (3.94 in. to 39.4 in. diameter).

|| *Z.V.d.I.* (1928), page 1493. " $C_d$  values for water, oil, steam, and gas."

and H. Peters.\* Their results are shown in Fig. 130, where the coefficients of discharge have been plotted to a base of Reynolds's numbers  $Vd/\nu$ , which enable the influence of velocity, diameter, and kinematic viscosity (and hence the kind of fluid) to be expressed. (See page 277.)

The results show that  $C_d$  lies between 0.95 and 0.985, and increases as Reynolds's number increases.

**Sharp-edged diaphragm orifices.** Tests show that the flow through sharp-edged diaphragm orifices is always less than that through well-rounded nozzles under the same conditions.

The lower curves in Fig. 128 show Weisbach's† results for his tests with air.  $C_d$  lies between 0.55 and 0.7. The increase with increasing

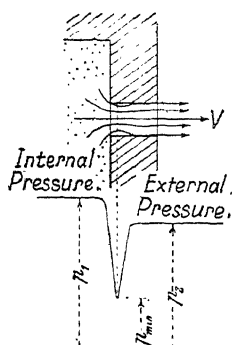


FIG. 131

pressure ratios is quite marked, but is slightly reduced for increased areas. Zeuner's results, obtained with a round, sharp-edged orifice, are also shown. His curve commences at the pressure ratio 1.5 and lies close to Weisbach's. The discharge coefficient increases with the pressure and continues to increase until after the acoustic velocity is attained, i.e. for values of  $p_1/p_a$  above 1.7. The abscissae values above 1.5 have been re-drawn to a smaller scale (bracketed abscissae values), for which the corresponding  $C_d$  curve is marked (Z.c.). As the air pressure rises from 1.5 to 4 atmospheres,  $C_d$  increases from about 0.65 to 0.83, but the discharge velocity remains constant.‡

In these tests the observed reduction in the rate of flow is due entirely to the jet contraction, so that

since  $C_v = 1$ . This follows, since the surface friction at the sharp edge must be less than in the case of nozzles which are rounded at inlet.

For very small pressure differences, giving at most  $\frac{p_1}{p_a} = 1.005$ ,

A. O. Müller§ found, in careful tests with air, that  $C_d = 0.597$ , which is appreciably higher than Weisbach's value of 0.54 at this pressure ratio. Weisbach's orifice, however, was not really sharp edged, as it consisted of a hole bored in metal 0.1 mm. thick. (See Fig. 143.)

**Flow through short cylindrical nozzles or bores.** Impulse and reaction measurement. The entrance edge is of importance in that it decides whether or not contraction will occur. If the edges of short cylindrical nozzles are well rounded at inlet the conditions are similar to the nozzles considered on page 256. The term "discharge area" means the area of the exit section of the nozzle.

When the edges at inlet are sharp, contraction occurs just beyond inlet both in parallel and in convergent bores. If the bore is deep enough,

\* *Z.V.d.I.* (1929), page 966. " $C_d$  values for standard nozzles."

† From Grashof's *Hydraulik*.

‡ The only reference to this interesting result is in the earlier edition of Zeuner's *Thermodynamik*.

§ *Forsch. Arb.* No. 49. "Measurement of the flow of gases by means of throttle plates."



the jet fills the full bore and leaves with a cross-sectional area equal to that of the bore. Weisbach has observed that, in the case of water and air, the pressure at the contraction and in its immediate neighborhood drops below the outlet pressure, so that the velocity  $V$  at this point is higher than corresponds to the pressure drop between inlet and outlet. Beyond the contraction, the jet velocity decreases in agreement with the increase in area, so that the kinetic energy of the jet decreases by  $\frac{V'^2}{2g} - \frac{V^2}{2g}$ , where  $V$  is the outlet velocity. Part of this energy decrease is utilized in increasing the static pressure and the remainder is converted

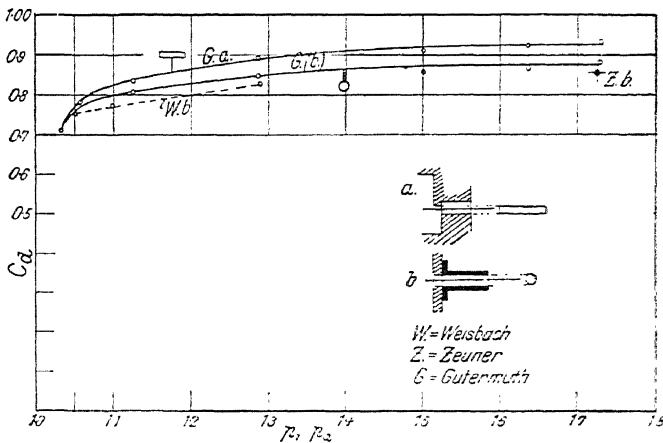


FIG. 132

to heat and must be regarded as a nozzle loss. Now this loss is considerably greater than the loss caused by friction on the walls of convergent nozzles, so that the velocity coefficient  $C_v$  is considerably less. The rates of flow are also less than with a well-rounded inlet, but greater than those obtained through a sharp-edged diaphragm orifice of the same diameter, notwithstanding the velocity reduction. Weisbach's and Zeuner's results with air, and Gutermuth's with dry steam, are shown in Fig. 132. In all these cases, and for all pressure ratios, the rates of flow are less than the theoretical.

Gutermuth's tests, with a circular passage, show slightly higher values than the others, while the rates of flow through a rectangular passage are definitely higher than those through a circular passage under the same conditions, probably because a flat plate was fitted in front of the orifice, causing an increase in the coefficient of contraction. It is not correct to regard the openings used in Gutermuth's tests as sharp-edged orifices, similar to those in diaphragms, as proved not only by the resulting discharge coefficients but also by Stodola's pressure measurements on similar openings, which show throughout the same pressure changes as in passages of the type shown in Fig. 131.

The values of the velocity coefficients have been determined in more recent tests, using parallel passages rounded at inlet and also convergent

and convergent-divergent sections. These are of importance in their application to turbine design, particularly with regard to the energy losses. The measurements were carried out either by the impulse method or by the reaction method. In both cases the impulse or reaction is given by

where  $V$  is the outlet velocity and  $W$  the mass flowing per second. By measuring  $P$  and  $W$ , the value of  $V$  is thus determined. See page 580 on the limits of application of this method.

The first impulse measurements with de Laval nozzles were probably carried out by E. Lewicki.\* Later tests of a similar nature were carried

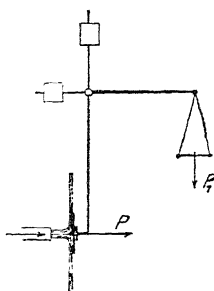


FIG. 133

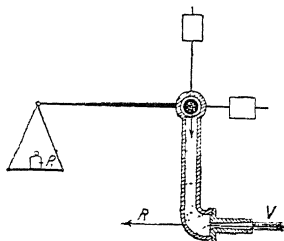


FIG. 134

out by Briling.† Lewicki showed, in these tests, that it was possible for the velocity, in de Laval nozzles, to exceed the acoustic velocity, as was previously established theoretically by Zeuner, although disputed later by other authorities partly as a result of tests. Lewicki's tests, moreover showed good agreement between the actual and experimental rates of flow both for saturated and superheated steam.

Using slightly superheated steam discharging through a cylindrical nozzle 0.254 in. diameter and 1.18 in. long, Briling found  $C_v = 0.95$  for velocities ranging from 500 to 1440 ft./sec. and  $C_v$  remained unchanged, using a rectangular section 0.157 in.  $\times$  0.394 in., under the same conditions. With a slightly convergent nozzle, having a minimum diameter of 0.338 in., the values of  $C_v$  ranged from 0.92 to 0.965 as the velocity was increased, i.e. as the pressure ratio was increased. (Fig. 128.)

A large number of reaction tests have been carried out by Josse and Christlein. Six different de Laval nozzles, as used in steam turbines, were tested, having sectional area ratios which increased from 1 : 1.185 to 1 : 12.95. Tests were also made on three turbine blade channels.

If the nozzles are used with the correct pressure ratio, the coefficient of velocity has its highest value, as would be expected, since the only loss is that due to friction, i.e. there is no impact loss. The values obtained in this way from these tests showed that the losses in nozzles with a relatively large increase in area, with a consequent high leaving velocity, were not so great as in nozzles with smaller area increases. The highest

\* *Z.V.d.I.* (1903), page 441. "Application of high superheat to steam turbines."

† *Forsch. Arb.* 68.

values of  $C_v$  were obtained at speeds between 2600 and 3600 ft. sec., where it amounted to about 0.95, while for a speed of 2100 ft. sec. it was 0.94. In the case of the guide blades the value was about 0.92 to 0.93.

Assuming a mean value of  $C_v = 0.945$  for de Laval nozzles, the jet loss is

$$\zeta = 1 - C_v^2 = 1 - 0.945^2 = 0.107 \text{ or } 10.7 \text{ per cent,}$$

and, for channels with slightly superheated steam,

$$\zeta = 1 - 0.923^2 = 0.148 \text{ or } 14.8 \text{ per cent.}$$

The high value of  $C_v = 0.92$  given above for guide blade channels is obtained only for steam velocities as high as the acoustic velocity. As the

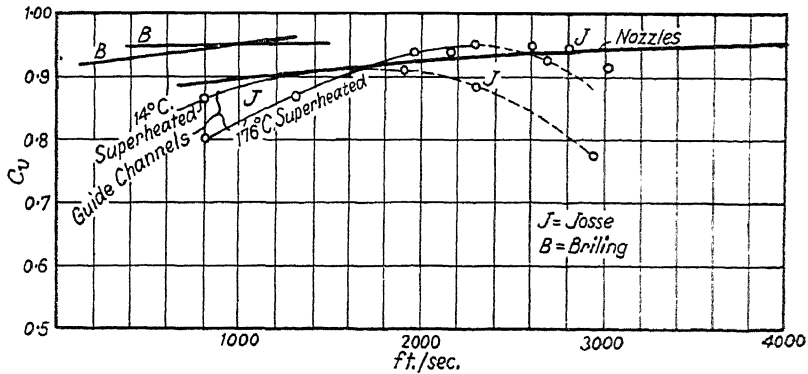


FIG. 135

velocity is decreased,  $C_v$  is also decreased, and at 800 ft. sec. becomes equal to 0.85, so that the energy loss is then

$$\zeta = 1 - 0.85^2 = 0.28 \text{ or } 28 \text{ per cent}$$

so that, at low speeds, the energy loss becomes serious.

When, at moderate speeds, the range of superheat is increased, the values of  $C_v$  are reduced, and it is only when the acoustic speed is reached that they become equal to those given by saturated steam (Fig. 135). The dotted portions of the curves in Fig. 135 have no real significance since they lie above the acoustic velocity.

Loschge, in measuring the rates of flow through similar channels, found that, for pressures at outlet below the critical pressure, the discharge coefficient was  $C_d = 0.92$ , and that, as the outlet pressure was increased,  $C_d$  was reduced, thus

with	$\frac{p_a}{p_1}$	=	0.7	0.8	0.9
	$C_d$	=	0.91	0.90	0.89

As in the case of convergent nozzles, the rates of discharge for superheated and dry steam were equal.

In testing de Laval nozzles with steam velocities above or below that corresponding to the area expansion ratio, the coefficients of discharge rapidly decrease.

In Fig. 136 will be found the results of the Institution of Mechanical Engineers Nozzle Research Committee on steam turbine nozzle tests,

which show, contrary to the other results, an increase in the velocity coefficient, as the velocity is decreased. The curve *R* refers to a Parsons reaction blading.

**Super-acoustic velocities in parallel or convergent nozzles.** As shown on page 222 *et seq.*, and also by tests, the discharge velocity from parallel or convergent nozzles cannot exceed the acoustic velocity  $V_s$ , corresponding to the state of the substance at the exit section. It follows, therefore, that the pressure at this section is higher than the back pressure  $p_a$ , when the ratio  $\frac{p_1}{p_a}$  (i.e. the inlet to the outlet pressure ratio) is above the critical value.

In apparent contradiction to this, E. Lewicki first showed, by means of impulse tests, that the discharge velocities from a convergent nozzle

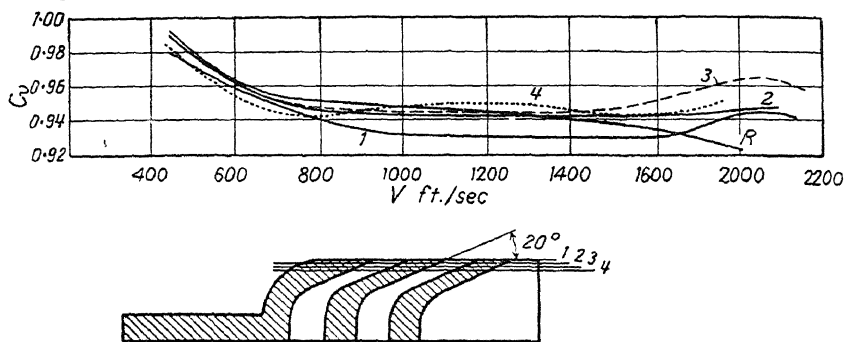


FIG. 136

were considerably greater than the acoustic velocities. The value of these super-acoustic velocities depends on the distance of the impact plate from the nozzle end (Fig. 133). The measured velocities have been plotted in Fig. 137.

The steam (saturated) inlet pressure was  $p_1 = 99$  lb./in.<sup>2</sup> abs., and this expanded through a nozzle into an air space, where the pressure was 14.40 lb./in.<sup>2</sup> abs., so that  $\frac{p_1}{p_a} = 6.87$ , which is well above the critical ratio of 1.732, for which the corresponding velocity is 1500 ft./sec. These tests show that the velocity increases (up to a certain limit) with the distance of the jet from the nozzle exit. This can only be caused by the excess pressure  $p_t - p_a$ , but the velocity increase cannot be calculated as if the jet were surrounded by a wall of the jet shape, in which case the jet velocity would rise to 2740 ft./sec., corresponding to the pressure

$p_a$ .

A certain limiting value of the final velocity of the free jet can, however, be determined as shown by Zerkowitz. The rate of flow from a convergent nozzle is independent of the outer pressure (provided this is below the critical), and is given by

$$= ba \sqrt{\frac{p_1}{v_1}}$$

$W$  is also independent of the processes occurring in the jet after it leaves the nozzle. Hence the velocity  $V_t$  at the nozzle exit must be given by

$$= V_n$$

as previously deduced.

Consider now the section of the free jet at which the static pressure within the jet becomes equal to the external pressure  $p_a$ . If it is assumed that no mixing with the surrounding air has occurred up to this section, so that the rate of flow is still  $W$  lb./sec., the following applies. The free external force used in accelerating the jet is  $a(p_t - p_a)$ , where  $p_t$  is the

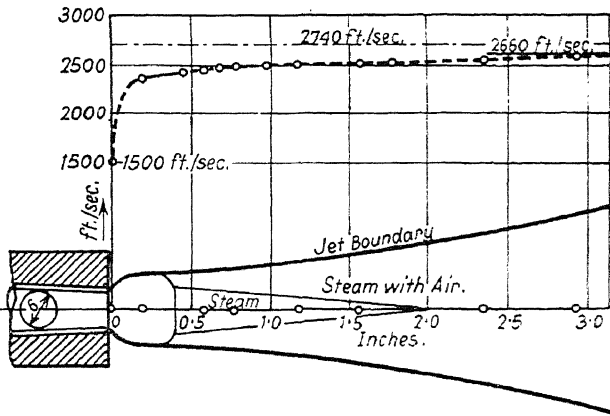


FIG. 137

pressure at the nozzle exit. The mass accelerated per second is  $W/g$ , and the velocity change along the nozzle axis is  $V - V_t$ , hence

$$(p_t - p_a) = \frac{W}{g} (V - V_t)$$

or 
$$V = V_t + \frac{W}{g} (p_t - p_a)$$

In the above test,  $W = 0.0608$  lb./sec.,  $p_t = 0.5775 \times 99 = 57.2$  lb./in<sup>2</sup> abs.,  $a = 0.0442$  in.<sup>2</sup>, so that

$$\begin{aligned} V &= V_t + \frac{0.0442 \times 32.2 (57.2 - 14.4) 144}{144 \times 0.0608} \\ &= V_t + 1000 \end{aligned}$$

With  $V_t = 1500$  ft./sec., we thus have

$$V = 1500 + 1000 = 2500 \text{ ft./sec.}$$

A considerable increase in velocity thus occurs outside the nozzle. The calculated values near the nozzle end, before mixing with the air occurs, agree well with the actual values measured by impact (Fig. 137).

Further on, however, the actual jet velocity is decreased owing to the work necessary to accelerate the air surrounding or mixing with it, and the determination of  $V$  from the impact equation

$g$

is no longer possible, since  $W$  is not now constant. If  $W$  were assumed to be constant the values of  $V$  would be too large. The true jet velocity is thus limited to the value calculated above.

It is important to notice that the super-acoustic velocities are only obtained outside a convergent or parallel nozzle.

**Two special cases.** Figs 138 and 139 represent special cases which have recently been investigated.\*

1. In the case represented in Fig. 138 discharge does not take place through a relatively small opening from a large vessel, as has been assumed in all the previous cases, but through a pipe having a diameter which is not much larger than the diameter of the orifice. Apart from the increase in velocity due to the approach velocity  $V_1$  and given by

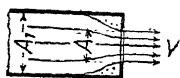


FIG. 138

$$\frac{V}{2g} = \frac{V_1^2}{2g}$$

another factor also tends to increase the rate of flow. Due to the flow, already established in the pipe, the stream lines in the neighbourhood of the opening now experience a smaller contraction, the amount of which depends on the ratio of the orifice area to the pipe area. If this ratio is increased (and the orifice is sharp edged), the contraction is reduced.

Thus, Müller found the following values, when air was discharged through circular sharp-edged orifices,

$A$	$\frac{1}{12.25}$	$\frac{1}{3.48}$	$\frac{1}{1.73}$
$C_d$	0.603	0.644	0.755

The pressure differences lay between 0.118 and 3.15"  $H_2O$ , which give

$$V = C_d A \sqrt{2gh}$$

and

$$W = C_d A \sqrt{2gh\rho}$$

2. Fig. 139 shows a pipe, fitted to a chamber, with a sharp-edged orifice at the pipe end. In this case, contraction is followed by an increase in area causing a retardation of the jet. The resulting energy loss is now greater, since the difference between the velocity in the pipe and that at the contraction is greater.

From Müller's tests with

$\frac{A}{A_1}$	$\frac{1}{12.25}$	$\frac{1}{3.48}$	$\frac{1}{1.73}$
$C_d$	0.632	0.685	0.764

where  $A$  is the area of the orifice.

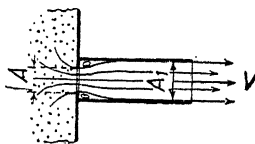


FIG. 139

\* *Forsch. Arb.* No. 49. A. O. Müller. "Measurement of gas flow with a throttle plate."

**Throttle plates for measuring rates of flow.** Quantities of gas or vapour flowing through a cylindrical pipe can be measured by means of a diaphragm having a concentric sharp-edged hole cut in it (Fig. 140). An orifice of this type has a throttling effect on the gas stream (see page 210), and a thrust is exerted on the side of the diaphragm facing the stream. If  $A_1$  is the cross-sectional area of the pipe,  $A$  the area of the orifice,  $V_1$  and  $V_2$  the mean gas velocities before and after the plate, and  $\rho_1$  and  $\rho_2$  the corresponding densities, we have

$$W = A_1 V_1 \rho_1 = A_1 V_2 \rho_2$$

so that  $\frac{V_1^2}{V_2^2} = \frac{\rho_2}{\rho_1}$

The gas flows through the orifice in the form of a closed jet, and shortly after passing the plate continuously increases in area until, at a certain section, it again completely fills the pipe. Between the orifice

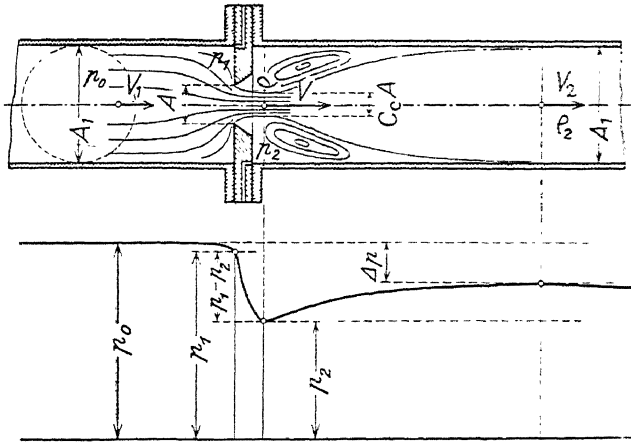


FIG. 140

and this section an irregular turbulent state exists in the space between the jet and wall surfaces (Fig. 140). The edge of the plate facing the stream must be sharp and the orifice should diverge rapidly at the back, so that the jet can develop freely, as in the discharge through an orifice in a thin wall. The narrowest jet section (*vena contracta*) then occurs at a short distance downstream from the orifice and is smaller than  $A$ . Its value is  $C_c A$  where  $C_c$  is the coefficient of contraction (see page 253). The velocity of the jet at this section is found from

$$W = C_c A V \rho$$

which, with  $A_1 V_1 \rho_1 = C_c A V \rho$ ,

gives

(299)

Since  $\frac{A_1}{A} > 1$ ,  $\rho_1 > \rho$ , and  $C_c < 1$ , it follows that  $V > V_1$ , i.e. the gas

velocity is always increased, due to the presence of the orifice. If, for example,  $\frac{A}{A_1} = \frac{1}{2}$ , the velocity is more than doubled. A pressure drop  $p_1 - p_2$  is required for this velocity increase, i.e. the pressure  $p_2$  at the *vena contracta* and in the space round the jet is less than the pressure  $p_1$  just before the orifice. Under these conditions (see page 219),

$$\frac{V_2^2}{2g} - \frac{V_1^2}{2g} = (p_1 - p_2)v_m = \frac{p_1 - p_2}{\rho_m} \quad (300)$$

where  $v_m$  is the mean specific volume, and  $\rho_m$  is the mean density, as given by the adiabatic  $p v$  expansion curve. If the pressure drop is relatively small,  $v_m = \frac{1}{2}(v_1 + v)$  or  $\rho_m = \frac{1}{2}(\rho_1 + \rho)$ .

From equations (299) and (300) and with  $\frac{A}{A_1} = m$ ,

$$\text{we have } p_1 - p_2 = \rho_m \frac{V^2}{2g} \left[ 1 - m^2 C_c^2 \left( \frac{\rho}{\rho_1} \right)^2 \right] \quad (301)$$

$$\text{so that } V = \sqrt{\frac{2g(p_1 - p_2)}{\rho_m} \times \frac{1}{1 - m^2 C_c^2 \left( \frac{\rho}{\rho_1} \right)^2}}$$

From this, with  $V' = C_c A V$  where  $V'$  is the volume flowing per second,

$$V' = \frac{C_c A}{\sqrt{1 - m^2 C_c^2 \left( \frac{\rho}{\rho_1} \right)^2}} \sqrt{\frac{2g(p_1 - p_2)}{\rho_m}} \quad (302)$$

$$\text{and writing } \frac{C_c}{\sqrt{1 - m^2 C_c^2 \left( \frac{\rho}{\rho_1} \right)^2}} = k \quad (303)$$

$$\text{gives } V' = k A \sqrt{\frac{2g(p_1 - p_2)}{\rho_m}} \quad (304)$$

$$\text{or } \frac{V'}{A} = k \frac{A}{A_1} \sqrt{\frac{2g(p_1 - p_2)}{\rho_m}} \quad (305)$$

Equation (304) is the usual discharge formula for an orifice of area  $A$  in a large vessel with an internal pressure  $p_1$  and an external pressure  $p_2$ , but the discharge coefficient  $k$  increases with  $m$ . The value of the volume flowing per second at the throttled conditions can be calculated from equation (304) when  $A$ ,  $\rho_1$ ,  $\rho$ ,  $\rho_m$ , and  $C_c$  are known and the pressure difference on the two sides of the plate is measured. If the pressure difference is small we can take  $\rho_1 = \rho = \rho_m$ . In measuring  $p_1 - p_2$  it should be observed that the points at which the pressures are read should be on the surface of the diaphragm and close to the pipe wall, as shown in Fig. 140.\* These points should, if possible, extend round the

\* See *Regeln für Leistungsversuche an Ventilatoren und Compressoren* (V.d.I., Verlag 1925), regarding the construction of these diaphragms.



inner edge. The method of taking the pressure measurement is just as important for the correct determination of the flow as the shape of orifice. If the plate be reversed the readings are no longer applicable.

The coefficient of contraction  $C_c$  of the sharp-edged orifice, as applied in equation (303), is not constant (see pages 257 and 264), since it increases as  $\frac{A}{A_1}$  increases. In addition,  $k$  depends on  $\frac{p_1}{p}$  or  $v_1$  i.e. on the amount of expansion to the *vena contracta*. The equation of the expansion curve is

which gives

If now  $p_1$  and  $(p_1 - p_2)$  are measured, and the value of  $k$  is determined from equation (303) by using the value of  $C_c$  from page 258,  $V$  is given by

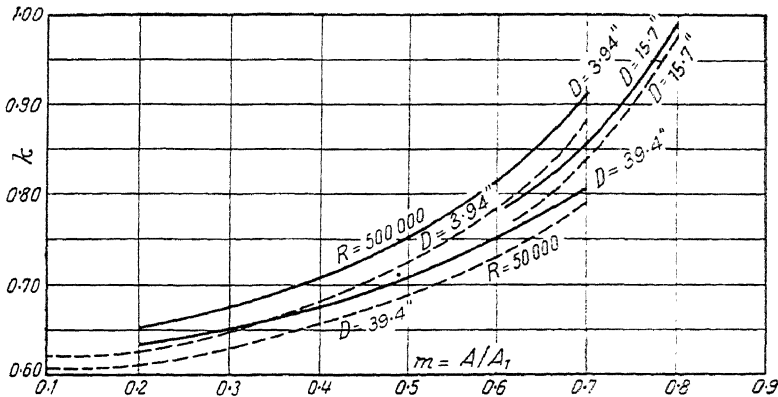


FIG. 141

equation (304). The values for  $C_c$ , however, are not extensive enough to suit all the cases arising in the measurement of flow. A large number of tests, however, on the direct determination of  $k$  in equation (304) have been carried out. The most extensive and reliable German tests are those by Jakob and Kretschmer,\* who measured the flow of air through pipes ranging from 4 in. to 40 in. in diameter. These tests served to show that  $k$  depends not only on the area ratio  $m$ , but also on Reynolds' number  $\left(\frac{V_1 d}{\nu}\right)$ . If, therefore, the values of  $k$  are plotted to a

base of  $m$  values, while keeping the Reynolds' numbers constant, a series of curves are obtained for the different pipe diameters. For a different Reynolds' number, another series of curves is obtained. Jakob's tests covered four such series with Reynolds' numbers of 50,000, 100,000, 300,000, and 500,000. In Fig. 141, the curves for  $d = 3.94$  in. and 39.4 in. and for Reynolds' numbers 50,000 and 500,000 alone, are shown.

\* *Forsch. Arb.* No. 311. M. Jakob and Fr. Kretschmer. "Coefficients of discharge for standard nozzles with pipe diameters between 100 and 1000 mm."

A comprehensive set of tests has also been carried out by R. Witte in the I. G. Farbenindustrie.\* Witte found that the dependence of  $k$  on the diameter breaks down when Reynolds' number is below  $5 \times 10^4$ , provided the throttle edge is very sharp and the pipe wall is smooth. For diameters above 300 mm., however, the effect of roughness of wall surface disappears. Witte obtained the following values of  $k$ —

(a) THROTTLE PLATES.

$m$	= 0.05	0.1	0.15	0.20	0.25	0.30	0.35
$k$	= 0.598	0.602	0.608	0.615	0.624	0.634	0.646
$m$	= 0.40	0.45	0.50	0.55	0.60	0.65	0.70
$k$	= 0.660	0.676	0.692	0.717	0.740	0.769	0.807

(b) I. G. NOZZLES.

$m$	= 0.1	0.2	0.3	0.4	0.5
$C_d$	= 0.989	1.00	1.017	1.045	1.098

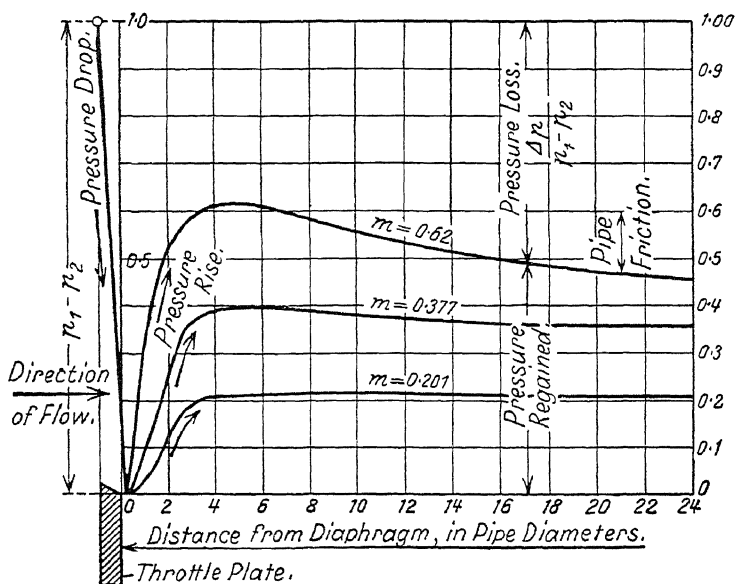


FIG. 142

These values are valid for small pressure drops. For larger pressure drops equation (271) on page 241 must be used. Witte allowed for this by introducing a correction factor  $C$ , which has to be multiplied by  $C_d$ . Thus, for the I. G. nozzle, with  $m = 0.41$  and

$$\frac{p_2}{p_1} = 1.00 \quad 0.95 \quad 0.90 \quad 0.85 \quad 0.80 \quad 0.75$$

he gives  $C = 1.00 \quad 0.964 \quad 0.928 \quad 0.892 \quad 0.856 \quad 0.820$

while, for a throttle plate with  $m = 0.37$  and  $p_2/p_1 = 0.95$  he gives  $C = 0.968$ . For values of  $m$  between 0.95 and 0.90,  $C$  remains constant and thereafter decreases till at  $m = 0.68$  it becomes equal to 0.88.

\* Z.V.d.I. "Discharge coefficients of nozzles and throttle plates."

See also the periodical *Tech. Mechanik und Thermodynamik*, vol. i. No. I (1930).

The resulting pressure drop  $\Delta p$  (Fig. 142) is of practical importance. By reconversion of the velocity  $V$  to pressure, only a portion of the pressure drop  $p_1 - p_2$  is regained. The remainder, i.e.  $\Delta p$ , represents a pressure loss. Jakob and Kretschmer's tests have confirmed a previous deduction of Kretschmer's, namely, that the pressure regained, expressed as a fraction of the pressure drop  $p_1 - p_2$ , is equal to the area ratio  $m$ , so that

$$\frac{\Delta p}{p_1 - p_2} = 1 - m$$

It should again be noted that the points at which the pressures are measured must not be chosen at random near the diaphragm, but must actually lie in the diaphragm itself, for which condition alone Fig. 141 applies. The reason for this is explained by referring to Fig. 142.\* In

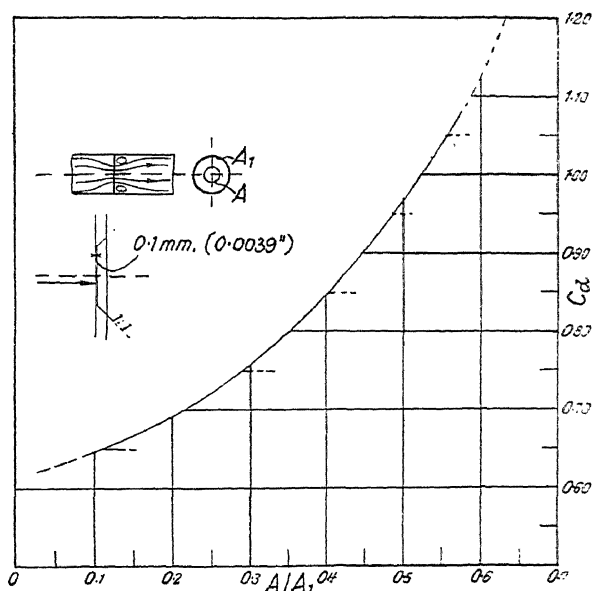


FIG. 143

A. O. Müller's tests the pressures were measured at points 1D and 5D before and after the diaphragm ( $D$  = pipe diameter), so that the pressure drop  $\Delta p$  was measured and not  $p_1 - p_2$ . The coefficients obtained by Müller, therefore, are greater than Jakob's in the ratio of  $\frac{1}{\sqrt{1-m}}$ , so that, for large values of  $m$ , the difference is considerable. Fig. 143 shows the values of  $C_a$  as found by Müller.

**Example 8.** With  $D = 3.937$  in. and  $\frac{A}{A_1} = 0.5$ , the value of  $k$ , as

\* See *Mitteilungen der Warmestelle Düsseldorf d. Ver. deutsch. Eisenhüttenleute*. No. 76. (Ausg. 2, 1928.) H. Jordan. "The measurement of gas, vapour, and liquid quantities in foundry work."



As opposed to this, however, an important development has recently taken place, as a result of practical steam turbine design. In simple non-divergent nozzles it has been found that phenomena occur which are only possible with velocities above the acoustic velocity. This apparent contradiction is due to the exit section of a steam turbine nozzle being cut obliquely, and not at right angles, to the nozzle axis. The conditions which determine whether or not the acoustic velocity will be exceeded are totally different in the two cases. In the exit region of the oblique section, i.e. in the space  $OAE$  in Fig. 145, a further expansion to a pressure below  $p_i$  occurs, so that the velocity is capable of rising above  $V_s$ . Actually the conditions of flow in this region are very complicated. They

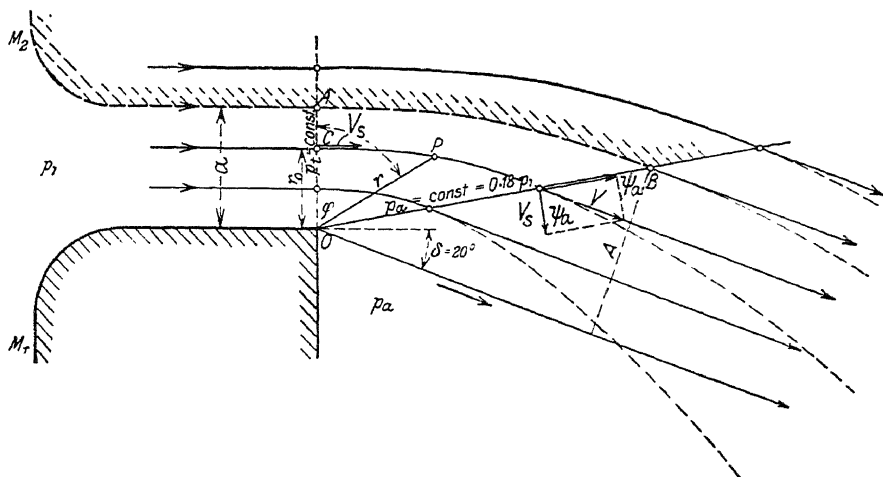


FIG. 144

can, however, be subjected to a general survey, and even lend themselves, to a certain extent, to calculative treatment, if use is made of the following case of stream line motion, which has been investigated by Prandtl and Th. Meyer.\*

Let a gas having the initial pressure  $p_i$  flow from a large space along a wall which is straight at the exit region (so that the flow is in parallel lines) into a second large space in which a constant pressure  $p_a$  exists (Fig. 144). An overflow occurs at the guide surface along the straight sharp edge  $O$ .

If now  $p_i$  is greater than  $p_{i\ cr}$ , then at  $O$  and at all points vertically above  $O$ ,  $V = V_s$ , and the pressure there is equal to the critical pressure as given by equation (307). Immediately to the right of  $O$  the pressure becomes equal to the external pressure  $p_a$ . This very sudden drop in pressure corresponds to a break in the stream line at  $O$ , and the direction of this stream line in the outer chamber is inclined at an angle  $\delta$  to the direction of the guide. This disturbance at  $O$  is communicated to all points on the perpendicular  $OA$ , since the stream velocity is  $V_s$ ; if it were greater than  $V_s$  and equal to  $V$ , say, the commencement of the

\* *Forsch. Arb.* No. 62. Th. Meyer. "On the two dimensional flow of gas with velocities above the acoustic."

change of direction for all the stream lines would be given by a straight line passing through  $O$ , inclined to the right at the angle  $\alpha$  to the direction of flow, where

$$\sin \alpha = \frac{V^s}{V}$$

$\alpha$  is called "Mach's angle."

The first case only will be considered here. All the stream lines at the points such as  $C$ , vertically above  $O$ , change their direction, so that from there onwards the distance between them continuously increases. A reduction in pressure and increase in velocity accompanies this enlargement in a filament bounded by two adjacent stream lines, and the initial velocity is equal to the acoustic velocity. Conditions are thus similar to those in the diverging portion of a de Laval nozzle. Points of equal pressure  $p$  lie on straight lines, such as  $OP$ , radiating from  $O$ , and the pressure becomes equal to the external pressure  $p_a$  along the ray  $OB$ . From here on, all the stream lines are again parallel to each other and to the broken stream line through  $O$ . The complete flow is thus deflected through the angle  $\alpha$  from its original direction.

On the assumption of adiabatic expansion, as given by the law

$$pv^\gamma = \text{constant},$$

Th. Meyer found that the polar equation for a stream line, with  $O$  as the pole, is

$$r = \frac{r_0}{\left[ \cos \left( \phi \sqrt{\frac{\gamma-1}{\gamma+1}} \right) \right]^{\frac{\gamma+1}{\gamma-1}}} \quad . \quad . \quad . \quad . \quad (308)$$

where  $r_0 = OC$  and the angle  $\phi = COP$ . This gives, for example, with  $\gamma = 1.405$  (gases at moderate temperature), the following values

$\phi$	= 0	30°	60°	90°	120°
$r$	= 1	1.15	1.77	4.00	12.85

The relation between  $\phi$  and the pressure  $p$  along the ray  $OP$  is expressed by

$$\phi = \frac{90}{\pi} \sqrt{\frac{\gamma+1}{\gamma-1}} \cos^{-1} \left[ (\gamma+1) \left( \frac{p}{p_i} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right] \quad . \quad (309)$$

By substituting  $p_a$  for  $p$  in equation (309) the angle  $\phi_a$ , at which the pressure becomes equal to the external pressure  $p_a$ , is obtained. The direction of flow at this ray is that of the tangent to the stream line at  $B$  (Fig. 144), so that the angle of deflection can be determined graphically.

Denoting the angle between this ray and the perpendicular to the direction of flow by  $\psi_a$  gives

$$\delta = \phi_a - \psi_a \quad . \quad . \quad . \quad . \quad . \quad . \quad (310)$$

$90^\circ - \psi_a$  is Mach's angle for the stream velocity at  $OB$ , and this velocity is given by the adiabatic drop between  $p_i$  and  $p_a$ .

$$\text{Now,} \quad \sin (90 - \psi_a) = \frac{V^s}{V} \quad . \quad . \quad . \quad . \quad . \quad . \quad (311)$$

where  $V_s$  is the acoustic velocity for the pressure  $p_a$ , i.e.

$$\frac{V}{V_s} = \dots \dots \dots (312)$$

This enables  $\psi_a$  to be found, after which  $\delta$  is found from equations (309) and (310).

Th. Meyer obtained the following values with  $\gamma = 1.405$

$\frac{p_a}{p_1} =$	0	0.01	0.03	0.05	0.1	0.2	0.3	0.4	0.5	0.527
$\delta =$	129° 19'	61° 14'	48° 17'	41° 15'	30° 28'	17° 58'	9° 52'	4° 6'	0° 25'	0°

Fig. 144, which is drawn to scale, shows the conditions for  $\frac{p_a}{p_i} = 0.18$  and gives  $\delta = 20^\circ$  and  $V = 1.7 V_a$  approximately.

The transition from this partly free flow to that which occurs in closed channels follows immediately by imagining a wall to be introduced

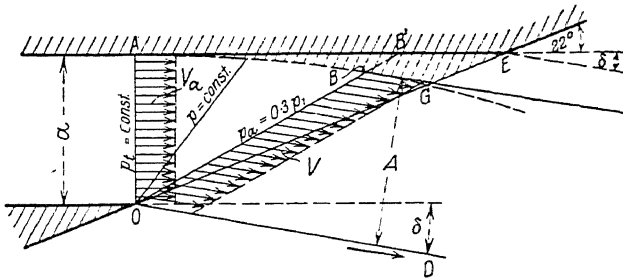


FIG. 145

which follows any stream line such as  $AB$  in Fig. 144. The flow conditions and pressures are unaffected by a wall of this type, so that the treatment given above applies to a channel having the lower boundary  $M_1O$  and the upper boundary  $M_2AB$ .

The application to oblique channels then follows, as shown in Fig. 145. Thus, take the angle of obliquity of the exit section as  $22^\circ$ . We have to find whether or not the expansion will proceed to the back pressure  $p_a = 0.3 p_i$ . Draw the Prandtl-Meyer line  $AB$  through  $A$ , as given by equation (308). With  $\frac{p_a}{p_i} = 0.3$ , the corresponding angle of deflection

is  $\delta = 9^\circ 52'$ , which gives the direction  $OD$  and the point  $B$ , although the latter is also given by equation (309). If now the upper wall were shaped as shown by  $ABG$  instead of  $AE$ , the pressure at all points along the ray  $OB$  would be  $p_a = 0.3 p_i$ , i.e. the external pressure (with  $V = 1.5 V_s$  approx.). Actually, however, the area at exit is slightly greater, being equal to that shown by  $OB'$  instead of  $OB$ . As a result the expansion ratio will be slightly greater than that given by calculation, and the outlet pressure will be reached at some point before  $B'$ . In addition the pressure distribution will not be given by straight lines, and the stream lines will not be quite parallel. On the whole, however, the general conditions are not seriously affected by these differences, since they are comparatively small. In particular, the angle of deflection

will be found to agree almost completely with that found by calculation.\* In the case of saturated steam these angles are increased.

### PRESSURE LOSSES IN PIPES

When gases or vapours flow in a pipe, a frictional resistance is experienced similar to that of a liquid such as water. Even if the pipe is perfectly uniform, horizontal, and free from bends, valves, or other obstructions, friction is still present, so that a continuous drop in pressure takes place along the pipe. If the initial and final pressures are  $p_1$  and  $p_2$  respectively, the pressure drop  $p_1 - p_2$  is utilized entirely in overcoming the resistance to flow, so that the existing initial velocity may be maintained throughout the length of the pipe. This pressure drop is not to be confused with a nozzle pressure drop, most of which is used in accelerating the flowing substance.

The resistance to flow depends, to a large extent, on the condition of the inner wall surface, and is thus similar to friction between two solids. There can be no question, however, of introducing a frictional coefficient in the same sense as is used for solids, since flowing gaseous substances are influenced by friction in a totally different manner and follow quite different laws. Even if the walls of a pipe be made as smooth as possible, appreciable frictional effects are still experienced.

In the majority of practical cases, the motion of the whole mass does not proceed along parallel streams, but in a sort of rolling movement called turbulence, and the energy required for this accounts for the greater part of the pressure drop.

If the velocity is reduced below a certain value, it is found that turbulence disappears, but even then a certain amount of frictional resistance remains, so that a force is still required to effect the motion. The force necessary to maintain unit relative velocity between two planes of unit area, and unit distance apart, is called viscosity ( $\eta$ ), and has a definite value for every fluid, including gases and vapours. In the case of flow through pipes, its action is such that the film at the wall is at rest. The film next to this one has to shear over it, and this continues to the centre of the pipe, where the greatest velocity is attained. This only occurs, however, at low velocities and in narrow pipes.† In most cases met with in practice, the flow is entirely turbulent and the viscous resistance is negligible compared with the force necessary to overcome the turbulence.

No simple calculative method, based on theoretical grounds, can be applied to determine the resistance set up by turbulence, but a simple type of formula can be deduced, based on the results of experiments.

The surface area with which a given weight (such as 1 lb.) is in contact, (Fig. 146) increases as the diameter of the pipe decreases, so that the resistance increases as the diameter decreases. The length of pipe required

\* See *Zeitsch. f. d. ges. Turbinenwesen* (1912), page 183. Christlein and Z.V.d.I. (1916), H. Baer. Also *Z.V.d.I.* (1916), page 770. A. Loschge.

† Regarding the measurement of viscosity by means of the flow through narrow pipes, see *Forsch. Arb.* No. 273, Speyerer. "Viscosity of steam." See also *Forsch. Arb.* No. 75, Ruckes. "Tests on the discharge of compressed air from capillary tubes"; and *Forsch. Arb.* No. 48, Becker, "Flow in circular pipes and its relation to Poiseuille's Law."



## FLOW OF GASES

per pound of the fluid is  $l = \frac{1}{\frac{\pi d^2}{4} \rho \frac{1}{4}}$ , where  $\rho$  is the density of the fluid in lb./ft.<sup>3</sup> The corresponding surface area is thus

The resistance to the flow per pound of fluid can be regarded as uniformly distributed over the surface  $S$  and acting in the opposite direction to the flow. If its value, as found by experiment, is  $F'$  lb. per ft.<sup>2</sup> of surface area, then

$$F = F'S = \frac{4F'l}{\rho d}$$

The work required to overcome this resistance, when 1 lb. flows along the length  $l$ , is

$$E = Fl = \frac{4F'l}{\rho d} \quad (313)$$

For non-circular cross sections,  $\frac{\pi d^2}{4}$  and  $\pi d$  are replaced by  $A$ , the area, and  $P$ , the perimeter, respectively, so that  $l' = \frac{1}{\rho A}$  and  $S' = Pl' = \frac{P}{\rho A}$

From which,

and 
$$E = \frac{F'Pl}{\rho A}$$

Hence the friction is proportional to the perimeter when the area is kept constant. In place of  $\frac{4l}{d}$  we now have  $\frac{Pl}{A}$ , or in place of  $\frac{l}{d}$ , we have  $\frac{Pl}{4A}$ .

Due to the pressure drop  $p_1 - p_2$ , caused by the flow, the volume of gases or vapours increases. The work delivered by means of this volume increase depends on the course of the  $pV$  expansion curve, i.e. on the nature of the state change. This work serves to overcome the resistance to flow. In the case of small pressure drops, however, the change in state is unimportant, so that the expansion work can be neglected, and only the work due to the pressure difference need be considered, as in the case of liquids.

This work is given by

$$W = (p_1 - p_2)v =$$

where  $v$  and  $\rho$  are the mean specific volumes and densities respectively.

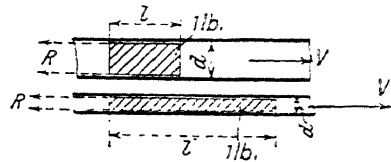


FIG. 146



The numerous tests which have been carried out to determine the values of  $f$ , or  $\beta$ , for liquids, as well as gases and vapours, show that the resistances in pipes or channels are not constant in the usual sense, but are functions of  $V$ ,  $d$ ,  $\rho$ , the viscosity  $\mu$ , and also, to a large extent, on the condition of the swept surface, i.e. whether smooth or rough. For this reason it is convenient to consider smooth and rough pipes separately. In the former case the influence of the walls can be neglected, so that the problem is considerably simplified, since widely different results are obtained with varying roughness of wall surface.

**Smooth pipes.** In smooth pipes  $f$  must be a function of  $V$ ,  $d$ ,  $\rho$ , and  $\mu$ . The problem is simplified if  $\rho$  is kept approximately constant, as is the case, for example, with water at different temperatures.  $f$  is then a function of  $V$ ,  $d$ , and  $\mu$  only. In the case of water, for which  $\mu$  changes rapidly with temperature, the influence of viscosity is thus predominant. In the case of air, however, at moderate temperature and varying pressures, the density  $\rho$  is of importance, since it changes with the pressure, while the effect of viscosity is not important, since it is not affected by pressure changes and but slightly for fairly wide temperature changes.

If, however, tests on water and air are to be compared with one another, all four magnitudes  $V$ ,  $d$ ,  $\rho$ , and  $\mu$  must be introduced in the expression for  $f$ .

The "kinematic viscosity"  $\nu$  is obtained from the coefficient of viscosity by means of the relation

Hence, in smooth pipes, the only variables which have to be considered are  $V$ ,  $d$ , and  $\nu$ , i.e.  $f$  is a function of these three magnitudes alone, so that

$$f = F(V, d, \nu) \quad . \quad . \quad . \quad . \quad . \quad (319)$$

Now in the fundamental equation (315) the quotient  $\frac{V^2}{2g}$  signifies a head in feet, while  $\frac{l}{d}$  is a pure number. Consequently  $f$  must also be a pure number (or dimensionless magnitude) if the expression on the left is to retain the dimension of a length.

The magnitudes  $V$ ,  $d$ , and  $\nu$ , therefore, in the unknown function of equation (319) must be so arranged that this function (whatever its form) still remains a pure number. This will be the case if the magnitudes are written as  $\frac{Vd}{\nu}$ \*, since  $\nu$  has the dimensions ft.<sup>2</sup>sec. Hence, in place of equation (319), we can now write more definitely

$$\left( \frac{Vd}{\nu} \right) \quad (320)$$

as first established by Reynolds.

This relation expresses the law of similarity for any flow with friction.

\* This is usually known as "Reynolds' number."

Two or more different gases, vapours, or liquids, flowing in pipes of different diameters, with different velocities, only exhibit geometrically or dynamically similar flow if, in all the pipes,  $\frac{Vd}{\nu}$  has the same value.

If this condition is fulfilled, the coefficient of friction  $f$  has the same value in all the pipes, provided they are smooth. If, therefore, the values of  $\frac{Vd}{\nu}$  are plotted against the experimentally determined values of  $f$ , it follows that the points for water and air, say, lie on a single curve.

If, however,  $f$  is plotted against  $V$ , for water and air, two widely separated curves are obtained, while if  $f$  is plotted for water at different temperature, to a base of  $V$  values, only a group of scattered points is obtained and these do not form a curve.

Using the numerous test results of the American engineers Saph and Schoder, Blasius\* demonstrated that a graph of  $f$  against  $\frac{Vd}{\nu}$ , for water, does actually give a single curve for different velocities of flow and different temperatures. This curve (*a*) is shown in Fig. 147. The points obtained by Blasius from Nusselt's tests on compressed air also lie on this curve. Ombeck's† curve (*b*) lies very close to this curve and was obtained from a large number of tests carried out at pressures varying from 1 to 10 atmospheres, and temperatures between 20° and 100° C.

These curves show that, for increasing values of  $\frac{Vd}{\nu}$ ,  $f$  first decreases very rapidly, but decreases slowly at high values of  $\frac{Vd}{\nu}$ . A lower limit for  $f$  at these high values has, however, not yet been found. The lowest measured value, given by Ombeck, is  $f = 0.0031$  or  $10^7\beta = 13.3$ . Jakob and Erk later gave slightly higher values in this region. Values of  $\frac{Vd}{\nu}$  below 2320 need not be considered here, since the turbulent flow then ceases and is replaced by the Poiseuille or laminar flow, for which the law of resistance is entirely different.‡

For a Reynolds' number of about 3000,  $f$  has its highest value of  $f_{max} = 0.011$  and  $10^7\beta = 47.5$ .

As shown by Reynolds, a straight line is obtained when  $\log \frac{Vd}{\nu}$  is plotted against  $\log f$ . Hence the curves *a* and *b* are hyperbolas. Ombeck gives the following relations,

$$f = \frac{0.0605}{\left(\frac{Vd}{\nu}\right)^{0.25}}$$

or

$$\frac{260}{Vd} \quad (322)$$

\* Z.V.d.I. (1912), page 639, H. Blasius. "Dynamical similarity in frictional flow."

† Forsch. Arb. Nos. 158 and 159. H. Ombeck. "Pressure losses in air flow."

‡ See Schiller's Forsch. Arb. No. 348.

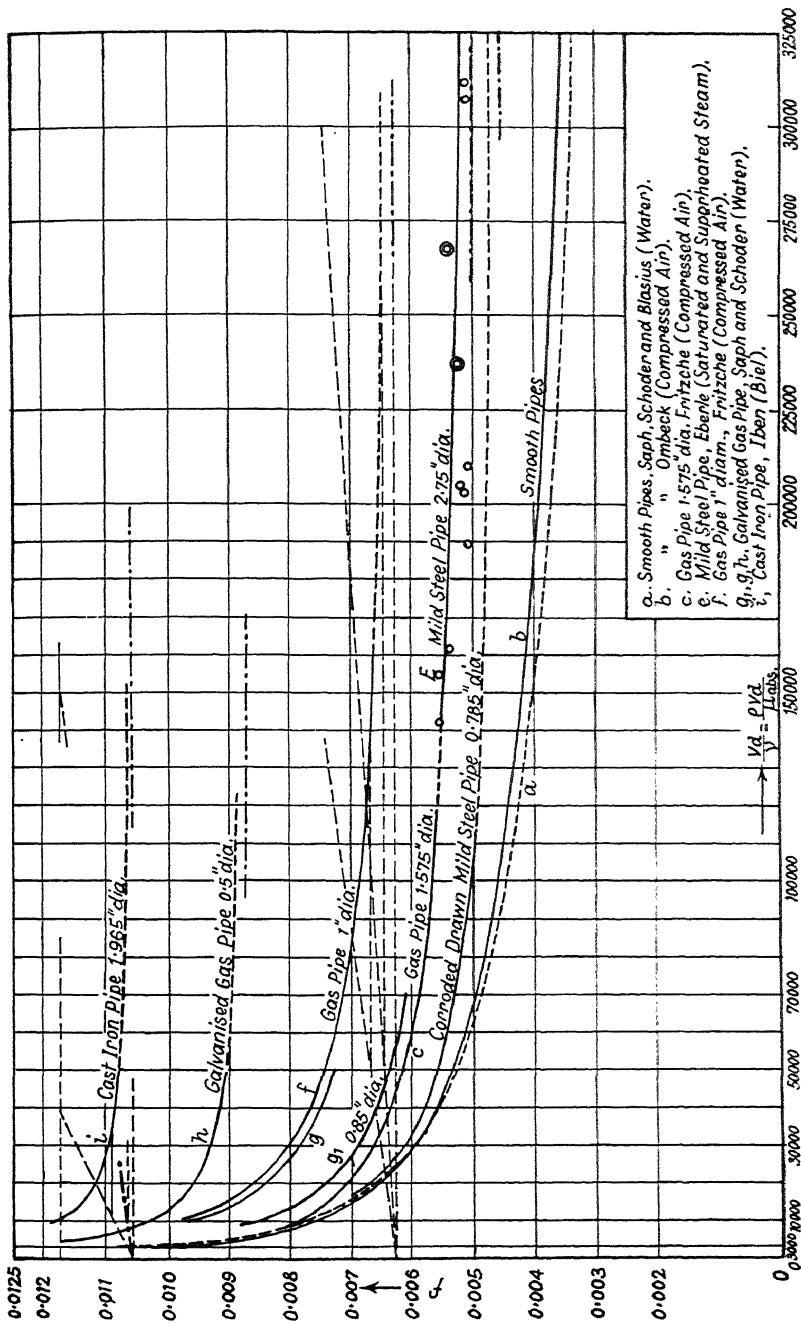


FIG. 1.47

Stanton and Pannell, however, from later tests gave

$$f = 0.0018 - 0.1526 \left( \frac{Vd}{\nu} \right)^{-0.35} \quad (323)$$

while Jakob and Erk's\* equation is

$$f = 0.00178 + 0.1526 \left( \frac{Vd}{\nu} \right)^{-0.35} \quad (324)$$

In order to apply these results to a practical case, it is necessary to know when a pipe may be regarded as smooth. In addition, the viscosity values for air and steam require to be known.

The results obtained by Saph and Schoder refer to drawn brass pipes of 3 mm. and 53 mm. diameter, and those of Ombeck to smooth drawn brass pipes from 20 to 40 mm. diameter and a smooth drawn steel pipe of 20 mm. diameter. Such pipes, along with drawn lead and copper pipes, may be regarded as smooth. A pipe made up of several lengths, however, must not be regarded as smooth, particularly when it contains a number of bends. Smooth machined short nozzles may be regarded as smooth pipes.

The viscosity  $\mu$  depends on the nature and temperature of the flowing fluid. The pressure appears to have but little effect on it when in the gaseous or liquid state. In physics  $\mu$  is always given in absolute units, but engineers generally measure  $V$ ,  $d$ , and  $\nu$  in technical or engineers' units.  $\frac{Vd}{\nu}$  itself is independent of the units employed, since it is a pure number.

The relation between the absolute and technical viscosity values is expressed by

$$\mu_{tech} = \frac{\mu_{abs}}{g}$$

$$\begin{aligned} \text{so that } \nu_{tech} &= \frac{\mu_{tech}}{\rho} = \frac{\mu_{abs}}{g} \times \frac{g}{\rho} \\ &= \frac{\mu_{abs}}{\rho} \end{aligned}$$

$$\text{This gives } \frac{Vd}{\nu} = \frac{\rho Vd}{\mu_{abs}} = \frac{\rho Vd}{g \mu_{tech}} \quad (325)$$

in which  $\rho$  is in lb. ft.<sup>3</sup>,  $V$  in ft. sec., and  $d$  in ft. This, therefore, enables  $\frac{Vd}{\nu}$  to be found for a particular case when  $\mu_{abs}$  is known. The following tables give values of  $\mu_{abs}$  for air and steam. In the case of steam, no difference is shown for saturated and superheated steam, since the value depends essentially on the temperature. The second table contains more exact values for steam and shows that the viscosity increases with the pressure when the temperature remains constant.

\* Z.V.d.I. (1924), page 583.

$10^7 \mu_{abs}$  VALUES FOR AIR AND STEAM  $\frac{\text{dals}}{\text{ft}^2}$ 

Temperature .	0°	10°	20°	30°	40°	50°	60°	70°	80°	90°
Air .	117.7	122.4	126.8	130.4	134.1	137.1	139.8	142.5	145.2	147.9
Steam .	60.51	63.33	66.09	68.91	71.73	74.49	77.31	80.14	82.96	85.72
Temperature .	100°	110°	120°	130°	140°	150°	160°	170°	180°	190°
Air .	150.6	152.6	155.2	157.3	159.5	161.7	163.8	165.9	168.1	170.2
Steam .	88.54	91.37	94.12	96.95	99.77	102.5	105.4	108.2	111.0	113.8
Temperature .	200°	210°	220°	230°	240°	250°	260°	270°	280°	290°
Air .	172.4	174.5	176.7	178.9	181.1	183.3	185.5	187.6	189.9	192.1
Steam .	116.6	119.4	122.2	125.0	127.8	130.6	133.4	136.2	139.0	141.8
Temperature .	300°	310°	320°	330°	340°	350°	360°	370°	380°	390°
Air .	194.3	196.5	198.7	200.8	203.0	205.2	207.3	209.6	211.7	213.9
Steam .	144.6	147.4	150.2	153.0	155.8	158.7	161.4	164.2	167.1	169.8

 $10^7 \mu_{abs}$  VALUES FOR SATURATED AND SUPERHEATED STEAM\*

Pressure plb./in. <sup>2</sup> abs. $t_s$ ° C.	14.22 99.1	28.44 119.6	56.9 142.9	85.3 158.1	113.8 169.6	142.2 179.0
$t_s$ ° C.	84.35	90.74	98.66	104.9	110.9	118.3
110	86.86	—	—	—	—	—
120	89.36	90.81	—	—	—	—
130	91.87	93.25	—	—	—	—
140	94.37	95.69	—	—	—	—
150	96.81	98.13	100.37	—	—	—
160	99.32	100.6	102.8	105.3	—	—
170	101.8	103.1	105.3	107.7	111.0	—
180	104.3	105.5	107.6	110.1	113.4	118.5
190	106.8	108.0	110.1	112.5	115.9	120.9
200	109.3	110.5	112.5	114.9	118.2	123.3
210	111.9	112.9	114.9	117.4	120.7	125.6
220	114.3	115.4	117.5	119.8	123.1	128.1
230	116.8	117.9	120.0	122.2	125.4	130.4
240	119.3	120.4	122.5	124.6	127.9	132.8
250	121.8	122.9	124.9	127.1	130.2	135.2
260	124.3	125.4	127.3	129.5	132.6	137.6
270	126.8	127.9	129.8	132.0	135.1	139.9
280	129.3	130.4	132.2	134.4	137.4	142.3
290	131.8	132.8	134.7	136.8	139.8	144.6
300	134.3	135.3	137.1	139.3	142.3	147.0
310	136.8	137.8	139.5	141.7	144.6	149.3
320	139.3	140.3	141.9	144.2	147.1	151.7
330	141.8	142.8	144.4	146.6	149.4	154.1
340	144.3	145.2	146.9	149.0	151.8	156.5
350	146.7	147.7	149.4	151.5	154.3	158.9

\* From *Speyerer, Z.V.d.I.* (1925). p. 749.

**Example 9.** Find the coefficient of resistance  $f$  when the velocity through a straight smooth pipe of 1.965 in. diameter is 131.2 ft./sec.—

- (a) For air at 20° C. and 14.22 lb. in.<sup>2</sup> abs.
- (b) For air at 20° C. and 142.2 lb. in.<sup>2</sup> abs.
- (c) For saturated steam at 142.2 lb. in.<sup>2</sup> abs.
- (d) For saturated steam at 1.422 lb. in.<sup>2</sup> abs.

For all four cases,

$$\frac{Vd}{\nu} = \frac{\rho_1 Vd}{\mu_{abs}} = \frac{131.2 \times 1.965 \rho_1}{12 \mu_{abs}}$$

$$= \frac{21.45 \rho_1}{\mu_{abs}}$$

- (a) With  $\rho = 0.0728$  and  $\mu_{abs} = \frac{126.8}{10^7}$ , we have

$$\frac{Vd}{\nu} = \frac{21.45 \times 0.0728 \times 10^7}{126.8} = 123,800$$

so that, from the curve  $b$  in Fig. 147,  $f = 0.00451$ .

- (b) With  $\rho = 0.728$ ,  $\frac{Vd}{\nu} = 1,238,000$ , and hence, from equation (321),

$$f = \frac{0.0605}{1238000^{0.224}} = \frac{0.0605}{23.17} = 0.00261$$

- (c) From steam tables at 142.2 lb. in.<sup>2</sup> abs.,  $t = 179^\circ$  C. and  $\rho = 0.315$ . so, from the second table of viscosity values,

$$\mu_{abs} = \frac{118.3}{10^7}$$

so that 
$$\frac{Vd}{\nu} = \frac{\rho Vd}{\mu_{abs}} = \frac{0.315 \times 131.2 \times 1.965 \times 10^7}{118.3 \times 12} = 572,000$$

Equation (321) now gives

$$f = \frac{0.0605}{572000^{0.224}} = \frac{0.0605}{19.5} = 0.0031$$

If, in place of equation (321), equation (323) is used, we have

$$f = 0.0018 + \frac{0.1526}{572000^{0.35}}$$

$$= 0.0018 + 0.00147 = 0.00327$$

- (d) Here,  $t = 45.4^\circ$  C.,  $\rho = 0.00414$ ,  $\mu_{abs} = \frac{73.11}{10^7}$

that 
$$\frac{Vd}{\nu} = \frac{0.00414 \times 131.2 \times 1.965 \times 10^7}{12 \times 73.11} = 12,150$$



Equation (321) now gives

$$0.0605 = 0.00737$$

or, from equation (324),

$$f = 0.0018 \frac{0.1526}{12150^{0.35}} = 0.0075$$

**Example 10.** Find the values of  $f$  for a circular nozzle of 0.787 in. diameter when saturated steam at 14.22 lb./in.<sup>2</sup> abs. flows through it at 328, 984, 1640, and 2296 ft./sec.

In this case,  $\rho = 0.0361$  lb./ft.<sup>3</sup>,  $t = 99.1$ , and  $\mu = 84.35$ . Hence, equation (325) gives

$$\frac{Vd}{\nu} = \frac{0.0361 \times 0.787 \times 10^7 V}{12 \times 84.35}$$

At

$V$	=	328	984	1,640	2,296
$\frac{Vd}{\nu}$	=	91,800	295,400	459,000	642,600
$f$	=	0.00468	0.00361	0.00326	0.00304

If steam at 1.422 lb./in.<sup>2</sup> abs. flows through the same nozzle, then

$f$	=	0.0074	0.0058	0.0052	0.0047
-----	---	--------	--------	--------	--------

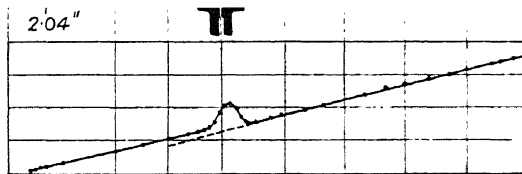


FIG. 148

**Rough pipes.** In the cases considered above, it has been assumed that the inner surface of the pipes is smooth. Actually, however, most pipes cannot be regarded as smooth. The majority of cast-iron pipes have a rough surface, while even mild steel pipes cannot be considered as smooth, particularly when galvanized or rusty. Welded pipes are also to be considered as rough, owing to the uneven surface caused by welding, while a row of rivet heads in a riveted pipe is equivalent to excessive roughness.

Every irregularity, such as a projection, indentation, joint, rivet head, or rough surface tends to obstruct the flow of the fluid and so increases the pipe resistance. A smooth pipe, made up of jointed lengths, has thus to be regarded as rough. Fig. 148, which is reproduced from Ombeck's measurements, shows the pressure changes in a length of pipe in which a joint occurs. Due to this joint a local pressure increase occurs. The

pressure beyond the joint is raised (in spite of the pressure lost at the joint), owing to the diameter being greater than that before the joint. In addition, the left-hand pipe was rough while the other section was smooth.

Hence, for rough pipes it is necessary to take account of the degree of roughness  $\varepsilon$  in addition to the quantities  $V$ ,  $d$ , and  $\nu$ . It is found by test, however, that the influence of  $\varepsilon$  becomes less as the diameter is increased, so that the important factor is  $\frac{\varepsilon}{d}$ , which can be called the roughness ratio.

If now  $f$  be plotted to a base of  $\frac{Vd}{\nu}$  (as in the case of smooth pipes), a single curve is obtained, provided the roughness ratio is constant, no matter what substance or state is considered. Thus, in Fig. 147, the resistance curve  $h$ , from Saph and Schoder's tests on a galvanized pipe of 0.484 in. diameter, is of this type. The curves  $g$  and  $g_1$  in the same figure refer to galvanized gas pipes of 0.63 in. and 1.05 in. diameter, so that the diameters are now larger but the absolute roughness is the same. The resistance coefficients for these pipes now show considerable differences and are smallest for the largest pipe. This shows, therefore, that the ratio  $\frac{\varepsilon}{d}$  has a marked effect on the results. The same applies to the curves  $c$  and  $f$ , which result from tests on gas pipes of 1.57 in. and 1 in. diameter and 65.5 ft. long.\* It will be seen that the curve  $f$  (diameter = 1 in.), which ought to be below the curve  $g$  (diameter = 0.63 in.), actually lies above it. It must be remembered, however, that the curve  $f$  refers to a jointed pipe, while curve  $g$  refers to a one-piece pipe. Most of this increased resistance in the case of built-up pipes is caused by the joints, but part of it may also be due to differences in the degree of roughness.

Fig. 147 shows also a curve (i)† for a cast-iron pipe of 1.965 in. diameter. Based on the diameters alone, this curve should lie below the curve  $c$ , for which the diameter is 1.575. Actually, however, it is considerably above it, so that the cast-iron pipe is much rougher than the gas pipe, i.e. in spite of its greater diameter the roughness ratio is considerably greater. The curve thus shows the marked effect of even moderate roughness on the resistance, which, for the curve  $i$  (rough pipe), is about double that for the curve  $b$  (smooth pipe). Roughness in small pipes is particularly disadvantageous. Soph and Schader found, for example, that  $f = 0.015$  and was practically constant for a gas pipe of 0.354 in. diameter.

Fig. 147 also serves to show that the variation in the value of  $f$  for rough pipes is relatively small, particularly for large roughness ratios. Hence, for these pipes, the velocity and viscosity are of secondary importance, especially at high Reynolds' numbers.

**Steam pipes.** The remarks given above apply also to vapours, such

\* *Forsch. Arb.* No. 60, A. Fritzsche. "Tests on flow resistance of gases in straight circular pipes." The pipes were joined by standard (German) flanges, separated by rubber packing 2 mm. thick. Each of the two lengths contained six of these joints. The ranges of velocity, pressure, and temperature were: 8.2 to 190 ft. sec., 2.85 to 158 lb. in.<sup>2</sup> abs., and 14° C. to 115° C. respectively.

† From Iben's tests, which, according to Biel, were carried out with extreme care and accuracy.

as steam, ammonia, carbon dioxide, etc., but some reference is necessary here regarding tests carried out by Eberle\* with saturated and superheated steam.

Eberle used a mild steel pipe line 2.75 in. diameter and 102 ft. long, having six flanges and two 90° bends. The steam speeds varied from 23 to 242 ft./sec., the pressures from 43 to 142 lb./in.<sup>2</sup> abs., and the temperatures from saturation to 225 and 270° C. respectively.

As given by Eberle, the value of  $f$  in all these tests varied only from 0.0049 to 0.0054, but no general law relating the variation in  $f$  with the pressure, velocity, or of superheat could be established. His values of  $f$  suggest that they can be regarded as constant over the whole explored region, both for saturated and superheated steam.

This constancy appears to be contrary to the considerable variation in  $f$  found by Fritzsche in his tests. If, however, Eberle's values are plotted, like the others, to a base of Reynolds' numbers, the apparent contradiction disappears. In Fig. 147, Eberle's points are shown, by small circles, for saturated steam, and small double circles, for superheated steam. These show at once that his tests have been carried out in a region within which  $f$  changes but slightly for smooth pipes and less still for rough pipes. A smooth curve drawn through the points slopes slightly downwards to the right, so that  $f$  gradually decreases as  $\frac{Vd}{\nu}$  increases. From the test points at  $\frac{Vd}{\nu} = 551,600$  and  $\frac{Vd}{\nu} = 656,000$  (not shown in Fig. 147) it can be assumed that a lower limit to the value of  $f$  has practically been attained, since values of  $f$  at these Reynolds' numbers are 0.0051 and 0.0054 respectively.

Hence, Eberle's conclusion that  $f$  is constant and equal to 0.0052 is justified so long as  $\frac{Vd}{\nu}$  is greater than about 150,000. This is usually the case in practice for high pressure steam, so long as the pipe diameter is not greatly different from 2.75 in. In the case of vacuum pipes, however, Fig. 147 shows that the values of  $f$  are considerably greater.

It will be observed that Eberle's curve  $E$  forms the continuation of Fritzsche's curve  $c$ , although the former is for a pipe having a diameter of 2.75 in., while the latter is for a pipe having a diameter of 1.575 in. Hence, the one curve completes the other, but the roughness ratio in Eberle's case must be greater than the other, due probably to the greater number of joints relative to the diameters and to the presence of the two bends, which have the same effect as increased roughness. The curve  $c$  can thus be applied to a mild steel pipe line of 2.75 in. diameter carrying steam.

For example, if steam at 0.71 lb./in.<sup>2</sup> abs. is exhausted from a turbine, through a pipe of 2.75 in. diameter at a speed of 131.2 ft./sec., we have, with  $\rho = 0.00217$  lb./ft.<sup>3</sup>,  $t = 32.5^\circ$  C. and  $\mu$  abs. =  $\frac{69.62}{10^7}$ ,

$$\frac{Vd}{\nu} = \frac{\rho Vd}{\mu_{\text{abs}}} = \frac{0.00217 \times 131.2 \times 2.75 \times 10^7}{12 \times 69.62} = 9370$$

so that, from Fig. 147,  $f = 0.008$ .

\* Z.V.d.I. (1908). Eberle. "Tests on heat and pressure losses in the transmission of saturated and superheated steam."

Eberle's value of  $f = 0.0052$  is not valid for diameters other than 2.75 in. For greater diameters  $f$  lies between the curve  $b$ , for smooth pipes, and the curve  $E-c$ . Ombeck's curve  $k$  refers to a smooth, drawn, mild steel pipe, the inner surface of which was roughened with acid. Before being roughened, this pipe gave the curve  $b$ . For diameters below 2.75 in. the higher curve  $c$  forms an upper limit, and shows that the value of  $f$  can be considerably greater than 0.0052. For example, the curve  $f$  can be used for steam pipes between 1.2 in. and 1.5 in. diameter, and curve  $h$  for steam pipes between 0.6 in. and 0.8 in. diameter.

**Lower limit of  $f$  for rough pipes.** In the case of a given smooth pipe in which a given substance is flowing,  $f$  appears to decrease continuously as  $\frac{Vd}{\nu}$  increases, as is substantiated by equation (321). In the case of rough pipes, however, there is a lower limit to the value of  $f$ , since, even with high velocities and very large diameters, the resistance to flow caused by the rough surfaces is never entirely absent. Although this appears to be self-evident, it should be remembered that the equations given by Fritzsche and Ombeck actually make the values of  $f$  decrease continuously as Reynolds' number increases. Hence, for large Reynolds' numbers, the values of  $f$ , as determined from these equations, are too low. Biel's\* conclusion that  $f$  tends to assume a constant value at some lower limit is basically sound, and the equations (323) and (324) substantiate this, as, from these, the lower limits for  $f$ , even for smooth pipes, are 0.0018 and 0.00178 respectively. The curves in Fig. 147, for rough pipes, have thus horizontal asymptotes at a distance  $f_x$  from the abscissa axis (as shown by the chain dot lines).

By means of Biel's collection of a large number of well-selected experimental points, the  $f$  curves for rough pipes were found to be rectangular hyperbolae, the horizontal axes of which were the asymptotes mentioned above. This is almost exact for high values of  $\frac{Vd}{\nu}$ , so that Fritzsche's curves are given directly by drawing rectangular hyperbolae. Curve  $E$  was obtained, for example, in this way.

In this way the equation of  $f$  for rough pipes becomes very simple. From the equation of a hyperbola we have

$$\left) \frac{Vd}{\nu}\right.$$

The constants, i.e.  $c$  and  $f_x$  are easily determined from the experimental curves. The curves for other roughness ratios are, however, different, but a fuller discussion, although along somewhat different lines, will be found in Biel's work. For most practical cases, Fig. 147 offers a sufficient number of points. Owing to the curves having been drawn to

\* *Forsch. Arb.* No. 44, R. Biel. "Pressure losses in the transmission of liquid and gaseous fluids." This excellent survey of the work then completed, including Fritzsche's and Ombeck's, deals not only with the difficult problem of rough pipes from basic principles, but also shows the effect of viscosity by means of the principle of dynamical similarity. This principle was but little known and applied at that time. In a later work (special edition of *Technische Mechanik*, des V.d.I. Verlags, 1925), called "Flow Resistance," Biel explains the principle of dynamical similarity.

a base of Reynolds' numbers, a greater number of experimental results are represented than is at first apparent.

**Smooth expansion pipes.** Tests were carried out (Fig. 149) by C. Bach and R. Stückle\* to determine the pressure losses in these pipes, when transmitting superheated steam. It was found, as in Eberle's tests, on straight pipes, that  $f$  was independent of the velocity (between 80 and 300 ft./sec.) and pressure of the steam. The steam temperatures ranged from 350 to 364° C. The following values of  $f$  were obtained for the different pipes—

1. Smooth expansion bends (unlagged),

$$d = 2.22 \text{ in.} \quad A = 41.8 \text{ in.} \quad B = 42.3 \text{ in.} \quad R = 12.75 \text{ in.} \quad r = 10.8 \text{ in.}$$

$$d = 2.28 \text{ in.} \quad A = 41.5 \text{ in.} \quad B = 43.6 \text{ in.} \quad R = 13.75 \text{ in.} \quad r = 8.9 \text{ in.}$$

gave  $f = 0.00872$  (varied between 0.00858 and 0.00893).

2. Rolled expansion bends

$$(a) \quad 2.16 \text{ in.} \quad A = 41.7 \text{ in.} \quad B = 42.9 \text{ in.} \quad R = 12.78 \text{ in.} \quad r = 10.8 \text{ in.}$$

gave  $f = 0.0187$  (unlagged) and  $f = 0.0178$  (lagged).

Another rolled bend with the same  $A$  and  $B$  values, but having a more uniform internal diameter, gave  $f = 0.0165$  to 0.0158.

$$(b) \quad d = 3.94 \text{ in.} \quad 55.8 \text{ in.} \quad B = 44.6 \text{ in.} \quad R = 19.1 \text{ in.} \quad r = 12 \text{ in.}$$

gave  $f = 0.0241$ .

This increased value of  $f$  is due to the much smaller radius of the bend relative to the pipe diameter.

The insertion of a rolled expansion bend in a pipe line gives the same resistance as that produced in a straight length, which is three to five times as long as the actual length of the expansion pipe.

**Exponential equations for the pressure loss.** In place of the usual equation for the pressure loss

$$p_1 - p_2 = \frac{4flV^2\rho}{2gd}$$

in which  $f$  is a function of  $V$ ,  $d$ ,  $\rho$ , and  $\mu$ , endeavours have been made to express the pressure loss by the equation

$$p_1 - p_2 = \frac{a\rho^q V^m}{d^n}$$

which gives the unknown function as

$$f = \frac{a\rho^{q-1}V^{m-2}}{4d^{n-1}}$$

since we can write

$$p_1 - p_2 = \left( \frac{a\rho^{q-1}V^{m-2}}{4d^{n-1}} \right) \times \frac{l}{d} \times \frac{1}{2g} \rho$$

From the accurate tests of Fritzsche it was first shown and later confirmed by Ombeck that  $m < 2$ ,  $n > 1$ , and  $q < 1$ , so that the pressure

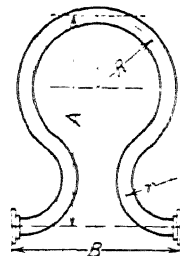


FIG. 149

\* Z.V.d.I. (1913), page 1136 f.

loss does not vary as the square of the velocity nor directly as  $p$ , and depends not only on  $\frac{l}{d}$  but also on the absolute value of  $d$ . It should be noted, however, that the values  $m$ ,  $n$ , and  $q$ , for the same pipe, are remarkably constant. Fritzsche found  $m = 1.852$ ,  $n = 1.269$ , and  $q = 0.852$ . By choosing other experimental values, other exponents are obtained, but the differences are not serious.

Fritzsche's values give, for air,

$$f = 0.00492$$

with  $V$  in ft. sec.,  $p$  in lb. ft.<sup>3</sup>, and  $d$  in ft.

Thus if  $V = 140.8$ ,  $p = 0.0728$ , and  $d = 0.23$ , we have  $f = 0.00518$ .

Brabée\* gives the following expressions, resulting from his extensive tests on hot water pipes with facet joints and diameters ranging from 0.55 in. to 1.93 in.

For water at 15° C.

$$p_1 - p_2 = 0.076 V^{1.751} L$$

For water at 70° C.

$$0.059 V^2 L$$

in which  $p$  is in inches of water,  $V$  in ft. sec.,  $d$  in inches, and  $L$  in feet. The expressions show the effect of the change in viscosity of the water due to the change in temperature. Brabée has applied these results successfully to the flow of hot water and steam installations.

The above methods of determining the coefficients of friction in no way alter the modern theory of dynamical similarity.

**Special cases of pressure loss.** The separate resistances caused in a pipe line by obstructions such as valves, cocks, dampers, branches, bends, etc., depend to a large extent on the design of these. Tests show, in all cases, that these pressure losses increase as the square of the velocity, from which it follows that the main factor in causing them is turbulence. Hence the pressure loss can be expressed by

$$\Delta p = \zeta \frac{V^2}{2g}$$

From the results of his own and other numerous tests on water, Brabée obtained the following values—

Ordinary valves  $\zeta = 6.5$  to 7

Knees (90°)  $\zeta = 1.2$  to 2

Bends (90°) when  $r > 5d$   $\zeta = 0$

For standard cast-iron bends  $\zeta$  is about 0.3.

\* *Z. V.d.I.* (1916), page 441.

The equivalent length  $l'$  of pipe follows from

$$\frac{4fl'V^2}{2gd} = V^2$$

giving  $l' =$  (326)

and hence depends on the pipe diameter. In Eberle's tests, for example, it was found, in the case of a pipe having a diameter of 2.75 in. and fitted with a stop valve, that  $l' = 54.8$  ft. For a similar valve in a pipe 5½ in. diameter and the same value of  $f$ ,  $l' = 2 \times 54.8 = 109.6$  ft. If a pipe line contains  $n$  valves, the equivalent length in the resistance equation is  $l + nl'$ , but it is preferable to use the coefficient  $\zeta$ .

**Examples on air and steam pipe lines.** The following problems can arise—

1. To find the pressure drop in a pipe line of given diameter  $d$  and given length  $l$  when the velocity is  $V$  ft./sec. (or the rate of flow is  $W$  lb./sec.).

The pressure drop is given by the sum of the friction loss and losses caused by valves, etc., i.e.

$$p_1 - p_2 = \left[ \frac{4f l V^2}{2gd} + \frac{\rho V^2}{2g} \Sigma \zeta \right] \quad (327)$$

In order to find  $f$  from Fig. 147, Reynolds' number has to be found from

$$Vd = \frac{\rho V d}{\mu_{abs}} \quad (328)$$

$\mu_{abs}$  is read from the table on page 281,  $\rho$  calculated from the given conditions, and the pressure drop  $p_1 - p_2$  is then calculated from equation (327).

2. To find the diameter of a pipe of given length  $L$  when the pressure drop for a given rate of flow  $W$  lb./sec. is not to exceed a certain value.

An approximate value for  $d$  must first be found from equation (327) by assuming a probable value for  $f$ .

From (329)

$$\frac{\pi d^2}{4} \rho$$

equation (327) gives, when the second term is neglected,

$$d^5 = \frac{f l W^2}{(g)} \quad (d \text{ in ft., } p_1 \text{ and } p_2 \text{ in lb./ft.}^2) \quad (330)$$

With this approximate value for  $d$ , Reynolds' number is found from equation (328) and from Fig. 147 this gives the approximate  $f$  value. If this does not agree with the value of  $f$  first assumed, a new value must be tried again in equation (330), and, with the new Reynolds' number thus obtained,  $f$  is again tested. If the resistances due to obstructions are high, equation (327) must also be checked again.

3. To find the rate of flow  $W$  lb./sec. through a pipe of diameter  $d$  and length  $l$  when the pressure drop is not to exceed  $(p_1 - p_2)$  lb./ft.<sup>2</sup>

If a value of  $f$  be assumed, a first approximation to  $V$  is found from equation (327). From this, an approximate value of  $W$  is given by equation (329). Reynolds' number can now be calculated, from which a value of  $f$  is obtained (in Fig. 147), which should agree with the assumed value. If it does not, the process is repeated until agreement is obtained.

When the pressure loss is considerable, the following should also be observed—

The pressure drop  $p_1 - p_2$  in the above examples should be only a small fraction of the initial pressure  $p_1$ . Due to the pressure reduction, an increase in the specific volume takes place, so that the velocity at the end of the pipe is always greater than that at the beginning. Due to this acceleration, there is an additional pressure drop  $\Delta p$ , which, for small pressure differences, is found approximately from

As long as  $\Delta p$  is only a small fraction of  $(p_1 - p_2)$  this method suffices. In pipe lines, however, which are very long compared with their diameters, the pressure drop is always considerable. In these cases, it is necessary to divide the pipe line into several parts and carry out the calculation step by step, or else use the method explained on page 292.

The resistances due to valves, bends, etc., must, of course, also be taken into account. In short pipe lines these resistances are of greater relative importance.

**Example 11.** An engine, on overload, uses 990 lb. of steam at 300° C. and 142.2 lb./in.<sup>2</sup> abs. per hour. The steam supply pipe is 164 ft. long and is fitted with two stop valves. Find the diameter of pipe required and the total pressure loss. Allow a steam velocity of about 100 ft./sec.

From steam tables, the density is  $\rho = 0.245$  lb./ft.<sup>3</sup>

The rate of flow is

$$W = \frac{990}{3600} = 0.275 \text{ lb./sec.}$$

Hence, with  $W = AV\rho$

$$\text{we have } A = \frac{0.275}{100 \times 0.245} = 0.01122 \text{ ft.}^2 = 1.618 \text{ in.}^2$$

$$\text{and } d = 0.1196 \text{ ft.} = 1.435 \text{ in.}$$

$$\text{Taking } d = 1.5 \text{ in. or } 0.125 \text{ ft. gives } V = 92 \text{ ft./sec.}$$

If  $f$  is assumed to be 0.00515, the pressure drop due to friction is

$$\begin{aligned} \Delta p_1 &= \frac{4 \times 0.00515 \times 164 \times 0.245 \times 92^2}{2 \times 32.2 \times 0.125} \\ &= 872 \text{ lb./ft.}^2 = 6.06 \text{ lb./in.}^2 \end{aligned}$$

The pressure drop through the valves is (see page 288)

$$\frac{2 \times 7 \times 92^2 \times 0.245}{2 \times 32.2} = 452 \text{ lb./ft.}^2 = 3.14 \text{ lb./in.}^2$$



The total pressure drop is thus

$$\Delta p = \Delta p_1 + \Delta p_2 = 9.2 \text{ lb./in.}^2$$

We now have

$$\frac{Vd}{\nu} = \frac{\rho Vd}{\mu_{abs}} = \frac{0.245 \times 92 \times 0.125 \times 10^7}{147} = 192,000$$

For this value of Reynolds' number the value of  $f$  can be regarded as constant and the assumed value is justified (Fig. 147).

**Example 12.** Air at  $20^\circ \text{C.}$  and  $142.2 \text{ lb./in.}^2$  abs. enters a long mild steel pipe of  $2.36 \text{ in.}$  diameter at a speed of  $98.5 \text{ ft./sec.}$  Find the pressures at  $328 \text{ ft.}$ ,  $656 \text{ ft.}$ , and  $984 \text{ ft.}$  from the inlet end.

The density of the air at inlet is

$$\rho = \frac{0.0808 \times 142.2 \times 273}{14.7 \times 293} = 0.726 \text{ lb./ft.}^3$$

and the viscosity is (page 281)

$$\mu_{abs} = \frac{126.8}{10^7}$$

$$\text{from which } \frac{Vd}{\nu} = \frac{\rho Vd}{\mu_{abs}} = \frac{0.726 \times 98.5 \times 2.36 \times 10^7}{12 \times 126.8} = 1,110,000$$

As shown in Fig. 147 the resistance coefficient is practically constant in this region and equal to  $0.0054$ , so that the pressure loss in the first length ( $328 \text{ ft.}$ ) of the pipe is

$$\begin{aligned} p_1 - p_2 &= \frac{4fLV^2\rho}{2gd} = \frac{4 \times 0.0054 \times 328 \times 98.5^2 \times 0.726 \times 12}{2 \times 32.2 \times 2.36} \\ &= 3950 \text{ lb./ft.}^2 = 27.4 \text{ lb./in.}^2 \end{aligned}$$

Hence, the pressure at this point is  $142.2 - 27.4 = 114.8 \text{ lb./in.}^2$  abs., the density, assuming the temperature to remain constant,

$$\text{is } \frac{0.726 \times 114.8}{142.2} = 0.585 \text{ lb./ft.}^3$$

and the velocity is

$$V_2 = \frac{98.5 \times 0.726}{0.585} = 122.1 \text{ ft./sec.}$$

Along the second length ( $328 \text{ ft.}$ ) the pressure loss is

$$\begin{aligned} p_2 - p_3 &= \frac{4 \times 0.0054 \times 328 \times 122.1^2 \times 0.585 \times 12}{2 \times 32.2 \times 2.36} \\ &= 4890 \text{ lb./ft.}^2 = 34 \text{ lb./in.}^2 \end{aligned}$$

Since, however, the pressure loss in equal lengths of the same pipe is inversely proportional to the density (see equation (330)), then, alternatively,

$$p_2 - p_3 = \frac{27.4 \times 0.726}{0.585} = 34 \text{ lb./in.}^2$$

Hence, at the second point, the air pressure is  $114.8 - 34.0 = 80.8$  lb./in.<sup>2</sup> abs., the density is

$$\frac{0.726 \times 80.8}{142.2} = 0.413 \text{ lb./ft.}^3$$

and the velocity is

$$V_3 = \frac{98.5 \times 0.726}{0.413} = 173.1 \text{ ft./sec.}$$

Along the third length the pressure loss is

$$p_3 - p_4 = \frac{27.4 \times 0.726}{0.413} = 48.1 \text{ lb./in.}^2$$

so that the final pressure is  $p_4 = 80.8 - 48.1 = 32.7$  lb./in.<sup>2</sup> abs.; the final density is

$$\frac{0.726 \times 32.7}{142.2} = 0.167 \text{ lb./ft.}^3$$

and the final velocity is

$$V_4 = \frac{98.5 \times 142.2}{32.7} = 429 \text{ ft./sec.}$$

Hence, along the last length of the pipe both the pressure loss and the velocity are higher than in the other two lengths.

The works required to accelerate 1 lb. of air in the three lengths are as follows—

$$\text{First length} \quad \frac{V_2^2}{2g} - \frac{V_1^2}{2g} = 80.8 \text{ ft. lb.}$$

$$\text{Second length} \quad \frac{V_3^2}{2g} - \frac{V_2^2}{2g} = 233 \text{ ft. lb.}$$

$$\text{Third length} \quad \frac{V_4^2}{2g} - \frac{V_3^2}{2g} = 2400 \text{ ft. lb.}$$

The approximate pressure drop in the third length caused by the acceleration work of 2400 ft. lb. is found from equation (331) and amounts

$$\begin{aligned} \text{to} \quad \Delta p &= \frac{2400}{\rho_3 - \rho_4} = \frac{2400}{4 \text{ lb./in.}^2} \\ &= \frac{2400}{0.167} \left( \frac{1}{2} \right) = 570 \text{ lb./ft.}^2 \end{aligned}$$

so that the final pressure will be about  $32.7 - 4 = 28.7$  lb./in.<sup>2</sup> abs. See next section regarding this problem.

**Very long pipe lines.** In pipe lines which are long compared with their diameters, a pressure loss  $p_1 - p_2$ , which is a considerable fraction of the initial pressure  $p_1$ , occurs along the length of the pipe. The velocity also increases considerably between the initial and final pipe sections, so that a considerable part of the pressure drop is required for the work

of acceleration. Due to this the equations of the previous section do not apply to very long pipes. In addition, a heat exchange usually takes place between the flowing fluid and the colder or warmer surroundings. In pipe lines which carry steam or hot gases there is always a considerable heat loss to the surroundings, even when the pipes are well lagged. On the other hand, where long pipe lines convey gases having the same temperature as the surroundings, as, for example, pipe lines for town gas, the temperature near the end of the pipe would be lower than that of the surroundings (due to the considerable pressure drop), were it not for the heat flow from the surroundings through the unlagged walls. In shorter pipe lines, in which the pressure drop is considerable, and through which the gases flow with a high velocity, so that the heat flow is reduced, the temperature drop in certain cases is considerable and the conditions of flow are totally different from those of long pipes under usual conditions. It will thus be recognized that there are two limiting cases for long pipes.

1. Pipe lines in which the temperature remains practically constant (i.e. isothermal flow). Gas mains belong to this class.

2. Pipe lines in which no heat exchange occurs with the surroundings (i.e. adiabatic flow).

**Long pipes in which the flow is isothermal.** Under steady conditions the weight flowing per second through every section is

$$W = \frac{AV_1}{v_1} = \frac{AV}{v} = \frac{AV_2}{v_2} \quad (332)$$

$$\text{or} \quad W = A\rho_1 V_1 = A\rho V = A\rho_2 V_2 \quad . \quad (333)$$

This gives  $\frac{V_1}{v_1} = \frac{V}{v} = \frac{V_2}{v_2}$ . (334)

$$\rho_1 V_1 = \rho V = \rho_2 V_2 \quad . \quad (335)$$

For gases at constant temperature

. (336)

so that  $\frac{V}{V_1} = \frac{v}{v_1} = \frac{p_1}{p}$ . (337)

$$\text{or} \quad V = \frac{1}{p} \quad . \quad (338)$$

The pressure loss along an element, of length  $dl$ , of the pipe is, from equation (314),

This, along with equations (337) and (338), gives

$2q \, d$

so that  $\frac{1}{2} dp^2 = 2q d$

and, integrating between the limits  $p_2$  and  $p_1$ , gi

$$\frac{1}{2}(p_2^2 - p_1^2) = \frac{-4f \rho_1 l p_1 V_1^2}{2g d} \quad (339)$$

from which

$$2g RT_1 \left( \frac{p_2}{p_1} - 1 \right) = \frac{4f l V_1^2}{d} \quad (340)$$

when  $\frac{\rho_1}{p_1}$  is replaced by  $\frac{1}{RT_1}$

The pressure drop is thus

$$-\sqrt{1 - \frac{4f l V_1^2}{2g RT_1 d}} \quad (341)$$

When the expression  $\frac{4f l V_1^2}{2g RT_1 d}$  is small compared with 1, this expression becomes the same as the usual equation for the pressure loss, namely

$$\frac{4f l V_1^2}{2g d} \quad (342)$$

The above deduction holds provided  $f$  is constant throughout the pipe. This is actually the case, since Reynolds' number  $\frac{Vd}{\nu} = \frac{\rho Vd}{\mu_{abs}}$  remains constant. This follows, since  $\rho V$ ,  $d$ , and  $T$  (and hence  $\mu_{abs}$ ) have the same values throughout.

**Example 13.** In the case of Example 12, find the pressures if the flow is assumed to be isothermal.

Here the pressure at the end of the first length is, from equation (341).

$$p_2 = p_1 \sqrt{1 - \frac{8 \times 0.0054 \times 328 \times 98.5^2 \times 12}{2 \times 32.2 \times 96.3 \times 293 \times 2.36}} = p_1 \sqrt{0.615} = 0.783 p_1$$

so that for  $p_1 = 142.5$ ,  $p_2 = 111.5$ , and  $p_1 - p_2 = 30.7$  lb./in.<sup>2</sup>

Working out the pressure drop in the same way with  $L = 656$  ft. gives  $p_2 = 0.479 p_1 = 69.2$  lb./in.<sup>2</sup> abs. against 80.8 lb./in.<sup>2</sup> abs. obtained in example 12.

If the pressure drop be worked out for the whole length of the pipe (984 ft.) the expression under the root sign in equation (340) is found to be negative, which means that the initial pressure of 142.2 lb./in.<sup>2</sup> abs. drops to atmospheric pressure before the air reaches the end of the pipe. Since  $p_2$  cannot drop below atmospheric pressure, when the pipe discharges to atmosphere, the limiting value of the initial velocity is given by equation (340), i.e.

$$14.7 = 142.2 \sqrt{\frac{8 \times 0.0054 \times 984 \times 12 V^2}{96.3 \times 293 \times 2.36 \times 2 \times 32.2}}$$

which gives  $V = 91.1$  ft./sec.

So that, if the inlet velocity is 91.1 ft./sec. the pressure at outlet will be atmospheric.

**Example 14.** A gas main 7.45 miles long and 5.90 in. diameter is to deliver gas at 14.96 lb./in.<sup>2</sup> abs. and 17° C. Find the volume of gas discharged per hour (at the outlet state) if the velocity at outlet is 103 ft./sec. Find also the necessary pressure of the gas at inlet to the main. Take the gas density at outlet as 0.0393 lb./ft.<sup>3</sup>

$$\text{Volume discharged per hour} = \frac{\pi}{4} \left( \frac{5.90}{12} \right)^2 \times 103 \times 3600 = 705,000 \text{ ft.}^3$$

at 14.96 lb./in.<sup>2</sup> abs. and 17° C.

The resistance of the pipe line is found from Fig. 147 when Reynolds' number is determined. For air, this would be

$$\frac{Vd}{\nu} = \frac{\rho Vd}{\mu_{abs}} = \frac{0.0393 \times 103 \times 5.9 \times 10^7}{12 \times 125.5} = 158,700$$

As given by Biell, we have  $\frac{\nu_{gas}}{\nu_{air}} = \frac{1.38}{1}$ , so that Reynolds' number for town gas is  $\frac{158700}{1.38} = 115,600$ .

For a smooth pipe, Fig. 147 gives  $f = 0.0045$  at this Reynolds' number. Since, however, a main of this length will have a considerable number of bends, a somewhat higher value, say 0.005, is required. From an equation given by Biell,

$$f = 0.0195^*$$

where  $Q$  is the flow of gas in ft.<sup>3</sup>/hour. This gives  $f = 0.0048$ .†

Equation (339) can be written as

$$p_1^2 - p_2^2 = \frac{8 f l \rho_2 V_2^2 p_2}{2g d}$$

so that

$$\begin{aligned} p_1^2 &= 14.96^2 + \frac{8 \times 0.005 \times 7.45 \times 5280 \times 0.0393 \times 14.96 \times 103^2 \times 12}{64.4 \times 5.9 \times 144} \\ &= 14.96^2 + 2150 \\ &= 2373 \end{aligned}$$

hence,  $p_1 = 48.6 \text{ lb./in.}^2 \text{ abs.}$

\* Taken from H. Kapper's *Handbuch der Brennstofftechnik*, Essen, 1928.

† Guman's work on "The determination of the friction coefficient in long distance gas mains," will be found in *Z.V.d.I.* (1930), page 107. From extensive tests on two Romanian mineral gas pipe lines, 25.2 miles and 3.32 miles long and each 10 in. diameter, Guman found the value of  $f$  to be 0.0044 for  $\frac{Vd}{\nu} = 980,000$  and 0.0041 for  $\frac{Vd}{\nu} = 1,460,000$ . These values of  $f$  include the effects due to the resistances caused by valves, water separators, and bends. The gas analysis showed over 99% methane. The absolute viscosity was assumed to be  $\mu_{abs} = 67.3 \times 10^{-7}$  at 0° C. The daily variations in temperature at the end of the pipe were very small. During the six months in which observations were made the limits of temperature were -5.2° C. and 13.5° C.





The change of state is thus expressed approximately by a rectangular hyperbola (i.e. an isothermal), so long as the velocity  $V$  is small compared with the acoustic velocity. For example, if  $V = 0.1 V_a$  (i.e.  $V = 100$  to  $130$  ft./sec. for air at moderate temperatures) and  $\gamma = 1.4$ ,

we have 
$$\frac{\gamma-1}{2} \left( \frac{V}{V_a} \right)^2 = 0.002$$

which is small compared with 1.

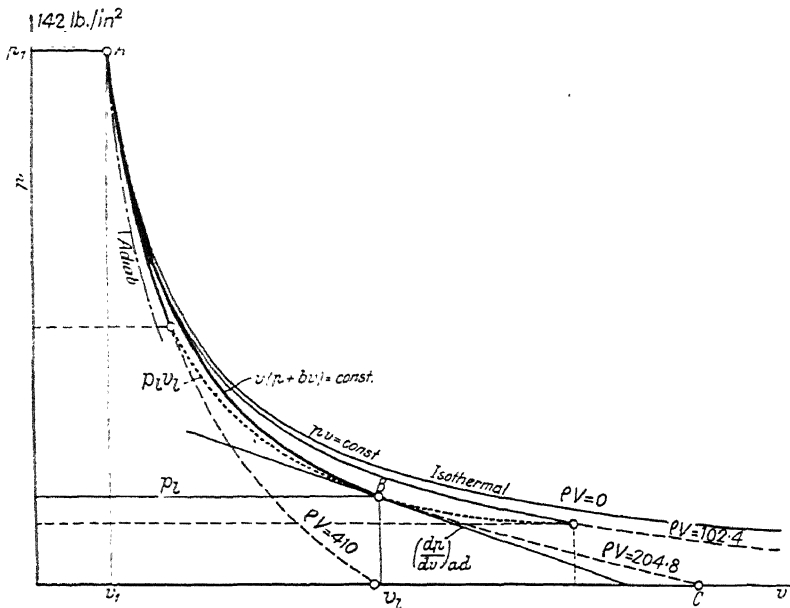


FIG. 151

Fig. 151 shows the state curves for the following values of

$$\rho V = 0, 102.4, 204.8, 410 \text{ lb./sec. ft.}^2$$

These refer to air at  $142 \text{ lb./in.}^2$  abs. and  $300^\circ \text{ C. abs.}$  The state curve  $ABC$  cuts the abscissa axis, since the volume has a finite value at  $p = 0$ , as shown by equation (354). This, however, means that where the pressure drops to absolute zero, the volume is still finite, which is impossible. Actually the curve loses its significance at an earlier point in the expansion. The limiting case of adiabatic flow with friction is that in which the friction is reduced to zero. The change of state is then expressed by  $p v^\gamma = \text{constant}$ . With friction the state curve can only lie above this curve, since the part of the flow energy required to overcome the resistances is converted to heat, and is equivalent to a supply of heat to the flowing substance (see page 310). Hence, as soon as the slope of any of the state curves in Fig. 151 becomes the same as that of the adiabatic the curve loses its physical significance.



This is also expressed by stating that, when flow is accompanied by friction, the entropy increases in the direction of flow. The equation of state ceases to be valid beyond the point at which it first shows a decrease in the value of the entropy, i.e. where  $d\phi = 0$ , or the change of state becomes an adiabatic. Fig. 153 shows the change of state on the  $T\phi$  field for air at 142 lb./in.<sup>2</sup> abs. and 90° C. This limiting point on the state curve is obtained as follows—

The slope of the adiabatic curve  $pv^\gamma = C$  is

$$\frac{dp}{dv} = -\gamma \frac{p}{v}$$

The slope of the actual state curve is found by differentiating equation (354), which can be written as

[illegible]

This gives

$$\frac{dp}{dv} = -\frac{p_1 v_1 + b v_1^2}{v^2} - b. \quad (357)$$

Equating this to the slope of the adiabatic curve gives the limiting values of volume and pressure, i.e.  $v_l$  and  $p_l$ . Thus

$$\gamma \frac{p_1}{v_1} = \frac{p_1 v_1 + b v_1^2}{v_1^2} + b . \quad . \quad . \quad . \quad . \quad . \quad (358)$$

which, along with equation (356), gives

$$v_i^2 = \frac{\gamma - 1}{b(\gamma + 1)} (p_1 v_1 + b v_1^2) . \quad . \quad . \quad . \quad . \quad . \quad (359)$$

If the value of  $b$ , as given by equation (353), be inserted in this, along with the substitution  $g\gamma p_1 v_1 = V_{a1}^2$ , then

$$\left(\frac{v_l}{v_1}\right)^2 = \frac{\gamma-1}{\gamma+1} \left\{ 1 + \frac{2}{\gamma-1} \left(\frac{V_{a1}}{V_1}\right)^2 \right\} . \quad . \quad . \quad . \quad (360)$$

This enables the maximum possible volume expansion ratio to be determined. Its value is reduced as the initial velocity  $V_1$  in the pipe approaches the acoustic velocity  $V_{a1}$  for the initial state. When  $V_1 = V_{a1}$  we have  $\frac{v_2}{v_1} = 1$ , so that no volumetric expansion is possible. If  $V_1$  is greater than  $V_{a1}$ , compression only is possible.

Equations (356) and (358) give the limiting pressure ratio as

$$\frac{p_i}{p_1} = \frac{V_1}{V_{a1}} \sqrt{\frac{2}{\gamma + 1} + \frac{\gamma - 1}{\gamma + 1} \left( \frac{V_1}{V_{a1}} \right)^2} \quad (361)$$

while equations (360) and (345) give the limiting velocity  $V_l$  at discharge

as 
$$V_i^2 = \frac{2}{\gamma + 1} V_{a1}^2 + \frac{\gamma - 1}{\gamma + 1} V_1^2 \quad . \quad . \quad . \quad (362)$$



i.e. the velocity at outlet from a uniform pipe cannot exceed the acoustic velocity of the gas referred to the outlet state.

The maximum heat drop convertible to kinetic energy is, from equations (347) and (363),

$$\left[ \frac{2}{\gamma-1} \frac{V_{a1}^2}{a_1^2} - 1 \right] \quad (365)$$

The friction work per pound of gas flowing in a cylindrical pipe is, from equation (318),

If  $V$  is changing continuously, this equation is only valid for an element  $dl$  of the pipe, so that

$$dE_f = \frac{4f V^2 dl}{2g d} \quad (366)$$

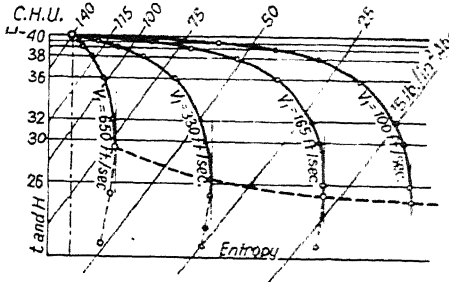


FIG. 153

As shown on page 277,  $f$  is not constant but a function of  $\frac{Vd}{\nu}$  or  $\frac{\rho Vd}{\mu}$  and also depends on the roughness of the pipe.

When conditions are steady, however, in a uniform pipe,  $\rho V$  is constant, so that  $f$  will then be a function of  $\mu$  alone. In the case of gases,  $\mu$  is independent of  $p$  and  $v$  and depends only on the temperature, with which it changes but slowly. If, therefore, the temperature changes along a pipe are not considerable,  $\mu$ , and therefore  $f$ , are almost constant, particularly at high Reynolds' numbers. The values of  $f$  for particular cases can be found from page 279.

As shown on page 309, we have, for adiabatic flow with friction,

$$-v dp = d \frac{V^2}{2g} + dE_f \quad (367)$$

which, with equation (366), gives

$$-v dp = d \frac{V^2}{2g} + \frac{4f dl V^2}{2g d} \quad (368)$$

In this expression the pressure can be replaced by the volume from equation (354), and the velocity by the volume from equation (345), so that

$$(p_1 v_1 + b v_1^2) \frac{dv}{v^3} = b \left( \frac{\gamma+1}{\gamma-1} \right) \frac{dv}{v} + \frac{4f b \gamma dl}{d(\gamma-1)} \quad (369)$$

Integrating between the limits  $v$  and  $v_1$ , and  $O$  and  $l$ , gives

$$-\frac{1}{2}(p_1 v_1 + b v_1^2) \left( \frac{1}{v^2} - \frac{1}{v_1^2} \right) = b \left( \frac{\gamma+1}{\gamma-1} \right) \log_e \frac{v}{v_1} + \frac{4f b \gamma l}{d(\gamma-1)}$$

or 
$$\frac{4f l}{d} = \frac{\gamma-1}{2\gamma} \left( 1 + \frac{p_1}{b v_1} \right) \left( 1 - \frac{v_1^2}{v^2} \right) - \frac{\gamma+1}{\gamma} \log_e \frac{v}{v_1} \quad (370)$$

This expression, along with equations (345) and (353), also gives

$$d = \gamma \left( \frac{V_1^2}{V^2} - 2.303 \frac{\gamma + 1}{\gamma} \right)$$

Equation (370) enables the volume ratio  $\frac{v}{v_1}$  to be found, while equation (371) gives the velocity ratio  $\frac{V}{V_1}$ , which is also equal to the volume ratio. Both these values occur at the distance  $l$  along the pipe.

The ratios  $\frac{V}{V_1} = \frac{v}{v_1}$ , as given by equation (371), have been plotted in Fig. 154 against the corresponding values of  $\frac{4fl}{d}$ . Each curve corresponds to a given initial velocity  $V_1$  (expressed as a fraction of the acoustic velocity  $V_{a1}$ ), and thus corresponds to a definite rate of flow  $W$  lb./sec., which also depends on the initial volume as shown by  $\rho_1 V_1 = \rho V = \frac{W}{A}$ . The abscissae values indicate how the velocity and specific volume increase along the pipe with different initial velocities. The absolute length  $l$ , which corresponds to a definite value of  $\frac{4fl}{d}$ , depends not only on  $d$  but also on  $f$ . Thus if  $f = 0.0075$ , then, for a pipe of 2 in. diameter and  $\frac{4fl}{d} = 20$ , we have  $l = \frac{20 \times d}{4 \times 0.0075} = 111$  ft.

If the pipe conveys air at an initial temperature of  $20^\circ \text{C.}$ , then  $V_{a1}^2 = g \gamma R T_1 = 32.2 \times 1.4 \times 96.3 \times 293 = 1,270,000$ , so that  $V_{a1} = 1127$  ft./sec. With  $V_1 = 0.1 V_{a1}$  we have  $V_1 = 112.7$  ft./sec., and the velocity ratio at the end of the pipe, as given by Fig. 154, is  $\frac{V}{V_1} = \frac{v}{v_1} = 1.19$ . If, in the same pipe, the initial velocity be increased to  $V_1 = 0.15 V_{a1} = 169$  ft./sec., the velocity ratio at the end becomes  $\frac{V}{V_1} = \frac{v}{v_1} = 1.7$ .

The curves flatten out as  $\frac{V}{V_1}$  increases and attain a maximum value of  $\frac{4fl}{d} = 1$ . This is found by differentiating equation (371) with respect to  $\frac{V}{V_1}$ , and is the value of  $\frac{4fl}{d}$  obtained by inserting the limiting values for  $\frac{v}{v_1}$  or  $\frac{V}{V_1}$  in equations (370) or (371).

This gives

$$\left( \frac{4fl}{d} \right)_{max} = \frac{1}{\gamma} \left[ \left( \frac{V_{a1}}{V_1} \right)^2 - 1 \right] - \left[ \left\{ 2.303 \frac{\gamma + 1}{2\gamma} \right\} \log \left\{ 1 + \frac{2}{\gamma + 1} \left( \frac{V_{a1}^2}{V_1^2} - 1 \right) \right\} \right] \quad (372)$$

Hence in a pipe of diameter  $d$  an initial velocity  $V_1$  (or rate of flow  $\frac{W}{A}$ ) is only possible up to a definite limiting value of  $l_{max}$ . The pressure ratio  $\frac{p}{p_1} = \frac{p_2}{p_1}$  is then such that the final velocity, as shown by equation

(362), approaches the acoustic velocity at the initial conditions. In the above example, where  $V_1 = 0.1V_{a1}$ , the maximum value of  $\frac{4fl}{d}$  is 67, so that  $L_{max} = \frac{67 \times 2}{12 \times 4 \times 0.0075} = 372$  ft., while, with  $V_1 = 0.15V_{a1}$ , we have  $\left(\frac{4fl}{d}\right)_{max} = 28$  and  $L_{max} = 155.5$  ft.

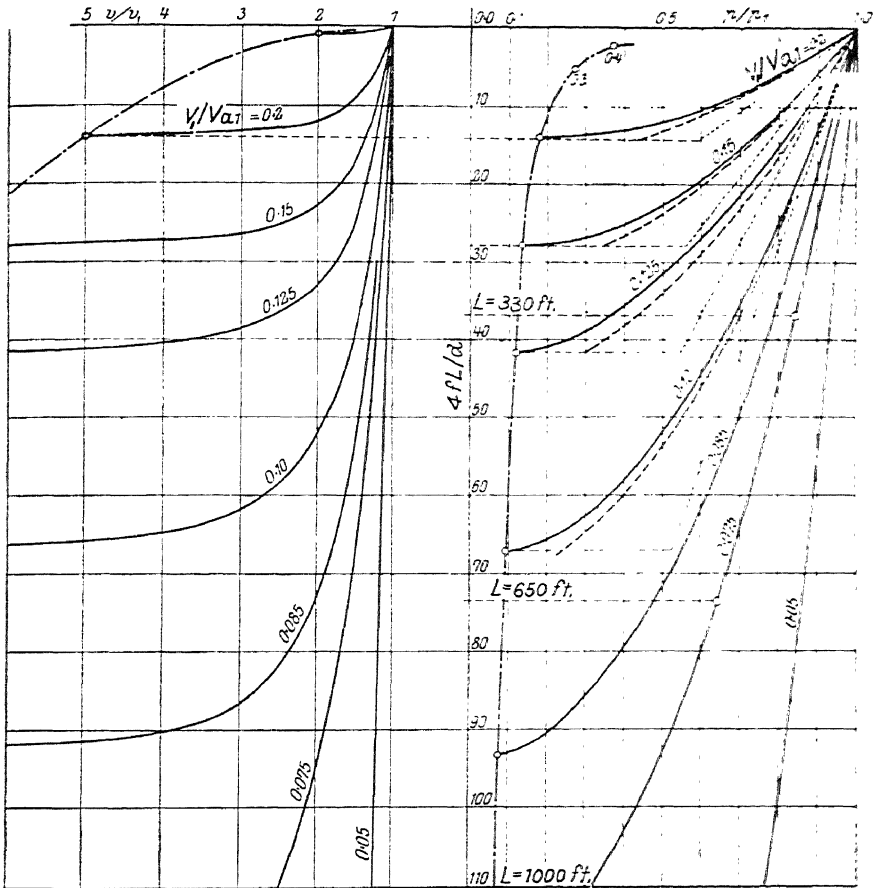


FIG. 154

As shown in Fig. 152, the final velocity ratio for both cases is about 0.91, so that  $V_t = 0.91V_{a1} = 1024$  ft./sec. The pressure ratios  $\left(\frac{p_1}{p_2}\right)$  in the two cases are 0.085 and 0.125 respectively. The rates of flow  $W$  lb./sec., corresponding to the velocities  $V_1 = 0.1V_{a1}$  and  $V_1 = 0.15V_{a1}$ , are also the maximum amounts which can flow through a pipe of this diameter, and no increase in  $W$  can be obtained by reducing the back pressure at the end of the pipe.

From equations (353) and (354) the relation between the pressure ratios and the volume ratios is given by

$$\frac{p}{p_1} \quad . \quad . \quad (373)$$

The values of  $\frac{p}{p_1}$  have been plotted against the  $\frac{x}{d}$  values on the right of Fig. 154. For a given value of  $\frac{V_1}{V_{a1}}$  the corresponding value of  $\frac{4fl}{d}$  was found from equation (371) and the value of  $\frac{p}{p_1}$  from equation (373). In this way the pressure changes along a pipe, in which the initial velocity is known, can be found. The curves again give a maximum value of  $\frac{4fl}{d}$  at which the smallest possible value of the end pressure  $p_1$  is obtained. If the pipe length be increased, the rate of flow  $p_1 V_1$  can no longer be maintained.

Where  $\frac{l}{x}$  is small compared with  $\left(\frac{l}{x}\right)$ , the pressure changes can be regarded as a linear function of the length. Thus, if  $V_1 = 0.1 V_{a1}$ , this applies for values of  $\frac{4fl}{d}$  between 0 and 20, if  $V_1 = 0.15 V_{a1}$  for values of  $\frac{4fl}{d}$  between 0 and 10, and if  $V_1 = 0.2 V_{a1}$  for values of  $\frac{4fl}{d}$  between 0 and 5. Beyond these lengths the pressure falls more rapidly, especially near the ends.

**Approximate equations.** In the numerical calculation of the values of  $\frac{4fl}{d}$  as given by equation (371), it will be observed that, for small values of  $\frac{V_1}{V_{a1}}$ , the second (logarithmic) term does not seriously affect the result. Thus, with  $V_1 = 0.15 V_{a1}$ , we have the following results—

$\frac{V_1}{V_{a1}}$	1.1	1.2	1.5	2	3	4	
First term	5.52	9.75	17.7	23.9	28.4	29.9	31.3
Second term	0.16	0.31	0.69	1.19	1.9	2.4	3.7
	5.36	9.44	17.01	22.71	26.5	27.5	27.6

The values of  $4fl/d$  are in this case, therefore, mainly dependent on the first term. Hence, by neglecting the second term, an approximate equation is obtained which can be regarded as valid for pipes having initial velocities between  $V_1 = 0$  and  $V_1 = 0.15 V_{a1} \doteq 200$  ft./sec. when  $V_{a1} = 1315$  ft. sec. If, further, the value  $\frac{\gamma-1}{2}$ , which is small compared with  $\left(\frac{V_{a1}}{V_1}\right)^2$ , be omitted in equation (371), the equation becomes

$$\frac{4fl}{d} = \frac{1}{\gamma} \left(\frac{V_{a1}}{V_1}\right)^2 \left\{ 1 - \left(\frac{V_1}{V}\right)^2 \right\} \quad . \quad . \quad . \quad . \quad (374)$$

$$\text{or} \quad \frac{V_1}{V} = \sqrt{1 - \frac{\gamma}{4fl} \left(\frac{V_{a1}}{V_1}\right)^2} \quad . \quad (375)$$

In flow of this type we can also assume, that  $pv = p_1v_1$ , so that with  $\frac{v_1}{v} = \frac{r_1}{r}$

$$\frac{p}{p_1} = \sqrt{1 - \frac{4 \gamma f l V_1^2}{d V_{a1}^2}} \quad (376)$$

Substituting the value  $g \gamma p_1 v_1$  for  $V_{a1}^2$  gives the pressure drop as

$$p_1 - p = p_1 \left( 1 - \sqrt{1 - \frac{4flV_1^2}{dg p_1 v_1}} \right) \quad . \quad . \quad . \quad . \quad (377)$$

which is the same as equation (341).

Finally, if the second term under the root sign is small compared with 1, we have

$$p_1 - p = \frac{4fl V_1^2}{2g d v_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad (378)$$

which is the usual expression for the pressure loss in pipes.

The changes in pressure and volume, as given by these approximate equations, are also shown in Fig. 154. Equation (378) applies only within a limited range of  $\frac{4fl}{d}$  values, but equation (377) shows close agreement to the correct values throughout.

**Example 15.** Find the pressures in the pipe of Example 12 if the flow is adiabatic instead of isothermal.

The acoustic velocity under the initial conditions is

$$V_a = \sqrt{g \gamma R T_1} = 1126 \text{ ft./sec.}$$

Hence  $\frac{V_1}{V_{a1}} = \frac{98.5}{1126} = 0.0875$

Fig. 154 shows (for  $\frac{V_1}{V_{a1}} = 0.1$ ) that the usual equation (378) is only valid for values of  $\frac{4fl}{d}$  up to about 30, while equation (377) is valid for values up to about 60. Now, with  $f = 0.0055$ , we have

$$\frac{df_l}{d} = \frac{4 \times 0.0055 \times 12L}{2.36} = 0$$

hence, with

$l$	=	328	656	984 ft.
$\frac{4fl}{\pi}$	=	36.7	73.5	110

Equation (377) is thus applicable to the first length of the pipe but not to the second and third lengths.

The smallest possible final pressure  $p_t$  is about  $0.07 p_1 = 10 \text{ lb./in.}^2$  abs., as given by equation (12) or by Fig. 154, and the final velocity is then the same as the acoustic velocity. The value of  $\frac{4fl}{d}$  at this point

is higher than 70, as shown in Fig. 154, but is given more closely by equation (372) as  $\left(\frac{4f l}{d}\right)_{max} = 92.9 - 4.0 = 88.9$ . This shows that in

the pipe length of 984 ft., for which  $\frac{4f l}{d} = 110.1$ , it is not possible for the initial velocity to be 98.5 ft./sec. This velocity applies only to a length which is not more than 656 ft., but the pressure loss for 656 ft. is very high.

From the approximate equation (376), we have

$$\frac{p}{p_1} = \sqrt{1 - 0.107 \times \frac{4f l}{d}}$$

so that for

$$l = \quad 328 \quad \quad 656 \text{ ft.}$$

we have

$$\frac{p}{p_1} = \quad 0.778 \quad \quad 0.462$$

or

$$p = \quad 111.0 \quad \quad 65.8 \text{ lb./in.}^2 \text{ abs.}$$

For the case in which  $V_1 = 0.1 V_{a1} = 112.6$  ft./sec. the result can be found directly in Fig. 154. This shows that the length 656 ft. is too long, while for 328 ft. the final pressure would be  $0.68 p_1 = 0.68 \times 142.2 = 97$  lb./in.<sup>2</sup> abs.

The conditions are improved when a smaller initial velocity, i.e. a smaller rate of flow, is adopted. Thus, with  $V_1 = 0.075 V_{a1} = 84$  ft./sec. Fig. 154 gives, with

$$\begin{array}{lll} l = & 328 & 656 \quad 984 \text{ ft.} \\ p = & 120 & 92 \quad 45.5 \text{ lb./in.}^2 \text{ abs.} \end{array}$$

The pressure drop is still high, however, and it is only when the initial velocity is reduced to  $V_1 = 0.05 V_{a1} = 56.3$  ft./sec. that the pressure drop becomes reasonable and equal to 35.6 lb./in.<sup>2</sup> abs., as shown by Fig. 154.

**Loss of available work due to friction.\*** In order to overcome the resistances experienced in any kind of flow (i.e. uniform, accelerated, or retarded) a quantity of work must be supplied at the expense either of the energy in the substance itself, or from some external source.

In the case of flow in uniform pipes, the pressure of the gas or vapour is reduced by friction. Thus the available work of the steam at the end of a long pipe is less than that at the boiler.

Again, in the case of accelerated flow through a nozzle, the steam velocity is reduced owing to friction, so that the actual work delivered at the turbine is less than the ideal work as given by the pressure drop and shape of the blades. The efficiency of the turbine is thus less than that of the ideal frictionless turbine.

As a result of flow resistances, the work required to compress air in turbo-blowers is considerably greater than that corresponding to the statical pressure difference.

\* See the general principles regarding irreversible flow as outlined on pages 200 to 228.



In all these cases, the work required to overcome the resistances is converted to heat and is returned as such to the substance. Hence no loss of energy occurs, so long as the heat generated is prevented from escaping by radiation or conduction, but the available work, in the case of the turbine, is reduced, while the work necessary for compression in the case of the turbo-blower is increased.

In the case of liquids, the whole of the heat developed by friction must be regarded as a loss of energy. In a water turbine, for example, the available work of the water, which is slightly increased in temperature by friction, is always reduced owing to the pressure loss. The same applies to rotary pumps, as the mechanical equivalent of the heat developed by friction means a considerable demand on the shaft power.

Conditions are different, however, when the working substance is a gas or vapour. Thus, when steam flows through a nozzle, friction heat is developed at the expense of part of the kinetic energy of flow. This heat remains in the steam and serves to increase the temperature and volume of superheated steam, or increase the dryness fraction of wet steam. In both cases the available work is increased, although the increase is less than that corresponding to the conversion by friction from kinetic energy to heat. In this respect, therefore, steam turbines are more efficient than water turbines.

The air temperature in turbo-blowers is higher than that given by adiabatic frictionless compression. This causes an increase in the volume of the air, i.e. for the same pressure ratio the volume ratio is reduced, so that an increase in shaft power is necessary, which must be regarded as a direct loss, even though the energy remains in the compressed air. As opposed to the turbine this heating of the working substance is not only useless but entails a supply of additional energy. The total driving work is greater than the shaft and frictional work owing to the additional work caused by the volume increase.

**Relation between the frictional work and loss of available work.** In the previous section it is shown that, as opposed to liquids, the work required to overcome friction in the case of gases and vapours is not necessarily the same as the loss of available work. Hence, in determining the losses, it is necessary to find the relationship between the friction work and the loss in the flow energy.

If, during the accelerating period (i.e. between inlet to, and outlet from, a nozzle or channel), the change of state were adiabatic and unaccompanied by friction, the relation between the pressure and volume is expressed by

The state line would be the usual adiabatic curve  $AB'$  (Fig. 155). Due to friction, however, the flowing substance is heated, so that the drop in temperature corresponding to the pressure drop is not so great as in frictionless adiabatic flow. This higher temperature means an increase in the volume above that shown by adiabatic expansion for the same pressure drop. The actual pressure-volume curve thus lies above the adiabatic. This also applies to vapours. If the vapour is wet its temperature remains constant at a given pressure, whether or not the flow is frictionless, but the heat developed by friction evaporates some



For an elementary change in state this gives

(350)

This equation now replaces equation (231) when the flow is accompanied with friction.

The work of acceleration is thus

$$E_a = E_0 - (E_R - E_z) \quad . \quad . \quad . \quad . \quad . \quad (381)$$

If now  $E_R > E_z$ , i.e. the total friction work is greater than the work regained by the increase in volume, then  $E_a < E_0$ , as must be the case if  $V$  is to be less than  $V_0$ . The loss in the energy of flow is now  $E_v = E_R - E_z$  and its fractional value, called the coefficient of resistance (see page 254) is

(382)

The actual kinetic energy at discharge is found from

$$E_n = \frac{V^2}{\hbar} = E_n - \zeta E$$

which gives

(383)

If the actual state curve  $AB$  is known, the frictional work  $E_R$  must be found, since  $E_0$  and  $E_z$  alone are given in Fig. 155. It is only in this way that  $\zeta$  can be found. On the other hand, if  $\zeta$  be given, the true final state can be determined.

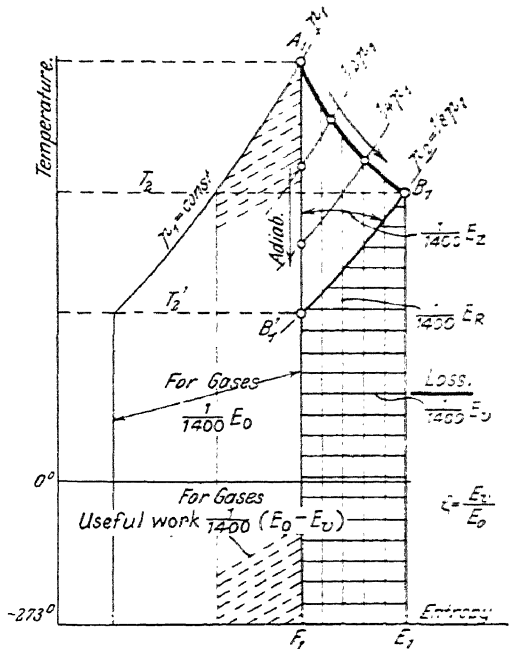


FIG. 156

The simplest solution is offered by means of the entropy diagram. The apparent amount of heat added during the actual change of state can be determined by transferring the true state line  $AB$  in Fig. 155 to the temperature-entropy field. If  $A_1B_1$  is the corresponding state curve in this field, the area  $A_1B_1E_1F_1$  represents the heat added.

Since no heat is supplied from outside, this heat can only be the friction heat  $AE_R$ , so that the area  $A_1B_1E_1F_1$  (Fig. 156) represents the total work of friction in C.H.U.

The loss of work is less than  $E_R$  and equals  $E_R - E_z$ , since the part  $E_z$  on the pressure diagram is regained, due to the frictional reheat.

This work  $E_z$  can also be represented by an area on the entropy diagram. In this field the curve  $A_1B_1$  corresponds to  $AB$  on the  $pv$  field

and  $A_1B'_1$  to the adiabatic curve  $AB'$ . By drawing in the constant pressure line for  $p_2$  (this is a horizontal for saturated vapours) through  $B'_1$  and  $B_1$  the line  $B'_1B_1$  corresponds to  $B'B$  on the  $pv$  field. The area  $A_1B_1B'_1$ , which corresponds to the work area  $ABB' = E_z$ , is equivalent to  $E_z$  in heat units. Hence the part  $AE_z$  of the friction heat reconverted to kinetic energy is represented by the triangular area  $A_1B_1B'_1$  in Fig. 156.

The lost friction work finally contained in the flowing substance as heat is thus equal to the area  $B_1B'_1F_1E_1$ . This quantity of heat, however, is also equal to the heat which would be necessary to raise the temperature from  $t'_2$  at  $B'_1$  (on the adiabatic) to the actual temperature  $t_2$ , while the pressure remains constant, so that for gases and vapours it is given by  $(c_p)_m(t_2 - t'_2)$ .

Hence, in order to find the lost work or the resistance coefficient  $\zeta$  it is sufficient to locate the points  $A_1$  and  $B_1$ , whereas the curve  $A_1B_1$  must be known if the complete frictional work is to be determined. Conversely, the point  $B_1$  can be located when the fractional loss  $\zeta$  is known.

Thus, for gases and superheated steam,

$$A\zeta E_0 = c_p(T_2 - T'_2)$$

$$\text{so that} \quad \zeta = \frac{c_p(T_2 - T'_2)}{AE_0} \quad . \quad . \quad . \quad . \quad . \quad . \quad (384)$$

and, for gases having constant specific heat,

$$AE_0 c = c_p(T_1 - T'_2)$$

$$\text{so that} \quad \zeta = \frac{T_2 - T'_2}{T_1 - T'_2} \quad . \quad (385)$$

Hence  $\zeta$  can be found when the initial temperature  $T_1$ , the final adiabatic temperature  $T'_2$ , and the final actual temperature  $T_2$  are known. The value of  $T_2$  depends on the amount of friction.

In the case of superheated steam it is preferable to use the total heats, since  $c_p$  changes very rapidly near the saturation state. The total heats are given directly in steam tables. We then have  $A = h_2 - h'_2$  and  $AE_0 = h_1 - h'_2$ , so that

$$\zeta = \frac{h_2 - h'_2}{h_1 - h'_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (386)$$

"1    " 2            "  $\phi$

where  $h_\phi$  is the adiabatic heat drop between the initial and final pressure (see page 461).

These relations show that  $\zeta$  is determined from the initial and final states, or if  $\zeta$  is known, the final state can be determined. Generally the final volume is required in order to determine the necessary cross-sectional exit area.

**Equation of flow.** The most general form of the equation of flow, which is applicable to any substance and any type of flow in which heat is neither supplied nor rejected, is found from the above relations as follows. In equation (383)

$$\frac{V^2}{2g} = E_0 - \zeta E_0$$





$$\text{and hence } -v dp = d \frac{V^2}{2g} + d E_R \quad . \quad . \quad . \quad . \quad . \quad (394)$$

which is analogous to equation (380) for expansion.

**Determination of the friction work.** The heat  $Q_R = A E_R$ , obtained by the conversion of this work, is found from the true state line  $AB$ , in the same way as would be done if the same heat were supplied to the compressed substance from the outside. We have, from the energy law, as expressed by equation (193)

$$Q_w = h_2 - h_1 - A \int_{p_1}^{p_2} v \, dp$$

$$\text{or} \quad Q_w = h_2 - h_1 - A(E_0 + E_z) \quad . \quad . \quad . \quad . \quad . \quad (395)$$

and hence, for gases, since  $h_2 - h_1 = c_p(T_2 - T_1)$

$$Q_w = A E_R = c_p(T_2 - T_1) - A(E_0 + E_z) \quad . \quad . \quad . \quad . \quad . \quad (396)$$

This, along with equation (391), gives the total work as

$$A E = A(E_0 + E_z) + c_p(T_2 - T_1) - A(E_0 + E_z)$$

or (for gases)

$$A E = c_p(T_2 - T_1) \quad . \quad . \quad . \quad . \quad . \quad (397)$$

or, more generally (since it also applies to vapours), with equations (395) and (391),

$$A E = h_2 - h_1 \quad . \quad . \quad . \quad . \quad . \quad (398)$$

The necessary work is thus merely the difference in total heats after and before compression, so that, in the case of gases, it depends on the temperature rise alone. The intermediate changes in state have thus no effect on the result.

Hence, in a diffuser, since

$$E = \frac{V_1^2}{2g} - \frac{V_2^2}{2g}$$

$$\text{we have} \quad A \left( \frac{V_1^2}{2g} - \frac{V_2^2}{2g} \right) = h_2 - h_1 \quad . \quad . \quad . \quad . \quad . \quad (399)$$

In uncooled centrifugal blowers, the work supplied to the shaft is greater than the above by the amount  $\frac{V_2^2}{2g}$  (i.e. the kinetic energy of the compressed gas), since the gas has a velocity  $V_2$  at discharge and has zero velocity initially, so that

$$A E_{shaft} = h_2 - h_1 + A \frac{V_2^2}{2g} \quad . \quad . \quad . \quad . \quad . \quad (400)$$

When the gas is compressed to a high pressure, however,  $\frac{A V_2^2}{2g}$  is generally negligible.

A clearer conception of these results is afforded by the  $T\phi$  chart. If the state line  $AB$  on the  $pv$  field of Fig. 157 be transferred to the  $T\phi$  field in Fig. 158, the resulting curve is  $A_1B_1$  and the area below  $A_1B_1$

represents the friction heat  $Q_R$  (or  $E_R$  in heat units). The additional area  $E_z$  on the  $pv$  field is represented here by the area  $A_1B_1B'_1$  and the total additional work is thus given by the complete area below  $B_1B'_1$ . This area, however, is also equal to the quantity of heat which would be necessary to raise the temperature of the gas at constant pressure from  $T_{2\phi}$ , i.e. the final adiabatic temperature to  $T_2$ , the true final temperature, since  $B'_1$  and  $B_1$  lie on the same pressure line. Hence the total additional work in heat units is

or, more generally,

$$A(E_z + E_R) = h_2 - h_{2\phi} \quad (401)$$

Hence, with equation (391) and with  $AE_0 =$  , we have

$$AE = h_{2\phi} - h_1 + h_2 - h_{2\phi} = h_2 - h_1$$

or, more generally,

$$AE = c_p(T_2 - T_1)$$

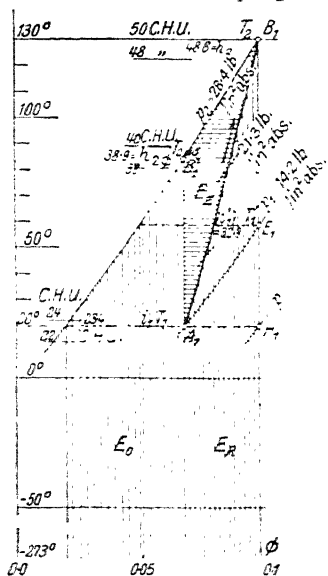


FIG. 155

$AE_0$  is represented by the area shown shaded with dotted lines on the  $T\phi$  field, so that the whole shaded area represents the complete work required.

In the  $pv$  field the area  $BHKG$  (Fig. 157) gives  $E$ .  $H$  is the point on the adiabatic through  $B$ , at which the temperature is again equal to  $T_1$ . Since, therefore, between  $B$  and  $H$  the temperature drops by  $T_2 - T_1$ , the area  $BHKG = c_p(T_2 - T_1) = E$ , as given on page 327.  $H$  is located as the point at which the isothermal through  $A$  cuts the adiabatic through  $B$ . For vapours, a line of constant total heat  $h_1$  has to be drawn in place of the isothermal. The pressure  $p'$  (which is only of geometrical significance) is given by

**Efficiency.** The frictionless adiabatic work is equal to  $AE_0$ , so that the adiabatic efficiency is

$$\eta_\phi = \frac{AE_0}{AE}$$

For gases, it is also

$$\eta_\phi = \frac{T_{2\phi} - T_1}{T_2 - T_1} \quad (402)$$

or, more generally,

$$\eta_\phi = \frac{h_{2\phi} - h_1}{h_2 - h_1} \quad (403)$$





so that  $AE_R = 8.3 \text{ C.H.U.}$

and  $AE_z$  is only about one-fifth of  $AE_R$ .

$$\text{Further, } \eta_{is} = \frac{0.4}{1.4} \times \frac{293}{110} \times 2.303 \log 2 = 0.53$$

$$\text{and } \eta_{th} = \frac{48.8 - 32.9}{25.4} = 0.63$$

## CHAPTER V

## DEVIATIONS FROM THE IDEAL GAS STATE

**Gases at very high pressures.** Experimental work on the compression of gases, at constant temperature, to pressures of several hundred atmospheres, has been carried out by various investigators. In particular, tests by Natterer (1855), Cailletet (1872), Pictet, Amagat (1873 to 1907), Wrablewski and Olszewski (1883), and Kamerlingh Onnes (1902 to 1918) have been carried out, mainly in an attempt to liquefy the gases. In addition, Holborn, Schultze, and Otto experimented on air, nitrogen, hydrogen, argon, neon, and helium with pressures up to 100 atmospheres and temperatures ranging from 183° C. to 400° C. These tests were performed in the German Imperial Institute.\*

These tests all agree in showing that both Boyle's Law for constant temperature

$$pv = \text{constant}$$

and Gay-Lussac's Law at constant pressure, along with the characteristic equation deduced from these

[illegible]

apply exactly only within certain limits of pressure and temperature. Generally the equations appear to be valid provided the temperature is not too low, nor the pressure too high. According to the kinetic theory of gases, they apply when the density is so small that the mutual attraction between the molecules no longer exists.

A representation of the nature and amount of the deviation, in the case of air, is obtained if a diagram be constructed which shows the values of  $pv$  to a base of  $p$ . Fig. 159 shows some special characteristics of these curves. If equation (406) applied throughout, the lines of constant temperature (i.e. the isothermals) would be straight and horizontal, since for  $T = \text{constant}$ ,  $pv$  is also constant. Actually, in water, the lines for  $T = \text{constant}$  are found to be curves which, at normal temperatures (e.g.  $7^\circ \text{C}$ . or  $280^\circ \text{C}$ . abs.), first fall and then rise as the pressures are increased.

It is only at temperatures above about 330° C. abs. or 57° C. that the curves are nearly horizontal up to about 10 to 26 atmospheres, so that, for this region, the ideal characteristic equation applies. The deviations, even at 0° C., in this pressure region are not serious and can be neglected in ordinary technical calculations. The gas constants in this region are  $R_0 = 96.0$ , as given by the mean molecular weight for air (see page 12), but for small pressures  $p \nu = 25,930$ , so that for  $T = 270^\circ \text{C. abs.}$

$$R_0 = \frac{25930}{270} = 96.0, \text{ which agrees with the above value.}$$

For the region between  $0^{\circ}\text{C.}$  and  $320^{\circ}\text{C.}$ , which alone is of importance

\* A comprehensive survey of this work and later work up to the year 1926 is contained in Vol. 279 of the *Bureau of Standards*, Washington, entitled "Relations between the temperatures, pressures, and densities of gases."

in the commercial compression of air, the product  $pv$  first slowly decreases with increasing pressures and then, at pressures between 50 and 80 atmospheres, increases slowly. Thereafter, however, the increases are

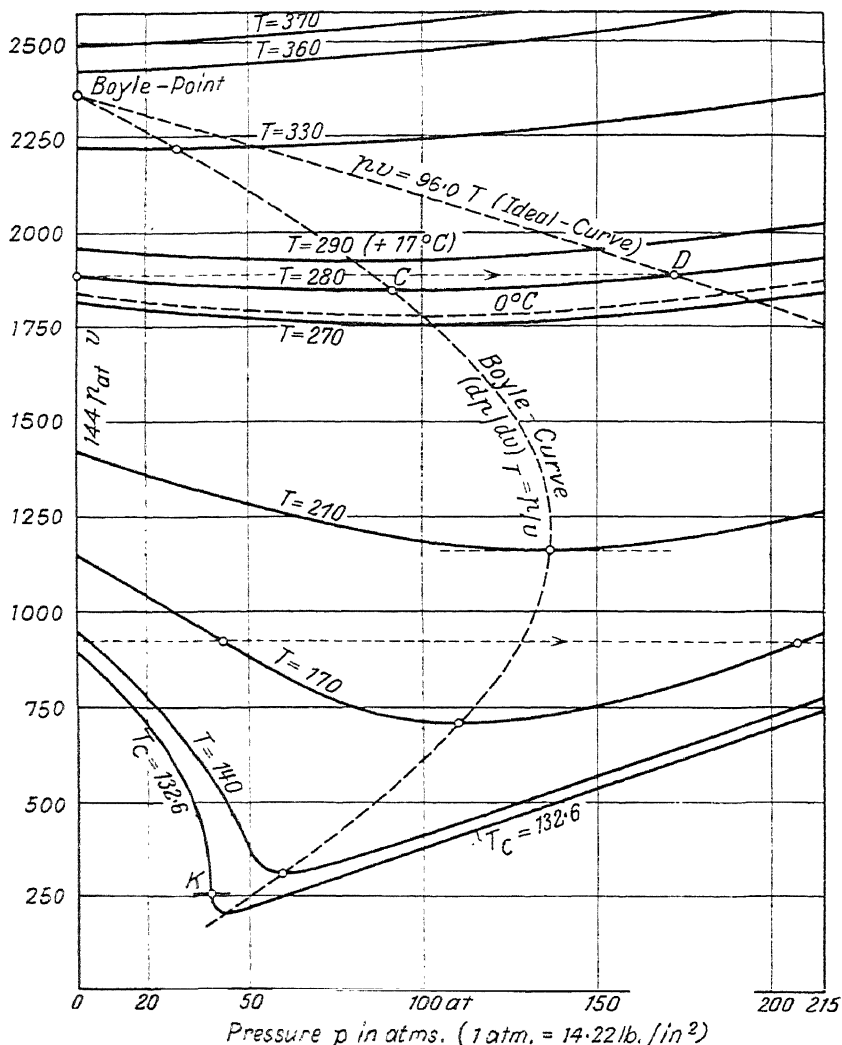


FIG. 159

rapid. Between  $17^{\circ}\text{C.}$  and  $47^{\circ}\text{C.}$  the quotients  $\frac{pv}{T}$  are only slightly different from 96.0 even at as high a pressure as 100 atmospheres. Thus, at  $290^{\circ}\text{C. abs.}$  it is  $\frac{27400}{290} = 94.5$  and at  $320^{\circ}\text{C.}$   $\frac{30850}{320} = 96.5$ , so that equation (406) and the normal gas constant can be regarded as applicable to ordinary calculations.

If, however, the final pressure is considerably above 100 atmospheres, as, for example, in the Linde air liquefaction process, where the final pressure is about 200 atmospheres, the differences are more serious and increase as the temperature rises. Thus, at 200 atmospheres, with

$$T = 290 \quad 300 \quad 320^\circ \text{ C. abs.}$$

$$t = 17 \quad 27 \quad 47^\circ \text{ C.}$$

$$p_2 v_2 = 28,600 \quad 29,800 \quad 32,200$$

while

$$p_1 v_1 = 27,800 \quad 28,800 \quad 30,800$$

$$\text{so that, with } \frac{p_2}{p_1} = 200,$$

$$\frac{1.029}{200} \quad 1.035 \quad 1.045$$

whereas, with the ideal gas,  $\frac{v_2}{v_1} = \frac{1}{200}$ . The volume at 200 atmospheres is thus from 3 to 4½ per cent greater than that of the ideal gas.

At low temperatures, however, a volume decrease occurs. Thus, with  $T = 250^\circ \text{ C. abs.}$  or  $t = -23^\circ \text{ C.}$  and  $p_1 = 1$  atmosphere, we have, with

$$p_2 = 50 \quad 100 \quad 150 \quad 200$$

$$p_2 v_2 = 23,000 \quad 22,550 \quad 22,600 \quad 23,300$$

$$p_1 v_1 = 24,050 \quad 24,050 \quad 24,050 \quad 24,050$$

which gives

$$\frac{v_2}{v_1} = \frac{0.956}{50} \quad \frac{0.938}{100} \quad \frac{0.940}{150} \quad \frac{0.980}{200}$$

while for the ideal gas we have the following ratios,

$$\frac{v_2}{v_1} = \frac{1}{50} \quad \frac{1}{100} \quad \frac{1}{150} \quad \frac{1}{200}$$

The volume at these pressures is thus between 4.4 and 2 per cent smaller than those of the ideal gas, and the differences increase as the temperature is reduced. At the critical temperature, i.e. at  $T_c = 132.6^\circ \text{ C. abs.}$ , we have, with  $p_1 = 1$  atmosphere and

$$\begin{array}{cccc} 50 & 100 & 150 & 200 \text{ atmos} \\ 2,958 & 5,410 & 7,710 & 10,050 \\ 12,650 & 12,650 & 12,650 & 12,650 \end{array}$$

so that

$$\frac{0.234}{50} \quad \frac{0.428}{100} \quad \frac{0.61}{150} \quad \frac{0.795}{200}$$

Hence the true gas volume at the critical temperature and at these pressures is between  $\frac{1}{4}$  and  $\frac{1}{3}$  of that given by Boyle's Law.

These relations can also be shown on the  $pv$  field (Fig. 160). At low temperatures the true isothermal  $ABC$  is at first not so steep as that ( $AD$ ) of the ideal gas, but later becomes steeper. At  $D$  the value of  $pv$  is the same as the initial value at  $A$ .  $ABCD$  lies entirely below the ideal isothermal passing through  $A$ . A still greater, but essentially similar, deviation is shown by the lower isothermal  $EF$ , which lies closer to the

vapour state. In this case, however, the point at which the hyperbola through  $E$  again cuts the curve is not shown, as it lies too far above the field of the figure. The points such as  $C$  on the curve  $ACD$  correspond to the lowest points on the  $pv$  curves in Fig. 159, and the curve obtained by joining these points is called the Boyle curve, since, at these points the slopes are equal to those of rectangular hyperbolae, so that the actual gases exhibit the properties of a perfect gas. The points  $D$  in Fig. 160

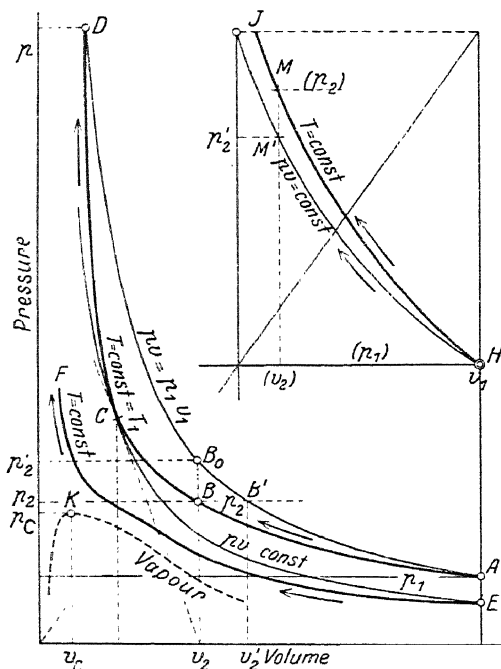


FIG. 160

correspond to the points  $D$  in Fig. 159, and are such that  $pv$  again assumes the same value as for the ideal gas.

The course of the isothermals at high temperatures is represented by the curve  $HJ$  on the  $pv$  field, and is steeper than the rectangular hyperbola through  $H$ .

The conditions shown above for air in the gaseous state apply also to other gases generally, except that the deviations occur at other temperatures. Thus hydrogen shows rising curves in the  $pv$ ,  $p$  diagram even at low temperatures, for which the air curves fall rapidly. On the other hand, methane shows falling curves between  $50^\circ \text{C.}$  and  $100^\circ \text{C.}$ , whereas the air curves rise in this region.

The following representation of the test results offers a clearer conception of the relations between the different gases. The quotient  $\frac{pv}{T} = R'$  has a value which varies with the pressure, when  $T$  is constant, as shown in Fig. 159, whereas for the ideal gas  $\frac{pv}{T} = R_0$  is constant.

The ratio  $\frac{p}{R_0}$  thus serves to indicate the deviation of the actual gas state from the ideal gas state. Hence, if the quotients  $\frac{p'}{T} = R'$ , corresponding to different pressures, are calculated from test results or from the Hausen diagram (Fig. 159), and the ratios  $\frac{R'}{R_0}$  are plotted to a base of pressures  $p$  (Fig. 161), the resulting diagram shows a group of curves passing through the point  $p = 0$  and  $\frac{R'}{R_0} = 1$ . In this diagram the horizontal line through  $\frac{R'}{R_0} = 1$  represents the ideal gaseous state. If

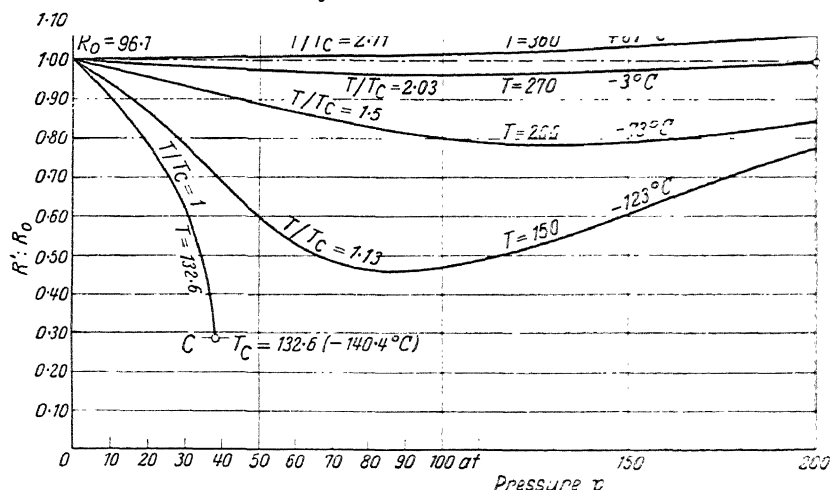


FIG. 161

the curves drop considerably below this line as the pressure is increased, the deviations from the ideal gas state are also considerable, in the sense that the actual gas shows a greater volume compression ratio than the ideal gas for the same pressure ratio (Fig. 160 points  $B$  and  $B'$ ). When, however, the curves lie above this, deviation also occurs, but in the opposite sense (points  $M$  and  $M'$ , Fig. 160). If now these curves be plotted for different gases, groups are obtained having approximately the same shapes and limits as those for air (Fig. 161).<sup>\*</sup> When such curves are available, the deviations of the individual gases from the ideal state and the gas volumes corresponding to given pressures and temperatures can be determined.

Since  $\frac{R_0'}{R_0} =$

we have

$$R_0 p_0 T_0$$

<sup>\*</sup> Curves of this type have been plotted in the publication of the Bureau of Standards, mentioned in the footnote on page 317, for the gases air, argon, helium, hydrogen, methane, neon, nitrogen, and oxygen.

where  $p_0$ ,  $v_0$ , and  $T_0$  refer to any initial state for which the ideal gas laws are applicable. We can, however, also compare any two states with one another by means of the relation

$$\frac{R_2'}{R_1'} = \frac{\rho_2 v_2 T_2}{\rho_1 v_1 T_1}$$

where

$$\frac{R_2'}{R_1'} = \frac{\frac{R_2'}{R_0}}{\frac{R_1'}{R_0}}$$

In this way the relations between the pressure, volume, and temperature are found without the use of a general form of characteristic equation. A diagram of the above type is shown in Fig. 162 for methane ( $\text{CH}_4$ ).

**Van der Waal's general equation of state for gases.** This equation shows the relation between  $p$ ,  $v$ , and  $T$ , and also shows the characteristics of a gas as described above. The equation is usually written as

$$. \quad (407)$$

in which  $a$  and  $b$  are constants for a given gas, while  $R$  is the gas constant.

The form of this equation is explained by the kinetic theory of gases as follows. The ideal characteristic equation can be deduced from the kinetic theory, provided the volume occupied by the gas molecules themselves is small compared with the volume between the molecules, and provided the mutual attraction of the molecules is negligible. If, however, the gas state is changed, so that it becomes relatively much denser either by increasing the pressure or lowering the temperature, these conditions are no longer fulfilled. By writing equation (407) in the form

$$p = \frac{RT}{v} \quad (408)$$

the quantities on the right represent pressures. In the case of an ideal gas,  $p = \frac{RT}{v}$  is caused purely by the reaction of the molecules which strike the wall, and thus depends on their velocity and density. If now the density  $\frac{1}{v}$  is so great that the space between the molecules is of the same order of magnitude as the volume of the molecules, then, for the same molecular velocity (and hence for the same temperature) the reaction (and hence the pressure) is increased in the ratio  $\frac{v}{v-b}$ . Due, however, to the mutual molecular attraction, which tends to reduce the pressure, the second term on the right of equation (408) must be introduced.

A series of isothermals can be drawn on the  $pv$  field by means of the equation, and these show general agreement with test results. A better



conception, however, is obtained if the pressures, volumes, and temperatures are represented as fractions of the critical pressure  $p_c$ , volume  $v_c$ , and temperature  $T_c$  respectively. (See page 396 regarding the critical state point, marked  $K$  in Figs. 159 and 160.)

From the relations

$$b = \frac{1}{3}v_c$$

$$a = 3p_c v_c^2$$

and

$$p = 8 p_c v_c$$

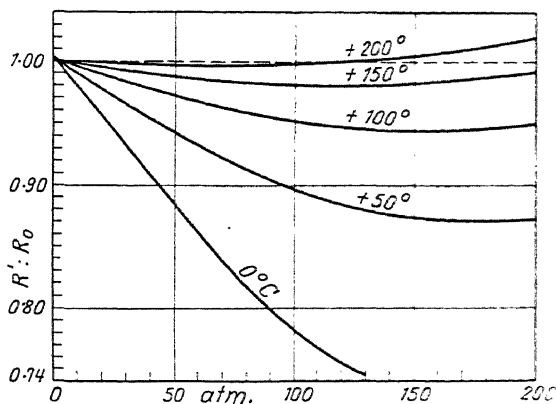


FIG. 162

which follow from equation (407), we have, for the characteristic equation

$$3v' - 1 = 8t' \quad (409)$$

where  $v' = \frac{p}{p_c}$ ,  $v' = \frac{v}{v_c}$  and  $t' = \frac{T}{T_c}$  (410)

If now a field be constructed having the ordinates  $p' = \frac{p}{p_c}$  and the abscissae  $v' = \frac{v}{v_c}$ , a series of isothermal curves which are common to all gases can be drawn. By means of equation (409), Fig. 161 can also be applied to represent conditions which are common to all gases. From equation (407) we have

$$8t' = \frac{3}{v'}$$

This enables the values of  $p'$  corresponding to different values of  $v'$  to be determined for constant values of  $t$ , so that  $\frac{p'v'}{t'}$  is also calculable.

We have, however,

$$\frac{p'v'}{t'} = \frac{pv}{T} \frac{T_c}{p_c v_c}$$

so that 
$$\frac{pv}{T} = \frac{8}{3} \frac{Rp'v'}{t'}$$

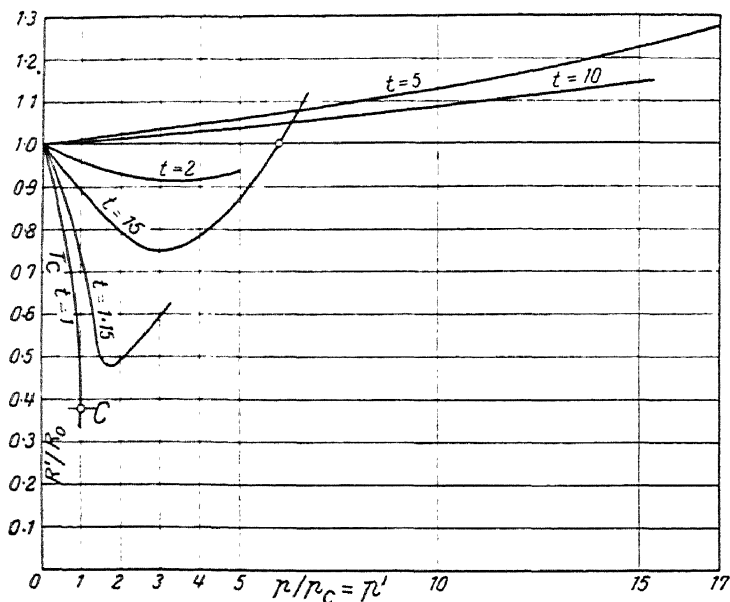


FIG. 163

Hence, by writing

$$\frac{pv}{T} = R'$$

and taking the ideal gas constant  $= R_0$ ,

we have 
$$\frac{R'}{R_0} = \frac{8}{3} \frac{p'v'}{t'}$$

Fig. 163 has been plotted by means of this relation. It will be observed that the shape of the curves and the actual values of  $\frac{R'}{R_0}$  are the same as in Fig. 161, which refers to air. Hence, Van der Waal's equation and the diagram (Fig. 163) derived from it, give an approximate representation of the deviations of a gas state from the ideal gas state. In this way, when the critical values  $T_c$  and  $p_c$  are known, the values of  $\frac{R'}{R}$  for any state of a gas can be found.

## CHAPTER VI

### SPECIAL APPLICATIONS OF THE GAS LAWS

**Working process in reciprocating air compressors.** During the outward stroke of the piston (Fig. 164) air is drawn into the cylinder through the suction valve at approximately atmospheric pressure  $p_0$  (line  $AB$ ). During the return stroke the air is compressed, so that the pressure rises continuously (line  $BC$ ). The discharge valve opens (point  $C$ ) as soon as the pressure in the cylinder becomes equal to, or slightly greater than, the pressure  $p$  in the receiver. From  $C$  the piston discharges the compressed air from the cylinder to the receiver at the constant pressure

$p$ . If the receiver pressure is  $p_1$  (i.e. less than  $p$ ) the discharge valve opens earlier at some point  $C_1$ , while for a receiver pressure  $p_2 > p$  it opens at a later point  $C_2$ . If the receiver contains no compressed air initially, it must be gradually filled, so that the discharge valve opens at a point close to  $B$  at first, and at continuously

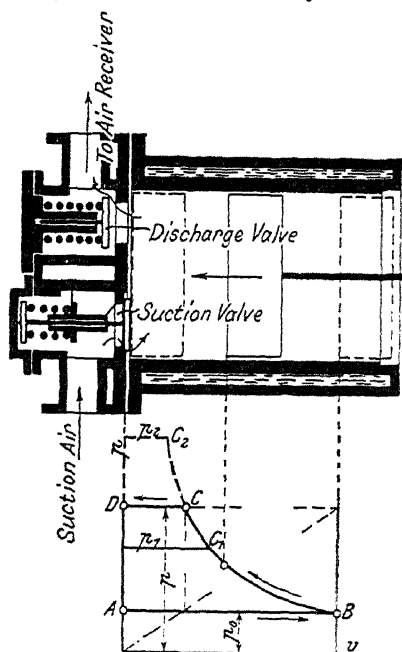


FIG. 164

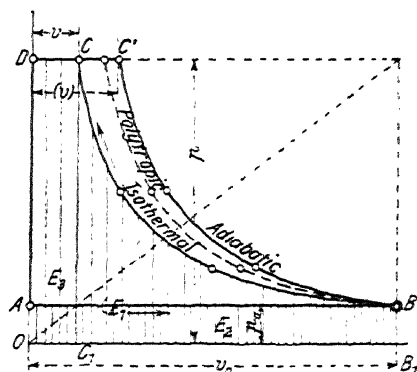


FIG. 165

increasing pressures thereafter. If the pressure in the receiver is to remain constant, the air withdrawn from it in a given time must be the same as the air delivered by the compressor in the same time.

The pressure changes in the cylinder during a working cycle are represented by the closed diagram  $ABCD$ , and the area of this diagram represents the work performed by the piston rod on the air per cycle. The magnitude of this area depends on the compression curve  $BC$ , as well as on the actual compression pressure  $p$ . The following are three special cases—

1. **Adiabatic compression**, i.e. in an uncooled cylinder. The curve  $BC$  then follows the law  $pv^\gamma = C$ , so that the temperature increases between  $B$  and  $C$ .



The isothermal work of compression is the same as the absolute isothermal compression work.

The heat which has to be withdrawn in isothermal compression is  $AE_1$  for 1 lb. of air (see page 105), so that it is also represented by the heat equivalent of the work done. This gives

$$Q = \frac{2.303}{\gamma - 1} p_0 v_0 \log \frac{p}{p_0} \text{ C.H.U.}$$

for 1 lb. of air, or

$$\frac{2.303}{\gamma - 1} p_0 \log \frac{p}{p_0} \text{ C.H.U.}$$

for 1 ft.<sup>3</sup> of air at suction conditions, or

$$Q = \frac{2.303}{\gamma - 1} p \log \frac{p}{p_0} \text{ C.H.U.}$$

for 1 ft.<sup>3</sup> of air at discharge conditions.

(b) *Adiabatic compression.* In this case

$$E_1 = \frac{1}{\gamma - 1} (pv - p_0 v_0) \text{ (See}$$

$$\text{and } E_3 = pv$$

$$\text{so that } E = E_1 + E_3 - E_2 = \frac{\gamma}{\gamma - 1}$$

$$\text{or } E = \gamma E_1$$

i.e. the work done is 1.4 times the absolute compression work. From page 109

$$\left( \frac{p}{p_0} \right)^{\frac{\gamma-1}{\gamma}}$$

so that the work is given by

$$(415)$$

Also, from page 108, we have

$$E = 1400 c_v (T - T_0)$$

or with  $t = T - T_0$  (i.e. the adiabatic rise)

$$E = 1400 \gamma c_v t \text{ or, since } \gamma = \frac{c_p}{c_v}$$

$$E = 1400 c_p t \quad (416)$$

$$\text{Now, since } \frac{T}{T_0} = \left( \frac{p}{p_0} \right)^{\frac{\gamma-1}{\gamma}}$$

$$\text{we have} \quad (417)$$

Equation (415), along with  $pv'^\gamma = p_0v_0'^\gamma$  gives

or per ft.<sup>3</sup> of compressed air at temperature  $T$ ,

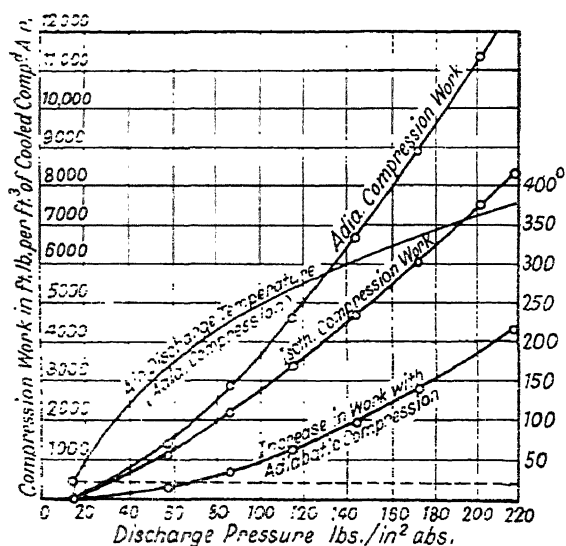


FIG. 166

The work per ft.<sup>3</sup> of air at suction temperature and at the compression pressure  $p$  is

$$\frac{p_0}{p} \left( \frac{p}{p_0} \right)^{\frac{1}{\gamma}} - 1 \quad (418)$$

since the volume of the air is reduced in the ratio  $\frac{T_0}{T}$ , i.e. 1 ft.<sup>3</sup> of air at  $T$  and  $p$ , becomes  $\frac{T_0}{T}$  ft.<sup>3</sup> at  $T_0$  and  $p$ .

Fig. 166 shows the work required to compress 1 ft.<sup>3</sup> of compressed cold air isothermally and adiabatically for pressures ranging from 1 to 15 atmospheres, and for an initial temperature of 20° C. For compression pressures above 5 atmospheres the temperature at the end of compression is above 200° C. This introduces difficulties due to lubrication and sealing at the gland. For a discharge pressure of 10 atmospheres abs., the adiabatic compression work is greater than the isothermal work by 2000 ft. lb./ft.<sup>3</sup> or 1.43 C.H.U./ft.<sup>3</sup>.

(c) *Polytropic compression.* If the compression is polytropic and follows the law  $pv^n = c$ , the work of compression is

$$E = \frac{n}{n-1} (pv - p_0v_0)$$

so that we again have

$$E = nE_1$$

As shown on page 114, we also have

$$E_1 = 1400 (c_v - c) (T - T_0)$$

where  $c = c_v \left( \frac{n-\gamma}{n-1} \right)$

This gives  $E = 1400n \left( \frac{c_p - c_v}{n-1} \right) (T - T_0)$

or, with  $c_p - c_v = \frac{R}{1400}$

$$E = \frac{n}{n-1} R (T - T_0) \quad . \quad . \quad . \quad . \quad . \quad (419)$$

Again, since  $\frac{T}{T_0} = \left( \frac{p}{p_0} \right)^{\frac{n-1}{n}}$

$$E = \frac{n}{n-1} RT_0 \left[ \left( \frac{p}{p_0} \right)^{\frac{n-1}{n}} - 1 \right]$$

and, since  $RT_0 = p_0v_0$

we also have  $E = \frac{n}{n-1} p_0v_0 \left[ \left( \frac{p}{p_0} \right)^{\frac{n-1}{n}} - 1 \right] \quad . \quad . \quad . \quad (420)$

By writing equation (419)

$$E = \frac{n}{n-1} RT \left( 1 - \frac{T_0}{T} \right)$$

we have  $E = \frac{n}{n-1} p v \left[ 1 - \left( \frac{p_0}{p} \right)^{\frac{n-1}{n}} \right] \quad . \quad . \quad . \quad (421)$

where  $p$  and  $v$  refer to the final state.

Equation (421) applies to 1 lb. of air. For any weight  $W$  lb. the work is given by

$$E = \frac{n}{n-1} p V \left[ 1 - \left( \frac{p_0}{p} \right)^{\frac{n-1}{n}} \right]$$

where  $V$  is the volume in ft.<sup>3</sup> of  $W$  lb. measured at the final conditions. Hence per ft.<sup>3</sup> of this air,

$$E = \frac{n}{n-1} p \left[ 1 - \left( \frac{p_0}{p} \right)^{\frac{n-1}{n}} \right] = \frac{n}{n-1} p \left( 1 - \frac{T_0}{T} \right)$$

If this air be cooled to the initial temperature  $T_0$ , we have (as under (b) above),

$$E = \frac{n}{n-1} p \left[ \left( \frac{p_0}{p} \right)^{\frac{n-1}{n}} - 1 \right] = \frac{n}{n-1} p \left( \frac{T}{T_0} - 1 \right) \quad (422)$$

For 1 ft.<sup>3</sup> of air at the suction conditions, we have, from equation (420)

$$E = \frac{n}{n-1} p_0 \left[ \left( \frac{p}{p_0} \right)^{\frac{n-1}{n}} - 1 \right] \quad . \quad . \quad . \quad . \quad (423)$$

The heat withdrawn per lb. is (page 114),

$$Q = c(T - T_0) = c_v \left( \frac{n-\gamma}{n-1} \right) (T - T_0)$$

From equation (419), this is also given by

$$Q = \frac{1}{1400} \frac{n-\gamma}{n(\gamma-1)} E$$

**POWER REQUIRED.** The horse-power required per  $V$  ft.<sup>3</sup> of air per hour is

$$H.P. = \frac{EV}{3600 \times 550} = \frac{EV}{1980000}$$

(In this expression all losses are neglected.)

Thus, per ft.<sup>3</sup> of air per hour the necessary power is, from equation (420),

$$(H.P.)_0 = \frac{n}{n-1} \frac{p_0}{1980000} \left[ \left( \frac{p}{p_0} \right)^{\frac{n-1}{n}} - 1 \right] \quad . \quad . \quad . \quad (425)$$

which applies also to adiabatic compression, when  $n$  is replaced by  $\gamma$ . In the case of isothermal compression,

$$(H.P.)_0 = \frac{2.303 p_0}{1980000} \log \frac{p}{p_0} \quad . \quad . \quad . \quad . \quad (426)$$

With efficient cooling, the mean value of the index of compression ranges from 1.2 to 1.3, and may be even smaller with slow running compressors. Actually  $n$  is not constant, i.e. the actual compression line is not polytropic in the sense used above. The exponent is usually greater at the beginning and smaller at the end than the mean index of compression. This is caused by the different quantities of heat flow due to the different temperature differences at the beginning and end of compression. Fig. 167 shows a typical indicator diagram of an air compressor.

**Effect of Cooling.** As shown in Fig. 165, the work required to compress air is considerably affected by the index of compression, and hence by the amount of cooling during compression. Cooling, however, has an additional advantage. Since isothermal compression is never carried out in practice, the walls of the compressor become heated during the compression period. The air drawn in during the following suction stroke



comes in contact with the warm walls, so that its temperature is raised from  $T_0$  to  $T'_0$ , and the specific volume is increased in the ratio  $\frac{T'_0}{T_0}$ . Now, from equations (411), (415), and (420) the total work is proportional to the product  $p_0 v'_0$ , no matter what the nature of the compression may be. Hence, due to the heating effect during suction, the work required is increased in the ratio  $\frac{T'_0}{T_0}$ . Due to cooling,  $T'_0$  can be reduced considerably below the corresponding adiabatic temperature, and it has been found that the gain in economy due to this is of the same order as that caused by lowering the index of compression, which alone is shown by the indicator diagram. Thus, if the suction air be heated from  $15^\circ \text{C}$ .

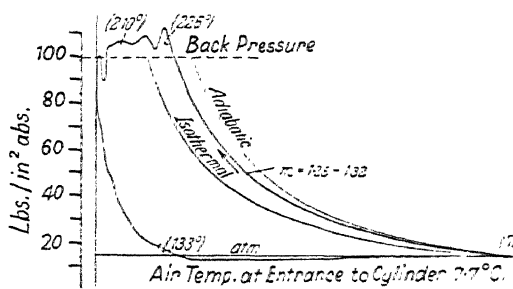


FIG. 167

to  $50^\circ \text{C}$ ., the ratio of the compression work to that with no heating is  $\frac{273 + 50}{273 + 15} = 1.12$ , i.e. an increase of 12 per cent. The effect is still more marked when the compression pressure is increased. Thus at 6 atmospheres pressure (gauge) the final adiabatic temperature is  $220^\circ \text{C}$ ., so that, if no provision is made for cooling, it is possible for the suction air temperature to be raised to  $100^\circ \text{C}$ . or more. The work required would then be increased in the ratio  $\frac{273 + 100}{273 + 15} = 1.3$ .

**Example.** Find the horse power required to compress  $1,000 \text{ ft}^3$  per hour of air (at  $60 \text{ lb./in}^2$  gauge pressure and  $15^\circ \text{C}$ .) from  $15 \text{ lb./in}^2$  abs. to  $60 \text{ lb./in}^2$  gauge, (a) isothermally, (b) adiabatically, (c) polytropically, with  $n = 1.22$ .

(a) From equation (414), we have

$$\begin{aligned} E &= 2.303 p \log \frac{p}{p_0} \\ &= 144 \times 2.303 (60 - 15) \log \frac{60}{15} \\ &= 17430 \text{ ft. lb./ft}^3 \end{aligned}$$

and, from equation (426),

$$(H.P.)_0 = \frac{1000 \times 17430}{1980000} = 8.8 \text{ h.p.}$$

Heat removed per hour is

$$Q = \frac{17430 \times 1000}{1400} = 12450 \text{ C.H.U.}$$

(b) From equation (418)

$$E = 3.5 p \left[ \left( \frac{p}{p_0} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right]$$

we have 
$$\begin{aligned} E &= 3.5 \times 144 \times 75 \left[ \left( \frac{75}{15} \right)^{\frac{0.4}{1.4}} - 1 \right] \\ &= 3.5 \times 144 \times 75 \times 0.585 \\ &= 22200 \text{ ft. lb. ft.}^3 \end{aligned}$$

and, from equation (425),

$$(H.P.)_0 = \frac{22200 \times 1000}{1980000} = 11.2 \text{ h.p.}$$

(c) From equation (422)

$$\begin{aligned} E &= \frac{n}{n-1} p \left[ \left( \frac{p}{p_0} \right)^{\frac{n-1}{n}} - 1 \right] \\ &= \frac{1.22 \times 144 \times 75}{0.22} \left[ \left( \frac{75}{15} \right)^{\frac{0.22}{1.22}} - 1 \right] \\ &= \frac{1.22 \times 144 \times 75 \times 0.34}{0.22} = 20400 \end{aligned}$$

and 
$$(H.P.)_0 = \frac{20400 \times 1000}{1980000} = 10.3 \text{ h.p.}$$

The temperature of the air at discharge from the compressor is

$$T = T_0 \left( \frac{75}{15} \right)^{\frac{0.22}{1.22}} = 1.34 T_0$$

so that, assuming  $t_0 = 20^\circ \text{C.}$ , we have

$$T = 1.34 \times 293 = 392^\circ \text{C. abs.} = 119^\circ \text{C.}$$

From equation (424) the heat removed per hour is

$$\frac{1}{1400} \times \frac{1.22 - 1.4}{1.22 \times 0.4} \times 20400 \times 1000 = -5380 \text{ C.H.U.}$$

If the rise in temperature of the cooling water is  $10^\circ \text{C.}$  the amount of water required per hour is

$$w = \frac{5380}{10} = 538 \text{ lb.}$$

Détermination of the work required to drive existing air compressors from the temperature rise of the air and cooling water.

The energy equation, when applied to compression accompanied by cooling, gives the absolute compression work as

for any compression law. The necessary work per lb. of air, however, is expressed by

$$E = E_1 + pv - p_0v_0$$

so that 
$$E = \frac{Q}{A} + \frac{c_v}{A} (T - T_0) + pv - p_0v_0$$

Now, since  $p_0v_0 = RT_0$  and  $pv = RT$

this gives 
$$E = \frac{Q}{A} + \left( \frac{c_v}{A} + R \right) (T - T_0)$$

or, since 
$$c_p = c_v + AR$$
$$E = 1400 Q + 1400 c_p (T - T_0)$$

For a given weight  $W$ , therefore,

$$E_w = 1400 QW + 1400 c_p W(T - T_0)$$

Hence, in order to find  $E_w$ , the following values have to be determined at the compressor.

1. The weight  $w$  and rise in temperature  $t$  of the cooling water used in a given time.

This gives  $QW = wt_w$

2. The supply and discharge air temperatures  $t_0$  and  $t$  respectively.

This gives  $T - T_0 = t - t_0$

3. The volume of the air supplied or discharged, i.e.  $V_0$  or  $V$  respectively.

This gives 
$$W = \frac{p_0 V_0}{R(273 + t_0)} = \frac{pV}{R(273 + t)}$$

where  $p_0$  and  $p$  are the respective absolute pressures in lb. ft.<sup>2</sup>.

The horse-power is now found from  $E_w$  by dividing it by 1,980,000.

The third quantity above is usually difficult to measure under ordinary conditions, so that, for reciprocating compressors, it is better to determine the power by means of an indicator. This method is, however, of advantage in the case of fans and blowers, since the indicator cannot then be used. (See *Z.V.d.I.* (1907), page 1669.) The measurement of the discharge gas temperature is, however, difficult.

**Two stage compression.** (Compound compressor.) An approach to isothermal compression, which depends on the efficiency of cooling of the cylinder, is rendered more difficult as the discharge pressure is increased. Hence two stages or more are employed when the discharge pressure is high. In the first cylinder, the air is compressed to some intermediate pressure and then discharged to an intercooler, where it

is cooled to about atmospheric temperature. The air then passes to the second stage, where it is compressed to the final pressure.

The  $p_v$  diagram in the L.P. cylinder, with adiabatic compression  $BC$ , is represented by  $ABCD$  in Fig. 168. The heated compressed air at pressure  $p_1$  and volume  $CD$  is cooled in the intercooler to the temperature at  $B$ , so that the volume is reduced to  $DE$  (curve  $BE$  is an isothermal). This is the volume of air drawn in at the H.P. cylinder (line  $DE$ ) and is compressed adiabatically as shown by  $EF$ . If the whole of the compression had been carried out

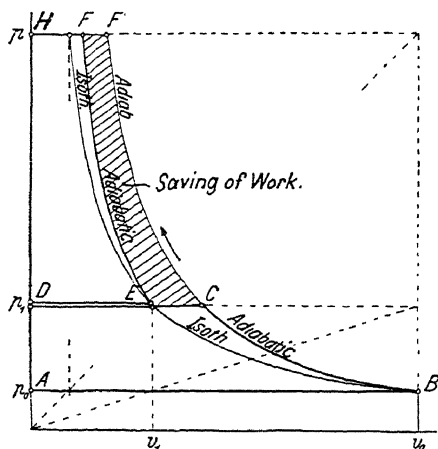


FIG 168

adiabatically in a single cylinder (i.e. in the L.P. cylinder) the work would have been represented by  $ABF'H$ , so that the saving in work is given by the area  $CF'FE$ . With polytropic compression the saving is still greater, since the compression curves would then lie to the left of  $BC$  and  $EF$ .

Here the saving in work is again greater than appears from the diagram, since the cooled suction air affects the necessary compression work as described above. This reduction of work is more marked in two stage compression than in single stage compression, since the air tem-

peratures for the same polytropic are considerably lower in the two stage cylinders. For pressures above about 100 lb./in.<sup>2</sup>, two or more stages are generally used.

The discharge pressure of the L.P. cylinder is such that the work done in the two stages is about equal, while for a preliminary calculation it is sufficient to assume isothermal compression. The L.P. work is then

$$p_0 v_0 \log_e \frac{p_1}{p_0} \text{ and the total work } p_0 v_0 \log_e \frac{p}{p_0}.$$

$$\text{This gives } p_0 v_0 \log_e \frac{p_1}{p_0} = \frac{1}{2} p_0 v_0 \log_e \frac{p}{p_0}$$

so that

$$p_1$$

The ratio of the cylinder volumes is the same as the pressure ratio,

so that

$$v_1 \quad p_0$$

If, for example, air is to be compressed to 170 lb./in.<sup>2</sup> abs. from a pressure of 15 lb./in.<sup>2</sup> abs., the pressure in the intermediate receiver should be

$$p_1 = 15 \sqrt[4]{\frac{170}{15}} = 15 \times 3.36 = 50 \text{ lb./in.}^2 \text{ abs.}$$



since the temperature of the air entering the air motor is assumed to be the same as that entering the compressor.

The difference in the areas of the two diagrams depends on the type of compression and expansion curves. This difference is greatest when both curves are adiabatics and is lessened when, as always occurs, both are polytropics. The area  $BCEF$  thus represents the greatest possible loss.

The efficiency of this type of power transmission is increased if the air is preheated before entering the motor. The point  $E$  then lies further to the right, so that the indicator diagram of the air motor is increased. This improvement, however, is only effected by an additional supply of

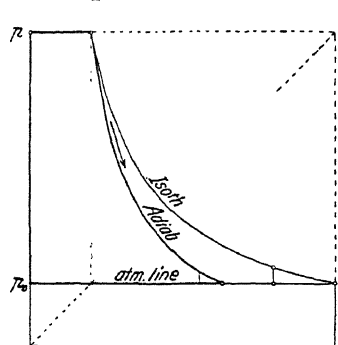


FIG. 170

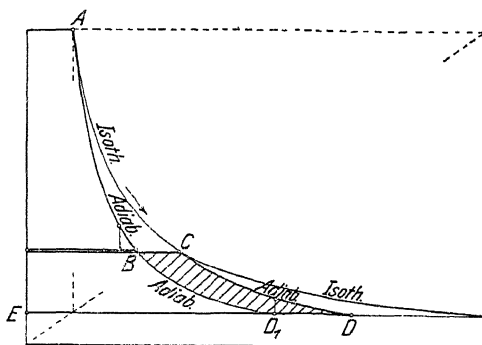


FIG. 171

heat, and the cost of this supply decides whether or not it is worth while preheating the air. It must be remembered, however, that a marked improvement in the running of the motor accompanies the preheating, since it avoids difficulties due to freezing of the moisture in the valve passages. Compression of the residual air on the return stroke has recently been adopted in order to increase the temperature of the walls.

Motors which operate non-expansively, or with only partial expansion, do not suffer from the effects of cooling, but the loss of available work is then considerable and represented by the area  $BCEG$  (when no preheating occurs). With preheating the extra work  $EE'G'G$  is gained, where

$t_0$  and  $t'$  are the temperatures before and after preheating. The proportional gain is thus

$$\frac{273 + t'}{273 + t_0}$$

The ideal cycle of the air motor has isothermal expansion, in which the air temperature is the same as that of the surroundings. This is effected, in theory at least, by supplying the jacket with heating water in place of the cooling water used in compressors, and allowing the piston to move very slowly. The work thus delivered is greater than the adiabatic work (Fig. 170).

By expanding the air in two stages (Fig. 171) a closer approach to

isothermal expansion is obtained, so that the work delivered is increased. In the first cylinder the pressure drops adiabatically, say, from  $p$  to  $p_1$ , as shown by the curve  $AB$ . The air, of volume corresponding to the

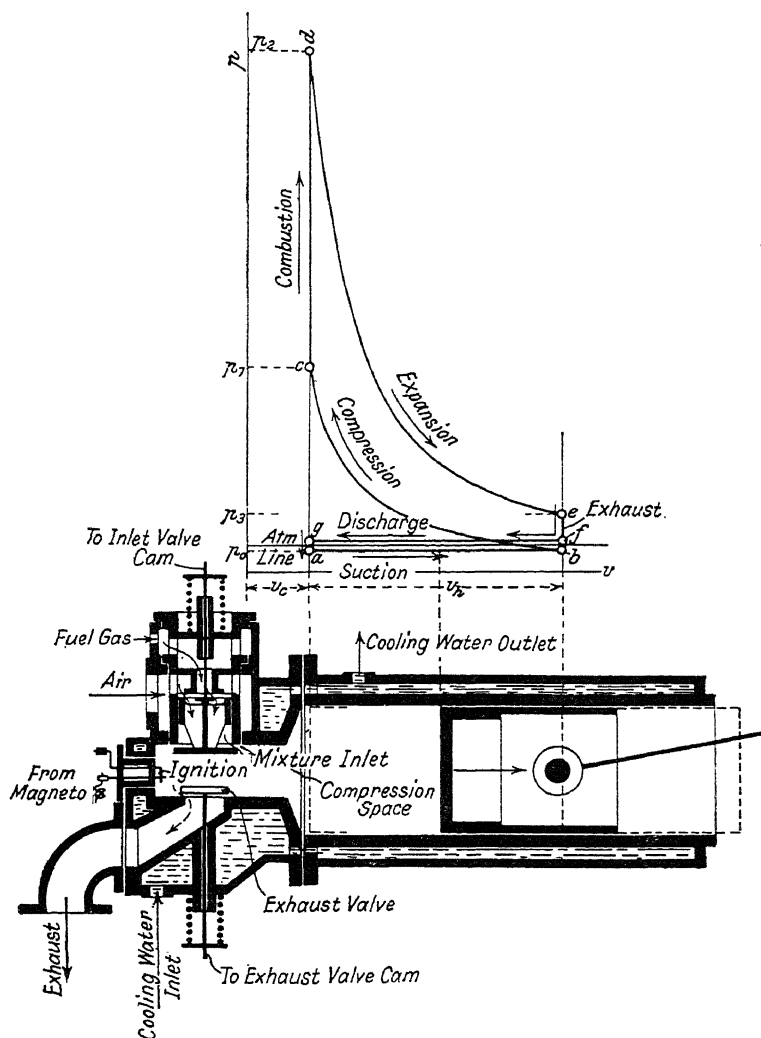


FIG. 172

point  $B$ , is removed to an interheater, where its temperature is raised to that of the surroundings. On entering the second cylinder, the volume is now increased, so that the state point is at  $C$  and expansion occurs from  $C$  to the outlet pressure. The shaded area below  $BC$  represents the gain due to interheating. This type of compound expansion has been applied successfully to locomotives in America.

As in the case of compressors, the actual gain due to two stage

expansion is greater than that indicated by the diagram, since the cooling effect of the walls is reduced on account of compounding.

The expressions for the work done by air motors with expansion down to the back pressure are the same as those already deduced for the air compressors (page 329).

### INTERNAL COMBUSTION ENGINES OPERATING ON THE OTTO CYCLE

Consider a mixture of air and gas (town gas, power gas, or petrol vapour, etc.) which has been drawn into the cylinder of a single or double acting engine (line  $ab$ , Fig. 172). After the inlet valve closes at the dead centre position, the mixture is compressed by the return movement of the piston (line  $bc$ ). The highest compression pressure  $p_1$  is attained at the end of this stroke. The

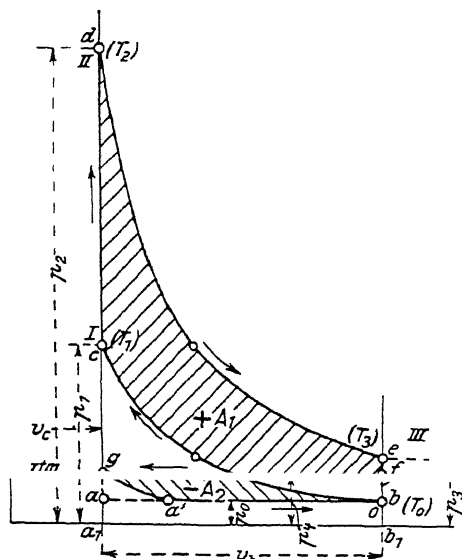


FIG. 173

pressure ratio  $\frac{p_2}{p_1}$  depends on the ratio of the sum of the clearance and stroke volumes ( $V_c + V_s$ ) to the clearance volume ( $V_c$ ). At or near the inner dead centre position the mixture is ignited and the pressure rises almost instantaneously (i.e. at constant volume) from  $p_1$  to  $p_2$  (line  $cd$ ). The value of  $p_2$  depends on the calorific value of the mixture and on the state  $p_1 T_1$  (see page 81). The high pressure glowing gases now expand and drive the piston outwards so that the pressure drops from  $p_2$  to  $p_3$ . The exhaust valve opens shortly before the end of this stroke, so that the products of combustion are expelled

from the cylinder and the pressure drops rapidly from  $p_4$  to  $p_0$  (line  $ef$ ). On the return stroke still more of the products are discharged from the cylinder at approximately atmospheric pressure (line  $fg$ ). The volume  $v_c$  of products remains, however, in the clearance space and mixes with the fresh incoming charge during the succeeding suction stroke. A working cycle thus requires four strokes, and an engine operating on this cycle is known as a "four stroke" engine.

**Two stroke engine.** The whole process can also be carried out in two strokes. In this case, however, the mixture must be delivered to the cylinder (by special air and gas pumps) near the outer dead centre position. Immediately before this, the waste products have to be removed by scavenging with air.

Although the constructional details in both two and four stroke



engines vary greatly, the following treatment applies to all engines operating on the Otto cycle.

The useful work transmitted by the piston is found as follows (Fig. 173). During compression ( $bc$ ) and expansion ( $de$ ) the work delivered to the piston rod is given by the area  $bcde = A_1$ , while during suction and exhaust the work done by the piston rod is given by the area  $abfg = A_2$ . The useful work is given by the difference of these two areas. In using the planimeter it is only necessary to follow the lines  $cdefga'bc$  in Fig. 173 to obtain the net work, as this gives  $+A_1 - A_2$ .

The useful work is now

$$E = a_1deb_1 - a_1cbb_1 - a_1gfb_1 +$$

With reference to the last area it should be noted that the suction pressure does not drop instantaneously from  $g$  to  $a$ , but gradually, as shown by  $ga'$ , due to the throttling of the air as it is drawn through the suction valve. In the theoretical cycle, however, this effect can be neglected.

It is unusual to find such a low suction pressure as that shown in Fig. 173 on full loads. With quantity governing at light loads, however, diagrams similar to that shown in Fig. 59 are obtained.

In the ideal diagram the expansion and compression curves are assumed to be adiabatics. Even under actual conditions and with water cooling, which is necessary for effective lubrication and to prevent pre-ignition, the curves do not greatly deviate from true adiabatics (see Fig. 58).

On the assumption of constant specific heat, we have, from page 106,

$$\text{Area } a_1deb_1 = 1400 c_v(T_2 - T_3)$$

where  $T_2$  and  $T_3$  are the absolute temperatures at  $d$  and  $e$  respectively.

Again, the area  $a_1cbb_1 = 1400 c_v(T_1 - T_0)$ , so that the useful work per pound of mixture during expansion and compression is

$$E = 1400 \{c_v(T_2 - T_3) - c_v(T_1 - T_0)\} \quad (427)$$

At normal loads, the negative work is relatively small, and as it represents a throttling loss it is omitted in considering the ideal case. Equation (427) therefore gives the ideal work of the cycle.

The heat liberated in the combustion along  $cd$  is

$$Q_v = c_v(T_2 - T_1)$$

and the mechanical equivalent of this is

$$E_0 = 1400 c_v(T_2 - T_1) \quad (428)$$

This represents the heat supplied in the process, and the highest possible fraction of it should be converted to work. This fraction, called the thermal efficiency of the ideal process, is

$$\eta_{th} = \frac{E}{E_0}$$

or, from equations (427) and (428),

$$\eta_{th} = \frac{E}{E_0} = \frac{c_v(T_2 - T_3) - c_v(T_1 - T_0)}{c_v(T_2 - T_1)} \quad (429)$$



The value of the mean molecular specific heat with a moderate amount of excess air is (from Fig. 23) about

$$C_{vm} = 5 + 0.5 \frac{t}{1000}$$

These give, for the above ratio,

$$5 + \frac{0.5t_2}{1000} \left) t_2 - \left( 5 + \frac{0.5t_1}{1000} \right) t_1 = \frac{1}{1 + \frac{t_1 + t_2}{10000}}$$

so that the improved expression for the thermal efficiency is

$$\frac{1}{1 + \frac{t_1 + t_2}{10000}} \left( 1 - \frac{1}{r_c^{\gamma-1}} \right) \quad . \quad . \quad . \quad . \quad (433)$$

With the compression ratios

$$r_c = \quad 3 \qquad 5 \qquad 7 \qquad 9$$

we have, with  $t_0 = 50^\circ \text{C.}$ ,

$$t_1 = \quad 225 \qquad 330 \qquad 415 \qquad 480$$

and, assuming a combustion temperature  $1500^\circ \text{C.}$  above the compression temperature,

$$t_2 = \quad 1725 \qquad 1830 \qquad 1915 \qquad 1980$$

The correction factors are, from these,

$$\frac{1}{1.19} \qquad \frac{1}{1.22} \qquad \frac{1}{1.23} \qquad \frac{1}{1.24}$$

The thermal efficiencies as given by equation (431) are reduced in these ratios and plotted in Fig. 181, where the resulting curves are marked "2nd approximation."

**Exact determination.** The exact determination of the thermal efficiency is obtained as follows. Denoting the internal energies at the four corners of the diagram by  $I_0$ ,  $I_1$ ,  $I_2$ , and  $I_3$ , we have

$$\text{the absolute expansion work} = I_2 - I_3$$

$$\text{the absolute compression work} = I_1 - I_0$$

$$\text{and the heat supplied} = Q = I_2 - I_1$$

$$\text{so that} \quad \eta_{\text{th}} = \frac{(I_2 - I_3) - (I_1 - I_0)}{I_2 - I_1}$$

$$= 1 - \frac{I_3 - I_0}{I_2 - I_1}$$

$I_3 - I_0$  is the heat remaining in the products at the end of expansion,

$$\text{that} \quad \frac{I_3 - I_0}{I_2 - I_1} = \frac{I_3 - I_0}{Q}$$

represents the fractional exhaust heat loss.

This value is easily obtained from Chart I for products. Fig. 174 illustrates the method of obtaining the values with  $Q = 28 \text{ C.H.U./ft.}^3$  at S.T.P.

For compression ratios of

$$r_c = 3 \quad 5 \quad 7 \quad 9$$

we have

$$\eta_{th} = 0.278 \quad 0.364 \quad 0.424 \quad 0.458$$

These values are also plotted in Fig. 181. It will be seen that the second approximation values show good agreement with these.

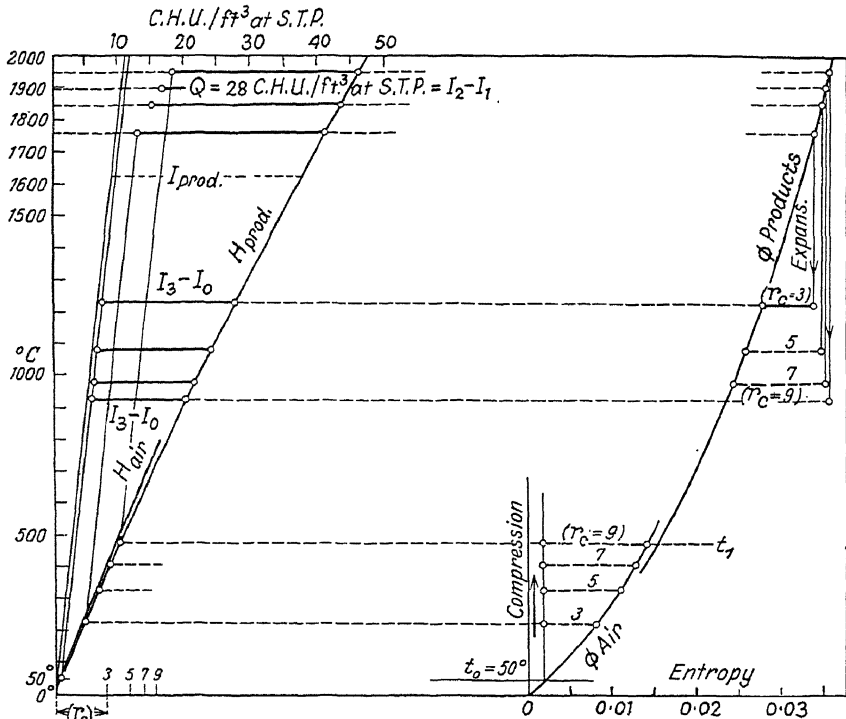


FIG. 174

Fig. 175 shows a theoretical diagram for  $r_c = 6.84$  and a heat supply of  $40.7 \text{ C.H.U./ft.}^3$  at S.T.P., which is considerably greater than in the above example. The diagram has been constructed with the aid of the energy chart and gives  $\eta_{th} = 0.45$ .

The actual thermal efficiencies, as obtained from indicator diagrams, are considerably smaller than the above values on account of heat and pressure losses in the engine, particularly during combustion and expansion, and may be about 75 to 80 per cent of the ideal values. For high compression ratios this gives thermal efficiencies between 0.30 and 0.33.

The fuel consumption  $C$  per h.p. hour is obtained from the calorific value  $H$  of the fuel and from the thermal efficiency as follows. With complete conversion of the heat to work, 1 lb. of the fuel would deliver



since such oils do not form a suitable air-fuel mixture at normal suction temperature. For these fuels, therefore, a different process, due to Rudolf Diesel, is adopted.

Diesel engines generally operate on four strokes, although for large powers two-stroke cycles are also employed, the practical difficulties

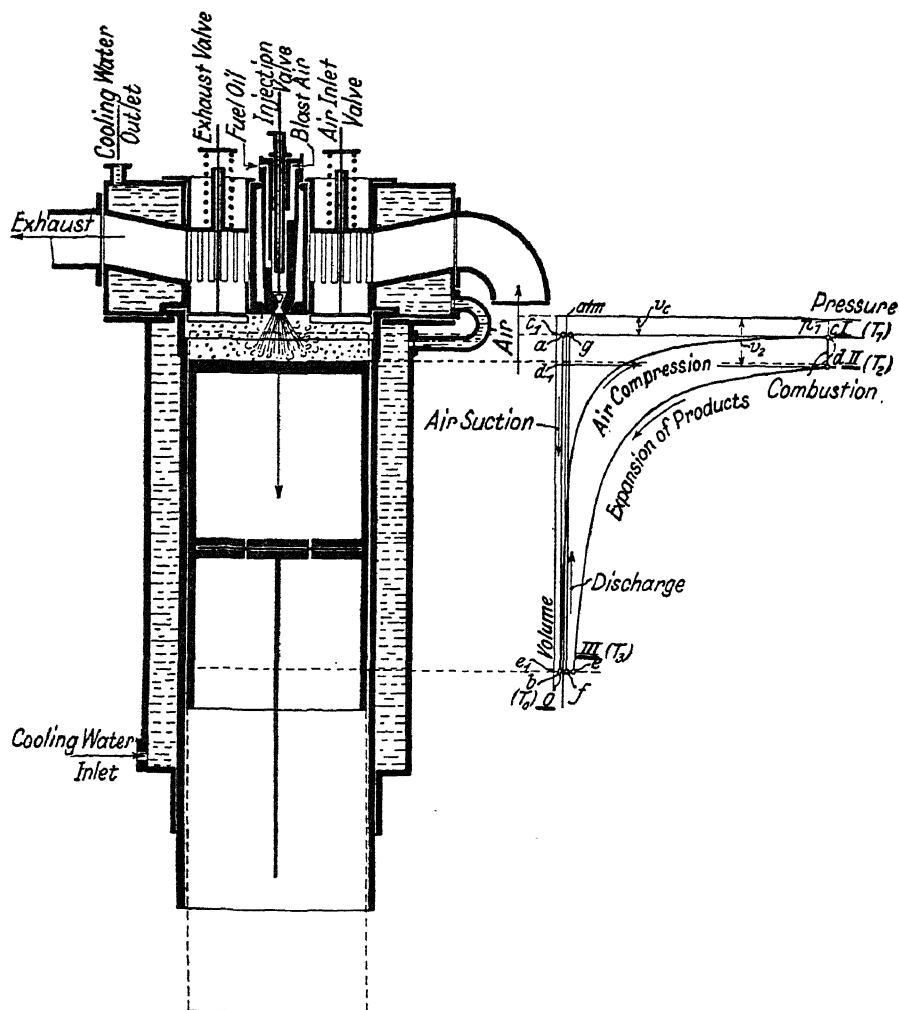


FIG. 176

being less than in two stroke gas engines. The essential difference between the Diesel and Otto cycles consists in the suction and compression strokes. During suction, in the Diesel engine, air and not an explosive mixture is drawn into the cylinder. On the return stroke this air is compressed to some pressure between 450 and 600 lb./in.<sup>2</sup> (which is considerably in excess of the compression pressure of gas engines), so that the temperature of the air is considerably increased (see page 111). At or near

the commencement of the third stroke, i.e. on attaining the maximum compression pressure, fuel oil is injected in the form of a fine spray, either with a small amount of compressed air at 750 to 900 lb./in.<sup>2</sup> pressure, or alone. In the former case an air compressor is required, in addition to the oil pump, in order to supply the blast air, while in the latter case this is dispensed with (solid injection engine).

The oil ignites as soon as it comes in contact with the glowing air, so that the temperature is increased. This increase depends on the combustion heat and the course of the pressure change. The rate of injection is arranged so that the pressure behind the outwards moving piston either remains constant during combustion or at least does not increase as rapidly as in explosion processes.\*

At the end of injection and combustion the products of combustion expand to about the end of the third stroke and are then released.

The indicator diagram of Fig. 176 is developed as follows. Suction of air occurs between  $a$  and  $b$  at a pressure slightly below atmospheric, while the piston moves from the top to the bottom dead centre position. This is followed by compression from  $b$  to  $c$ , so that the pressure rises to between 450 and 600 lb./in.<sup>2</sup> and the temperature to about 700° C., while the piston moves from the bottom to the top dead centre position. Injection and combustion of the oil commences at  $c$  and ends at  $d$ . The line  $cd$  is horizontal in the ideal cycle, but in the actual cycle there is first a pressure rise followed by a gradual merging with the expansion line. The hot products expand from  $d$  to  $e$ , while the piston drops to the lower dead centre position. Exhaust commences at  $e$  (in the actual engine it commences earlier) and  $ef$  shows the rapid drop of pressure caused by releasing the high pressure products. The cylinder contents are exhausted along  $fg$  as the piston moves upwards.

The useful work delivered to the piston can be taken as equal to the area  $b c d e$ , since the pressure during suction is nearly atmospheric. This area is made up of the constant pressure area  $a_1 = c_1 c d d_1$ , the expansion area  $a_2 = d_1 d e e_1$ , and the compression area  $a_3 = b c c_1 e_1$ , such that

$$A = a_1 + a_2 - a_3$$

Considering these separately, we have

$$a_1 = p_1(v_2 - v_c)$$

$$\text{or, with } p_1 v_2 = RT_2 \quad p_1 v_c = RT_1$$

$$a_1 = R(T_2 - T_1)$$

which applies to 1 lb. of the cylinder contents.

Again, from page 106,

$$a_2 = 1400 c_v(T_2 - T_3)$$

$$\text{and } a_3 = 1400 c_v(T_1 - T_0)$$

where  $c_v$  is taken provisionally as constant.

\* The pressure curve very largely depends on the air blast pressure and method of opening the fuel valve. The indicator diagram in Fig. 60, which shows an abnormal combustion curve, gives first a rapid pressure drop followed by approximately constant pressure burning. Probably the air injection pressure was too low or the opening of the injection valve too late. Fig. 177 shows normal diagrams for various loads, as given by K. Kutzbach in *Z.V.d.I.* (1907), page 521.

We then have

$$E = R(T_2 - T_1) + 1400\{c_v(T_2 - T_3) + c_p(T_3 - T_0)\}$$

In this expression  $T_1$  is the temperature at the end of adiabatic compression and beginning of combustion,  $T_2$  is the temperature at the end of combustion, and  $T_3$  is the temperature at the end of adiabatic expansion.

The temperature rise during combustion ( $cd$ ) depends on the heat liberated by the combustion of the fuel oil with 1 lb. of air. Now, from page 24, the amount of air theoretically required for the combustion of 1 lb. of oil is 14.5 lb., while the actual air is at least 25 per cent in excess of this, giving a minimum weight of  $14.5 \times 1.25 = 18.2$  lb. of air. Hence, in practice, no more than  $\frac{1}{18.2}$  lb. of oil can be used per pound of air. This weight of oil liberates about

$$= 605 \text{ C.H.U. of heat} = Q_p$$

Hence, this is the amount of heat liberated at constant pressure. (With more excess air,  $Q_p$  is reduced.)

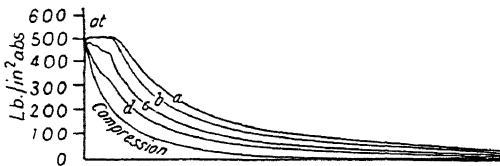


FIG. 177

The temperature rise  $T_2 - T_1$  is now given by

$$Q_p = c_p(T_2 - T_1)$$

if  $c_p$  is first assumed to be the same for products and air.

The absolute work equivalent to this heat is  $E_0 =$

$1400 Q_p$  ft. lb., and the thermal efficiency of the Diesel cycle is

$$\eta_{th} = \frac{AE}{Q_p}$$

Substituting the above values of  $E$  and  $Q_p$  gives

$$\begin{aligned} \eta_{th} &= \frac{\frac{R}{1400} (T_2 - T_1) + c_v [T_2 - T_3 - (T_1 - T_0)]}{c_p(T_2 - T_1)} \\ &= \frac{R}{1400c_p} + \frac{1}{\gamma} \frac{(T_2 - T_3) - (T_1 - T_0)}{T_2 - T_1} \end{aligned}$$

or, since  $\frac{R}{1400c_p} = \frac{\gamma - 1}{\gamma}$  (see page 100)

$$\eta_{th} = 1 - \frac{1}{\gamma} \frac{T_3 - T_0}{T_2 - T_1}$$

Denoting the compression ratio by  $r_c = \frac{V_s + V_c}{V_c}$  and the ratio of the greatest to the least combustion volumes by  $\rho$ , we have, for the adiabatic state changes (page 108),

$$\frac{T_1}{T_0} = r_c^{\gamma-1} \text{ and } \frac{T_2}{T_3} = \left(\frac{r_c}{\rho}\right)^{\gamma-1}. \quad (435) \text{ and } (436)$$





With  $t_0 = 50^\circ \text{C.}$ ,  $\gamma = 1.38$ , and

$r_c =$	8	10	12	15
we now have				
$t_1 =$	438	503	556	630 ° C.
and, with a temperature rise of $1200^\circ \text{C.}$ during combustion,				
$t_2 =$	1638	1703	1756	1830 ° C.
$\rho = \frac{T_2}{T_1} =$	2.64	2.54	2.45	2.33
$\eta_{th} =$	0.434	0.490	0.523	0.571
(from equation (438).)				

These values are plotted in Fig. 181 as a first approximation. The correction factors are

$\frac{1}{1.13}$	$\frac{1}{1.14}$	$\frac{1}{1.15}$	$\frac{1}{1.17}$
------------------	------------------	------------------	------------------

so that the second approximation values are

$\eta_{th} =$	0.381	0.436	0.453	0.490
---------------	-------	-------	-------	-------

These are also plotted in Fig. 181.

**Exact determination.** The exact value of  $\eta_{th}$  is found as follows. Let  $I_0$ ,  $I_1$ ,  $I_2$ , and  $I_3$  be the internal energy values of the air charge and products at the four corners of the diagram Fig. 176. With these

the absolute expansion work  $= I_2 - I_3$

the absolute full pressure work during combustion  $= Ap_1(v_2 - v_c)$

the absolute compression work  $= I_1 - I_0$

and the heat supplied

$$Q_p = H_2 - H_1 = I_2 - I_1 + Ap_1(v_2 - v_c)$$

This gives the thermal efficiency as

$$\eta_{th} = \frac{I_2 - I_3 + Ap_1(v_2 - v_c) - I_1 + I_0}{I_2 - I_1 + Ap_1(v_2 - v_c)} = 1 - \frac{I_3 - I_0}{Q_p}$$

$I_3 - I_0$  is the internal energy remaining in the exhaust gases.

The values of  $I_3 - I_0$  are now found from Fig. 178 with a heat supply of  $Q_p = 28 \text{ C.H.U./ft.}^3$  at S.T.P.

For

$r_c =$	10	12	15
$I_3 - I_0 =$	16.6	16.0	15.0

so that

$\eta_{th} =$	0.410	0.430	0.465
---------------	-------	-------	-------

These are considerably less than the values given by equation (438), but agree well with the second approximation values as given by Fig. 181.

A theoretical diagram, drawn to scale, is shown in Fig. 179. The actual thermal efficiencies, as obtained from indicator diagrams, are less than the above theoretical values, due to heat and pressure losses. The theoretical values represent an upper attainable limit. With an indicated efficiency ratio of 80 per cent the attainable indicated thermal efficiency

is about  $0.482 \times 0.8 = 0.386$ , and with a mechanical efficiency of 0.88 the brake thermal efficiency is  $0.88 \times 0.386 = 0.34$ .

As shown in Fig. 181, the Diesel engine performance is superior to that of the gas engine. This, however, is due solely to the much higher compression ratios adopted in the Diesel. In order to have  $\eta_{th} = 0.45$ , the gas engine must operate with  $r_c = 8.5$  and the Diesel with  $r_c = 12$ .

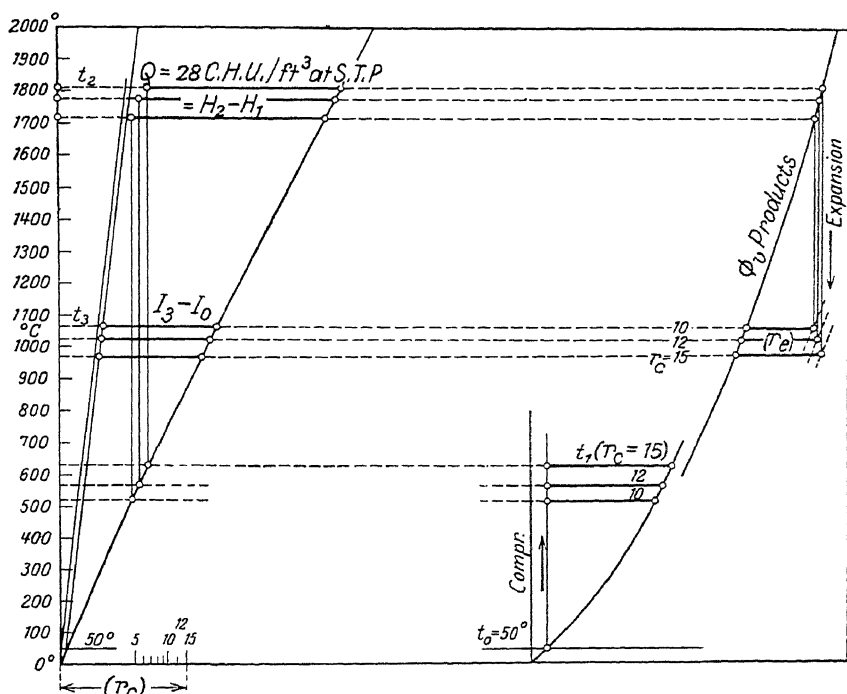


FIG. 178

A Diesel engine with  $r_c = 15$  gives  $\eta_{th} = 0.48$ , but the compression ratio of the gas engine, to give this efficiency, is impracticable owing to the pre-ignition which would follow during compression.

Diesel engines which operate with combustion partly at constant volume (as in modern solid injection engines) show thermal efficiencies somewhat higher than those in Fig. 181, and are about the same as those for gas engines having the same compression ratio.

### EXPLOSION GAS AND OIL TURBINES

The working cycle is described on page 198. As shown in Fig. 101, the fuel and air are compressed isothermally, either as a mixture or separately,

from the pressure  $p_0$ . The ratio  $\frac{p_0}{p_2} = r_c$  is also the volume compression ratio in the same sense as that used in gas and oil engines. The theoretical thermal efficiency of the process is deduced in the following, in the same way as was done for the gas and oil engines.

The useful work is found from the separate areas in Fig. 101 as follows—

$$\text{Area } p_1CDE = +c_p(T_1 - T_2) \quad (\text{See page 177.})$$

$$\text{Area } p_1CBF = - \dots$$

$$\text{Area } ABFE = -ART_0 \log_e \frac{p_0}{p_2}$$

This gives the useful work, per pound of mixture, as

$$AE = c_p(T_1 - T_2) - Av_1(p_1 - p_0) - ART_0 \log_e \frac{p_0}{p_2} \quad (439)$$

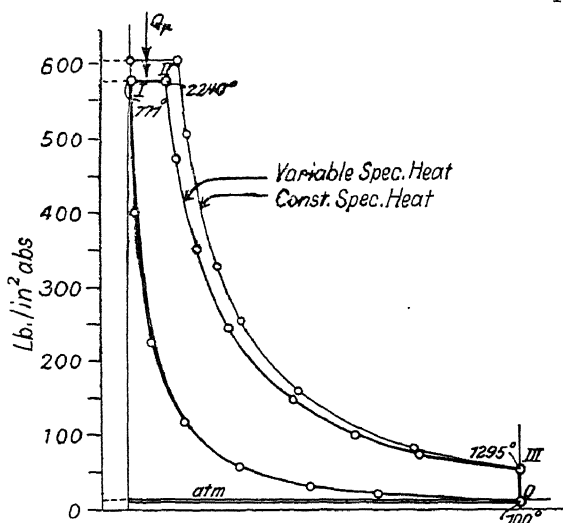


FIG. 179

$$\text{Now with } v_1 = \frac{RT_1}{p_1}, \frac{p_1}{p_0} = \frac{T_1}{T_0} \text{ and } AR = c_p - c_v$$

$$\begin{aligned} \text{we have } AE &= c_p(T_1 - T_2) - (c_p - c_v)(T_1 - T_0) - (c_p - c_v)T_0 \log_e \frac{p_0}{p_2} \\ &= c_v(T_1 - T_0) - c_p(T_2 - T_0) - (c_p - c_v)T_0 \log_e \frac{p_0}{p_2} \end{aligned}$$

The heat supplied during combustion is

$$Q_v = c_v(T_1 - T_0) \quad (440)$$

that the thermal efficiency is

$$\begin{aligned} \eta_{th} &= \frac{c_v(T_1 - T_0) - c_p(T_2 - T_0) - (c_p - c_v)T_0 \log_e \frac{p_0}{p_2}}{c_v(T_1 - T_0)} \\ &= 1 - \frac{c_p(T_2 - T_0) + (c_p - c_v)T_0 \log_e \frac{p_0}{p_2}}{c_v(T_1 - T_0)} \end{aligned}$$

with  $\frac{c_p}{c_v} = \gamma$ , this gives

$$\eta_{th} = 1 - \frac{\gamma \left( \frac{T_2}{T_0} - 1 \right) + (\gamma - 1) \log_e \frac{p_0}{p_2}}{\frac{T_1}{T_0} - 1} \quad (441)$$

With the relations

$$\frac{T_2}{T_0} = \frac{T_2}{T_1} \times \frac{T_1}{T_0}, \quad \frac{T_1}{T_0} = \frac{p_1}{p_0} \text{ and}$$

$$\frac{T_2}{T_1} = \left( \frac{p_2}{p_1} \right)^{\frac{\gamma-1}{\gamma}} = \left( \frac{p_0}{r_c p_1} \right)^{\frac{\gamma-1}{\gamma}}$$

we have 
$$\eta_{th} = 1 - \frac{\gamma \left( \frac{p_1}{p_0} \right)^{\frac{1}{\gamma}} \frac{1}{r_c^{\frac{\gamma-1}{\gamma}}} - 1 + \frac{\gamma-1}{\gamma} 2.303 \log r_c}{\left( \frac{p_1}{p_0} - 1 \right)} \quad (442)$$

The thermal efficiency thus depends on both the compression ratio  $r_c$  and the pressure ratio  $\frac{p_1}{p_0}$ , but chiefly on the former. With  $r_c = 1$ , i.e. with no compression,

$$\eta_{th} = 1 - \gamma \frac{\left( \frac{p_1}{p_0} \right)^{\frac{1}{\gamma}} - 1}{\frac{p_1}{p_0} - 1}$$

Taking the same heat supply as in the gas and oil engine examples, namely 28 C.H.U./ft.<sup>3</sup> at S.T.P., the temperature rise during combustion is about 1600° C., so that with  $t_0 = 50^\circ$  C. or  $T_0 = 323^\circ$  C. abs.,  $T_1 = 1600 + 323 = 1923^\circ$  C. abs., and  $\frac{T_1}{T_0} = \frac{p_1}{p_0} = 5.95$ .

With this value, and compression ratios ranging from 1.5 to 10, equation (442) (with  $\gamma = 1.3$ ) gives the thermal efficiencies plotted in Fig. 181, which are to be regarded as first approximations only.

As shown in the case of the gas and oil engines, these can be improved by multiplying by the correction factor

$$\frac{C_v(t_1 - t_0)}{C_{vm1}t_1 - C_{vm0}t_0}$$

Since  $\gamma = 1.3$ , we have  $C_v = \frac{1.985}{0.3} = 6.62$

and  $C_{vm} = 5 + \frac{0.5}{1000} t$

and the correction factor becomes

$$6.62 \\ 0.5 \quad 0.75 + \frac{v}{13000}$$

Since, in this case,  $\gamma$  is taken as 1.3 instead of 1.4, the correction factor is larger than that required for the gas and oil engine. Its value for the above example is  $\frac{1}{0.88}$ .

The exact determination is found as follows. The first area above is more correctly expressed by  $H_1 - H_2$  (page 176). The other areas remain

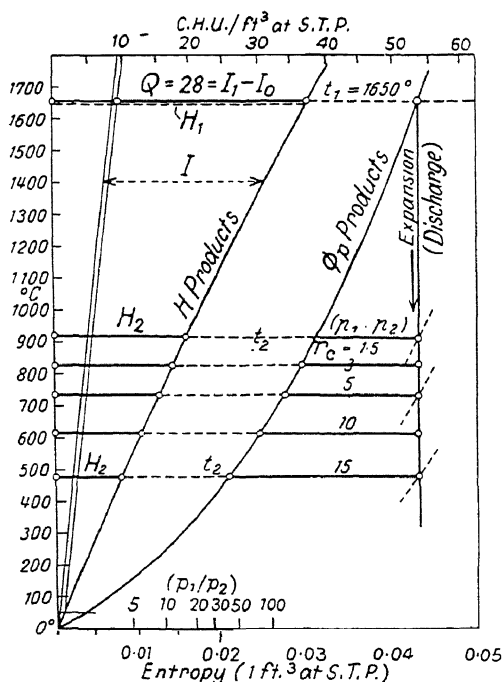


FIG. 180

the same. Expressing all the quantities per ft.<sup>3</sup> at S.T.P., i.e. per  $\frac{m}{354}$  lb. (for which quantity the chart has been drawn), we have, for the useful work,

$$AE = H_1 - H_2 - 0.0056T_1 \left( 1 - \frac{T_0}{T_1} \right) - 0.0129T_0 \log$$

and the thermal efficiency is

$$\frac{AE}{Q}$$

The values are found from Fig. 180 with  $t_0 = 50^\circ \text{C.}$  and  $Q = 28.1$  C.H.U./ft.<sup>3</sup> at S.T.P., and

$$r_c = 1.5 \quad 2 \quad 3 \quad 5 \quad 10 \quad 15$$

which, with  $p_2 = 1$ , gives

$$p_0 = 1.5 \quad 2 \quad 3 \quad 5 \quad 10 \quad 15 \text{ atm. abs.}$$

We also have

$$p_2 \quad p_0 \quad p_2 \quad p_0 \quad v$$

From Fig. 180 the combustion temperature is  $t_1 = 1650^\circ \text{C.}$ , so that

$$\frac{p_1}{p_0} = 5.95, \text{ and}$$

$$\frac{p_1}{p_2} = 8.92 \quad 11.90 \quad 17.85 \quad 29.75 \quad 59.50 \quad 89.25$$

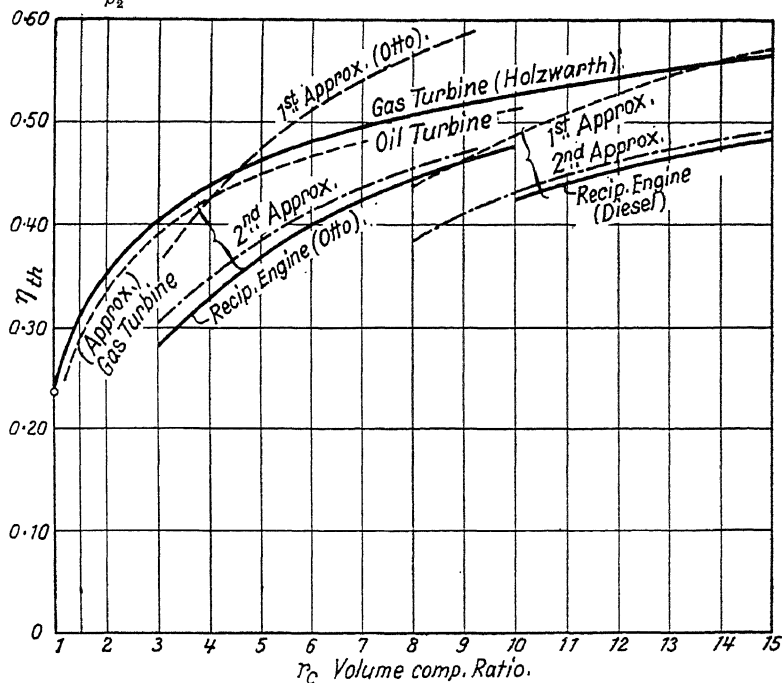


FIG. 181

Using the method explained on page 146, the entropy curve  $\phi_p$ , along with the portable pressure scale, gives the temperature  $t_2$  at the end of expansion, for which the corresponding total heats are

$$H_2 = 19.5 \quad 17.9 \quad 15.6 \quad 13.0 \quad 10.0 \quad 8.3$$

With  $H_1 = 37.8$ ,  $0.0056 (T_1 - T_0) = 8.95$ , and the compression works as

$$0.725 \quad 1.24 \quad 1.97 \quad 2.88 \quad 4.12 \quad 4.85$$

we have, finally,

$$\begin{aligned} EA &= 8.65 & 9.8 & 11.3 & 13.0 & 14.8 & 15.8 \\ \eta_{th} &= 0.31 & 0.35 & 0.40 & 0.46 & 0.53 & 0.56 \end{aligned}$$

These values have also been plotted in Fig. 181. They are considerably in excess of those for the gas and oil engines.

## CHAPTER VII

### STEAM AT NORMAL PRESSURES

#### ✓ SATURATED STEAM

THE properties of steam will be readily followed if its generation from liquid water be considered.

Let water, contained in an open vessel, i.e. at atmospheric pressure, be supplied with heat. The temperature first rises to  $212^{\circ}\text{F.}$ , after which boiling occurs. The pressure of the steam generated is equal to that of the atmosphere ( $14.7\text{ lb./in.}^2\text{ abs.}$ ) and its temperature is the same as the water, i.e.  $212^{\circ}\text{F.}$  However great the heat flow, the temperature does not exceed this.

If the pressure on the surface of the water be more than atmospheric, as when water is contained in a boiler, the temperature at which boiling commences is higher, as is also the steam temperature. The feed water supplied to the boiler must first be raised to this temperature before it is changed to steam. At pressures below  $14.7\text{ lb./in.}^2\text{ abs.}$ , e.g. in open vessels on high hills, or in closed vessels in which a partial vacuum exists, boiling commences at temperatures below  $212^{\circ}\text{F.}$  Thus water at  $68^{\circ}\text{F.}$  boils if the pressure to which it is subjected be reduced to  $0.685\text{ " Hg.}$

The generation and existence of steam at a definite pressure ( $p$ ) corresponds, therefore, to a definite temperature, called the "saturation temperature."\* Conversely, any given steam temperature corresponds to a definite saturation pressure. Steam in the state in which it is developed from water is called saturated, and the discussions of this section deal only with such steam.

The relation between the steam pressure  $p_s$  and steam temperature  $t_s$  in the saturated state is shown in Fig. 185. Here the pressures have been plotted to a base of temperatures. The steam tables at the end also show the corresponding values of  $p_s$  and  $t_s$ .

The latest and probably most accurate tests on these have been carried out at the German Imperial Institute.† Previous to this, Regnault's values were generally adopted, and these are not greatly different from the latest results. Other tests have been carried out by Battelli, by Ramsay and Young, by Caillatet, and also in the Technical Physical Laboratory, Munich (Knoblauch, Linde, and Klebe).‡ See page 395 regarding high pressure steam.

\* Evaporation is a steam generating process in which the free surface of the water alone takes part. It is dependent on the partial pressure of the vapour in the surrounding air and only ceases when this pressure becomes equal to the saturation pressure corresponding to the water temperature (when, therefore, the surrounding air is saturated with water vapour). In open vessels, therefore, evaporation commences long before boiling, due to the low vapour pressure in the air.

† In the region between  $0^{\circ}\text{C.}$  and  $50^{\circ}\text{C.}$  by Scheel and Heuse, between  $50^{\circ}\text{C.}$  and  $200^{\circ}\text{C.}$  by Holborn and Henning, and between  $200^{\circ}\text{C.}$  and  $374^{\circ}\text{C.}$  by Holborn and Baumann.

‡ See, e.g. Chwolson, *Lehrbuch der Physik, Wärme*; Winkelmann, *Handbuch der Physik*; Zeuner *Technische Thermodynamik*. Also F.A., *Z.V.d.I.* No. 21.



A simple general equation which accurately satisfies the test results between the range 32° F. to 705.2° F. (critical) has not yet been established, but there are numerous formulae applicable to a small range which agree satisfactorily with the test results. One such, which is also sufficiently simple for technical purposes, is

$$A \quad B^*$$

The exactitude and limits of this equation can easily be found by plotting  $\log p_s$  against  $\frac{1}{T_s}$  or  $\frac{1}{460 + t_s}$ . The region within which an almost straight line is thus given serves to indicate the limits of the equation. For the three regions between 70° F. and 212° F., between 212 and 400° F., and between 400° F. and 660° F. the following can be applied.

(a) Between 70° F. and 212° F.,

$$\log p_s = 7.1308 - \frac{4003.9}{T_s} \quad (p_s \text{ in lb./in.}^2 \text{ abs.})$$

For very small pressures it is sometimes more convenient to deal with inches of mercury. For this

$$\log p_s = 7.4397 - \frac{4003.9}{T_s} \quad (p_s \text{ in inches Hg})$$

(b) Between 212° F. and 400° F.,

$$= 6.8015 - \frac{3782.0}{T_s}$$

(c) Between 400° F. and 660° F.,

$$\log p_s =$$

An equation which applies to the whole saturated region up to the critical point has lately been established by Callendar.† With  $p$  in mm. Hg, it runs

$$\begin{aligned} \log p = \text{const.} - \frac{2000}{T} - 4.7174 \log T \\ + 0.20956 \log \left\{ \frac{1+z}{1-z} \right\} + 0.001138 \frac{p}{T} \end{aligned}$$

in which the constant on the right is determined for a pressure of 760 mm. and a temperature of 100° C. See page 413 regarding the significance of  $z$ . The equation is theoretically valid but too cumbersome for practical calculations.

\* This is deduced on purely theoretical reasoning by Van der Waal. See also *Z. V.d.I.* (1905). Dieterici: Die kalor Eigenschaften des Wassers und seines Dampfes. In addition to these, there are numerous other formulae, which, however cannot be given here.

† Engineering, 1928., 2 Nov., p. 567. H. L. Callendar, "Steam Tables and Equations."

**Specific volume  $v_s$  and specific weight  $\rho_s$ .** As the pressure of steam is increased its density increases, i.e. the volume of a given weight is reduced. If the pressure, volume, and temperature of saturated steam followed the laws of a perfect gas, then

$$\text{or} \quad p_s v_s = RT_s$$

This equation shows the relations with fair accuracy. By taking the value of  $R$  for gaseous steam, namely 85.85, we have

$$v_s = 85.85 \frac{T_s}{p_s} \quad (p_s \text{ in lb./ft.}^2)$$

If the volume be calculated from this, using corresponding  $T_s$  and  $p_s$  values, the result obtained is always less than that given by test results or the correct value from Clapeyron's equation (page 426). Hence the density of saturated steam is always above that obtained by assuming the steam to behave as a gas, having the molecular weight of water. In the diagram II, at the end, the volume thus calculated is called the ideal gas volume. (Left half of the sheet, the ordinates are temperatures, and the abscissae specific volumes.)

Numerous tests have been carried out to determine the specific volume of saturated steam. An exact direct determination presents special difficulties, since the least withdrawal of heat through the walls is accompanied by condensation of the saturated steam to water.

The latest and most exact tests for pressures up to 11 atmospheres (156 lb./in.<sup>2</sup>) have been carried out in the Munich Technical Physical Laboratory.\*

These values have been used as the basis of the steam tables in the Appendix.

The volume values in Zeuner's older steam tables were calculated from Regnault's tests on the pressure, temperature, and latent heat, using the Clapeyron-Clausius equation. They agree well with the Munich values. Using the same method, Henning, and more recently Jakob (up to 600° F.), found the saturation volumes of steam. They made use of the latent heats determined calorimetrically.

The following equation by Mollier† holds with fair accuracy,

$$v_s = \frac{332.64}{p^{15/16}} \quad (p \text{ in lb./in.}^2 \text{ abs. and } v_s \text{ in ft.}^3/\text{lb.})$$

$$\rho_s = 0.003006 p$$

\* See Fig. 207, taken from *Forsch. Arb.*, vol. xxi (Thermal properties of saturated and superheated steam). Part I by Knoblauch, Linde, and Klebe; and Part II by R. Linde. A very complete critical survey of other test results will also be found in this volume. See also *Z.V.d.I.* (1911), W. Schüle (Properties of steam from the latest tests); and *Forsch. Arb.* 220, Eichelberg (The thermal properties of steam within technically important limits). Also Schüle's Vol. II, *Tech. Ther.* (Fourth Edition, Section 63). The results of these and other tests in Munich have been collected and used in *Tabellen und Diagrammen für Wasserdampf*, by O. Knoblauch, E. Raisch, and H. Hansen (Munich, 1923, Oldenbourg). The tables extend to 850 lb./in.<sup>2</sup> and 450° C.

† R. Mollier, *New Tables and Diagrams for Steam*. (Berlin, J. Springer, 1906.) Fifth Edition. The tables extend to 3560 lb./in.<sup>2</sup> abs. and 1060° F.

Within moderately wide limits, therefore, the density is almost proportional to the pressure.

In Fig. 182 the steam pressures have been plotted to a base of specific volumes and densities. The volume line is hyperbolic in nature, while the density line passes through the origin and is almost straight. See also page 399.

**Wet steam.** All that has so far been given on steam refers to the dry saturated state. In the normal generation of steam, however, small drops of water are carried up into the steam space and thus form a part

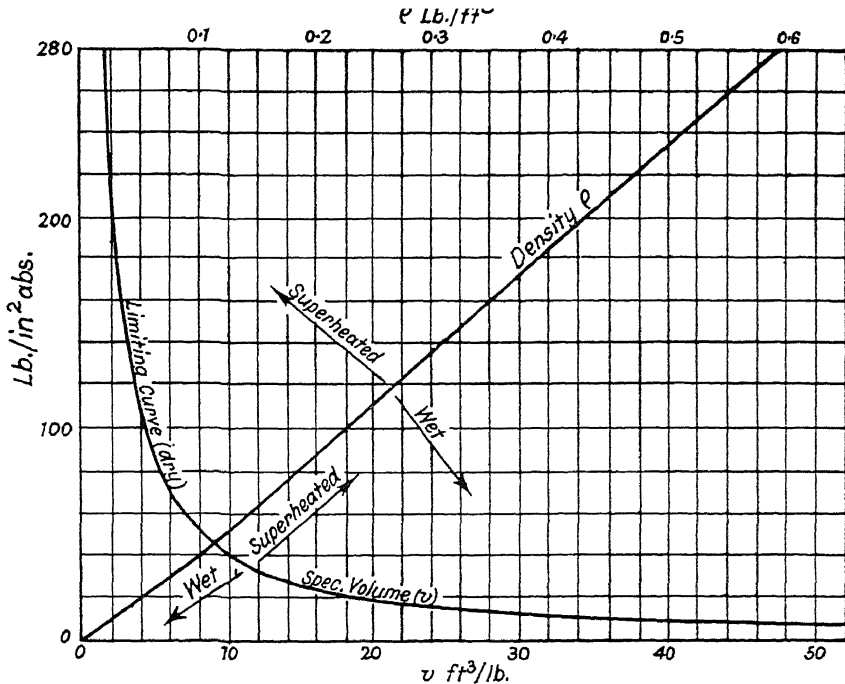


FIG. 182

of the steam mass. Even if this be avoided, drops of water are formed by condensation in the conducting pipes, due to the loss of heat. This can also occur in the engine cylinder. Since, on the formation of steam from water, heat may be absorbed without a rise in pressure and temperature, it follows that when heat is rejected by the steam, the pressure and temperature can remain constant, but a weight of steam corresponding to the amount of heat given up is reconverted to water. (See under "Latent Heat" regarding this.) Hence, in practice, saturated steam always consists of a mixture of pure steam and liquid. It is then known as "wet steam." Its composition is given in fractional parts. The weight  $q$  of pure steam in 1 lb. of wet steam is called the dryness fraction (or specific steam weight). The weight of liquid in 1 lb. of the mixture is then  $(1 - q)$  lb. and is known as the wetness or wetness fraction. Both  $q$  and

$(1 - q)$  are frequently given as percentages. Thus  $q = 0.9$  and  $(1 - q) = 0.1$  corresponds to 90 per cent pure steam and 10 per cent wetness.

The volume  $v$  of 1 lb. of wet steam is smaller than the volume  $v_s$  of dry steam, for if a part of the latter be condensed at constant pressure, a reduction in volume occurs. This follows since the volume occupied by the condensed water is much smaller than that of the steam from which it is formed. The volume occupied by  $q$  lb. of dry steam is  $qv_s$ , while the remaining  $(1 - q)$  lb. occupies the volume  $0.01602(1 - q)$  ft.<sup>3</sup> (1 lb. water occupies 0.01602 ft.<sup>3</sup>). Hence the volume occupied by 1 lb. of wet steam is

$$v = qv_s + (1 - q) 0.01602$$

If, as is always the case in practice, the steam content be high, then  $(1 - q) 0.01602$  is very small compared with  $qv_s$ . Thus if  $q = 0.75$ , which indicates a very considerable wetness, for steam at

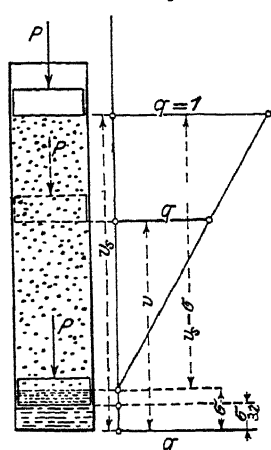


FIG. 183

$p =$	2	100	200 lb./in. <sup>2</sup> abs.
$v_s =$	174	4.43	2.29
$qv_s =$	130.5	3.32	1.72
$0.01602(1 - q) =$	0.004	0.004	0.004

Even in the case at 200 lb./in.<sup>2</sup> abs., the volume of the liquid is not  $\frac{1}{4}$  per cent of the total volume. Hence it is sufficiently accurate to take the specific volume of the wet steam as

$$v = qv_s$$

$$\text{or the specific weight } \rho = \frac{\rho_s}{q}$$

The wet steam density varies inversely as the weight of dry steam content. The pressure and temperature of the wet steam are the same as that of the dry saturated steam.

### Heat quantities involved in the generation and condensation of steam.

The conditions under which water is converted to steam, or steam to water, affect the amounts of heat used or liberated. In the following it will be assumed that both processes occur at the boiling pressure  $p_s$ . Let the cold liquid water be subjected to the required boiling pressure and then boiled by supplying heat. This is similar to what occurs in a boiler, where the cold water is first brought to the boiler pressure by the feed pump and then supplied with heat. The work required to deliver the water from the atmosphere to the boiler will—as a fraction of the total energy of steam generated—first be neglected.

An example of the reverse process, i.e. condensation of steam, is supplied by the condenser of a steam engine. In this case steam (at low pressure) is converted to water, which in turn is further cooled. During the complete change the pressure remains constant.

The generation of steam from cold water thus really involves two processes—heating of the water to the saturation temperature and boiling (Fig. 183). In the same way during condensation there occurs first the conversion of steam to water, followed by the cooling of the water.

(a) **Heating to the saturation temperature.** The heat required to raise 1 lb. of water from 32° F. to the saturation temperature  $t_s$  is called the liquid or sensible heat  $h$ . If the specific heat be denoted by  $c$ , then

$$h = c(t_s - 32)$$

The specific heat  $c$  can be assumed to be independent of pressure, but increases continuously with the temperature from about 70° F. up to the highest temperature at which water can exist (705.2° F.).

Between 32° F. and 212° F. the differences are very slight. The mean specific heat  $c_m$  in this region is therefore practically the same as the

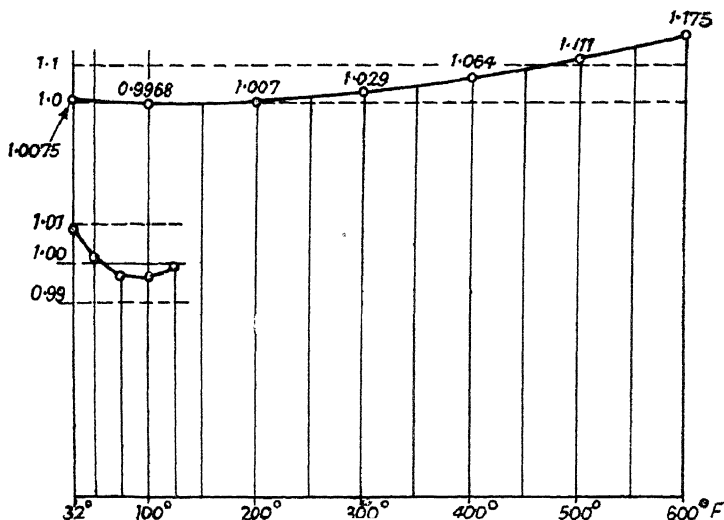


FIG. 184

true specific heat  $c$ . For technical purposes, therefore, we can take in this region

$$c = c_m = 1$$

From this it follows that  $h = (t - 32)$ .

For higher temperatures, however,  $h > (t - 32)$  although the differences for the usual temperatures adopted are not large. At 400° F., for example,  $h = 375.3$  B.Th.U./lb.

Fig. 184 shows the variation of  $c$  with temperature between 32° F. and 600° F. (1243 lb./in.<sup>2</sup> abs.). The values of  $h$  are given in Tables I and II at the end for temperatures ranging from 32° F. to 482° F.

Dieterici\* gives, for the true (instantaneous) specific heat of water between 122° F. and 570° F.,

$$c = 1.0007 - 0.00009855t + 0.0000006398t^2$$

and the mean specific heat,

$$c_m = 1.0007 - 0.00004928t + 0.0000002133t^2$$

\* Z.V.d.I. (1905), page 362.

The values found from tests in the German Imperial Institute\* differ slightly from these, as shown by the following—

32° F.	41	50	59	68	77	86	95	104	113
$c = (1.005)$	1.0030	1.0013	1.0000	0.9990	0.9983	0.9979	0.9979	0.9981	0.9996

The volume  $\sigma$  of water also increases when heat is supplied to it.† At 212° F. the increase is 4.33 per cent of the volume at 39.2° F. (4° C.), at 400° F. 16.4 per cent, and at 572° F. as much as 38.7 per cent. The mean coefficients of expansion per ° F. between these limits is thus  $\frac{0.0433}{172.8} = \frac{1}{3980}$ ;  $\frac{0.164}{360.8} = \frac{1}{2240}$ ;  $\frac{0.387}{532.8} = \frac{1}{1380}$  and hence show that the volume of the water increases considerably as the temperature rises.

The difference between the volume  $v_s$  of the saturated steam at  $t_s$  and that of the water at 32° F. ( $\sigma_{32}$ ) gives the increase in volume caused by the total supply of heat while the volume increase due to boiling alone is  $v_s - \sigma$ . In the usual cases met with in practice it is sufficiently accurate to take  $\sigma = 0.01602$ , but for pressures over 45 lb./in.<sup>2</sup> abs. and below 225 lb./in.<sup>2</sup> abs., where great accuracy is required,  $\sigma$  may be taken as 0.0176. For very high pressures see page 399.

The proportional volume increase in steam formation is  $v_s/\sigma$ . At 212° F. this is  $\frac{26.9}{\bar{n}} = 1610$ , and at 400° F.  $\frac{1.8}{\bar{n}} = 100$ . Due to the

increase in volume of the water during heating under saturation pressure, the water delivers the expansion work  $144 p_s (\sigma - \sigma_{32})$ , or, in Fig. 183, the work done by the piston as the volume changes from  $\sigma_{32}$  to  $\sigma$ . This part, which, after heating, does not remain as heat in the water but is stored externally as potential energy, forms only a very small fraction of the liquid heat. Even at 401° F. (250 lb./in.<sup>2</sup> abs.) it is only

$$Ap_s(\sigma - \sigma_{32}) = \frac{144 \times 250}{778} (0.0187 - 0.0160) = 0.786 \text{ B.Th.U./lb.}$$

whereas the sensible heat is  $h = 376 \text{ B.Th.U./lb.}$

(b) **Boiling.** When the water is raised to the saturation temperature, boiling commences. All the additional heat added goes to form steam, without any change in temperature. The heat  $L$  required to completely boil 1 lb. of water at the saturation temperature is called the latent heat.

During boiling all the heat  $L$  is converted to work. On the one hand, the external work performed in overcoming the pressure  $p_s$  as the volume increases from  $\sigma$  to  $v_s$  is  $144 p_s (v_s - \sigma)$  ft. lb. (in Fig. 183 this is the work of the piston as the volume changes from  $\sigma$  to  $v_s$ ). The corresponding heat equivalent is  $144 Ap_s(v_s - \sigma)$ , which is known as the external heat of formation.

This amount, however, as shown by test, is only a small fraction of  $L$ . Hence the amount

$$L - \frac{1}{778} p_s (v_s - \sigma) = \rho$$

disappears in the steam. This can only be accounted for by the work required to change the state of aggregation of the molecules, which is thus  $778\rho$  ft. lb.

\* Holborn, Scheel, and Henning. *Heat Tables*, page 60.

† This statement holds except between 0° and 4° C., where the volume decreases on the addition of heat.

The latent heat has thus to be regarded as the sum of the external and internal work of formation, i.e.

Of these two parts, the internal heat of formation is by far the greater, in spite of the considerable increase in volume during evaporation. This is clear from Fig. 185. The dotted line divides  $L$  into the two parts.

The total amount of heat required to produce 1 lb. of dry saturated steam from water at 32° F. under a constant saturation pressure is thus

$$H = h + L$$

This quantity is called the "Total Heat." It is the amount of heat

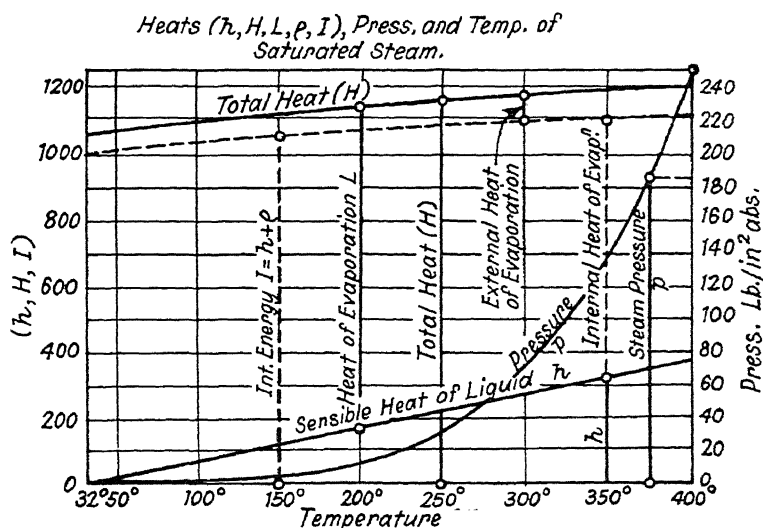


FIG. 185

which has to be transferred from the furnace of a boiler to every pound of feed water when this enters at 32° F. and leaves as dry saturated steam.

If, as is always the case, the feed water enters the boiler at a temperature above 32° F., the heat which has to be delivered is less than the above, the difference being equal to the sensible heat  $h_0$  at the feed temperature  $t_0$ , so that

$$H' = H - h_0$$

If the steam is wet on leaving the boiler, a further amount has to be deducted. See below.

**Values of the latent and total heats.** The most reliable of the older values of  $H$  were those given by Regnault. They were adopted in Zeuner's steam tables and are closely given by

$$H = 1082 + 0.305t, \text{ B.Th.U./lb.}$$

The latent heat is given from this, on assuming  $h = (t_s - 32)$ , by

$$L = 1114 - 0.695t_s$$

It will be seen from this that  $L$  is large compared with the sensible heat.

For  $t = 212^\circ \text{F.}$  we have, for example,

$$h = 180$$

$$L \doteq 967$$

i.e. the heat required to evaporate 1 lb. of water at  $212^\circ \text{F.}$  is 5.37 times greater than that required to heat water from  $32^\circ \text{F.}$  to  $212^\circ \text{F.}$ , and the total heat necessary to produce 1 lb. of steam from water at  $32^\circ \text{F.}$ , and at constant atmospheric pressure, is about 1147 B.Th.U.

In addition, Regnault's tests showed that the latent heats decreased as the steam temperatures, and consequently pressures, were increased. At  $400^\circ \text{F.}$   $L = 836$  and is thus 131 B.Th.U., less than the value at  $212^\circ \text{F.}$

On the other hand,  $H$  increases steadily up to pressures of about 500 lb./in.<sup>2</sup> abs., although the increases are relatively small. At  $400^\circ \text{F.}$ ,  $H = 1204$ , against 1147 at  $212^\circ \text{F.}$  Hence, within the limits of pressure normally used in boiler practice, the variation in  $H$  is slight, i.e. the heat required to generate 1 lb. of steam at these pressures is almost constant. See page 405 regarding high pressures.

The latest and most accurate experimental work on latent heats carried out in Germany is that of the Physical Technical Reichsanstalt.\*

They agree closely with Regnault's total heat values, which are only about 0.4 per cent smaller for temperatures above  $212^\circ \text{F.}$ , less than 0.1 per cent smaller between 160 and  $212^\circ \text{F.}$ , and 0.6 to 1 per cent greater below  $160^\circ \text{F.}$  The steam tables at the end show the Reichsanstalt values for  $L$  (up to  $550^\circ \text{F.}$ ), and the  $H'$  values are obtained from these and  $h$ , using Dieterici's values for the specific heat of water. See page 397 for high pressures.

**Feed pump work and thermal potential at constant pressure.** The heat energy required to pump the water into the boiler is, neglecting frictional losses,

where  $p_0$  is the atmospheric pressure.

This quantity is exceedingly small compared with the heat required to form the steam. For  $p = 300 \text{ lb./in.}^2 \text{ abs.}$  it amounts to only

$$\frac{144}{778} (300 - 15) 0.016 = 0.85 \text{ B.Th.U./lb.}$$

Since the feed pump energy is only indirectly—and sometimes not at all—supplied by the heat in the boiler steam, and since actually it is much greater than the ideal value† when expressed in terms of the heat

\* F. Henning. Latent heat of water and the specific volume of steam between  $30^\circ \text{C.}$  and  $180^\circ \text{C.}$  *Z.V.d.I.* (1909), page 1768. Also *Z.V.d.I.* (1929), page 504. M. Jakob. Latent heat of water and the specific volume of saturated steam up to  $210^\circ \text{C.}$  and (page 629) between  $210^\circ$  and  $250^\circ \text{C.}$

† E.g. when steam is taken from the boiler to operate a feed pump, or when the feed pump is driven from the engine shaft.



from the furnace, it is customary, and advisable for the sake of clarity, not to include it in the energy required to generate the steam.

By thermal potential is meant

$$H' = I + Apv$$

and since  $I = h + \rho$  and  $v = v_s$  for dry steam

$$\begin{aligned} \text{we have } H' &= h + \rho + Apv_s \\ &= h + \rho + Ap(v_s - \sigma) + Ap\sigma \end{aligned}$$

The sum of the first three values is  $H$ , so that

$$H' = H + Ap\sigma$$

The thermal potential is thus greater than the total heat by the small amount  $Ap\sigma$ , which at high pressures is practically the same as the feed pump heat.

**Internal energy of steam.** During boiling the work  $Ap(v_s - \sigma)$  is delivered to the outside (external heat of formation), while the portion  $\rho$ , required to perform internal work, remains in the steam. The sensible heat  $h$  also remains in the steam after boiling. Hence, of the total heat supplied to the steam, the quantity  $h + \rho$  remains as energy in the steam.

This quantity

$$h + \rho = I$$

is known as the energy or internal energy of the steam.

This value is of importance in the case of the adiabatic expansion of steam, the work of which comes entirely from  $I$  (page 379), and in the case of condensation of steam at constant volume (page 385). The values of  $I$  for dry saturated steam at different temperatures are shown in Fig. 185.

The heat  $Ap(v_s - \sigma)$  delivered to the outside as expansion work during boiling is not to be regarded as lost, for if the steam be later condensed, at the constant pressure  $p_s$ , the heat equivalent of the work done as the volume contracts is redelivered by this external pressure in the same way as when a gas is cooled (page 92). See also Fig. 183. It is only when condensation is carried out in this way, i.e. under a constant pressure equal to that at which evaporation occurred, that the heat given up by the steam (or the heat required to liquefy it) is exactly the same as that given to it during its formation.

If, however, the evaporation occurs at constant pressure (i.e. with increasing volume) and condensation at constant volume (i.e. with decreasing pressure) then the heat liberated is, in this case, less by the amount  $Ap_s(v_s - \sigma)$ . For this reason a pressure below atmospheric exists in the steam space by virtue of which the external air pressure could deliver extra work.

**Wet steam.** The heat per unit weight contained in wet steam is less than that in dry steam. Since 1 lb. of dry steam requires the total heat  $h + L$  for its formation, the heat required for  $q$  lb. of dry steam is  $q(h + L)$  B.Th.U., and the heat required for  $(1 - q)$  lb. of moisture is  $(1 - q)h$  B.Th.U. Hence the total heat of 1 lb. of wet steam, reckoned from 32° F., is  $q(h + L) + h(1 - q)$ , or

$$H_w = h + qL$$

If the volume of the steam is  $v$ , we have also

$$H_w = h + qp + Ap(v - \sigma)$$

since  $h$  is the same whether the steam be wet or dry, while the internal heat of formation is only  $qp$  B.Th.U. and the external only  $Ap(v - \sigma)$  B.Th.U.

The total heat of wet steam is  $h + L - L(1 - q)$ , and is thus  $L(1 - q)$  B.Th.U. less than that of dry steam. If now the temperature of the feed water is  $t_0$ , the total heat supply to produce 1 lb. of wet steam of quality  $q$  is

$$H' = H - h_0 - (1 - q)L$$

in which  $H$ ,  $h_0$ , and  $L$  are obtained from steam tables.

Thus, for steam at 150 lb./in.<sup>2</sup> abs., with  $q = 0.95$  and a feed temperature of 60° F.,

$$\begin{aligned} H' &= 1193.1 - 28 - 0.05 \times 862.2 \\ &= 1122.0 \text{ B.Th.U./lb.} \end{aligned}$$

The energy of wet steam is given by the sum of the liquid heat  $h$  and the internal latent heat  $qp$ , i.e.

**Example 1.** A steam pressure gauge on a boiler shows a pressure of 107 lb./in.<sup>2</sup> Find the steam temperature if the barometric pressure is 28" Hg.

The absolute steam pressure is  $107 + 0.491 \times 28$

$$= 107 + 13.75 = 120.75$$

Hence, from the steam tables, the temperature is

$$t = 341.2 + \frac{.75}{5} \times 3.3 = 341.2 + 0.5 = 341.7$$

(Interpolation.)

**Example 2.** Find the weight of 350 ft.<sup>3</sup> of wet steam at 100 lb./in.<sup>2</sup> abs. with  $q = 0.88$ .

From the tables, 1 ft.<sup>3</sup> of dry steam at 100 lb./in.<sup>2</sup> abs. weighs 0.2260 lb.

With  $q = 0.88$ , therefore, the weight of 1 ft.<sup>3</sup> of the wet steam is  $0.2260/0.88 = 0.257$  lb. and the weight of 350 ft.<sup>3</sup> is 90.0 lb.

**Example 3.** The consumption of a 1000 h.p. steam turbine is 15.4 lb. of saturated steam per h.p. hour. The steam has to leave the guide blades, both in the high pressure and low pressure sections, with an axial velocity of 325 ft./sec. Find the cross-sectional areas ( $a$ ) in the high pressure section with  $p = 105$  lb./in.<sup>2</sup> abs. and  $q = 0.95$ , ( $b$ ) in the low pressure section with  $p = 1.0$  lb./in.<sup>2</sup> abs. and  $q = 1$ .

$$\text{Steam per hour} = 1000 \times 15.4 = 15400 \text{ lb.}$$

$$\text{Steam per sec.} = 15400/3600 = 4.28 \text{ lb.}$$

( $a$ ) Volume of 1 lb. at 105 lb./in.<sup>2</sup> abs. is 4.212 ft.<sup>3</sup> for dry steam, and for the wet steam  $4.212 \times 0.95 = 4.00$  ft.<sup>3</sup>/lb. Hence the volume of

$4.28 \text{ lb.} = 4.28 \times 4.00 = 17.12 \text{ ft.}^3/\text{sec.}$  For a velocity of  $325 \text{ ft./sec.}$  the area required is  $\frac{17.12}{325} \text{ ft.}^2 = \frac{144 \times 17.12}{325} = 7.58 \text{ in.}^2$

(b) At  $1 \text{ lb./in.}^2$  abs. the specific volume of dry steam is  $333.0 \text{ ft.}^3/\text{lb.}$

Hence the necessary area is  $\frac{333.0 \times 4.28 \times 144}{325} = 631 \text{ in.}^2$ , i.e. about 83 times greater than in the high pressure section.

**Example 4.** A boiler supplies steam at  $115 \text{ lb./in.}^2$  abs. to an engine developing 100 h.p. with a steam consumption of  $20 \text{ lb./h.p. hour.}$  Find the heat transferred from the furnace to the boiler water per hour. Temperature of the feed,  $50^\circ \text{ F.}$

Steam used per hour, 2000 lb. From the tables, the total heat of the dry steam for feed water at  $32^\circ \text{ F.}$  is 1189.4, and therefore for feed water at  $50^\circ \text{ F.}$  it is  $1189.4 - (50 - 32) = 1189.4 - 18 = 1171.4 \text{ B.Th.U./lb.}$  Hence the steam takes up  $1171.4 \times 2000 = 2,342,800 \text{ B.Th.U./hour.}$  For a boiler efficiency of 70 per cent, and coal having a calorific value of 13,000 B.Th.U./lb., this corresponds to a coal consumption of

$$\frac{2342800}{0.7 \times 13000} = 258.0 \text{ lb. per hour.}$$

**Example 5.** In the cylinder of an engine which is supplied with dry saturated steam at  $140 \text{ lb./in.}^2$  abs. the steam on entering becomes 15 per cent wet, due to the cooling effect of the cylinder walls. How much of the heat carried in by the steam is transmitted to the walls?

The total heat in 1 lb. of dry steam at  $140 \text{ lb./in.}^2$  abs. is 1192 B.Th.U./lb. Hence the minimum heat given to the walls is the latent heat in  $0.15 \text{ lb.}$  of steam, which, from the tables, is  $0.15 \times 867 = 130 \text{ B.Th.U.}$  or  $\frac{130}{1192} \times 100 = 10.9$  per cent of the total heat.

**Example 6.** A heating system using low pressure steam at  $16 \text{ lb./in.}^2$  abs. requires 400,000 B.Th.U. per hour. Find the weight of steam supplied to the heating element per hour—

(a) If the condensate is returned to the boiler at saturation temperature.

(b) If the condensate leaves the heating element at  $176^\circ \text{ F.}$  and drops a further  $50^\circ \text{ F.}$  on its passage to the boiler.

(a) The latent heat  $L$  is required for each pound of steam condensed. For steam at  $16 \text{ lb./in.}^2$  abs. this is 968.5 B.Th.U. Hence the heating element requires  $\frac{400000}{968.5} = 414 \text{ lb. steam per hour.}$  If no heat loss occurs

between the boiler and heating element, and from there back to the boiler, all the steam heat is employed usefully and the efficiency of heating is 100 per cent. The heat taken up by the steam in the boiler per hour is 400,000 B.Th.U.

(b) The heat delivered by the steam in the heating element is  $968.5 + (216 - 176) = 968 + 40 = 1008 \text{ B.Th.U./lb.}$  The feed water, however, enters the boiler at  $126^\circ \text{ F.}$ , so that the heat necessary there is  $968 + 216 - 126 = 1058 \text{ B.Th.U./lb.}$

Weight of steam required

$$= \frac{400000}{1008} = 397 \text{ lb.}$$

$$\text{Efficiency} = \frac{1008}{1052} =$$

**Example 7.** A low pressure turbine uses 44.2 lb. of steam at 16 lb./in.<sup>2</sup> abs. per kw. hour. The moisture content of the steam is 24.5 per cent. What would be the consumption if the steam were initially dry?

The total heat of dry steam at 16 lb./in.<sup>2</sup> abs. is 1153 B.Th.U./lb., and for feed water at 60° F. this becomes 1125 B.Th.U. The total heat of wet steam with  $1 - q = 0.245$  is  $0.245 \times 968.5 = 233$  B.Th.U. less, and is thus  $1125 - 238 = 887$  B.Th.U.

Hence the required consumption is  $\frac{44.2 \times 887}{1125} = 34.2$  lb. per kw. hour.

**Example 8.**  $W_1$  lb. of cold water at  $t_0$ ° F. enter a water purifying unit and mix there with  $W_2$  lb. of dry saturated steam supplied by the boiler. The purified water at  $t$ ° F. is now supplied to the feed pump of the boiler. Express the weights  $W_1$  and  $W_2$  in terms of the total weight  $W = W_1 + W_2$ .

The fresh water supply is heated from  $t_0$  to  $t$ ° F. and thus takes up  $W_1(t - t_0)$  B.Th.U. of heat. The steam condenses and the condensate cools to  $t$ ° F., so that the heat given up is  $W_2 \{H - (t - 32)\}$ . These two heat quantities are equal, giving

$$W_1(t - t_0) = W_2 \{H -$$

Again, since

$$W_2 = W - W_1$$

it follows that

$$W_1 = \frac{W \{H -$$

and

$$W_2 = \frac{W \{t -$$

For example, with  $t_0 = 60$ ° F.,  $t = 140$ ° F.,  $H = 1192$  B.Th.U./lb. (for  $p = 140$ ),

$$\frac{1 - (140 - 32)}{1192 - (60 - 32)} = .932 W$$

$$W_2 = .068 W_1$$

**Example 9.** In a jet condenser the ratio  $n$  of the weight of cooling water to the weight of steam condensed is found as follows—

At the end of expansion in the engine cylinder let the steam pressure be  $p_e$  and the quality  $q_e$ , so that the total heat is  $H_e - (1 - q_e)L$ .

If now the steam enters the condenser without performance of work, its thermal potential does not alter. This means that the total heat also remains practically constant. Hence, each pound of steam brings in the

heat  $H_e - (1 - q_e)L_e$ , and since it is condensed it gives up this heat, less the sensible heat at  $t_w$ , i.e.  $H_e - (1 - q_e)L_e - (t_w - 32)$ . The cooling water takes up the heat  $n(t_w - t_0)$ , and, since the two heat quantities are equal,

$$H_e - (1 - q_e)L_e - (t_w - 32) = n(t_w - t_0)$$

giving

$$n =$$

The pressure  $p_e$  lies between 10 and 20 lb./in.<sup>2</sup> abs., so that  $H_e$  lies between 1144 and 1158 (average about 1150) and  $L_e$  between 983 and 961 (average 970).

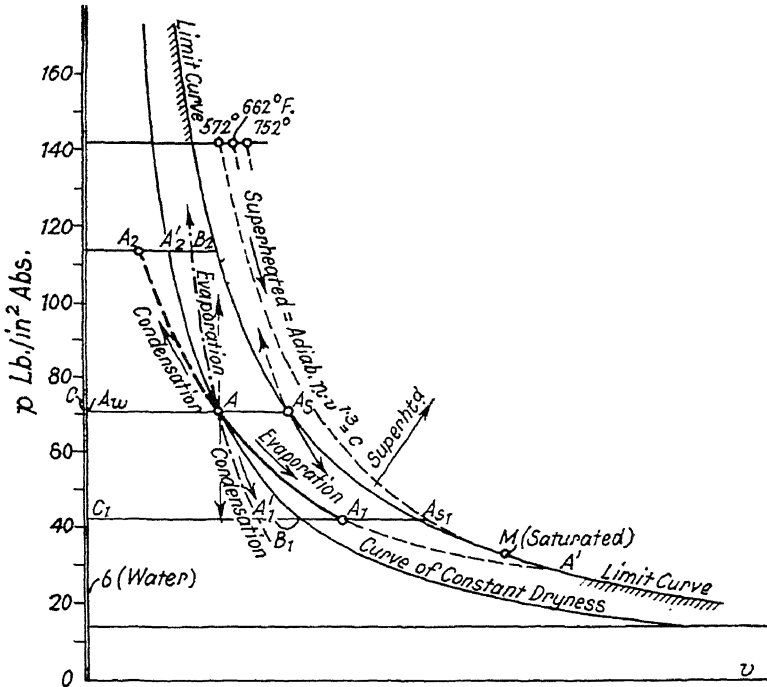


FIG. 186

$$\text{This gives } n = \frac{1150 - (1 - q_e) 970 - (t_w - 32)}{t_w - t_0}$$

If, for example,  $t_0 = 95^\circ \text{ F.}$  and  $t_w = 130^\circ \text{ F.}$

$$\text{then } n = \frac{82 + 970q_e}{35}$$

Assuming the steam dry on entering the condenser, i.e. if  $q = 1$ , we have  $n = 30 \text{ lb.}$

If, on the other hand, it is wet with  $q_e = 0.75$ , then  $n = 23$  lb.

The two assumptions give results which differ considerably. In a preliminary investigation it is customary to make the first assumption, which errs on the safe side. In actual tests of engines the true value of  $q_e$  will have to be introduced.

**Changes in the dryness fraction of steam caused by changes of state. Curves of constant quality.** The state of wet steam is determined by its pressure (saturation pressure) and by its quality ( $q$  or  $1 - q$ ). Since the specific volume is determined by the latter, it follows that the state of the steam is also given by the pressure and volume.

By plotting simultaneous values of  $p$  and  $v$  on a co-ordinate field (Fig. 186) the point  $A$  for wet steam lies within the field bounded by the dry saturated curve, since wet steam occupies a smaller volume ( $CA$ ) than dry steam ( $CA_s$ ) for the same pressure. The dryness fraction is given by the ratio  $\frac{CA}{CA_s}$  and the wetness by  $\frac{AA_s}{CA_s}$ .

Or, more exactly, from page 358,

$$v = qv_s + (1 - q)v_w$$

giving 
$$q = \frac{v - v_w}{v_s - v_w}$$

If the specific volumes  $\sigma$  of the water corresponding to the different pressure or temperature values be plotted in Fig. 186, then

$$q = \frac{A_w A}{A_w A_s}$$

The difference for normal steam pressures is negligible.\* The curve of water volumes is called the lower limit curve.

If the steam expands in any arbitrary way, such as shown by the line  $AA_1$ , the quality generally changes. Thus the quality at  $A_1$  is higher than at  $A$ , and is given by  $\frac{C_1 A_1}{C_1 A_{s1}}$ . Hence between  $A$  and  $A_1$  a partial evaporation of the moisture has occurred.

If a line  $AA'_1$  be plotted in Fig. 186 in such a way that  $A'_1$  divides  $A_{s1}$  in the same ratio as  $A$  divides  $CA_s$ , then the quality is constant.  $AA'_1$  in Fig. 186 has been drawn for a moisture content of 35 per cent ( $q = 0.65$ ). From this line it can be seen at once whether steam, in expanding from  $A$ , alters its quality in any way. Thus along the dotted line  $AB_1$  increase in wetness occurs. The same takes place along the vertical below  $A$  (cooling at constant volume). Along the horizontal from  $A$  to the right evaporation obviously occurs.

With compression from  $A$  along  $AA_2$  the wetness increases, while the reverse occurs along  $AB_2$ .

Hence, when wet steam expands or is compressed, either evaporation or condensation can occur, depending on the course of the state change.

If, starting from  $A$ , expansion occurs according to a definite law, with an increase in moisture content, then, on compressing from  $A$ ,

\* See Fig. 221 for  $\text{CO}_2$  and Fig. 210 for high pressure steam.

according to the same law, a decrease in moisture content occurs, and *vice versa*.

Steam originally dry, as at  $A_s$ , will, on expansion, always become wet if the state curve runs below the dry saturated limit curve; in compression it can become superheated (e.g. in adiabatic compression).

Curves of constant quality are similar to the dry saturated curve, and hence also follow the law  $pv^{1.6/15} = C$ .

In Fig. 193, which shows the diagram taken from a condensing steam engine operating on saturated steam, the volume curve of dry saturated steam is plotted from the weight of steam used per stroke. The true volume curve shows the steam to be wet throughout expansion. The qualities  $q$  have been plotted to a base of piston position. See also page 513, example (a).

### SUPERHEATED STEAM

If the dry saturated steam be removed from the boiler to a separate unit and heated there, its volume increases and its temperature rises above the saturation temperature corresponding to the boiler pressure. Let the superheater space be in free communication with the steam space of the boiler (as always occurs in practice) then the pressure in the superheater will equal that of the boiler.

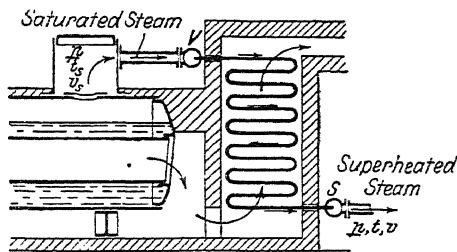


FIG. 187

Fig. 187 shows the arrangement with the necessary valves, etc., omitted. Saturated steam is supplied from the dome of a boiler to a header  $V$ , to which are fitted a series of pipe coils. These form the superheater proper and finish in another header  $S$ , from which the steam supply is fed. The steam enters the header  $V$  at the boiler temperature  $t_s$ , is heated as it flows downward, and enters the header  $S$  at the temperature  $t > t_s$ . Since its volume simultaneously increases from  $v_s$  to  $v$ , it leaves the superheater with a velocity greater than that at  $V$ .

Whether the steam be heated by the waste gases of the boiler or by a special furnace, the properties of the hot steam, which alone are dealt with here, remain the same.

Superheated steam can, therefore, be defined as steam possessing a temperature above its saturation temperature. Thus, if steam at 125 lb./in.<sup>2</sup> abs. has a temperature of 600° F., its range of superheat is 600 - 344 = 256° F.; if for the same temperature the pressure be 75 lb./in.<sup>2</sup> abs., then the range of superheat is 600 - 308 = 292° F.

The superheated state can also be recognized from the specific volume, since the volume of steam increases during superheating. Steam is superheated if, at a given pressure, a definite weight occupies a greater volume than that occupied by the same weight of dry saturated steam, or if, in a given volume, the weight of steam is less than would be occupied by dry saturated steam at the same pressure. Thus atmospheric vapour is usually superheated, and it is only when clouds or mist are formed that the vapour becomes saturated. (Superheated steam is also called unsaturated.)

**Total heat.** In order to produce superheated steam there must be a supply of dry saturated steam at the same pressure, hence the heat first required is  $h + L$ . To this is added the superheat.

The general relations of steam in the superheated region are closely related to those of a gas. Part of the superheat is required to effect the rise in temperature, and the remainder to perform the expansion work. The total amount of heat required to raise the temperature of 1 lb. of

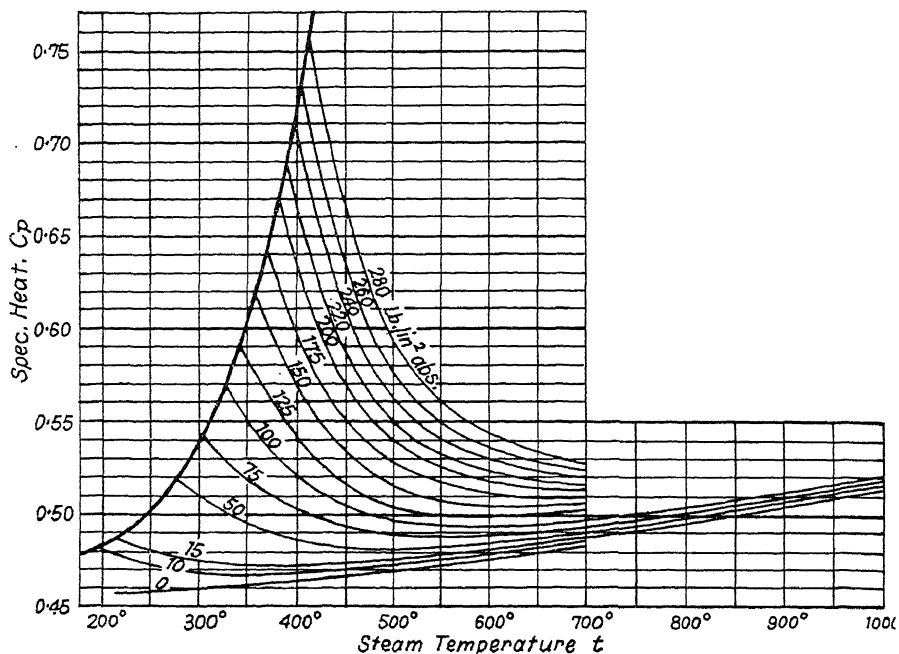


FIG. 188

superheated steam through  $1^{\circ}$  F. is called—as in the case of gases—the specific heat at constant pressure  $c_p$ .

It was previously assumed that  $c_p$  was independent of the temperature and pressure and had the value 0.48. It is now known—from the tests of Knoblauch and Jakob, and later tests carried out in Munich—that this assumption is not valid.  $c_p$  increases with the pressure when the temperature is kept constant. If the pressure is kept constant,  $c_p$  first decreases from the saturation point with increasing temperatures, and then slowly increases. (See Fig. 188.)

The superheat can be found from the mean specific heat value and the temperature rise, as shown for the case of variable specific heat dealt with on page 47. The values of  $(c_p)_m$  have been found from the curves of Fig. 188, by Knoblauch, for the different pressures and temperatures. Compare the following table. The values hold between the saturation temperature (given below the pressures in second line) and the superheated steam temperatures (given in the first vertical column).



The total heat can now be found from

$$H' = h + L + (c_p)_m (t - t_s)$$

MEAN SPECIFIC HEAT VALUES  $C_{pm}$  FOR SUPERHEATED STEAM

$p_s \frac{\text{lb.}}{\text{in.}^2} \text{ abs.} =$	10	15	30	60	90	120	150	180	210	240	270	300
$t_s$ °F. =	193.3	213.0	250.3	292.6	320.4	341.2	358.5	373.1	386.1	397.6	407.7	417.4
$C_{ps}$ =	0.482	0.487	0.500	0.528	0.555	0.584	0.612	0.643	0.675	0.708	0.743	0.779
250° F.	0.477	0.484	—	—	—	—	—	—	—	—	—	—
300° F.	0.473	0.480	0.493	0.523	—	—	—	—	—	—	—	—
350° F.	0.470	0.475	0.489	0.516	0.545	0.578	0.611	—	—	—	—	—
400° F.	0.469	0.474	0.486	0.508	0.533	0.560	0.589	0.622	0.662	0.705	—	—
450° F.	0.469	0.472	0.483	0.503	0.525	0.545	0.569	0.595	0.622	0.654	0.687	0.724
500° F.	0.469	0.472	0.482	0.499	0.518	0.537	0.556	0.577	0.598	0.621	0.646	0.670
550° F.	0.469	0.472	0.481	0.497	0.514	0.529	0.546	0.565	0.582	0.602	0.620	0.639
600° F.	0.471	0.473	0.481	0.495	0.510	0.525	0.539	0.555	0.570	0.586	0.602	0.618
650° F.	0.471	0.474	0.481	0.494	0.509	0.520	0.534	0.548	0.561	0.574	0.589	0.602
700° F.	0.471	0.475	0.482	0.495	0.507	0.518	0.531	0.542	0.553	0.566	0.578	0.592
750° F.	0.471	0.477	0.484	0.495	0.506	0.516	0.529	0.537	0.549	0.560	0.571	0.586
800° F.	—	—	0.485	0.495	0.506	0.516	—	—	—	—	—	—
850° F.	—	—	0.486	0.496	0.506	0.515	—	—	—	—	—	—
900° F.	—	—	0.487	0.497	0.506	0.515	—	—	—	—	—	—
950° F.	—	—	0.488	0.498	0.506	0.514	—	—	—	—	—	—
1000° F.	—	—	0.490	0.499	0.507	0.513	—	—	—	—	—	—

**The equation of state.** In the case of saturated steam each pressure  $p$  has a corresponding temperature and, if dry, a corresponding specific volume. Superheated steam, on the other hand, can have any volume and any temperature above the saturation values. The three magnitudes of state  $p$ ,  $v$ , and  $T$  are related to one another—like gases—by an equation of state or characteristic law. The usual characteristic equation for gases,  $pv = RT$ , however, does not show agreement with the results obtained by test, particularly near the saturation region.

From the Munich tests an equation, valid within the region investigated and showing the inter-dependent relationships with great accuracy, has been established. As given by Linde (*Forsch. Arb.* No. 21),

$$pv = RT - p(1 + ap) \left[ C \left( \frac{671.4}{T} \right)^3 - D \right]; \quad (p \text{ in lb./ft.}^2 \text{ } T \text{ in } ^\circ\text{F. abs.})$$

where  $R = 85.85$ ,  $a = .00001$ ,  $C = 0.4966$ ,  $D = 0.0833$

R. Linde suggests the following approximate equation as suitable for practical work, especially in finding the specific volume or density from the pressure and temperatures.

$$pv = 85.85T - 0.2563p$$

or  $p(v + .2563) = 85.85T$  (for  $p$  in lb./ft.<sup>2</sup> abs.)

$p(v + .2563) = .5962T$  (for  $p$  in lb./in.<sup>2</sup> abs.)

The equation in this form is similar to the gas equation, the only difference being the additional term  $0.2563p$ .

The specific volume at a given pressure and temperature is thus

$$v = \frac{1}{p} - 0.2563 \quad \text{or} \quad -0.2563 \quad (\text{for } p \text{ in lb./in.}^2 \text{ abs.})$$

and the density is

$$\rho = \frac{1}{v}. \quad \text{For } p \text{ in lb./in.}^2 \text{ abs. this gives}$$

$$\rho = \frac{144}{85.85T - 36.9p}$$

These relationships lose their validity entirely when the steam, on changing its state, enters the saturation region. This is the case whenever  $p$  and  $v$ , or  $p$  and  $T$ , or  $v$  and  $T$  assume values which correspond to those given in the saturated steam tables. In Fig. 182 the curve given by the corresponding  $p_s$  and  $v_s$  values from steam tables separates the superheated field from the wet field, and is therefore called the limiting curve. Its equation is given very closely by

$$p^{15/16} v_s = 332.64$$

**Example 10.** Find the quantity of heat required to superheat 1 lb. of dry saturated steam at 150 lb./in.<sup>2</sup> abs. to a temperature of 700° F. Find the percentage excess heat in this superheated steam over that of the dry saturated steam.

The temperature of the saturated steam is 358.5° F., so that the range of superheat is  $t - t_s = 700 - 358.5 = 341.5$ ° F. The specific heat between 358.5 and 700 and for  $p = 150$  is  $(c_p)_m = 0.531$ , hence the superheat is  $0.531 \times 341.5 = 181$  B.Th.U.

The total heat of the dry saturated steam is 1193.1 B.Th.U. so that the total heat of the superheated steam is  $1193.1 + 181 = 1374.1$  B.Th.U. The percentage excess of the latter is thus  $\frac{181}{1193.1} \times 100 = 15.15$ .

**Example 11.** As a rule the temperature of superheated steam, as supplied to engines, does not exceed 650° F. Find how much greater the volume of this steam is compared with the volume of dry saturated steam at 15, 60, 120, and 180 lb./in.<sup>2</sup> abs.

Since  $v = \frac{0.5962T}{p} - 0.2563$  and  $T = 460 + 650 = 1110$ ° F. abs.  
for

$p =$	15	60	120	180
$v =$	43.84	10.76	5.25	3.42

For dry saturated steam,

$v_s =$	26.27	7.159	3.710	2.522
---------	-------	-------	-------	-------

Hence the volume increases are

$\frac{v}{v_s} =$	1.67	1.50	1.42	1.35
-------------------	------	------	------	------

As a rough check, the superheated steam can be treated as a gas, so that  $\frac{v}{v_s} = \frac{T}{T_s}$ . Thus, taking a pressure of 120 lb./in.<sup>2</sup> abs., for which

$$T_s = 460 + 341.2 = 801.2$$

$$\frac{v}{v_s} = \frac{1110}{801.2} = 1.39$$

**Example 12.** Superheated steam at  $p$  lb./in.<sup>2</sup> abs. and  $t^\circ$  F. is used to heat an equal weight of wet steam at  $p'$  lb./in.<sup>2</sup> abs. having a quality  $q$ . How much moisture is converted to steam if the temperature of the superheated steam drops to  $t_1^\circ$  F.? Assume that  $p$  and  $p'$  remain constant.

Let  $(c_p)_m$  be the mean specific heat of the superheated steam between the saturation temperature  $t_s^\circ$  and the temperature  $t^\circ$  and  $(c_p)_{m1}$  the mean specific heat between  $t_s^\circ$  and  $t_1^\circ$ .

The heat given up by the superheated steam is then

The heat required to increase the dryness fraction of the wet steam from  $q$  to  $q_1$  is  $L(q_1 - q)$ .

$$\text{Hence } q_1 - q = \frac{(c_p)_m(t - t_s) - (c_p)_{m1}(t_1 - t_s)}{L}$$

Taking, for example,  $p = 150$  lb./in.<sup>2</sup> abs.,  $t = 650^\circ$  F.,  $t_1 = 550^\circ$  F.,  $p' = 14.7$  lb./in.<sup>2</sup> abs., gives  $t_s = 358.5$ ,  $t - t_s = 291.5$ , and  $t_1 - t_s = 191.5$ . Also  $(c_p)_m = 0.534$ ,  $(c_p)_{m1} = 0.546$ , and  $L = 971.4$ .

$$\begin{aligned} \text{Hence } q_1 - q &= \frac{0.534 \times 291.5 - 0.546 \times 191.5}{971.4} \\ &= 0.053 \end{aligned}$$

The dryness is therefore increased by 5.3 per cent.

For practical work  $c_{pm}$  can be taken as 0.534 between 650 and 550° F., giving

$$q_1 - q = \frac{0.534 \times 100}{971.4} =$$

**Entropy of steam.** On page 123 *et seq.* the conception of entropy was established from the known behaviour of a perfect gas. The treatment in the case of vapours, however, has to be reversed. It is only by first assuming that the conception of entropy is known and applicable to vapours that the changes of state of these substances can be theoretically followed. The general significance of the conception of entropy, and its validity for all types and states of substances, followed from the second law of thermodynamics (see page 167 *et seq.*). The statement that the conception of entropy, as established previously for gases, applies also to vapours in any state, is merely equivalent to an expression of the second law as applied to changes of state of vapours. These changes were only completely understood after Clausius had developed the entropy conception from this law.

(a) **Saturated steam.** As shown on page 167, the amount of heat received or rejected by unit weight (of gas or vapour) in any small change of state is

$$dQ = Td\phi$$

where  $T$  is the instantaneous absolute temperature and  $d\phi$  the small change of entropy.

The entropy  $\phi$  is a magnitude which depends only on the instantaneous state of the steam, and is, therefore, defined by the values  $p$ ,  $v$ , and  $T$  corresponding to this state. The path by which the steam arrives at this state has no effect on  $\phi$ . Hence, when it is not possible to determine  $\phi$  from the general relations between  $p$ ,  $v$ ,  $T$ , and  $Q$  owing to the exact relationship between these being unknown, use may be made of any particular change of state, the course of which is known. It is usual to take the entropy of water at 32° F. or 492° F. abs. as zero, and hence the entropy in any other state can be found from the processes of heating and boiling at constant pressure, for which exact data are furnished by test.

The growth of entropy for a small part of the change of state is, in general,

$$d\phi = \frac{dQ}{T}$$

The cold water, which is subjected to the boiler pressure, has first to be raised to the boiling temperature  $T_s$ . Assuming the specific heat of water to be constant and equal to unity gives

$$dQ = cdT = dT \text{ B.Th.U.}$$

The corresponding increase in entropy is

$$d\phi = \frac{dT}{T} = \frac{dT}{T}$$

By summing the successive increments  $\frac{dT_1}{T_1}$ ,  $\frac{dT_2}{T_2}$ ,  $\frac{dT_3}{T_3}$ , from 492° F. abs. to  $T_s$ , the total entropy increase is

$$\phi_w = \log_e \frac{T_s}{492}$$

(entropy of water), or

$$\phi_w = 2.303 \log \frac{T_s}{492}$$

Hence the water line on the temperature entropy diagram is represented by a logarithmic curve (as in the heating of a gas at constant pressure or volume), Fig. 189. The area below  $A_0A_1$  is the sensible heat  $h$ .

During the ensuing evaporation at constant pressure the temperature remains constant, and this isothermal change of state is represented by the straight line  $A_1A_2$  corresponding to an entropy increase  $\phi'$ . The heat absorbed during this change, i.e. the latent heat  $L$ , is given by the rectangular area below  $A_1A_2$ . This is equal to  $T_s\phi'$ , so that

and

(Entropy increase during boiling.)

The total increase in entropy from water at 32° F. to dry steam at  $T_s = t_s + 460$  is thus

$$\phi = 2.303 \log \frac{T_s}{492} + \frac{L}{T_s}$$

**Entropy of the dry steam.** The entropy of 1 lb. of wet steam having the quality  $q$  is found by considering the heat required to produce it from

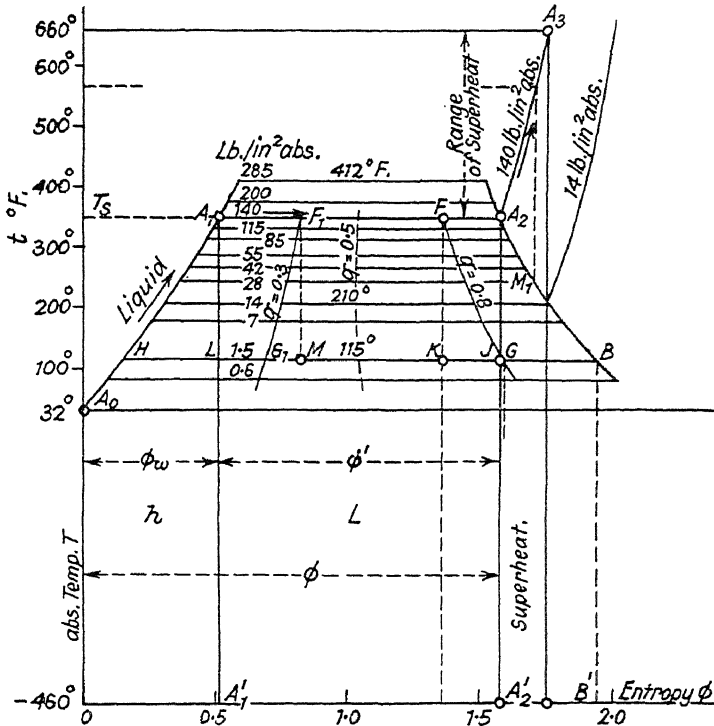


FIG. 189

water at 32° F. The whole pound of water has to be heated to  $T_s$  while  $q$  lb. of steam are formed, hence the entropy is

$$\phi_w = 2.303 \log \frac{T_s}{492} + \frac{qL}{T_s}$$

In Fig. 189 the ratio of the distances  $A_1F = qL/T_s$  and  $A_1A_2 = L/T_s$  gives the dryness fraction.

By calculating the entropy of dry steam in this way and plotting their values against  $T$  as ordinates, the curve  $A_2B$  is obtained. (Saturation curve on  $T\phi$  diagram.)

In addition, by dividing the horizontal distances between the water and saturation curves in the same ratio, curves of constant quality are obtained. Three such curves are shown in Fig. 189 with  $q = 0.8, 0.5$ , and  $0.3$ .

In Fig. 198 the saturation curve has been plotted to a larger scale. Curves of constant quality are shown and intermediate quality curves can be interpolated. The water curve has been omitted owing to the larger scale used.

The area below any curve on the  $T\phi$  diagram down to the abscissae axis represents—as in the case of gases—the heat added or withdrawn. Thus the area  $OA_0A_1A'_1$  (Fig. 189) gives the sensible heat at  $T_s$ ; the rectangle  $A_1A_2A'_2A'_1$  gives the latent heat  $L$ ; the area  $OA_0A_1A_2A'_2$  the total heat  $H_s$  of the dry steam; the area under  $OA_0A_1F$  the total heat of wet steam with  $q = 0.8$ ; the area under  $A_0HK$  the total heat of wet steam at the state  $K$  (for which  $q = HK/HB$ ); the area under  $FG$  the heat added as the steam expands at constant quality from  $F$  to  $G$  or the heat withdrawn as the steam is compressed at constant quality; in the same way the area  $A_2BB'A'_2$  is the amount of heat which has to be supplied to the steam initially at  $A_2(T_2)$  if it remains dry on expanding to  $B$ .

If any curve on the  $pV$  field be transferred to the  $T\phi$  field, the area below the  $T\phi$  curve represents the heat added or withdrawn during this change of state.

**More exact value of the water entropy.** In drawing down the entropy diagram, use must be made of the reliable values of the water entropy, since all the remaining curves are built up from this.

If the specific heat of the water is  $c$ , then

$$d\phi = \frac{dT}{T}$$

As given by Dieterici, from  $104^\circ \text{ F.}$  upwards,

$$c = 1.0007 - 0.00009855t + 0.0000006398t^2$$

From this, with  $t = T - 460$

$$c = 1.811 - 0.0006872T + 0.0000006398T^2$$

$$\text{Hence } d\phi = 1.811 \frac{dT}{T} - 0.0006872 dT + 0.0000006398T dT$$

Integrating between the limits  $T$  and  $460 + 104 = 564$  gives

$$\log_e - 0.0006872 (T - 564)$$

$$+ 0.0000003199 (T^2 - 564^2)$$

or, with  $\log_{10}$  instead of  $\log_e$  and  $t$  in the non-logarithmic terms,

$$\phi_w - \phi_{104} = 0.037399 + 2.71962 \log \frac{T}{564} - 0.0003929t + 0.0000003199t^2$$

From the specific heat values given in Fig. 184, between  $32^\circ \text{ F.}$  and  $104^\circ \text{ F.}$ , the entropy at  $104^\circ \text{ F.}$  can be found graphically and is

$$\phi_{104} = 0.13689^*$$

$$\text{Hence } \phi_w = 0.17429 + 2.71962 \log \frac{T}{564} - 0.0003929t + 0.0000003199t^2$$

reckoned from  $32^\circ \text{ F.}$  and valid between  $104^\circ \text{ F.}$  and  $572^\circ \text{ F.}$

\* From the Munich tables 0.13675 and Mollier 0.1367.

The specific heat equation given by Dieterici does not give the entropy values between  $32^\circ \text{ F.}$  and  $104^\circ \text{ F.}$  These entropy values given in the diagram have been found graphically by a step-by-step method.

For example, for

212	392	572
0.3124	0.5572	0.76

The liquid curve shown in the diagram has been drawn from values of  $\phi_w$  obtained in this way.

(b) **Superheated steam.** If the specific heat of superheated steam were constant, the entropy change for a constant pressure change of state could be determined directly.

With  $dQ = c_p dT = T d\phi$   
the entropy change would be

$$d\phi = c_p \frac{dT}{T}$$

Taking the dry saturated steam to be the initial state, the entropy change up to the superheated temperature is found by summation and gives

$$\phi - \phi_s = c_p \ln \frac{T}{T_s}$$

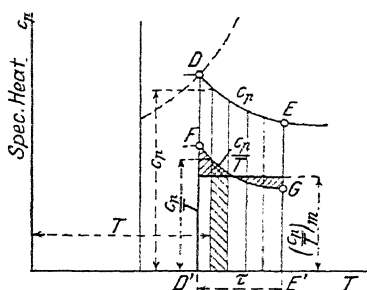


FIG. 190

On the entropy diagram, therefore, the line  $A_2A_3$  (Fig. 189) would be a logarithmic curve, as for gases and the liquid curve  $A_0A_1$ . It would be about twice as steep as the latter, since  $c_p$  for superheated steam at moderate pressures is about half that for water.

Actually this assumption regarding  $c_p$  is not valid. Near the saturation curve the specific heat  $c_p$  first decreases rapidly with increasing superheat and then increases (page 370). The above equation gives, therefore, only approximate values. Since the variation of  $c_p$  is complicated and cannot be represented by any simple equation, it follows that the entropy values also cannot be deduced from any simple equation. The test results supplied by Knoblauch and Jakob, and the further Munich values of  $c_p$ , enable the quantitative values of the entropy to be determined for all the pressures and temperatures likely to be used, without the aid of an equation. For a small change of state, whether  $c_p$  is constant or not,

$$dQ = c_p dT = T d\phi$$

hence 
$$d\phi = c_p \frac{dT}{T}$$

If now  $DE$  (Fig. 190) is the curve showing the variation of  $c_p$  at constant pressure, the values of  $c_p/T$  can be found for any number of points on this curve. By plotting these to a base of temperatures, the curve  $FG$  is obtained.

The area below this curve between two ordinates gives the quantity  $\frac{c_p}{T} dT$ , which is the elementary change of entropy.

The area  $FGE'D'$  is thus the total change of entropy in superheating through  $t^\circ$ . This area can be found either by means of a planimeter or

by dividing it into narrow strips and summing. If its mean height is

$$\left( \frac{c_p}{T} \right)_m \text{ the entropy increase during superheating through } t^\circ \text{ is}$$

$$= \left( \frac{c_p}{T} \right)_m t$$

The superheat curves at constant pressure in diagrams II and III have been drawn in this way.

**Expansion and compression of saturated steam at constant quality.**

On page 356 it was shown that the law connecting the pressure and volume under this condition is

$$p^{1.67} v_s = \text{constant}$$

For initially wet steam with quality  $q_0$ , since

$$v = q_0 v_s$$

$$p^{1.67} v = q_0 \times \text{constant} = \text{constant}$$

The heat quantities which have to be supplied or withdrawn in order to maintain the condition of constant quality during expansion or compression are easily found with the aid of the entropy conception.

Choosing the point  $A_2$  (Fig. 189) as the initial point, the curve of expansion for  $q = 1$  is  $A_2B$ . Hence the entropy increases during expansion. The heat which has to be supplied is given by the area below  $A_2B$ , i.e. the area  $A_2BB'A'_2$ . This heat is considerable for a large number of expansions.

If this heat be completely or partially withheld some of the steam condenses.

If, on the other hand, the steam be compressed from  $B$  to  $A_2$ , the entropy decreases by the amount  $B'A'_2$ . The area below  $BA_2$  represents the heat which has to be withdrawn. If this heat is not withdrawn during compression the steam becomes superheated. If more heat than this be withdrawn the steam becomes wet during compression.

From the above consideration it is improbable that steam in an engine cylinder expands or is compressed at constant quality.

At first sight it appears contradictory that steam, on expanding from 150 lb./in.<sup>2</sup> abs. to 1.5 lb./in.<sup>2</sup> abs. or less at constant quality (as in Fig. 189), should have heat supplied to it, for the dry saturated steam at 1.5 lb./in.<sup>2</sup> abs. contains less heat than that at 150 lb./in.<sup>2</sup> abs. This, however, is explained by the fact that the heat added and a part of the heat contained in the steam itself is converted to mechanical work during expansion and disappears from the steam.

For initially wet steam of quality  $q_0$  (point  $F$ ) the state curve at constant quality on the entropy diagram is  $FG$ . This line divides the horizontal intercepts between the water and saturation curves in the constant ratio  $q_0$ , since,

from page 375,  $q_0 = \frac{A_1F}{A_1A_2}$ , and this ratio is to remain constant as the

pressure drops. The area below  $FG$  represents the heat which has to be added during expansion. In compression ( $GF$ ) the same heat has to be removed. The line  $FG$  has been drawn for  $q_0 = 0.80$ , and the line  $F_1G_1$  for  $q_0 = 0.3$ , i.e. for very wet steam. In the latter case the entropy



decreases as the steam expands, which is contrary to the case with  $q_0 = 0.8$ . Hence, when the steam becomes very wet, heat has to be withdrawn and not added for expansion at constant quality. The curve for  $q = 0.5$  is almost parallel to the  $T$  axis, and hence represents approximately the limiting case where heat is neither added nor withdrawn. The entropy diagram enables a closer analysis to be made of each change in state.

### EXPANSION AND COMPRESSION OF STEAM IN AN INSULATED VESSEL. (ADIABATIC CHANGE OF STATE)

(a) **Saturated steam.** From the last section it will be clear that moderately wet steam becomes wetter on expanding adiabatically and dryer on being compressed adiabatically. This follows since no heat is supplied in expansion or compression. Hence, referring to page 375, it follows that the adiabatic for expansion falls more rapidly than the curve of constant quality and rises more rapidly in compression.

The entropy diagram shows the changes in quality and volume for a given pressure change. An adiabatic change is shown on this field by a vertical line (constant entropy, see page 127), e.g. the line  $A_2G$  (Fig. 189) represents an adiabatic expansion from 140 lb./in.<sup>2</sup> abs. to 1.5 lb./in.<sup>2</sup> abs.

The moisture content finally is given by  $1 - q \doteq \frac{JB}{BH} = 0.2$ . The line  $FK$  represents adiabatic expansion from 140 to 1.5 lb./in.<sup>2</sup> with an initial quality  $q = 0.8$ . The moisture content finally is  $1 - q = \frac{KB}{BH} = 0.326$ .

For intermediate points the qualities are found in the same way.

Starting, however, at an initial point  $A_1$  on the water curve, the expansion  $A_1L$  causes evaporation. The volume increase, with adiabatic expansion, amounts to about 0.2 of the dry steam volume. In all three cases the temperature drops from 358.5° F. to 115.9° F.

In addition, steam formation occurs when the initial state  $F_1$  is chosen ( $q = 0.3$ ), since the curve of constant quality  $F_1G_1$  lies to the left of the adiabatic vertical  $F_1M$ , and not to the right as at  $FG$ . The steam is dryer at  $M$  than at  $G_1$  and the quality increases from 0.3 at  $F_1$  to  $\frac{HM}{HB} = 0.38$  at  $M$ .

The pressure-volume curve, corresponding to the expansion along  $FK$ , can easily be constructed. If the initial volume is  $v_0 = q_0(v_s)_0$  and  $v = qv_s$  is any later volume, the proportional volume increase is

$$\frac{v}{v_0} = \frac{q}{q_0} \frac{v_s}{(v_s)_0}$$

The specific volumes  $v_s$  and  $(v_s)_0$  of the dry steam correspond to the initial and final temperatures  $t_0$  and  $t$ , and are given by the steam tables, while  $q_0$  is given and  $q$  is read from the entropy diagram. When, therefore, a temperature entropy diagram is available, any adiabatic can easily be drawn in the  $pv$  field by plotting the values

$$v = v_0 \frac{q}{q_0} \frac{v_s}{(v_s)_0}$$

against the corresponding pressures.

The curve  $AB$  (Fig. 191) represents an adiabatic expansion from initially dry steam at 150 lb./in.<sup>2</sup> abs. to 7.0 lb./in.<sup>2</sup> abs. If the steam remained dry the volume increase between 150 lb./in.<sup>2</sup> abs. and 45 lb./in.<sup>2</sup> abs. would be  $\frac{9.37}{\sqrt{2}} = 3.12$ .

From the entropy diagram, however, the quality of the steam after adiabatic expansion to 45 lb./in.<sup>2</sup> abs. is  $q = 0.925$ . Hence its volume is

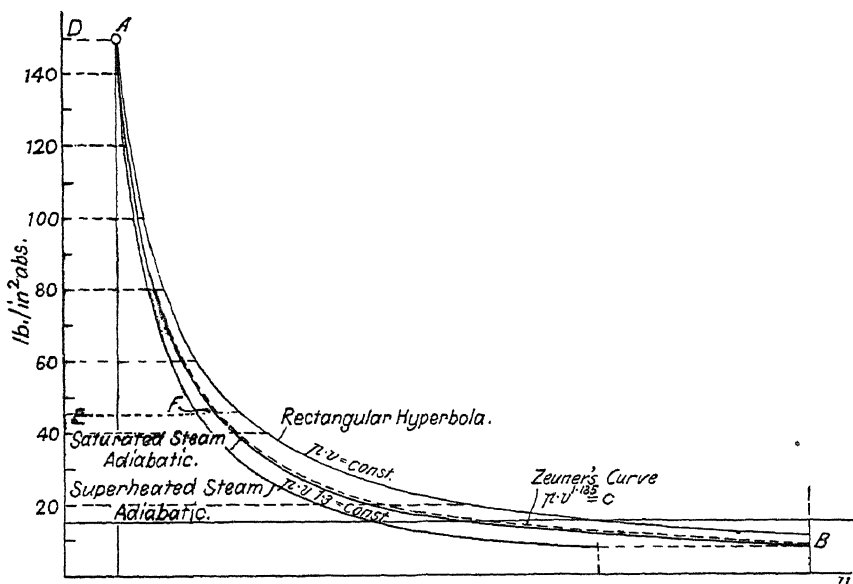


FIG. 191

smaller in the same ratio (see page 358), so that the expansion ratio is  $0.925 \times 3.12 = 2.886$ , and hence  $EF = 2.886 DA$ .

The absolute expansion work is obtained at the expense of the internal energy of the steam, since no heat is supplied. From page 363 the initial energy is  $I_0 = h_0 + q_0 \rho_0$  and the final energy  $I = h + q\rho$ . The difference  $I_0 - I$  gives the heat equivalent of the steam work.

$$\text{Hence } E = 778 \cdot [h_0 + q_0 \rho_0 - (h + q\rho)] \text{ B.Th.U./lb.}$$

$h_0$ ,  $\rho_0$ ,  $h$ , and  $q$  can be found from the tables,  $p_0$  and  $p$  are given. As shown above, the final quality  $q$  is found from the entropy diagram. The same process holds for the compression work.

The area below  $AB$  in Fig. 191 gives the expansion work between 150 and 7 lb./in.<sup>2</sup> abs. With  $q_0 = 1$ , the entropy diagram gives  $q = 0.849$ . Taking the other values from the tables gives

$$\begin{aligned} E &= 778 [330.9 + 779.4 - (144.5 + 0.849 \times 923.4)] \\ &= 778 \times 182.3 \end{aligned}$$

Hence 182.3 B.Th.U. are converted to work, i.e.  $E = 142,000$  ft. lb.

**Zeuner's approximate equations.** For steam having an initial quality  $q$  the adiabatic equation is given closely by

$$pv^{1.035 \div 0.1} q = \text{constant}$$

For steam initially dry, therefore, ( $q = 1$ )

$$pv^{1.135} = \text{constant}$$

With  $q = 0.8$ ,

$$pv^{1.115} = \text{constant}$$

It is obvious that the exponent of the adiabatic curve must be greater than that of the constant quality curve (1-0667), since in the adiabatic case the volume increases less rapidly as the pressure drops. With increasing initial dryness, however, the constant quality curves on the entropy field become steeper for pressures above 15 lb./in.<sup>2</sup> abs., i.e. the curves of constant quality tend to have the same slope as the adiabats. This explains the reduction of Zeuner's adiabatic exponent as the initial steam qualities are increased.

The equations are applicable down to about  $q = 0.7$  and for pressure ratios as high as 20. Fig. 191 shows a Zeuner curve (dotted) for dry steam. The steam work is—as shown previously for gases—with  $n = 1.035 \pm 0.1q$

$$E = \frac{144 p_0 v_0}{n-1} \left[ 1 - \left( \frac{p}{p_0} \right)^n \right]$$

$$= \frac{144 p_0 v_0}{n-1} \left[ 1 - \left( \frac{v_0}{v} \right)^{n-1} \right]$$

In the above example we have, from this,

$$= \frac{144 \times 150 \times 3.002}{0.135} - \left( \frac{7}{150} \right)^{0.119} \Bigg]$$

against 142,000 ft. lb. given above.

(b) **Superheated steam.** In an adiabatic change of state the pressure volume curve is given closely by the law

[illegible]

This relation (with the exponent 1.33) was previously established by Zeuner from his approximate equation of state of superheated steam and the assumption of constant specific heat. From Callendar's more accurate equation of state, Mollier has deduced the same equation. Although this deduction is also based on the assumption of constant specific heat for low pressures and high temperatures, the equation is sufficiently accurate to satisfy practical requirements. See also page 408.

From equation (444) it follows that the slope of the adiabatic for superheated steam is steeper than that of saturated steam and not so steep as that of gases, since the exponent 1.3 lies between 1.4 (for gases) and 1.135 (for saturated steam). The superheat adiabatic is also drawn in Fig. 191. As in the case of a gas, the temperature drops in an adiabatic



from  $A$ , the work area below the adiabatic curve with its steeper slope (Fig. 191) is smaller. If, however, the weight of steam in the two cases is the same, the adiabatic work, as shown in the above example, is greater, due to the higher value of  $v_0$  in the product  $p_0 v_0$ .

**Transition from the superheated to the saturated state.** In an adiabatic expansion, superheated steam approaches the saturated state as the pressure drops. As soon as the steam becomes saturated the equation  $pv^{1.3}$  no longer holds, and in its place the equation  $pv^{1.135}$  has to be applied.

In Fig. 186 an adiabatic for steam initially at 142 lb./in.<sup>2</sup> abs. and 572° F. is shown. It cuts the saturation curve at  $M$ . Hence, at 32.8 lb./in.<sup>2</sup>

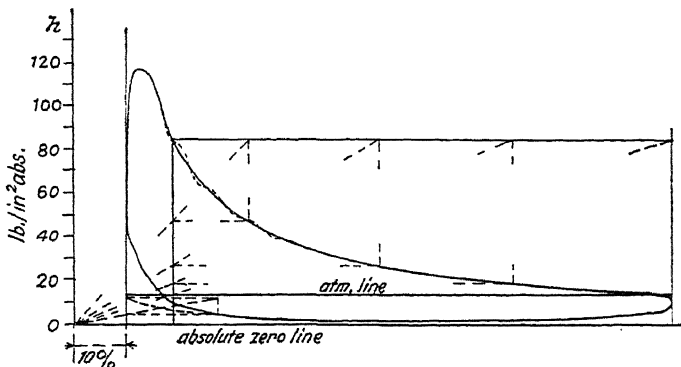


FIG. 192

abs. the superheat is zero. For higher initial temperatures the period of superheat is increased.

The same result is shown more simply by the entropy diagram (Fig. 189). Expansion begins at the point on the line  $A_2A_3$  where the temperature is 572° F. and is shown by the dotted vertical, which finishes at  $M_1$  on the saturation curve, where the pressure is 28 lb./in.<sup>2</sup> abs. and the temperature 248° F.

### ✓ ACTUAL STATE CHANGE IN THE EXPANSION AND COMPRESSION OF STEAM IN AN ENGINE CYLINDER

(a) **Engine using saturated steam.** The indicator diagrams of engines operating on saturated steam show an expansion which, in general, is expressed by

$$pv = \text{constant (rectangular hyperbola)}$$

from which the volume increases inversely as the pressure. It happens quite frequently that the expansion line follows this law exactly. Fig. 192,\* for example, shows a steam engine diagram in which this occurs. In order to analyse the expansion it is necessary to know the proportional clearance volume. The indicator spring number is also required along with the barometric pressure, so that the absolute zero pressure line can be drawn. Further, all the valves and the piston must be steam tight. In Fig. 192 these conditions have been fulfilled.

\* Cylinder diameter 12.2 in., stroke 20.5 in., 115 rev./min. Covers and cylinder steam jacketed.

For the same engine it is found that the expansion line depends on the weight of the steam supplied. With a greater supply it lies below, and with a smaller supply above, the rectangular hyperbola. In addition, exhaust at atmospheric pressure or to a condenser affects this, as well as steam jacketing.

In using saturated steam, it is always found that the steam is wet (frequently 20 per cent and more) at the beginning of expansion. Even if the steam in the supply pipe be dry, partial condensation occurs when it enters the cylinder, since the walls of the latter are colder than the live steam. Assuming a quality  $q = 0.8$ , the expansion law in a non-conducting cylinder would be the adiabatic  $pv^{1.115}$ . The rectangular hyperbola falls more slowly than this curve, hence it follows that the steam takes up heat from the walls during expansion.

If the quality were to remain constant during expansion, which means a supply of heat (as shown on page 376), the law of expansion

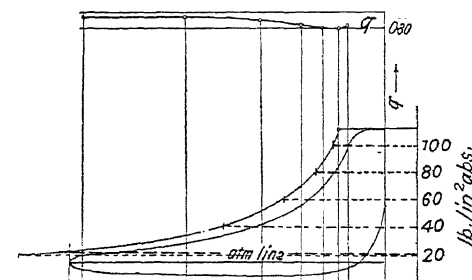


FIG. 193

would be  $pv^{1.067}$ , which is still below the rectangular hyperbola. Hence the steam becomes continuously dryer when the indicator shows an expansion line following the flatter curve  $pv = c$ . In Fig. 193 this is shown quantitatively for the above engine.

The condensate formed on the walls, as the steam enters the cylinder, maintains its initial temperature longer than

the steam, which becomes quickly cooled by the expansion. Since the pressure simultaneously decreases, a rapid evaporation of the condensate occurs, which is further aided by the walls heated during the inflow.

The compression line of the diagram, however, is very rarely a portion of a rectangular hyperbola. If no heat flow occurred during compression, the law of compression would be  $pv^{1.135} = \text{constant}$ , and this gives a quicker rise than the hyperbola. Hence the latter would require a removal of heat during compression. Since, however, the wall temperature, as shown by Callendar and Nicholson's tests, does not drop much below the mean temperature (even during the previous exhausting) it follows that heat is supplied by them during compression, particularly at the beginning. Hence a considerably steeper compression curve is obtained than that given by  $pv = \text{constant}$ . In most cases the compression line does actually lie above the hyperbola, as shown by the much higher end compression pressure obtained. (Figs. 192 and 193.)

An expansion line following the law  $pv = c$  is sometimes described as an isothermal, although it is well known that as the pressure of wet steam drops the temperature also drops. This term, which has been carried over from the theory of gases, should be avoided, since an isothermal for saturated steam is represented by an horizontal line on the  $pv$  field.

(b) **Superheated steam.** The actual expansion line for superheated steam falls more rapidly than that of dry saturated steam (Fig. 194).

This agrees with the fact that a given volume increase in an insulated cylinder shows a greater pressure drop for superheated steam than for saturated steam. If now a heat exchange occurs between the steam and walls and so changes the conditions, this difference still remains.

The expansion line of saturated steam lies above the adiabatic. The same holds for superheated steam. In trying to find a fractional exponent which will fit the general case of expansion, as shown by indicator cards, it is found that it lies between 1.25 and 1, but never assumes the adiabatic value 1.3.

In discussing the actual expansion line it should be noted that the temperature of superheated steam at the beginning of expansion is considerably lower than that in the supply pipe. While the steam is entering the cylinder it is cooled by the walls. If the range of superheat is low

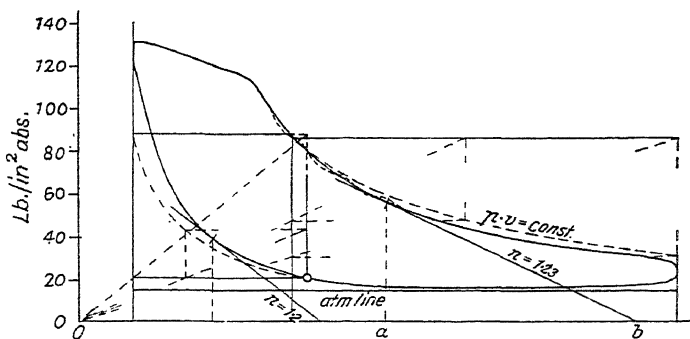


FIG. 194

initially, the steam may become saturated or wet at the beginning of expansion. In such cases the expansion line may be almost exactly a rectangular hyperbola.

When the range of superheat in the supply pipe is high, the steam will be superheated at the beginning of expansion. The range of superheat at this point depends on the weight of steam supplied, on the back pressure, and on whether the engine is a single cylinder or compound.

It rarely happens that the steam remains superheated to the end of the expansion. The transition point (as dealt with on page 383) where the steam becomes saturated depends on the conditions. In general, the treatment of the compression line is the same as for saturated steam. See Fig. 194, which refers to an engine using superheated steam without condenser.

### CHANGE OF STATE AT CONSTANT VOLUME

(a) **Saturated steam.** If dry or wet saturated steam, contained in a closed vessel of constant volume, gives up heat to the outside, partial condensation occurs and the pressure and temperature drop.

Assuming the pressure or temperature drop to be known, the heat given up  $Q$ , and the weight of steam condensed, are easily determined as follows.

Let  $p_0$  be the initial pressure and  $q_0$  the initial quality. The initial volume of 1 lb. is

$$v_0 = q_0 (v_s)_0$$

If  $q$  is the quality after cooling, the final volume is

$$v = q v_s$$

Since the volume and weight remain constant,

$$v_0 = v$$

so that 
$$q = q_0 \frac{(v_s)_0}{v_s}$$

Hence, for every pound of the total steam, the amount

$$(q_0 - q) \text{ lb.}$$

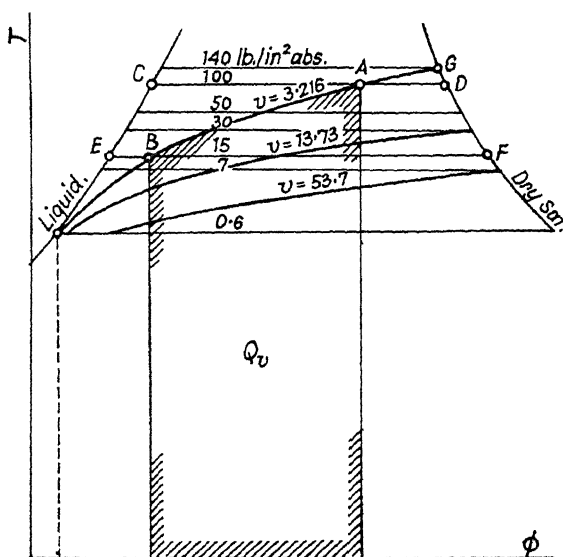


FIG. 195

is reconverted to water. The values  $v_s$  and  $(v_s)_0$  are the dry steam volumes as given by the steam tables at  $p$  and  $p_0$ .

Since, during cooling, no volume change occurs, no external work is done on or by the steam, so that no part of the heat is converted to mechanical work. Hence the heat withdrawn is taken entirely from the sensible heat  $h$ , and the internal latent heat  $\rho$ .

The sum of these is known as the internal energy of the steam. The initial value of this is

and the final value



The difference gives the heat withdrawn

Since  $q$  is found from the equation above, and  $h_0$ ,  $\rho_0$ ,  $h$ , and  $\rho$  are given by the tables,  $Q$  can be determined.

This change of state is represented on the entropy diagram as follows (Fig. 195). The point  $A$ , corresponding to the initial state of the steam of quality  $q_0$ , lies on the temperature level  $T_0$  and its position on this line is given by

$$q_0 = \frac{CA}{CD} \quad (\text{see page 375})$$

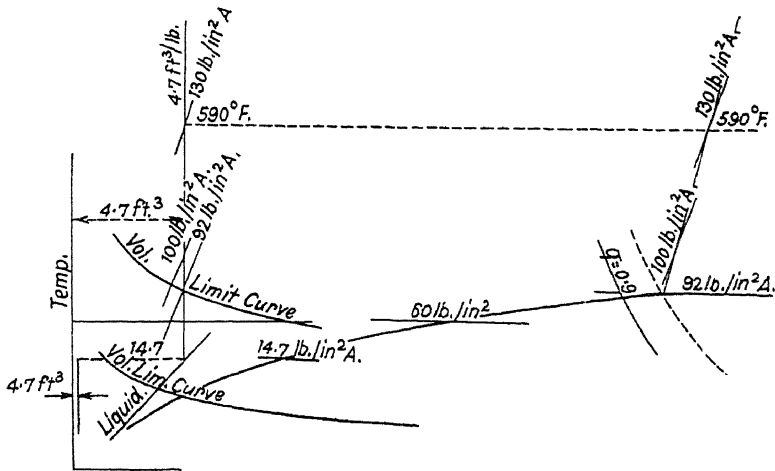


FIG. 196

The point  $B$ , at any lower temperature  $T$ , divides  $EF$  in the ratio

$$q = \frac{EB}{EF}$$

Hence, with the above value for  $q$ ,

$$\frac{EB}{EF} = \frac{CA}{CD} \frac{(v_s)_0}{v_s}$$

In this way any point  $B$  lying below  $A$  can be found.

The area below  $AB$  gives the heat  $Q_1$ .

A few constant volume curves have been drawn in Fig. 195. If  $AB$  is continued to lower pressures, it approaches the water curve, but never cuts it, since, even with a large removal of heat, a certain weight of steam remains, although this weight can be made negligibly small.

If  $AB$  is continued to higher pressures, it cuts the saturation curve. At  $G$  all the moisture has disappeared. If more heat be supplied the steam becomes superheated.

The  $TV\phi$  diagram offers a ready means of drawing the entropy curve  $v = \text{constant}$ . As shown in Fig. 196, it is only necessary to drop a



steam, the line is also straight, but instead of passing through  $O$  it now passes through the point  $O_1$  lying farther to the right, and

$$O_1D = \alpha^\circ$$

The line  $AB$  has been drawn for  $d = 0.1873$  lb./ft.<sup>3</sup> and  $EF$  in the same way for  $d = 0.3122$  lb./ft.<sup>3</sup>, i.e. for denser steam. For the latter value  $\alpha$  is also different and the point of intersection  $O_2$  lies nearer  $D$ .

The following table of  $\alpha$  values has been constructed from a graphical interpolation of the Munich values.

Density lb./ft. <sup>3</sup>	Saturation Pressure lb./in. <sup>2</sup> abs.	$\alpha^\circ$ F.
0.03	11.0	448.0
0.06	24.0	422.3
0.09	38.0	399.8
0.12	52.0	379.1
0.15	65.5	362.0
0.18	79.0	345.8
0.21	93.0	331.4
0.24	107.0	320.6
0.27	121.0	310.7
0.30	136.0	301.7
0.33	151.0	293.6

On the  $TV\phi$  diagram the pressure rise can be determined directly in the  $TV$  portion. The entropy curve can also be drawn by projecting horizontals across to cut the lines of pressure (Fig. 196).

**Example 14.** Find the heat given up by 1 ft.<sup>3</sup> of dry saturated steam at 150 lb./in.<sup>2</sup> when it is cooled, at constant volume, to 140° F. Find also the final pressure and quality.

In the initial state the specific volume of the dry steam is  $(v_s)_0 = 3.002$  ft.<sup>3</sup>/lb. In the final state the corresponding volume is 123.1 ft.<sup>3</sup>. Hence the final quality is

$$\frac{3.002}{123.1} = 0.02439$$

Hence the moisture content is  $100 - 2.44 = 97.56$  per cent. Its pressure, as shown by the tables, is 2.89 lb./in.<sup>2</sup> abs.

The initial internal energy for 1 lb. is  $330.9 + 779.4 = 1110.3$  B.Th.U. and, finally, it is

$$h + qp = 108 + 0.02439 \times 947.6 = 131.11 \text{ B.Th.U.}$$

Hence the heat given up per pound is

$$Q = 1110.3 - 131.1 = 979.2 \text{ B.Th.U.}$$

The weight of 1 ft.<sup>3</sup> at 150 lb./in.<sup>2</sup> abs. is 0.3331 lb. Hence the heat given up is

$$0.3331 \times 979.2 = 326 \text{ B.Th.U.}$$

If cooling had taken place at constant pressure (usual case) the steam would give up  $H_0 - h = 1193.1 - 108 = 1085.1$  B.Th.U./lb., or, for the weight in the vessel,  $0.3331 \times 1085.1 = 361$  B.Th.U.

**Example 15.** Find by how much the pressure of superheated steam at 185 lb./in.<sup>2</sup> abs. and 716° F. decreases when it is cooled to 446° F. at constant volume.

The specific volume in the initial state is given by

$$\begin{aligned} v_a &= \frac{.5961T}{185} - 0.2563 \\ &= \frac{.5962 \times 1176}{185} - 0.2563 = 3.79 - .0256 \\ &= 3.534 \text{ ft.}^3/\text{lb.} \end{aligned}$$

Hence the density is

$$d = \frac{1}{3.534} = 0.283 \text{ lb./ft.}^3$$

The  $a$  value corresponding to this from the table of values (p. 389) is

$$a = 306.8$$

$$\text{Hence } \frac{p}{p_0} = \frac{306.8 + 446}{306.8 + 716} = \frac{752.8}{1022.8} = 0.735$$

$$\text{and } p = 185 \times 0.735 = 136 \text{ lb./in.}^2 \text{ abs.}$$

The pressure drop is  $185 - 136 = 49 \text{ lb./in.}^2$

**Example 16.** Dry saturated steam at 136 lb./in.<sup>2</sup> abs. remains in a superheater when steam is shut off to the engine. Find the rise in pressure if hot gases at 1022° F. flow over the superheater tubes, while the superheated steam remains shut off from the boiler and engine.

The steam is heated at constant volume, and the value of  $a$  corresponding to 136 lb./in.<sup>2</sup> abs. is 301.7. If the steam temperature rises to 1022° F. the pressure rise ratio is

$$\frac{p}{p_0} = \frac{301.7 + 1022}{301.7 + 350.7} = 2.03$$

The pressure therefore becomes  $136 \times 2.03 = 276 \text{ lb./in.}^2 \text{ abs.}$

The pressure rise is much more dangerous when liquid or wet steam remains in the superheater, since the steep pressure rise shown in Fig. 197 for saturated steam continues so long as any moisture is present. Thereafter the pressure increase is much slower, as shown by the straight superheat lines ( $EF$ ).

**Steam diagrams.** 1. The  $T\phi$  diagram (Fig. 198) contains curves of constant pressure and constant quality for saturated and superheated steam within the limits of pressure and temperature normally dealt with. The curves in the superheat field have been drawn from the Munich test results.

Volume changes in the superheat region can be calculated by means of the Linde-Tumlirz equation given in the top left corner of the diagram and those in the saturated region from steam table values.

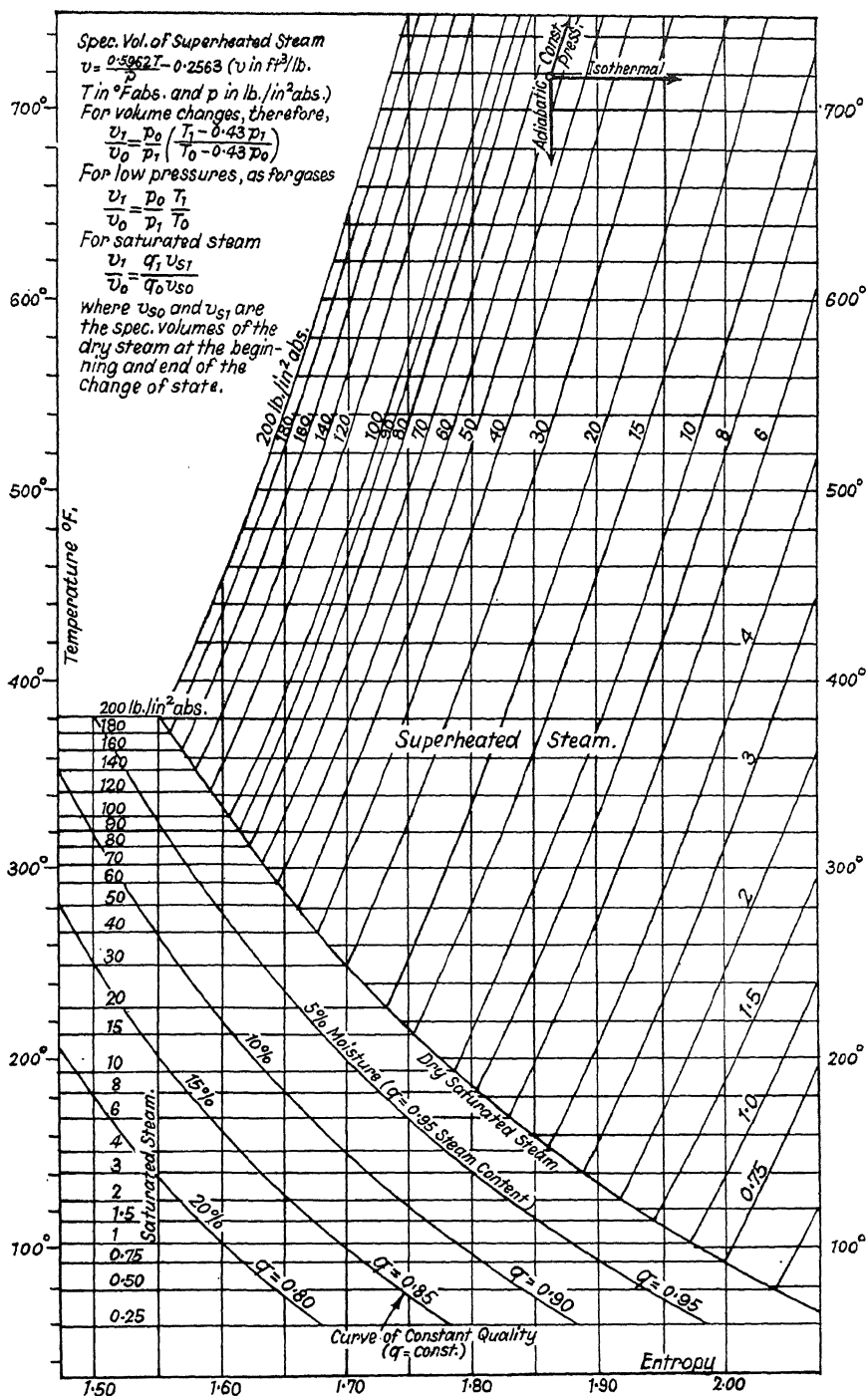


FIG. 198

The liquid curve is not shown in this diagram, but diagram II embraces the complete water and steam region from the smallest to the highest pressures and up to temperatures of 850° F.

2. The temperature volume entropy ( $TV\phi$ ) diagram (diagram II\*). In this diagram the temperatures are plotted as ordinates against volumes on the left side and entropy on the right as abscissae. Two diagrams are thus embodied in one, a state diagram  $TV$  and a heat diagram  $T\phi$ . For the first, three different volume scales are required in order to represent the volumes between the critical pressure region and 0.6 lb./in.<sup>2</sup> abs. The three regions are, from 0.6 to 15 lb./in.<sup>2</sup> abs., from 15 to 600 lb./in.<sup>2</sup> abs., and from 600 to the critical pressure 3200 lb./in.<sup>2</sup> abs. The three

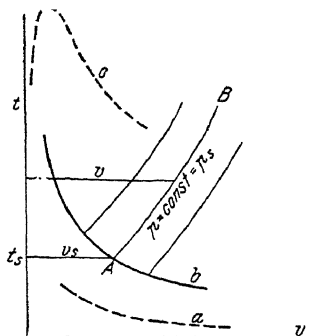


FIG. 199

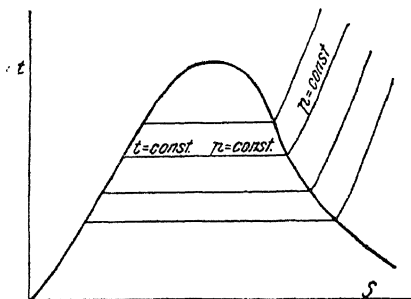


FIG. 200

groups of volume curves are shown on the left. Figs. 199 and 200 represent the essential features of the two diagrams. No explanation is required for the  $T\phi$  diagram. It starts at water at 32° F. on the one side and dry saturated steam at 0.6 lb./in.<sup>2</sup> abs. on the other, and rises to the critical point where the two curves join, and thus shows the entire liquid and saturated curve. In the superheat field the diagram extends to 850° F. and 1700 lb./in.<sup>2</sup> A few curves at higher pressure are also included. Lines of constant pressure are plotted in the superheat field, the entropy values of which are those given by the earlier Munich tests on the specific heats. The volume curves have been omitted both in the wet and in the superheat fields. This is because the volume curves are given on the left as a function of the temperature, so that, for each state of the steam, the temperature, volume, and entropy are represented by a length, while the pressure values are read on the labelled curves.

The volume-temperature curves form a state diagram, which gives the relations between the pressure, volume, and temperature in the dry, wet, and superheated state without reference to the entropy.

In order that the superheat region may be included, curves of constant pressure are plotted in the  $TV$  field. These curves slope upwards to the right and start from the dry saturated curve.

At any point *A* on the saturation curve (Fig. 201) the volume and temperature are given by the co-ordinates themselves, while the pressure

\* This was first published by the author in *Z. V.d.I.* (1911), page 1506. The diagram is on a smaller scale, and contains no constant total heat curves and fewer volume curves. Regarding the changes due to the later test results, see page 416.

is read on the horizontal pressure line running across the complete field. The co-ordinates of the curve  $AB$  indicate how the volume of superheated steam increases with temperature when the pressure is kept constant. Every such curve corresponds to a particular pressure, which runs into the saturation pressure and which is labelled. These superheat curves range from 0.85 to 350 lb./in.<sup>2</sup> abs., and are based on the Munich tests on superheated steam volumes.\* See page 417 regarding the plotting of the volume curves in the high pressure region.

Fig. 201 shows how the specific volume of superheated steam on expanding (at constant pressure) deviates from that of a gas. The abscissa of  $A'$  would give the gas volume in the saturated state according to the gas equation

with  $R = 86$  as the gas constant for steam. The true volume  $v_s$  at  $A$  is smaller. On being supplied with heat the gas volume changes, as shown by the straight line  $A'B'$ , while the superheated steam volume changes as shown by the curved line  $AB$ .

The volume curves for superheated steam, which extend as far as the top edge of the diagram, belong to the intermediate pressure region, i.e. from 15 to 600 lb./in.<sup>2</sup> abs. The volume scale has been marked along the 700 lb./in.<sup>2</sup> abs. pressure line; the scale is 1 ft.<sup>3</sup> = 0.8 in. The volume scale of the lowest region is 1 ft.<sup>3</sup> = 0.02 in., and is drawn along the lower edge.

The saturation curve for the critical region is drawn in the superheat field of the intermediate region, but has a volume scale ten times greater, i.e. 1 ft.<sup>3</sup> = 8 in.

Two curves of constant quality  $q = 0.5$  and  $0.9$  are drawn in the entropy field. By their aid the quality  $q$  at any other point in the wet field, where the temperature is  $T$ , is easily determined, while the corresponding volume is found by marking off this quality on the  $TV$  field, i.e. the distance to the saturation curve at the level  $T$  is divided in the ratio  $q$ . For very wet steam the volume has to be increased by the amount  $0.016(1 - q)$ , i.e. the volume of the water.

In the superheat field any point on the entropy diagram can be transferred to the volume field by drawing an horizontal across to the same pressure line.

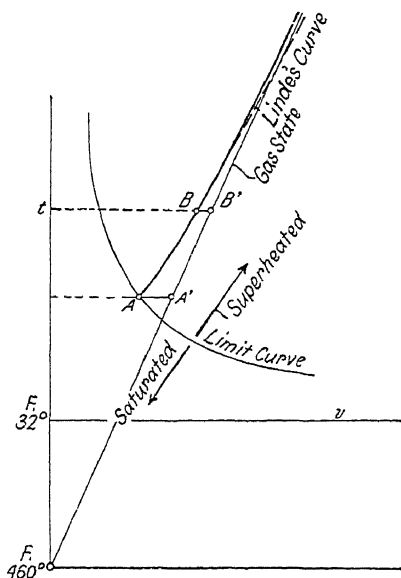


FIG. 201

\* See *Forschungsarbeit*, Vol. 21. The thermal properties of saturated and superheated steam between 100° and 160° C. Part I by Knoblauch, Linde, and Klebe; and Part II by R. Linde.

In this way any change of state in the entropy field can be transferred to the volume diagram. In addition to the volume and temperature, the pressure is also given. In the reverse process, changes in the volume field can be transferred to the entropy field, so that the heat supplied or withdrawn per pound can be determined.

From the volume field alone the change in volume with temperature, at constant pressure, is given by the constant pressure lines. The change in pressure and temperature at constant volume is given by the vertical lines and the change in pressure and volume at constant temperature by the horizontal lines.

The changes of pressure with temperature in an adiabatic change of state are given by the perpendiculars in the entropy diagram. The corresponding volume changes are given by the points of intersection of the horizontals (constant temperature) with the lines of constant pressure in the volume diagram. In the wet field the points with the same quality have to be located in the two fields as mentioned above.

For the adiabatic  $pV$  curve, Zeuner gives

$$pv^{4/3} = \text{constant}$$

while Callendar's equation is

$$p(v - 0.01602)^{1.3} = \text{constant}$$

or, approximately,  $pv^{1.3} = \text{constant}$

On trying out the adiabatics in diagram III, good agreement is obtained with Callendar's equation, while the exponent given by Zeuner appears too high. See page 415.

The adiabatics in the wet field are applied in the same way from any pressure up to the critical, but space prevents a discussion here regarding suitable equations.



## CHAPTER VIII

### STEAM AT HIGH PRESSURES

UP to the year 1921 steam pressures above 300 lb./in.<sup>2</sup> were not of general technical interest, since the maximum pressures used for steam engines or turbines at that time lay between 230 and 300 lb./in.<sup>2</sup>, while the normal pressures adopted were between 180 and 200 lb./in.<sup>2</sup> At this time W. Schmidt—who carried out the pioneer work, 25 years earlier, of introducing superheated steam for operating prime movers—published the test results obtained from a 150 h.p. quadruple expansion engine working with steam at about 825 lb./in.<sup>2</sup> abs. and at a temperature of 840° F. After this engine had proved that pressures of this order, along with a high initial temperature, meant a considerably reduced heat supply per unit of work obtained, and that the construction of steam prime movers and boilers was possible for these conditions, general interest was aroused in high pressure steam, particularly in Germany and the United States. In addition, it had the effect of intensifying scientific research in this field, which has led to an important completion of our knowledge of the properties of steam.

### SATURATED STEAM

**General relations of state.** The relations of state for high pressure steam will be best understood by considering the generation of steam under constant pressure with successively increasing pressures, as represented in Fig. 202.

At the freezing point (32° F.) let the specific volume of the water be  $\sigma_{32}$  and its pressure  $p_1$ . When heat is supplied, with this pressure constant, the volume of the water increases with the rising temperature until the saturation temperature  $t_1$  is reached, the volume of the water then being  $\sigma_1$ . During evaporation  $t_1$  remains constant, while the volume of the steam increases in accordance with the value of the quality  $q$  and attains the value  $v_{s1}$  for  $q = 1$ ,  $v_{s1}$  being greater than the volume of the water by the amount  $v_{s1} - \sigma_1$ . For a higher pressure  $p_2$  the volume of the liquid  $\sigma_2$  is greater at the boiling point and the volume  $v_{s2}$  of the dry saturated steam smaller. Hence the difference between the steam and water volume is also smaller, as shown by the small triangle in Fig. 202.

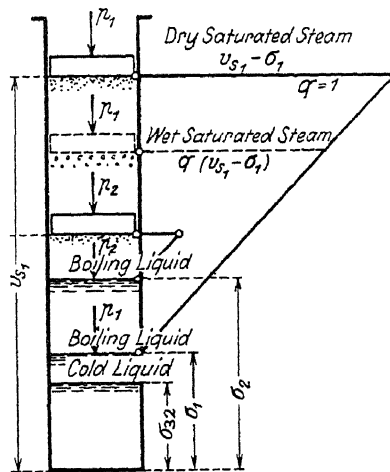


FIG. 202

A limit is reached at a still higher pressure  $p_c$ , where the volume of the steam is found to be the same as the volume of the water at the boiling

point. This means that no evaporation, with its attendant volume increase, occurs. At this pressure, which is called the critical pressure, and at the corresponding (critical) boiling temperature the heated liquid is found to have passed over to the steam state without passing through

the wet steam state. If heat be supplied at the constant pressure  $p_c$  to the water after it reaches the critical state, the temperature immediately rises above the critical. This is contrary to what occurs in evaporation at sub-critical pressures, where the temperature only increases after all the water has evaporated. At pressures below the critical, the volume of the steam only commences to increase according to the expansion law for superheated steam after evaporation is complete, whereas at the critical pressure this commences immediately the critical temperature is exceeded.

By plotting the volumes of liquid and saturated steam against pressures (Fig. 203, *b* and *c*) two curves are obtained, which meet at the critical point. The curve just at the junction may be continuous, as in Fig. 203*b*, or broken, as in Fig. 203*c*. It was formerly assumed that the curve was continuous, but from Callendar's latest tests the broken curve is obtained. For this, however, the water must be perfectly free from air. This condition is not likely to be fulfilled under actual working conditions.

#### The pressure temperature curve.

The relation between the pressures and saturation temperatures up to the critical state has been determined by various investigators. Early tests were carried out by

Cailletet and Colardeau, Batteli, and by de Laval, while the latest and most accurate are those of Holborn and Baumann.\* The course of the pressure temperature curve is drawn diagrammatically in Fig. 203*a*. Fig. 204 gives the latest test results to scale. The critical pressure is 3200 lb./in.<sup>2</sup> abs. and the critical temperature 705° F.

\* Tests by Keyes and Smith at the Massachusetts Institute of Technology showed almost complete agreement with these.

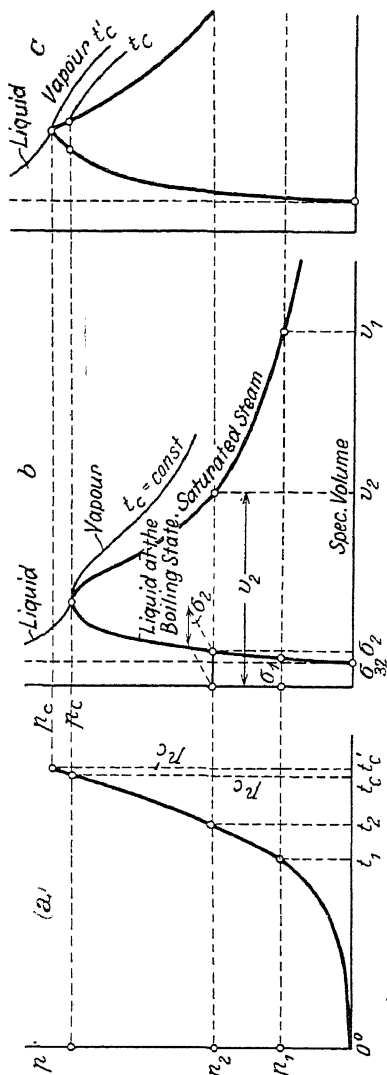


FIG. 203

Callendar was the first to confirm that at 705° F. and 3200 lb./in.<sup>2</sup> abs. the meniscus, but not the separating surface between the steam and water, disappears. Hence the surface tension is then zero, as has been shown by Traube and Teichner. As opposed to Holborn and Baumann's results, however, Callendar has managed to raise the saturation pressure to 3655 lb./in.<sup>2</sup> and the saturation temperature to 717° F., provided all trace of air be removed from the water. In addition, Callendar found that, with the same provision, the volume of the liquid still increases between 705 and 717° F. and the steam volume still decreases. In this region, steam and water can, as in the lower pressure regions, exist in any proportion. The state of the mixture, however, appears to be unstable and the equilibrium is disturbed by the slightest addition of gases.

**Latent heat, sensible heat, and total heat.** Since at the critical state no evaporation occurs, the latent heat is zero and the total heat supplied to generate steam from cold water consists of the sensible heat. This state, according to Callendar, does not occur for air-free water till the temperature is 717° F., so that at 705° F.—the previously

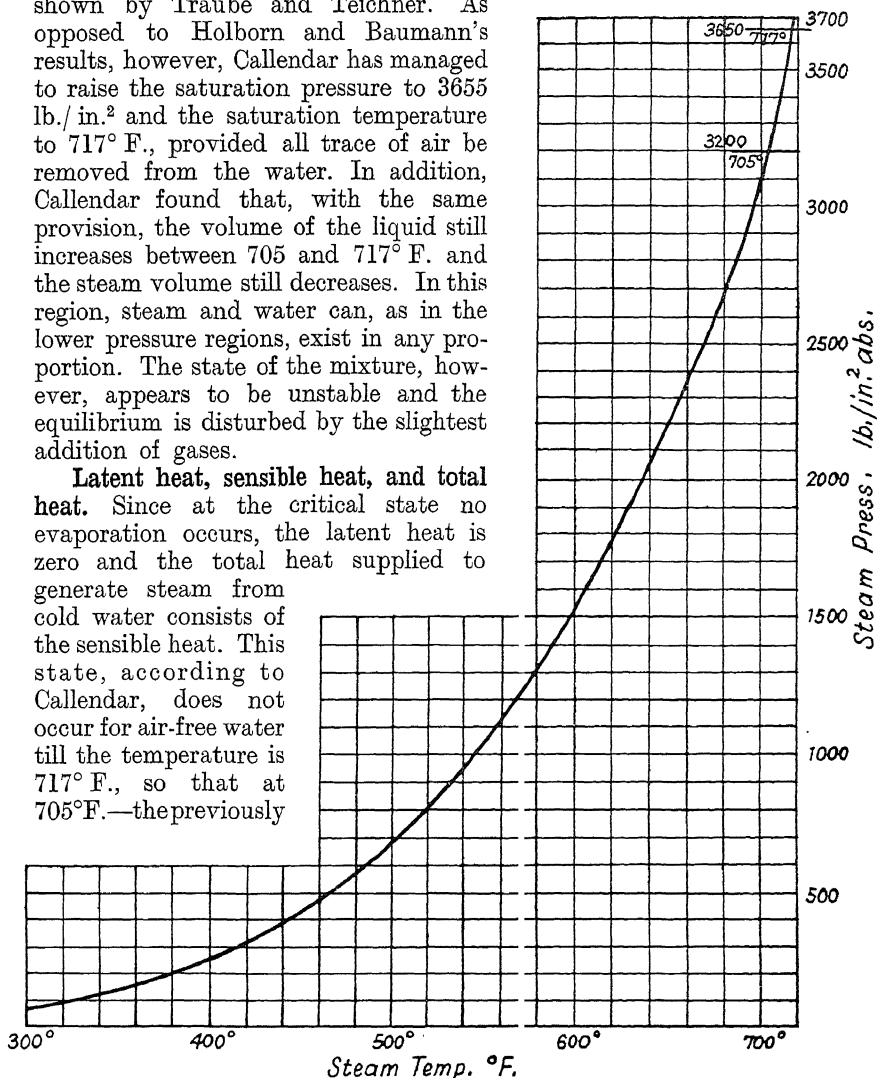


FIG. 204

accepted critical temperature—the latent heat is still appreciable and amounts to 130 B.Th.U./lb.

The latent heats between 410° F. and 480° F. have recently been determined calorimetrically by means of very careful tests by Jakob in the German Reichsanstalt. (See *Forschungsarbeiten*, No. 310.) The table below shows these values along with the specific volumes of the dry saturated

steam obtained by Jakob from them. They serve to complete the older calorimetrically determined values of Holborn and Henning above 360° F., where the deviation commences to be appreciable. In Fig. 205 Jakob's and Holborn and Henning's values have been plotted along with the curve constructed by the author in 1911, from the earlier volume measurements of Ramsay and Young and of Batteli, between 360° F. and the critical point. At the same time attempts were made to keep as close to the Holborn-Henning test results as possible.

TABLE OF LATENT HEATS AND SPECIFIC VOLUMES OF SATURATED STEAM BY JAKOB

$t$	$L$ B.Th.U./lb.	$v_s$ ft. <sup>3</sup> /lb.	$t$	$L$ B.Th.U./lb.	$v_s$ ft. <sup>3</sup> /lb.
80	1047.6	633.1	280	924.5	8.63
100	1036.8	350.9	300	909.9	6.459
120	1025.6	203.4	320	894.4	4.908
140	1013.9	123.1	340	878.4	3.785
160	1002.6	77.35	360	861.7	2.957
180	990.7	49.64	380	843.8	2.335
200	978.5	33.62	400	825.5	1.865
220	966.1	23.13	420	806.4	1.502
240	952.4	16.32	440	785.9	1.218
260	938.7	11.76	460	763.6	0.995
			480	740.0	0.817

In addition, Callendar's latent heat values in the critical region are also plotted (between 705 and 716° F.), and only assume a zero value when the temperature becomes 716° F.

The author's earlier (1911) curve is evidently too high, due to the uncertain volume measurements of Ramsay and Young in the saturated region. In addition, Holborn and Henning's values were evidently too high. From Eichelberg's thermodynamic calculations (*Forschungsarbeiten* No. 220, 1920), which are based on the Munich specific heat values, it later became obvious that the latent heats in this region should be reduced, and this has been confirmed by Jakob's tests. In the actual critical region, however, the earlier curve agrees with Callendar's but finishes at 705° F.

In the same diagram the liquid heats are also plotted. Those up to 590° are from Dieterici's specific heat values, and those in the critical region are due to Callendar.\* On the assumption that water at the saturation temperature contains an equal volume of steam in solution, Callendar has established the equation for the sensible heat as

$$h = ct + L - \sigma$$

which agrees well with the experimental values.

\* The recent test results supplied by the Bureau of Standards between 32° F. and 520° F. and published in *Mech. Eng.* (1929), No. 2, page 126, are plotted in Fig. 205. They agree closely with Dieterici's values up to 400° F. and then become somewhat larger. These measurements have been extended recently. (*Mech. Eng.*, February, 1930.)

Finally, the total heat values  $h + L$  are plotted in Fig. 205. These attain a maximum value of 1202.4 B.Th.U. at 464° F. and decrease from there to 993.6 at 705° F and 838.8 at 716° F. (See also page 406.)

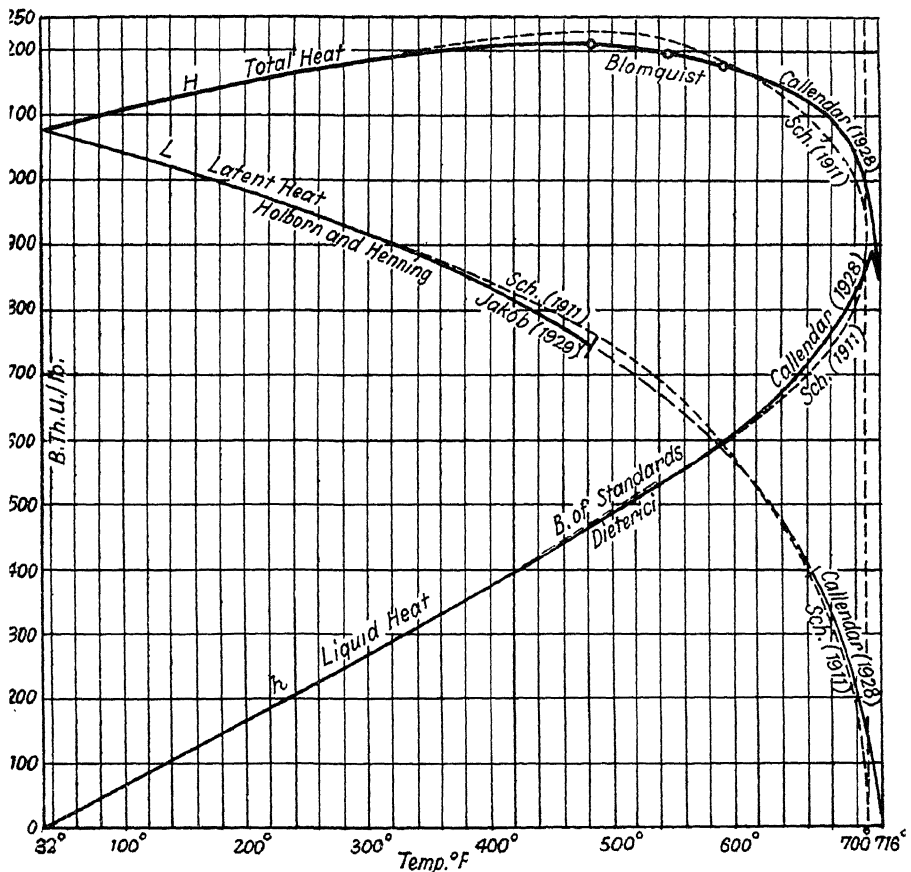


FIG. 205

In Fig. 206 are shown Callendar's total heats of liquid and steam to a base of saturation pressures.\* At 716° F. Callendar found that the mixture was in the state of a liquid with all the steam condensed.

**Specific volume and density of saturated steam and water.** In the region between 360° F. and 480° F. (about 140 to 576 lb./in.<sup>2</sup> abs.) the specific volumes of the steam have been calculated by Jakob from his accurate measurements of the latent heats, using the Clapeyron-Clausius equation along with the Holborn-Baumann steam pressure curve. There can be no doubt that Jakob's values are the most accurate at present. (See the table on page 398.)

Callendar's values of  $\rho_s = \frac{1}{v_s}$  and  $\rho_w = \frac{1}{\sigma}$  for steam and water have

\* Eng. (October, 26th, 1928). Steam tables and equations.

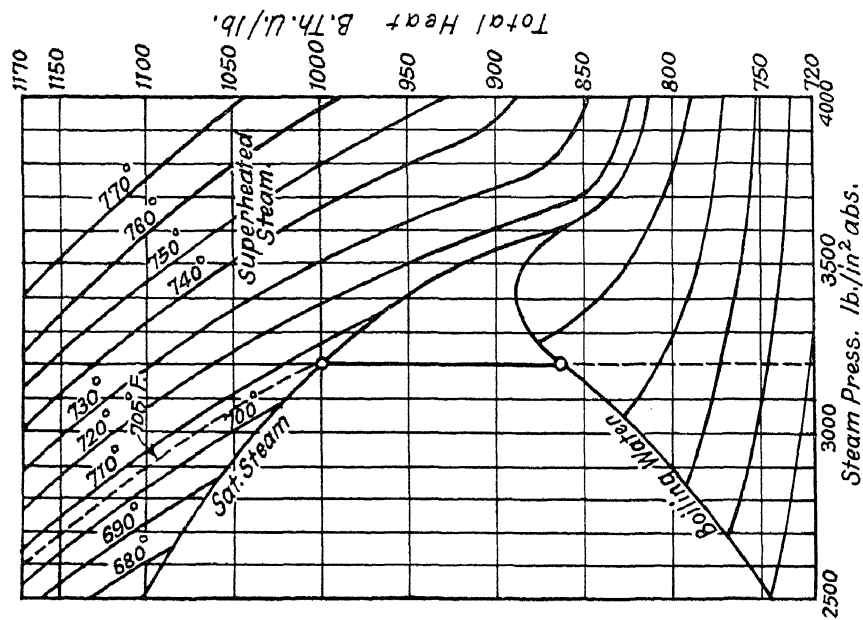


Fig. 206

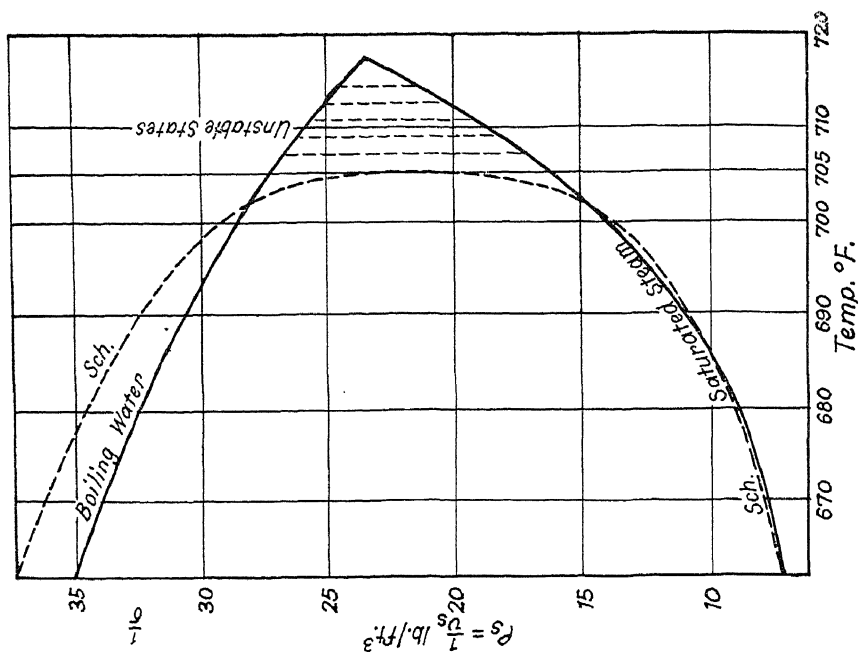


Fig. 207

been plotted in Fig. 207. The author's earlier values are also shown (dotted). It will be seen that the agreement for the steam densities up to 702° F. is good. In the liquid region the deviations are greater, since the curve marked "Sch." is extrapolated from Ramsay and Young's values, which extend to 518° F., whereas Callendar's curve is based on Waterson's values, which are lower and extend as far as 600° F. (Vol. ii, 4th Edition, Fig. 130.)

### SUPERHEATED STEAM

**Specific volume.** Early tests on the direct determination of the specific volume of superheated steam between 350° F. and 500° F. were carried

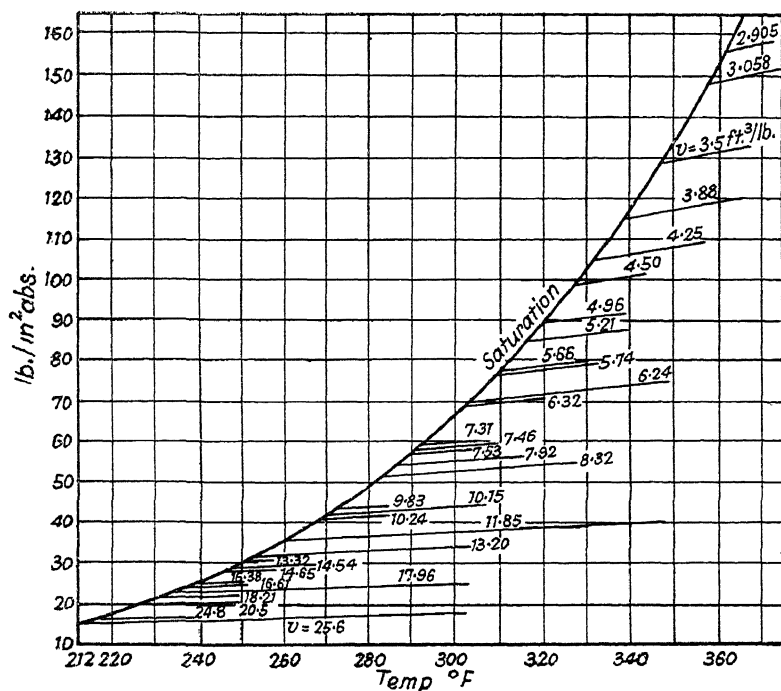


FIG. 208

out by Ramsay and Young, and up to the critical state by Battelli. It was found later, however, that the results in the saturation region were inaccurate, due to the effect of the walls of the measuring tubes. Tests, in which this was avoided by heating the walls and using a vessel in place of a pipe, were first carried out in the Technical Physical Laboratory at Munich (*F.A.* No. 21, Knoblauch, Linde, and Klebe). The steam was heated at constant volume and the curves of constant volume shown in the *pt* diagram (Fig. 208) were obtained, from which the equation of state (given on page 371) was deduced by Linde. In addition, the steam pressure curve was re-determined, so that, by continuing the curves of constant volume backwards, the saturation volumes would be determined. These tests lay within the region 15 to 155 lb./in.² abs.

Tests of a similar nature have recently been carried out between 1280 and 3700 lb./in.<sup>2</sup> in the Massachusetts Institute of Technology by Smith. The results are shown in Fig. 209 along with their saturation

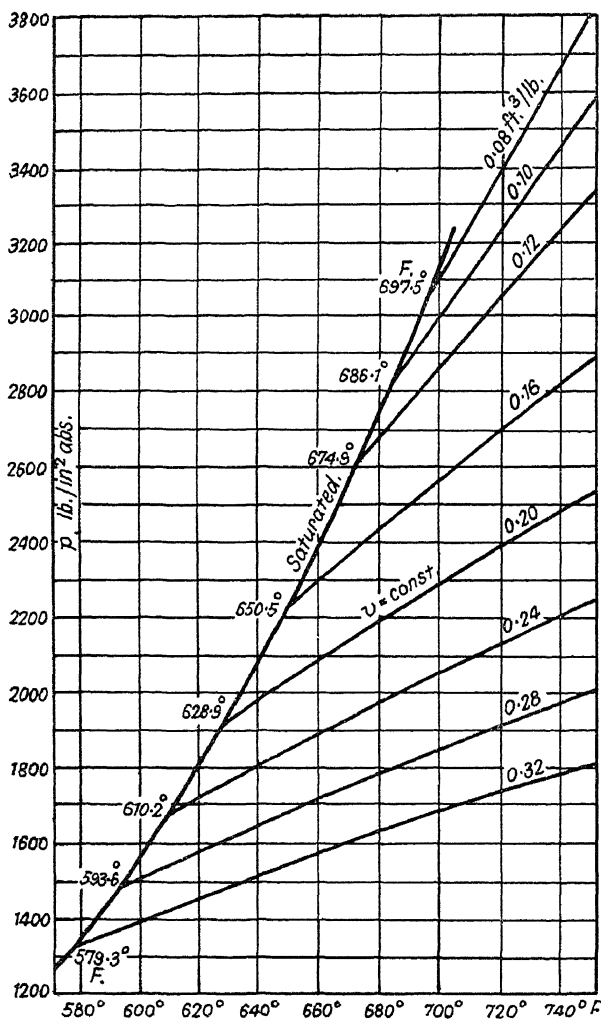


FIG. 209

pressure and temperature curve, which agrees well with Holborn and Baumann's, and from which the saturation volumes are also given.

In Fig. 210 these results, along with Callendar's, have been plotted with pressures as ordinates to a base of volumes, the limits of pressure being 1400 lb./in.<sup>2</sup> and 3700 lb./in.<sup>2</sup> and temperature 626 to 752° F.

**Pressure temperature measurements in adiabatic expansion.** The first measurements in the expansion of superheated steam at pressures of a few atmospheres were carried out by Hirn and Cazin, who allowed steam



to flow from a closed vessel and determined the pressure and temperature changes of the residual steam in the vessel. In rapid discharge the expansion of these residuals is adiabatic.

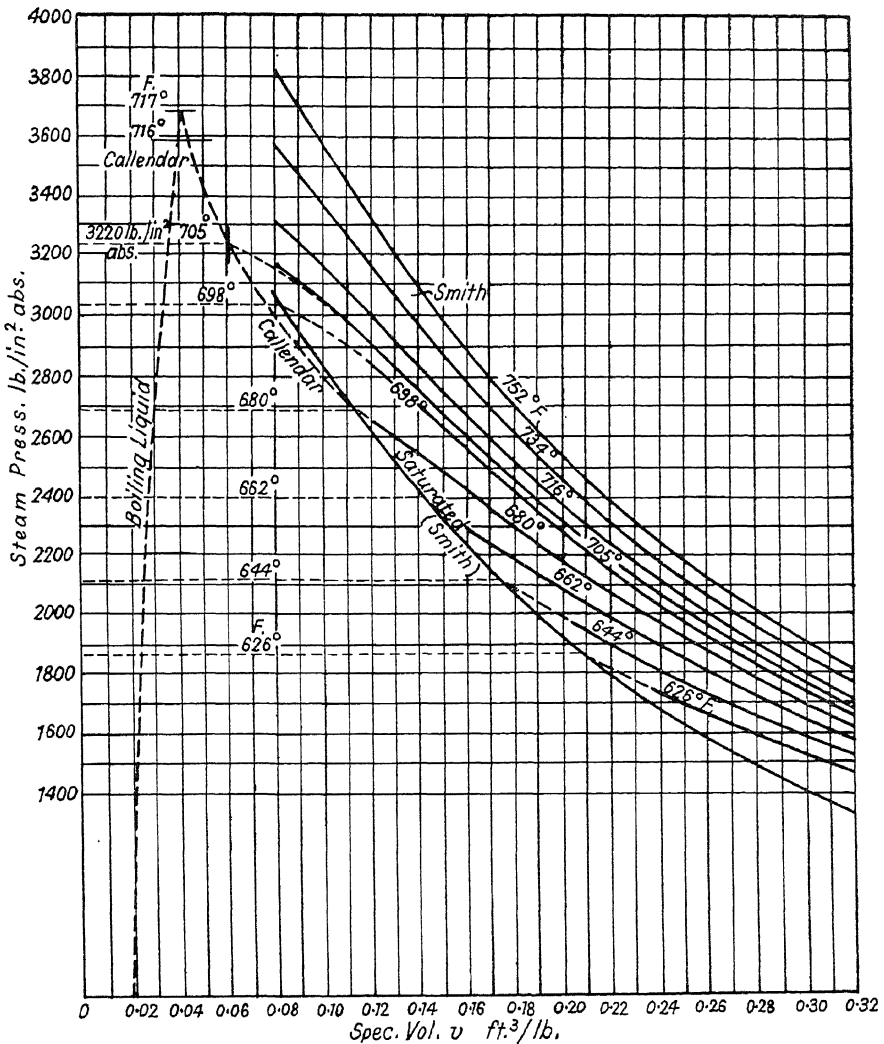


FIG. 210

This gave the simple relation

$$\frac{T}{T_0} = \left( \frac{p}{p_0} \right)^{0.236}$$

Callendar found later, using a similar method, that

$$\left( \frac{T}{T_0} \right)^{3/13} \quad \text{or} \quad \frac{p}{p_0} = \left( \frac{T}{T_0} \right)^{13/3}$$

Since the exponent given by Hirn is  $0.236 = 12.7/3$ , both exponents are about the same.

According to Callendar\* this relation holds up to the critical pressure and not, as was previously assumed, within the normal pressure region alone. From this Callendar also shows that the adiabatic pressure volume

relation

$p(v-b)^{1.3} = \text{constant}$  is valid up to maximum pressures.

Finally, the following relation also holds,

$(v-b) T^{10/3} = \text{constant}$   
or, approximately,  
 $= \text{constant}$

These three relations hold within the superheat region, but break down when, in the course of adiabatic expansion, the steam becomes saturated.

Callendar's method, therefore, for finding adiabatic pressure, volume, and temperature changes in the superheat field up to the highest pressures and temperatures likely to be met with in practice, is the same as for gases, except that the exponent is  $\gamma = 1.3$ . The same holds in finding the work done, and for technical calculations forms a useful simplification.

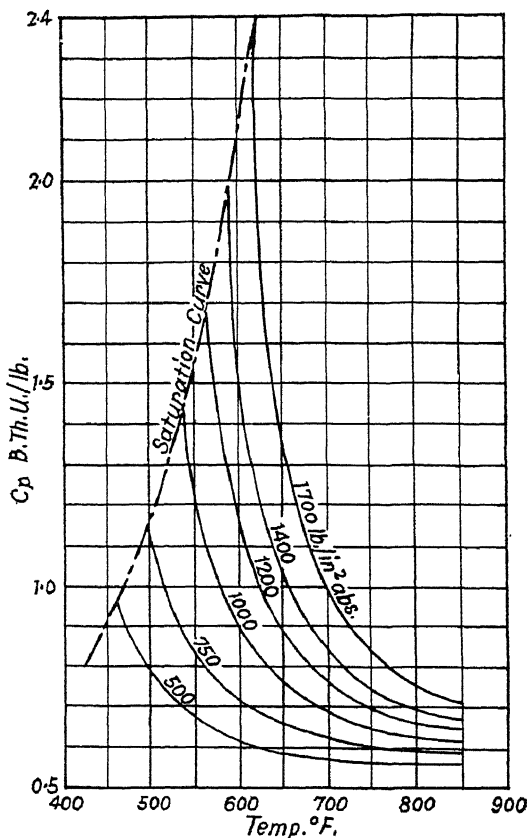


FIG. 211

**Specific heat at constant pressure.** The specific heat  $c_p$  has been determined for the complete pressure region up to 1700 lb./in.<sup>2</sup> abs. and up to 842° F. in the Munich Technical Physical Laboratory under the guidance of Professor Oscar Knoblauch, by direct calorimetric test. The latest of this extended series of tests is given in Fig. 211, from which it will be seen that the previously known dependence of  $c_p$  on pressure and temperature at low temperatures still holds for pressures up to 1700 lb./in.<sup>2</sup> abs. Further tests up to 3500 lb./in.<sup>2</sup> abs. and in the critical region are being carried out in the same place.†

\* *Engineering* (1928), page 530. Steam tables and equations. Extended by direct experiment to 4000 lb./in.<sup>2</sup> and 400° C.

† O. Knoblauch and We. Koch, *Z.V.d.I.* (1928), page 1733. The specific heat of superheated steam from 30 to 120 atmospheres, and from the saturation temperature to 840° F.

From Fig. 211 the specific heat at 1400 lb./in.<sup>2</sup> near the saturation curve is 2, and is thus double that of water at normal temperatures and four times as great as that of superheated steam at 15 lb./in.<sup>2</sup> abs.

**Determination of total heat by throttling tests.** The total heat of saturated or superheated steam at pressure  $p_1$  and temperature  $T_1$  can be determined directly by condensing the steam at constant pressure  $p_1$  in a calorimeter and measuring the heat liberated, or by heating water electrically and measuring the total amount of heat required to produce the superheated steam.\*

The total heat can, however, be determined indirectly by making use of the property that, in throttling steam from the pressure  $p_1$  to the

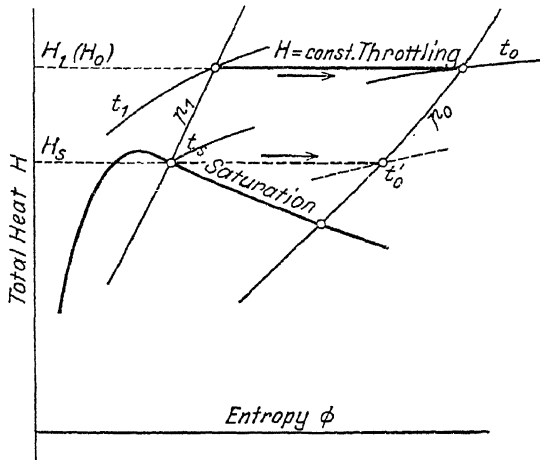


FIG. 212

lower pressure  $p_2$ , the total heat remains constant (page 211). Hence, when steam is throttled from a pressure  $p_1$  down to atmospheric pressure  $p_0$  by discharging it to a vessel in free communication with the atmosphere, its total heat, after throttling, is the same as the initial value  $H_1$ . By designing the discharge vessel in the form of a constant pressure calorimeter, the value of  $H_1$  at atmospheric pressure is thus determined.

If, however, the total heats of superheated steam at atmospheric pressure and different temperatures are already known, all that is required is the temperature  $t_0$  of the throttled steam, which is smaller than  $t_1$ . The total heat  $H_0$  at the state  $p_0 t_0$  is then equal to the required total heat  $H_1$  at the state  $p_1 t_1$  (Fig. 212).

Griessman† was the first to adopt this method. Later, Davis and Kleinschmidt, in America, carried out experiments in the same way for pressures up to 600 lb./in.<sup>2</sup> abs. and temperatures to 680° F. On the same basis Havlíček‡ determined the total heat of superheated steam at pressures of 740, 1420, 2200, 2870, and 3600 lb./in.<sup>2</sup> and temperatures up

\* In this way Jakob determined the latent heats up to 590° F.

† F.A. 13 (1906), Beitrag zur Erzeugungswärme des überhitzten Wasserdampfes.

‡ Festsch., Prof. Stodola, page 202, Neuzeitliche Versuche über die phys. Eigenschaften des Hochdruckdampfes.

to 900° F. Callendar's values of the total heats shown in Fig. 206 were also obtained in this way and show close agreement with Havliček's values.

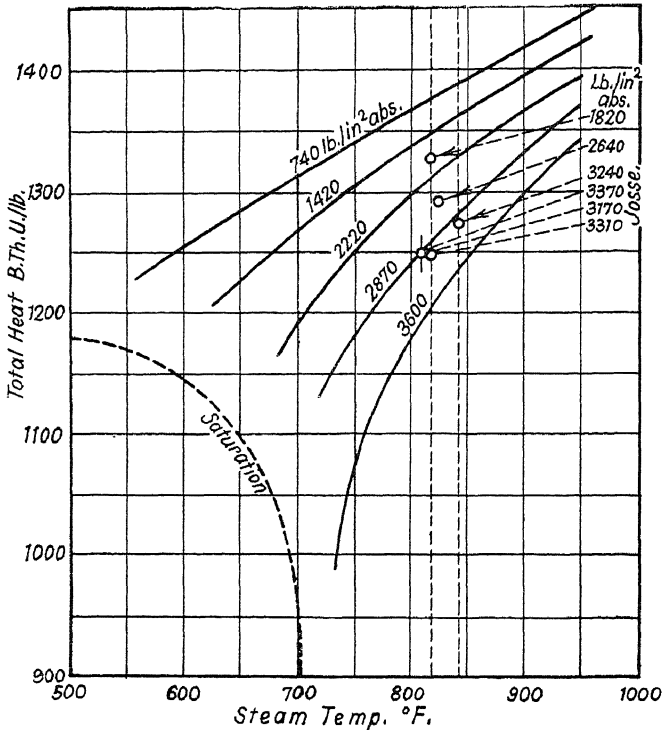


FIG. 213

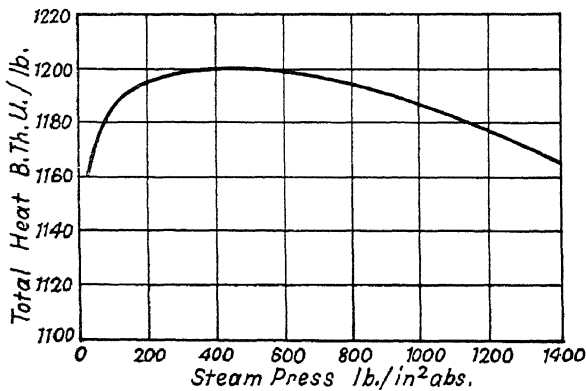


FIG. 214

This method is also valid for dry saturated steam, since the steam is superheated after throttling. Blomquist found in this way the total heat of saturated steam for pressures up to 1400 lb./in.<sup>2</sup> His values are shown

in Fig. 213.\* The values at  $p = 700, 1000,$  and  $1400 \text{ lb./in.}^2$  have been read from this diagram and transferred to Fig. 214, and show close agreement with the total heat curve for saturated steam.

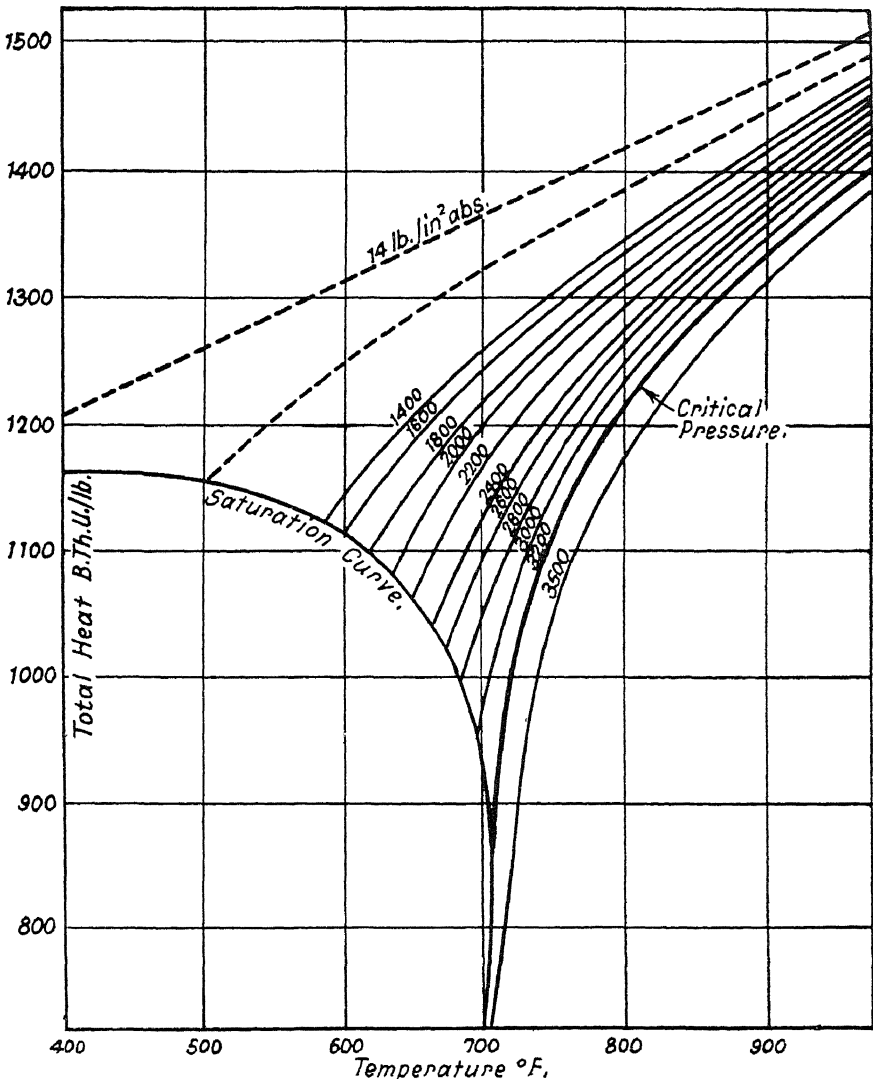


FIG. 215

For pressures above about  $1850 \text{ lb./in.}^2$  abs. steam is wet after being throttled to atmospheric pressure, as is shown immediately by the  $H\phi$  diagram. In this case the measurement of  $t_0$  alone is insufficient.

Josse† has recently determined the total heat of superheated steam

\* Sonderheft II der *Z.V.d.I.*, über Hochdruckdampf, etc. The tests were carried out in Göteborg on an Atmos boiler with steam at  $1560 \text{ lb./in.}^2$  pressure.

† *Z.V.d.I.* (1929), page 1815. E. Josse. Untersuchungen am Benson-Kessel.

calorimetrically in the neighbourhood of the critical state. The steam or water was removed at various points in the pipe system of a Benson boiler. In this way the table of results given below was obtained. They, along with the remaining known test results, were used by Josse in constructing a new *HT* diagram for pressures between 1400 and 3500 lb./in.<sup>2</sup> abs. (Fig. 215).

TOTAL HEATS OF SUPERHEATED STEAM AT HIGH PRESSURES FROM TESTS BY JOSSE

Pressure lb./in. <sup>2</sup> abs.	Temperature ° F.	Total Heat B.Th.U./lb.
3527	716.0	921.6
3470	714.2	936.0
3442	712.4	916.2
3286	705.2	864.0
2773	681.8	896.4
2062	640.4	941.4

**Characteristic equation for superheated steam.** The general relation between the pressure  $p$ , the volume  $v$ , and the temperature  $t$  for highly superheated steam and at low pressure tends to approach that of a gas

$$pv = RT, \text{ or } v = \frac{RT}{p}$$

e.g. for steam contained in the products of combustion of an internal combustion engine. At higher steam pressures and with less superheat, such as are used in steam engines and turbines, and particularly near the saturated region, the volume of the steam—as shown by direct measurements and indirect calculations—is always smaller than that obtained from the gas equation for given values of  $p$  and  $T$ . Hence the general equation can be written as

$$v = \frac{RT}{p} - \Delta v$$

in which  $\Delta v$  represents the difference between the gas volume and the actual volume of the steam. This difference depends on the state of the steam and can be shown graphically on the  $vT$  diagram, as in Fig. 201, page 393, where the distance  $BB' = \Delta v$  and the straight line passing through the origin shows the change in volume with temperature on the assumption that the steam behaves as a gas, while the plotted curve gives the actual steam volumes. The greatest deviation of the steam volume from the gas volume occurs at the saturation curve with  $\Delta v = AA'$ .

The quantity  $\Delta v$  is a function of the pressure and temperature, i.e.

$$\Delta v = f(pT)$$

It might be assumed, however, that  $\Delta v$  is a function of the pressure or of the temperature alone, as was done in many of the earlier proposed equations of state. In Zeuner's characteristic equation, for example,

$$pv = RT - Cp^m$$

$\Delta v$  depends only on the pressure and not on the temperature. In the later equation of Tumlirz and R. Linde

$$pv = RT - Cp$$

the value  $\Delta v$  is independent of both pressure and temperature, and has a constant value. ( $C = \Delta v = 0.2563$ .)

In the equation, however, given by Callendar, which was also used by Mollier (with different constants),

$$pv = RT - C \frac{p}{T^{10/3}}$$

the value  $\Delta v = \frac{C}{T^{10/3}}$

is a function of the temperature alone.

From the second equation given by R. Linde (page 371)

$$\Delta v = (1 + ap) \left[ C \left( \frac{671.6}{T} \right)^3 - D \right]$$

and is thus a function of temperature and pressure.

This equation suitably represents the Munich tests on the volumes, from which it was established, but breaks down at a particular temperature, since the quantity in the square brackets can be negative at high temperatures. This is impossible, as  $\Delta v$  would then be negative. The temperature at which the equation ceases to hold is found from

$$C \left\{ \frac{671.6}{T} \right\}^3 - D = 0$$

With  $D = 0.0833$  and  $C = 0.4966$   
 $T = 1220^\circ \text{ F. abs.}, t = 760^\circ \text{ F.}$

Hence the equation holds up to about  $760^\circ \text{ F.}$  From it, however, it is clear that  $\Delta v$  is a function of  $p$  and  $T$ .

In the equation

$$pv = RT - f(pT)$$

or  $\frac{pv}{T} = R - \frac{1}{T} f(pT)$

by writing  $R = \frac{1}{T} f(pT) = R'$

$$\frac{pv}{T} = R'$$

it is seen that, for superheated steam, the value  $R'$ , which is equal to the quotient  $\frac{pv}{T}$  is not constant but depends on  $p$  and  $T$ , and deviates from the gas constant for steam, the deviation increasing as the steam approaches the saturated state.

If simultaneous values of  $p$ ,  $v$ , and  $T$  are known by measurement, or if they can be determined by calculation without actually knowing the

equation of state, and if the values of  $\frac{pv}{T}$  be plotted to a base of  $p$  (or  $T$ ), a useful representation of the deviation of superheated steam from the gas state is obtained. This diagram was first adopted by Jakob, who used a graphical method of determining the volumes from the

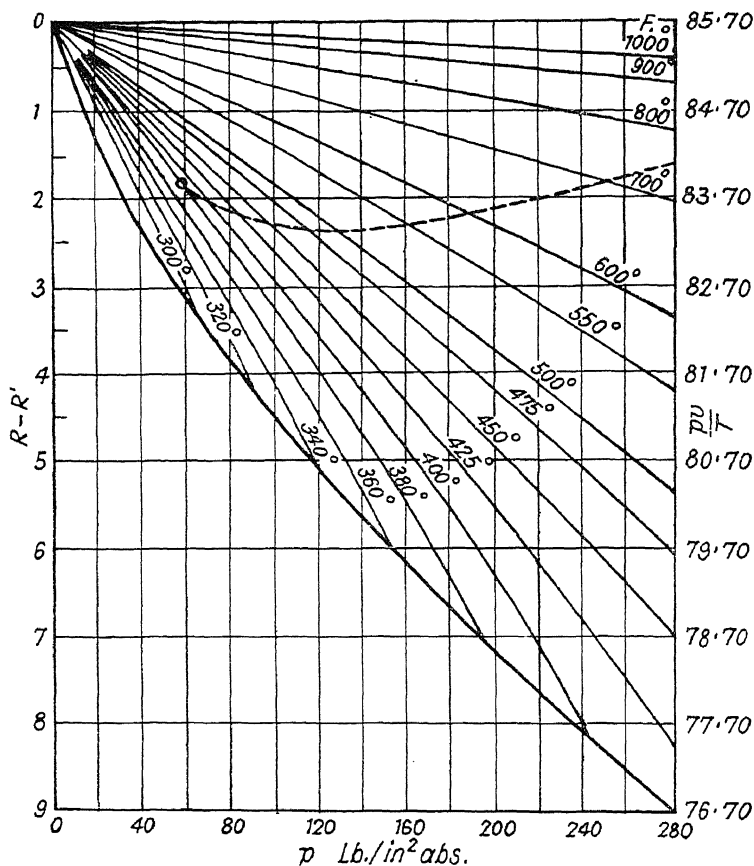


FIG. 216

Munich specific heat values and then constructed the diagrams with  $\frac{pv}{T}$  as ordinates against  $p$  or  $T$  as abscissae. The former is shown to scale in Fig. 216.

This diagram shows immediately that  $R'$  changes with the pressure and temperature and decreases as the pressure increases or the temperature decreases. The relation between  $\Delta v$  and  $R'$  is expressed by

$$R' = R - \Delta v \frac{p}{T}$$

or

$$\Delta v = (R - R') \frac{T}{p}$$



in which  $R - R'$  are the ordinates of the curves in Fig. 216 lying below the abscissae axis through  $R = 85.7$ . From this  $\Delta v$  can be found for any state of the steam. For example, at 200 lb./in.<sup>2</sup> abs. and 650° F.,

$$R - R' = 2$$

$$\text{Hence} \quad \Delta v = \frac{2 \times (460 + 650)}{200 \times 144} = 0.0771$$

The ratio  $\frac{\Delta v}{v}$  is given by

$$\frac{\Delta v}{v} = \frac{R - R'}{R'}$$

so that, for this state,

$$\frac{\Delta v}{v} = \frac{2}{84} \text{ or } 100 \frac{\Delta v}{v} = 2.4 \text{ per cent}$$

Again, with the same pressure but at 400° F.,

$$R - R' = 6.57, \quad R' = 79.4, \quad 100 \frac{\Delta v}{v} = 8.26 \text{ per cent}$$

$$\text{and} \quad \Delta v = \frac{6.57 \times 860}{800 \times 144} = 0.196$$

To find now if  $\Delta v$  is a function of  $T$  or  $p$  or both, proceed as follows. From the above

$$\frac{R - R'}{p} = \frac{\Delta v}{T}$$

The quotient  $(R - R') \frac{1}{p}$  gives the inclination ( $\tan \alpha$ ) of the line joining the initial point of the group of curves in Fig. 216 to any point on the curve for  $T = \text{constant}$ . If now  $\Delta v$  is a function of  $T$  alone, then, for this curve,  $\frac{\Delta v}{T}$  is constant and with it the angle of inclination  $\alpha$ . Hence the curve itself is a straight line, and for  $\Delta v$  to be a function of  $T$  alone, the diagram shown in Fig. 216 would consist of a group of straight radial lines. This, however, is not the case, although the deviation for each curve up to a particular pressure marked on the diagram is not serious. From this it will be seen that, in the region below this limit,  $\Delta v$  depends only on the temperature and not on the pressure as demanded, e.g. by Callendar's original equation. Above this limit, however, the lines are definitely curved so that  $\Delta v$  in this region depends on both  $T$  and  $p$ .

The values of  $R'$  in this region become smaller, and the values  $R - R'$  greater, than those given by the straight rays, as the pressure at a particular temperature increases. For example, at 430° F. and up to 100 lb./in.<sup>2</sup> abs., the deviation from the straight line is negligible, at 200 lb./in.<sup>2</sup> abs. it is about 0.36 units of  $R'$ , and at 280 lb./in.<sup>2</sup> abs., 0.9 units.

The first Jakob diagram extended to only 300 lb./in.<sup>2</sup> abs. A similar

diagram (Fig. 217) extending to 3500 lb./in.<sup>2</sup> abs. and 750° F. has lately been supplied by J. H. Keenan,\* and is based on the American measurements on the specific volume mentioned above. The general trend of the curves in Keenan's diagram is the same as in the original Jakob diagram, but show increased divergence at the higher pressures, e.g. at

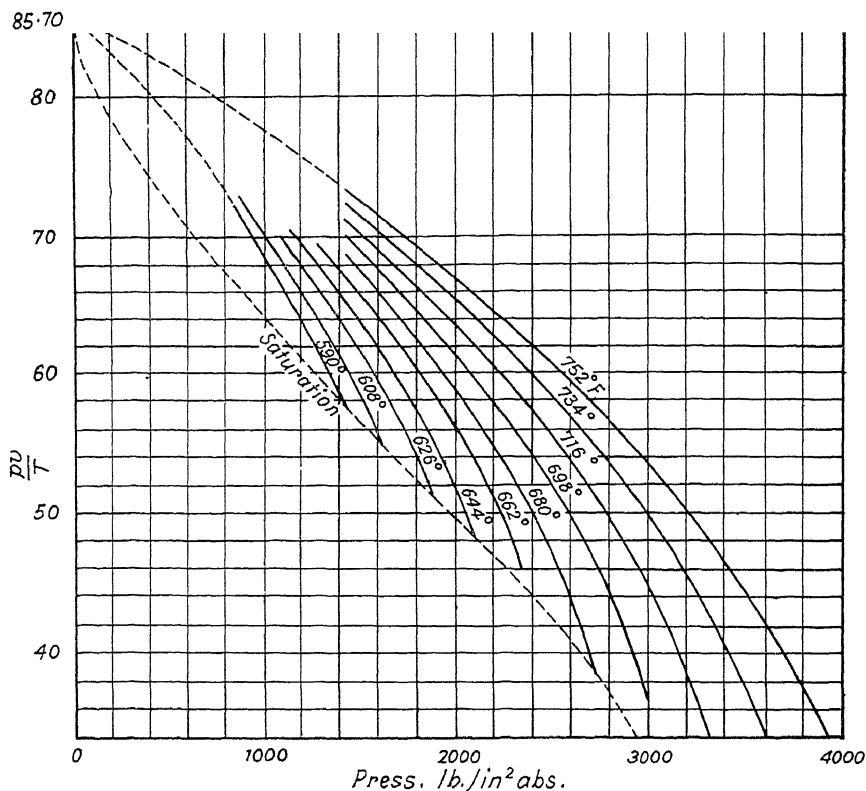


FIG. 217

1500 lb./in.<sup>2</sup> abs.  $R'$  has decreased to values between 62 and 70, and at 3500 lb./in.<sup>2</sup> abs. to values between 35 and 44. Hence, in this region, the deviation from the gas law is very considerable since  $\frac{100 \Delta v}{v}$  for  $R' = 62$  is equal to 38.5 per cent and for  $R' = 35$ , 135 per cent, i.e. the ideal volumes are 38.5 and 135 per cent greater than the actual steam volume. This is shown also in Diagram II.

**Adiabatic change of state in the Jakob state diagram and Callendar's state equations.** Callendar's relation between adiabatic pressure and temperature changes for any state is

$$P = \frac{P_0}{T_0^{13/3}} = \text{constant}$$

\* *Mech. Engineering* (1928), page 157. In *Mech. Engineering* (February, 1929), Keenan has enlarged this diagram and completed it from the latest tests.

In the lower state region the original Callendar characteristic law is

$$\frac{p(v-b)}{T} = R - C \frac{p}{T^{13/3}}$$

Since now in an adiabatic change the quotient  $\frac{p}{T^{13/3}}$  is constant, the expression on the right is also constant for an adiabatic change, and hence  $\frac{p(v-b)}{T} = \text{constant}$

or, for moderate pressures with sufficient accuracy,

$$\frac{pv}{T} = \text{constant}$$

as established by Callendar.

From this it follows that adiabatic expansion or compression is represented on the Jakob diagram by horizontal lines parallel to the  $p$  axis. In addition, the adiabatic pressure and temperature changes are found very simply from this diagram in the lower state region, and the volume changes are given, from these, by

$$\frac{v_2}{v_1} = \frac{p_1}{p_2} \frac{T_2}{T_1}$$

Callendar\* takes account of the influence of pressure on  $\Delta v$ , which becomes important at high pressures by multiplying the expression for  $\Delta v$  in the lower state region

$$\Delta v_0 =$$

by a correction factor in the form of a geometrical series, giving

$$\begin{aligned} \Delta v &= \Delta v_0 \left( 1 + \frac{1}{z^2} + \frac{1}{z^4} + \dots \right) \\ &= \Delta v_0 \frac{1}{1 - z^2} \end{aligned}$$

In this  $z$  is so chosen that the relation for an adiabatic change of state

$$p = \text{constant}$$

which holds also in the upper state region, is satisfied.

This is the case if  $z$  is written as

$$z = k \Delta v_n \frac{p}{T}$$

The characteristic law in the upper region then takes the form

$$v - b = \frac{RT}{p} - \Delta v_0 \frac{1}{1 - \frac{1}{n^2}} \quad . \quad . \quad . \quad . \quad (448)$$

\* H. L. Callendar. Steam tables and equations. *Engineering* (26th October and 2nd November, 1928).

For adiabatic changes, since

$$\Delta v_0 \frac{p}{T} = \frac{C_p}{T^{13/3}}$$

and since  $\frac{p}{T^{13/3}} = \text{constant}$ , the expression in the denominator of the second term on the right in equation (448) is constant. By re-writing this equation as

$$T = R - \Delta v_0 \frac{p}{T}$$

it will be seen that, for adiabatic changes of state, the complete expression on the right is constant, so that again, as in the lower state region,

$$\frac{p(v-b)}{T} = \text{constant}$$

or, if sufficiently removed from the critical volume,

$$\frac{pv}{T} = \text{constant}$$

It follows, therefore, that in the upper state region, as given by the American tests (Fig. 217) in the  $pv/T$  diagram, the adiabatic changes of state are represented by horizontal straight lines.

A further determination is required, e.g. with the aid of the entropy diagram, which could be constructed from the Munich specific heats, in order to find if this simple relation actually holds in the critical region. If this is the case the adiabatic changes of state over the whole region of superheated steam can be followed graphically in a Jakob diagram or expressed by the simple relations which hold for the low pressure region.

The author found, on calculating values of  $\frac{pv}{T}$  for a series of pressures and using Callendar's coefficients ( $k = 4.012$  for lb/in.<sup>2</sup>, ft.<sup>3</sup>/lb), that the results were higher than those given by the American tests (Fig. 217). Since, however, the validity of the latter has not yet been confirmed, the question still remains open.

**The characteristic equation based on the specific heats  $c_p$ .** According to thermodynamic laws, a close relationship exists between the equation of state of superheated steam and its specific heats. This is represented by the differential\*

$$\left( \frac{dc_p}{dp} \right)_T = -AT$$

The left side gives the change in specific heat with pressure when the temperature is constant, and on the right side the only magnitudes involved are  $T$  and  $v$ . The method which was first employed by Jakob

\* Vol. ii, Fourth Edition, Article 6.

of finding the relation between the volume and temperature at constant pressure graphically can also be employed analytically. For this, it is necessary to express the specific heats—as given by the Munich tests—as an analytical function of  $p$  and  $T$ . Calculations of this nature have been carried out by R. Planck,\* by Eichelberg,† by Goodenough in America, and by Knoblauch, Raisch, and Hausen, who have published tables and diagrams for steam (1923). Mollier also, in the later editions of his tables and diagrams for steam, has adopted the same method.

The characteristic law is also affected by the form of the analytical relation represented by the Munich  $c_p - T$  curves. Eichelberg succeeded thus in establishing not only a simple characteristic equation but in expressing all the other state values by simple relationships. This is particularly useful in finding the total heats and entropies. His characteristic equation runs

$$pv = 0.5956T - \frac{106.40p}{\left(\frac{T}{100}\right)^3} - \frac{698000000p \left\{ \frac{p}{14.223} + 2 \right\}^{2.7}}{\left(\frac{T}{100}\right)^{14}}$$

( $p$  in lb./in.<sup>2</sup> abs.,  $v$  in ft.<sup>3</sup>/lb.,  $T$  in ° F. abs.)

The second term of this equation confirms Callendar's equation in the lower state region, while the third takes account of the effect of pressure on the deviation from the gas law in the upper state region. The question as to how far this applies still remains open.

In addition, Eichelberg found that the law

$$pv^\gamma = \text{constant}$$

for adiabatic expansion held, with but slight deviations, in the value of the exponent  $\gamma = 1.3$  over the whole of the region covered by his experiments, which thus agrees with Callendar, who arrived at this conclusion in a totally different manner.

Taking advantage of this, Mollier then found that the Munich measurements of  $c_p$  could be represented as a function of Callendar's expression

$\frac{p}{T^{13/13}}$ . Mollier gave, finally, the characteristic equation as

$$\frac{pv}{T} = 0.5961 - \frac{2.2732}{100} \frac{10.1592p^3}{\left(\frac{T}{100}\right)^{13/13}}$$

( $p$  in lb./in.<sup>2</sup> abs.,  $v$  in ft.<sup>3</sup>/lb.,  $T$  in ° F. abs.)

or, multiplying by  $T$ ,

$$2.2732 \quad 10.1592p^3$$

$$\left(\frac{T}{100}\right)$$

\* *Z. V.d.I.* (1916), page 187.

† *Forsch. Arb. V.d.I.*, No. 220 (1920). Also given in Vol. ii, Fourth Edition, Article 9.

This equation agrees with Eichelberg's in form and in the values of the exponents of  $T$  and  $p$ , and shows only at high pressures a slight difference in the numerator of the third term, since Eichelberg takes account of the variation in the specific heat  $c_p$  with temperature at very low pressures (using an equation by Goodenough), whereas Mollier neglects this.

Regarding the Munich characteristic equation, reference should be made to the steam tables by Knoblauch, Raisch, and Hausen about to appear in a new edition. The steam pressure range is greatly extended.

**Steam charts in the high pressure region.** Two diagrams for high pressure steam, which extend in the saturation region to the critical state and in the superheat region to 1100 lb./in.<sup>2</sup> abs. and 850° F., were first published by the author in 1911 (in *Z.V.d.I.*). One diagram, which is described on page 390 and is contained in the pocket at the end with necessary additions, is really a double diagram on one sheet, since the temperatures are plotted as ordinate to a base of volumes and of entropies. The second diagram is an  $H\phi$  or Mollier chart, which is given here with corrections according to the latest results. (Chart III.)

The superheat field curves were based on the Munich tests on specific heats and later completed in accordance with recent results. The saturation region was constructed from the numerous test results available at the time. The diagrams thus represented the results of direct experiment, as far as these extended, and, in the unexplored region, use was made of thermodynamic relations in combination with the results of tests. Arbitrary extrapolation was in all cases avoided. The values in the saturated steam region were collected in the form of steam tables, and these were obtained without reference to the tables for normal pressures and temperatures existing at that time.\*

During the past six years considerable advancement has taken place in experimental work on steam, as shown above, and as a result of this the two steam diagrams have been further improved and redrawn without the use of formulae.

At present a large amount of experimental work is being carried out in Germany and America, embracing the critical and super-critical region, so that final diagrams may be constructed. In England, Callendar has already constructed a diagram—based on his own experiments—which extends to 4000 lb./in.<sup>2</sup>† In Germany, Mollier has produced the *Neuen Tabellen und Diagramme für Wasserdampf* in a Sixth Edition (1928), which extend to 250 atmospheres and 550° C.‡ A new edition of the Munich tables and diagrams is in course of preparation up to 120 atmospheres, and will give the Munich  $C_p$  values above the critical state, from which it will be possible to confirm Callendar's observations on the specific heat in the critical state.§

Regarding the alterations, mention must first be made of the

\* It was with the aid of these tables and the two charts that the thermodynamic calculations for the high pressure steam engine of Wilhelm Schmidt were carried out.

† See *Engineering* (May, 1929).

‡ Note by translator. An English edition of these tables has been edited by Dr. Moss and published by Pitman.

§ Keenan has recently published an extensive  $H\phi$  diagram based on the American measurements. (*Mech. Eng.*, February, 1929.)

saturation curve above 150 lb./in.<sup>2</sup> abs. The change was rendered necessary by the later tests by Jakob on the latent heats mentioned above, which show considerable differences from the previously accepted values of Holborn and Henning and the older volume measurements of Ramsay and Young.

The water curve also deviates from the earlier curve, but only above 590° F. From there on to the critical state it has been constructed from Callendar's test values by finding the entropy changes according to

$$d\phi = \frac{dh}{T}$$

It is remarkable that this line deviates but slightly from the earlier curve, the construction of which was provisionally effected by means of tests on the specific heats of the liquid at the boiling point. These specific heats increase rapidly with temperature in this region, and thus cause considerable difficulty. (See Schüle's *Technische Thermodynamik*. Bd. II, page 337.)

In the critical region itself, which, according to Callendar, lies between 676° F. and 716° F., the Callendar test results have been used. As shown by the diagram, the top of the  $T\phi$  curve is not rounded, as was previously assumed, but shows a broken peak. Both the liquid branch (left) and the saturation branch (right), which start at this peak and join the liquid and saturation curves, were obtained from Callendar's liquid and latent heat values.

The saturation curve between 480° F. and 660° F. was constructed from the latent heats as given by the combined Jakob and Callendar curve of  $L$  values (Fig. 205).

In the superheat field, the curves of constant pressure between 400 and 1700 lb./in.<sup>2</sup> abs. have been reconstructed from the Munich  $c_p$  values according to the relation

$$d\phi = \quad = 2.303 c_p d \log T$$

The values of  $c_p$  were plotted to  $\log T$  as base, and the areas below these curves gave the entropy changes.

In the critical region the three curves of constant pressure for the saturation temperatures 680, 698, and 717° F. have been plotted from Callendar's measured values of the total heat of superheated steam, making use graphically of the relation

$$d\phi_p = \frac{dH}{T}$$

In the volume field of the diagram, the volumes of the liquid and of the steam in the critical region have been plotted from Callendar's measurements. The resulting junction of the liquid and steam curves is not continuous but broken, showing unstable transition. Between 150 and 550 lb./in.<sup>2</sup> abs. the curve of volumes, which is based on Jakob's values, deviates but slightly from the original curve. Between 480° F. and 650° F. the volume curve (which at about 600° F. must coincide with the old curve, since at this point the latent heats also coincide) continuously approaches the Callendar and Jakob curve. Finally, the

saturation volume curve was completed from the American tests between 380° F. and 698° F. In the superheat field the volumes were plotted at increasing temperatures and at constant pressures from the American measurements (Fig. 209).

In the  $H\phi$  diagram changes have also been made, e.g. in the saturation line above 115 lb./in.<sup>2</sup> abs. and in the superheat field above the same pressure. The curves of constant pressure have been plotted from the  $T\phi$  entropy values and the total heat values given by the Munich  $c_p$  measurements, while Callendar's total heats have been used in the critical region.

In the critical region the characteristic changes given by Callendar's new measurements cannot be clearly shown, as Callendar himself remarked. The actual critical point is not marked in the diagram.\*

\* After the two diagrams had been constructed, the "Skeleton table for steam, with explanations" resulting from the International Steam Table Conference in London (July, 1929) were published. These are given at the end and enable comparisons to be made with the diagram and tabulated values of total heats, and specific volumes of water and of saturated and superheated steam from 32° F. to 1000° F., and from 15 to 3500 lb./in.<sup>2</sup> abs.



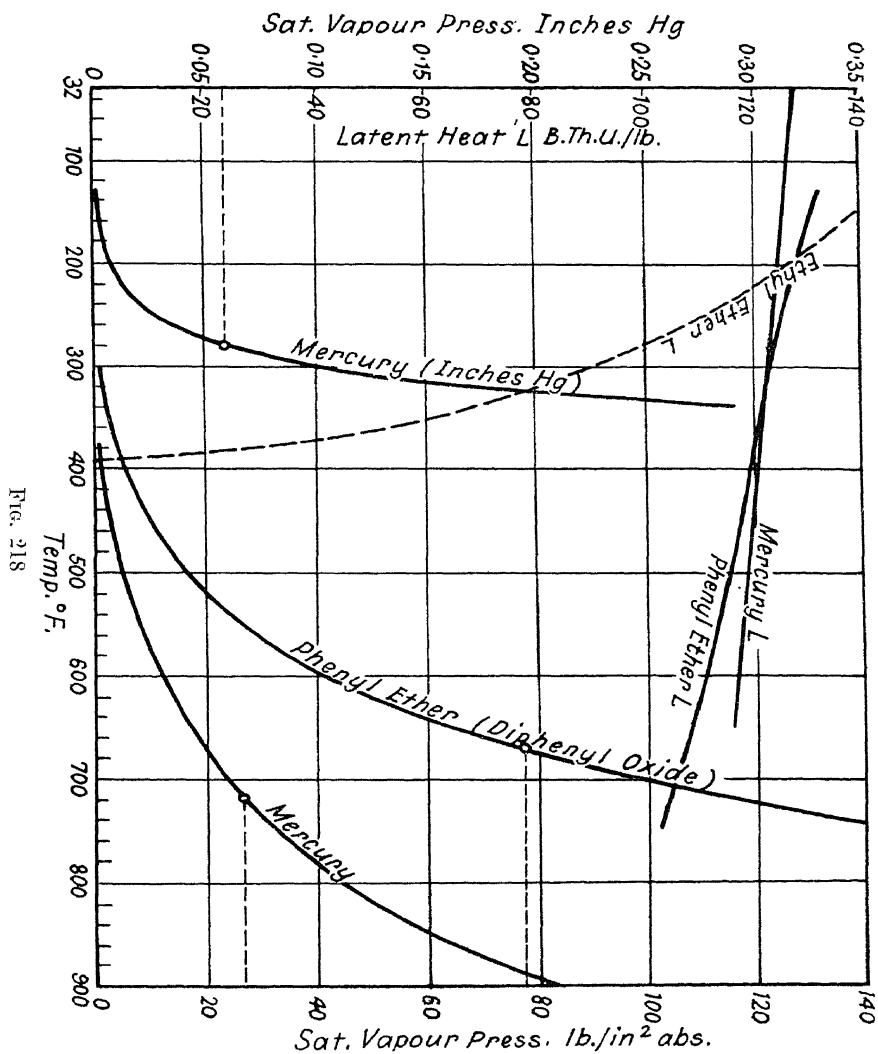
## CHAPTER IX

### GENERAL RELATIONS FOR VAPOURS

**Hot and cold vapours, gases.** As a rule, vapours are generated from liquids such as water, oil, alcohol, ether, mercury, etc., by an evaporation process. In order to convert such substances—which at normal pressures and temperatures (1 atmosphere and 60° F.) exist as liquids—to vapour, the liquid must first be raised to the boiling temperature  $t_s$ , which depends on the pressure to which the liquid is subjected. After this the liquid can be transformed to saturated vapour by supplying an amount of heat which varies with the nature of the substance. If the pressure be kept constant during this supply of heat the temperature will also be constant, so long as the vapour is in contact with the liquid. For water, the boiling temperature at 14.7 lb./in.<sup>2</sup> abs. is 212° F., but if the pressure be raised considerably higher temperatures are attained. Thus at 150 lb./in.<sup>2</sup> abs. the temperature for water is 388° F. and for mercury 750° F. Vapours of this type can be described as hot vapours. The treatment given in Chapters VII and VIII for steam applies equally well to all other chemically uniform vapours, which are non-dissociated at the saturation temperatures. The numerical values of the saturation temperatures, pressures, sensible heats, latent heats, and total heats are, however, different for the different substances. Fig. 204 shows the saturation curve for steam. In Figs. 218 and 219 the saturation curves for mercury\* and phenylether (diphenyloxide  $C_6H_5OC_6H_5$ ) have been plotted. These two substances have lately acquired a certain importance, owing to their use as working substances in prime movers (page 603). When a saturated vapour is heated above its saturation temperature (at constant pressure) it becomes superheated, and the higher the range of superheat the more closely does it assume the properties of a perfect gas.

As opposed to hot vapours, saturated cold vapours consist of substances which, at normal pressures and temperatures (1 atmosphere, 60° F.), exist in the gaseous state, e.g.  $CO_2$ ,  $NH_3$ ,  $SO_2$  (the saturation curves of which are given in Fig. 220), also  $CO$ , air,  $O_2$ ,  $N_2$ ,  $H_2$ . For such substances, vapour formation is effected in the opposite way, i.e. by removing heat from them so as to bring them down to the saturated state, after which, by a further removal of heat, they can be liquefied. This process, however, is not so simple as that involving the generation of hot vapours, since bodies at a sufficiently low temperature to liquefy these substances, at atmospheric pressure, do not exist in nature, and the normal cooling water temperature of about 50° F. is not low enough to liquefy or vaporize most of these substances, even if the pressures be raised indefinitely. These conditions have been clarified by tests on the liquefaction of the permanent gases. In particular the liquefaction of carbon dioxide by Andrews (1869), and the further tests of Amagat with

\* From the *Phy. Zeitschr* (1926), page 265. F. Bernhardt. Saturation pressures of mercury up to 2000 atmospheres.



the same substance have served to illustrate the phenomena connected with this problem.

At  $p = 14.7$  lb./in.<sup>2</sup> abs. and  $t = 59^\circ$  F.  $\text{CO}_2$  exhibits the properties of a gas and cannot be liquefied by cooling water at this pressure. If, however, the gas be compressed to 734 lb./in.<sup>2</sup> abs. and cooled to  $59^\circ$  F., it is found to be saturated and, on a further removal of heat, can be completely liquefied. In the  $pv$  diagram the isothermal compression curve for  $59^\circ$  F., according to Andrew's and Amagat's results, is as shown in Fig. 221. At  $B$  the  $\text{CO}_2$  is dry saturated, and the distance  $B_0B$  gives the specific volume  $v_g$  of the vapour at 734 lb./in.<sup>2</sup> abs., while  $B_0B' = \sigma$  gives the specific volume of the liquid. If the  $\text{CO}_2$  be compressed at the higher temperature  $86^\circ$  F., it is not until the pressure is raised to about

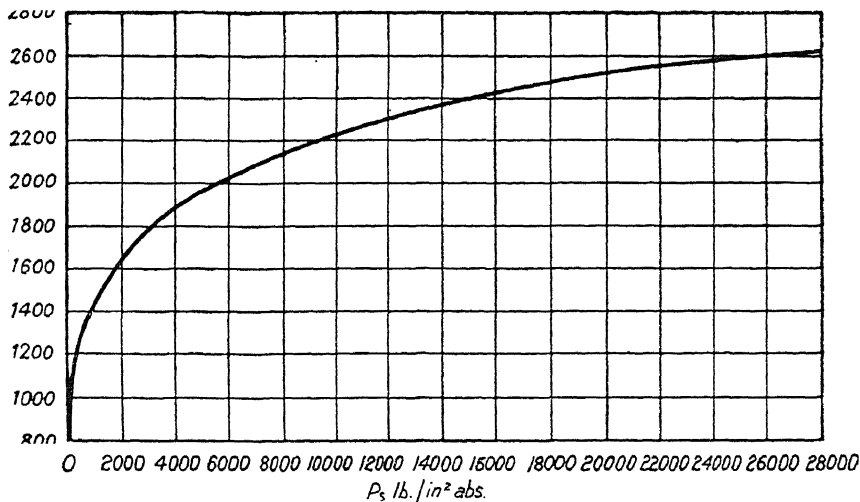


FIG. 219

1040 lb./in.<sup>2</sup> abs. that the saturated condition is attained. The specific volume of dry saturated vapour at this pressure is smaller, while that of the liquid is larger than the corresponding values at  $59^\circ$  F.

As shown for steam on page 395, the vapour volumes approach the liquid volumes as the pressure rises, until, at the point  $C$  (Fig. 221), they become equal. While the liquid and the vapour can exist in any ratio below this point, the difference between the two at the critical pressure and temperature disappears. Also, at all temperatures above the critical, no difference is to be found between the gas, vapour, or liquid, however high the pressure may be. This is shown by the isothermals for  $95^\circ$  F. and  $122^\circ$  F.

**Limit curves.** As in the case of steam, vapours which are found to be at temperatures above the saturation are said to be superheated.

If any superheated vapour at the state  $A$  (Fig. 221) be compressed isothermally until the saturated state  $B$  is reached,  $BB_0$  gives the specific volume of the dry saturated vapour. If these specific volumes, obtained by test, be plotted against their corresponding saturation pressures, the resulting curve  $GG$  is obtained. This curve, as shown

already for steam, is called the saturation curve. By means of it the condition of vapour, i.e. whether superheated, dry, or wet, in any state of pressure and temperature can be determined. The point  $A$  on the right of the curve denotes superheated vapour,  $B$  on the curve dry vapour and  $B_1$  on the left wet saturated vapour. In the latter case the quality or dryness is given by the ratio  $B'B_1 : B'B$  and the wetness by  $B_1B : B'B$ .

Again, if, in the same figure, the specific volumes of the liquid be plotted against the corresponding saturation pressures or temperatures,

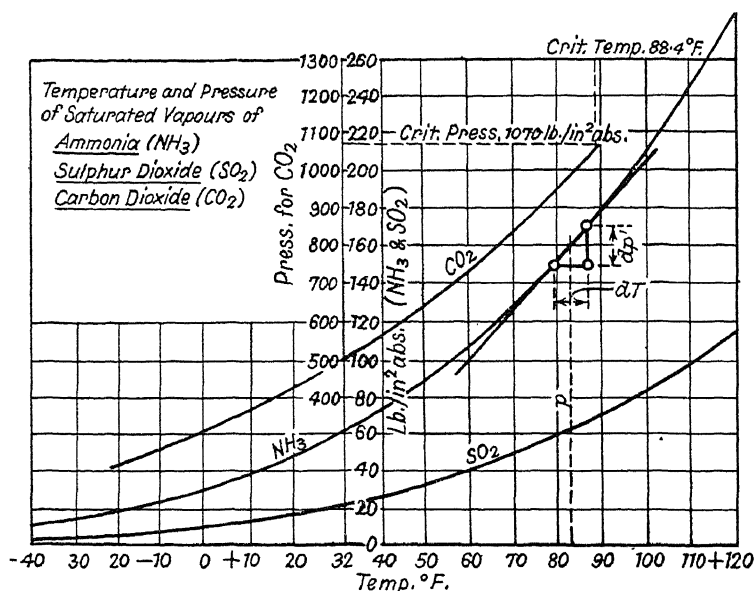


FIG. 220

the lower or liquid limit curve  $G'G'$  is obtained. This separates the vapour-free liquid from the wet vapour.

As shown in Fig. 221 the two curves continuously approach one another. As the point  $C$  is approached along the liquid curve  $G'G'$ , the liquid heats increase as well as the pressure and temperature. Starting at the lower end of the curve  $CG$ , the pressures and temperatures increase and the volumes decrease as  $C$  is approached. At  $C$  itself the values of  $T$ ,  $p$ , and  $v$  are the same for the liquid and the dry saturated vapour.

The process of liquefying a vapour from the superheated state is as follows (Fig. 221). Starting at  $A$ , the vapour is first compressed along any curve  $AB$  (e.g. isothermally, i.e. with vigorous heat removal). In this way the saturated state  $B$  is reached, the position of which depends on the compression path  $AB$ . From  $B$  onwards no further compression is required. In its place a cooling medium is required to withdraw the latent heat  $L$ . The amount of this depends on the particular vapour under consideration. As liquefaction proceeds, the volume decreases, so that the piston, previously used in the compression, continues to move in the same way, while the pressure, along with the temperature, of the

mixture of liquid and vapour remains constant. In practice, the compression also is effected in a cylinder, while the cooling is carried out in a separate vessel (condenser) at constant pressure. As soon as the mixture assumes the volume  $B_0B'$  the whole of the vapour is converted to liquid at the pressure corresponding to the saturation temperature.

The liquid on the isothermals to the left of the liquid curve is said to be undercooled\* compared with saturated vapour at the same pressure.

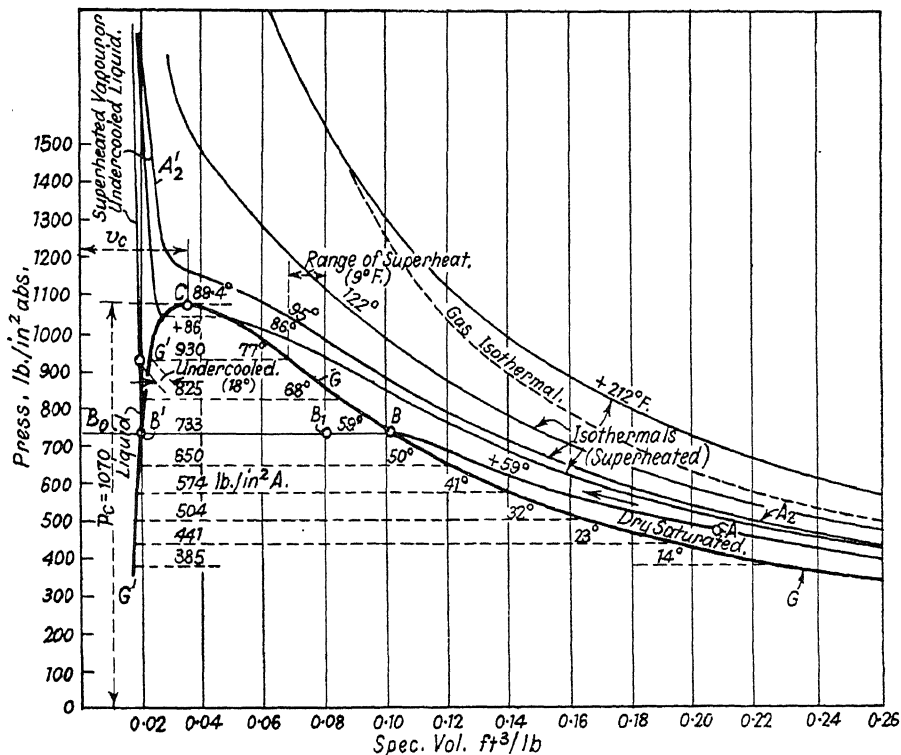


FIG. 221

Thus water, at normal temperatures and at all pressures above about 0.3 lb./in.<sup>2</sup> abs., and CO<sub>2</sub> above 750 lb./in.<sup>2</sup> abs. are found to be undercooled. It is possible to bring undercooled liquids to the liquid limit curve by heating at constant pressure, provided this pressure lies below the critical. If, however, the pressure is above the critical the liquid passes over to the superheated state without passing through the saturated state. So long as its temperature is below the critical it can be described as undercooled liquid, and above this as superheated liquid. In the latter case it is equally correct to describe it as superheated vapour or gas. There exists no limit between the liquid and the vapour in this region.

\* An essentially different state, namely, when a substance exists at temperatures below the freezing point without solidifying, is also described as undercooled.

In compressing superheated vapour or gas from a point such as *A* it is possible that the saturation curve may not be reached. This depends on the initial state and on the path of the compression line, which is conditioned by the nature of the cooling effect employed. An example of this is the compression of  $\text{CO}_2$  at normal temperatures with insufficient cooling. If in the compression the temperature remains above  $88^\circ \text{F.}$ , then, however high the pressure may be raised, it is impossible to liquefy the  $\text{CO}_2$ , since the limit curve is never reached. Hence, at temperatures above the critical, which depends only on the nature of the vapour, liquefaction is impossible, no matter what pressure is attained.

If, therefore, a vapour, whose critical temperature lies below the normal cooling water temperature, is to be liquefied, special means must be adopted to reduce the temperature below the critical.

As the temperatures of superheated vapour are lowered below the critical, the corresponding necessary liquefying pressures are also reduced. The table below contains critical values for  $\text{Cl}_2$ ,  $\text{CO}_2$ ,  $\text{NH}_3$ ,  $\text{SO}_2$ , and  $\text{H}_2\text{O}$ .

**Conditions governing the liquefaction of gases.** Before the idea of a critical state was conceived and the nature of the saturation curve understood by tests on  $\text{CO}_2$  and other vapours, attempts to liquefy the permanent gases were unsuccessful. If, however, these gases are really vapours far removed from the saturated state, then, by sufficiently lowering the temperatures, it should be possible in all cases to liquefy them. The only practical difficulty lies in the fact that the critical temperatures of such gases as oxygen, nitrogen, air, and hydrogen are exceedingly low (see table below). Before such low temperatures could be produced it was impossible to liquefy gases.

#### CRITICAL STATE VALUES

(AND BOILING TEMPERATURES  $t_s$  AT ATMOSPHERIC PRESSURE)

Substance	$t_c$ ° F.	$p_c$ lb./in. <sup>2</sup> abs.	$v_c$ ft. <sup>3</sup> /lb.	$t_s$ ° F. at 760 mm.
$\text{Cl}_2$ . . .	285.8	1193	—	-33.9
$\text{CO}_2$ . . .	87.8	1069	0.0346	-108.4
$\text{NH}_3$ . . .	271.2	1653	0.0836	-28.7
$\text{SO}_2$ . . .	312.8	1159	0.0307	+17.6
$\text{H}_2\text{O}$ . . .	705.2	3200	0.0464	212.0
$\text{O}_2$ . . .	-181.8	730.4	—	-361.4
$\text{N}_2$ . . .	-232.8	492.0	0.0515	-320.1
$\text{H}_2$ . . .	-403.6	156.4	—	-423.0
Air . . .	-221.1	574.6	0.0458	-311.8
$\text{CO}$ . . .	-221.8	529.1	—	310.0

In July, 1908, Professor Kamerlingh Onnes, of Leiden, succeeded in liquefying helium,\* so that it can now be claimed that all the known gases have been liquefied. Onnes found for helium that the critical temperature was  $-450.4^\circ \text{F.}$ , and the critical pressure  $32.7 \text{ lb./in.}^2 \text{ abs.}$  Under atmospheric pressure the gas is liquefied at  $-451.3^\circ \text{F.}$  During

\* H. Kamerlingh Onnes. "Untersuchungen über die Eigenschaften der Körper bei niedrigen Temperaturen, welche Untersuchungen unter anderen auch zur Herstellung von flüssigem Helium geführt haben." (*Commuc. Leiden Labor. Supply. No. 35. Nobelprisnede.*)

evaporation, the temperature dropped to  $-454.2^{\circ}\text{F}$ . The lowest pressure at which the helium still remained liquid was 0.15 mm. Hg, at which the temperature was only  $2.1^{\circ}\text{F}$ . above absolute zero.

The process of evaporation at constant pressure of cold vapours is the same as that of water (page 354). The absolute values of  $h$ ,  $L$ ,  $\rho$ , and  $H$  are, however, different for different substances. Thus, while the latent heat  $L$  of steam is about 1000 B.Th.U./lb., the value for ammonia (at about the same distance from the critical point) is only about 550, for  $\text{SO}_2$  about 150, and for  $\text{CO}_2$  about 110 B.Th.U./lb.

Regarding the latent heat  $L$ , it should be noted that, as the critical point is approached,  $L$  continuously decreases, until at the critical point it becomes zero. Fig. 222 shows the decrease in latent heat with increasing temperature for  $\text{CO}_2$ , and Fig. 218 for some other substances.

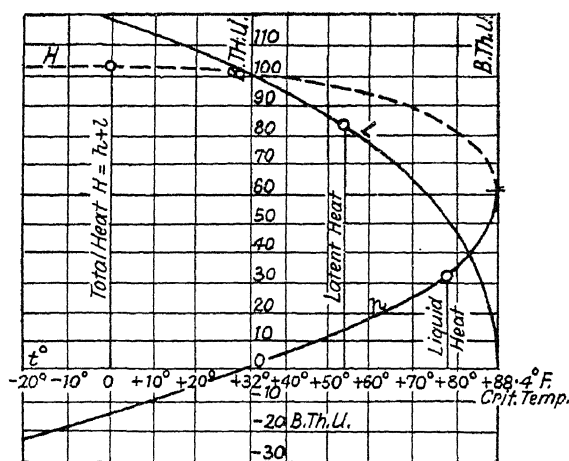


FIG. 222

The reverse occurs in the case of the sensible heats, which attain their highest value at the critical point. The total heat attains a maximum value before the critical point and decreases thereafter until, at the critical point, it becomes equal to the sensible heat (Fig. 222).

This is made clear by Fig. 221 and by a consideration of the discussion on the critical state. The volume increase during evaporation continuously decreases until at the critical point it becomes zero. At this point, the external evaporation heat is zero. The internal evaporation heat, however, is also zero here, since the fluid maintains its aggregation. The total heat is then equal to the sensible heat.

The same conditions are exhibited by steam near the critical point (cf. Fig. 205).

#### Cold vapour used as working substance in waste heat engines.

A cylinder filled with  $\text{SO}_2$  liquid and heated with exhaust steam from a steam engine at about  $140^{\circ}\text{F}$ . is capable of delivering  $\text{SO}_2$  vapour at a pressure of 150 lb./in.<sup>2</sup> abs.

Use may be made of this in obtaining useful work from some of the otherwise wasted heat contained in the exhaust steam. Instead

cooling water in the surface condenser of the steam engine, liquid  $\text{SO}_2$  is employed as the cooling medium. This causes the  $\text{SO}_2$  to evaporate and the pressure obtained may be as high as 200 lb./in.<sup>2</sup> abs., depending on the temperature of the exhaust steam. This high pressure vapour may be used in a single cylinder engine called a waste heat engine.

The exhaust from this engine has to be condensed at constant pressure and requires no compressor. If the available cooling water is at 50° F., the condensation pressure may be about 50 lb./in.<sup>2</sup> abs. Hence the heat drop is that given by  $\text{SO}_2$  between 200 and 50 lb./in.<sup>2</sup> abs. In spite of the considerable reduction in heat for a given load effected by this combination, however, it is seldom found to be economical, due to the high initial cost of the plant. For further details reference should be made to the works of Josse. (*Mitteilungen aus dem Maschinenlaboratorium der Königlichen Technischen Hochschule. Berlin II, III.*)

The energy contained in the exhaust steam from a reciprocating engine is now utilized in the steam turbine, the operation of which is totally different from the above. The steam works in the turbine at very low pressures. These turbines are called exhaust or waste heat turbines, and operate in the same manner as any other steam turbines.

### THE CLAPEYRON-CLAUSIUS EQUATION

A relationship exists between the latent heat  $L$ , the volume increase during evaporation, and the slope of the vapour pressure-temperature

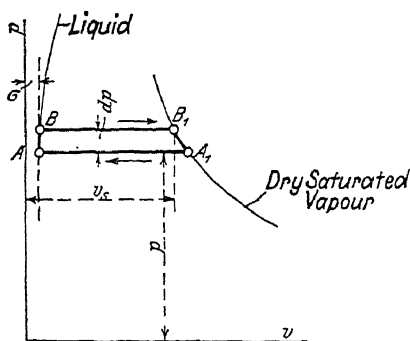


FIG. 223

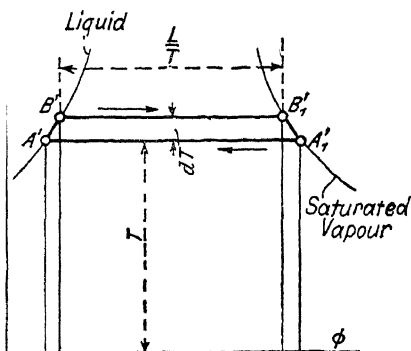


FIG. 224

curve, which is of fundamental importance, not only in evaporation but also in melting and sublimation processes.

At  $A$  in Fig. 223 let the pressure of 1 lb. of water be  $p$ , corresponding to the saturation temperature  $T$ . Also let its volume be  $\sigma$ . The pressure is now increased by  $dp$  by supplying sufficient heat, and the corresponding temperature becomes  $T + dT$ . The change of state  $AB$  occurs along the liquid curve. The heat supplied is, therefore, the difference in sensible heats corresponding to the pressures  $p + dp$  and  $p$ . This is represented on the  $T\phi$  field (Fig. 224) by the area below  $A'B'$ . The liquid is now completely evaporated at the pressure  $(p + dp)$ . During this change the volume increases by the amount  $BB_1 = v_s - \sigma$ , and the entropy by the amount  $B'B_1 = \frac{L}{T}$ . The latent heat  $L$  supplied is the area below  $B'B_1$ .



Thereafter the steam expands from the volume  $v_s$ , such that it remains dry until the point  $A_1$  is reached, where the pressure is equal to the initial pressure  $p$ .  $B_1A_1$  is thus an element of the saturation curve, and the heat added is the area below  $B_1A_1$ .

From  $A_1$  the steam is completely condensed to water, so that the final state coincides with the initial state of the liquid at  $A$ . During the condensation the heat withdrawn is the rectangular area below  $A_1A'$ .

In the cyclic process  $ABB_1A_1A$  described by the steam, a quantity of work equal to the area  $ABB_1A_1$  is delivered, and the heat equivalent of this work is the area  $A'B'B_1A_1$  (page 183). The work area is  $(v_s - \sigma)dp$  when the small triangles below  $AB$  and  $A_1B_1$ , which in the limit disappear, are neglected. The heat converted to work is  $\frac{L}{T}dT$  and its mechanical equivalent  $777.8 \frac{L}{T}dT$ .

This gives

$$(v_s - \sigma)dp = 777.8 \frac{L}{T}$$

or

$$\frac{1}{777.8} T \frac{dp}{dT} \quad . \quad (449)$$

This is the Clapeyron-Clausius equation.

The quotient  $dp/dT$  is the slope of the  $pT$  curve at the pressure  $p$  (Fig. 220), and is thus obtained from this curve. If, in addition to this, the volume increase  $(v_s - \sigma)$  during evaporation be known, the value of the latent heat  $L$  is obtained (without test) from the equation; or, if  $L$  be known by test, then  $(v_s - \sigma)$  can be calculated. This was the method adopted by Zeuner in constructing his steam tables from Regnault's test results.

If, by test, all the quantities of the equation are known, they should satisfy the equation. This is one of the most rigid proofs of the validity of the first and second laws of thermodynamics. The most reliable results show agreement with the equation as shown by comparing the volumes\* found from the  $L$  and  $\frac{dp}{dT}$  values with the volumes found by experiment.†

The above equation gives a general relation between the magnitudes  $p$ ,  $T$ , and  $v$  in the dry saturated state and the magnitudes  $L$  and  $\sigma$ . It offers a means, therefore, of establishing a proof of empirical equations which have been established from test results, as, for example, the relation between  $p$  and  $T$  in the saturated state. For the latter, taking the approximate gas equation as

$$pv_s = RT_s$$

and neglecting  $\sigma$  as small compared with  $v_s$ , gives

$$\frac{dp}{p} = \frac{L}{AR} \frac{dT}{T^2}$$

\* *Z.V.d.I.* (1909), page 1768. Holborn and Henning. Latent heat of steam, etc.

† *F. A.* Heft 21 (1905). Knoblauch, Linde, and Klebe. The thermal properties of saturated and superheated steam. A comparison of the values is given in *Z.V.d.I.* (1911), page 1506 *et seq.*

Also, by taking  $L$  as constant,

$$\log p = \text{constant} - \frac{L}{AR} \frac{1}{T_s}$$

which is the same as the equation given on page 355. Due to the simplifying assumptions on which this equation is based, it is obvious that it applies within a limited region only. By a more rigid treatment, as, for example, taking account of the variation in  $L$  with temperature, and taking a more exact relation between  $p$ ,  $v$ , and  $T$ , a pressure-temperature equation holding over a greater range can be derived.

### EQUILIBRIUM BETWEEN THE VAPOUR, LIQUID, AND SOLID STATE

**Liquid and vapour.** From liquid water at  $t > 32^\circ \text{F.}$ , which completely fills the space in a cylinder (Fig. 225a) closed by an air-tight piston, and which is maintained at a constant temperature, it is found that on suddenly raising the piston to a height  $h_1$ , vapour continues to be formed until the pressure in the space evacuated by the piston reaches a definite value, called the saturation pressure  $p_s$ . It is only when this pressure is attained that the formation of vapour ceases and thereafter equilibrium exists between the liquid and vapour. If the piston be again suddenly raised to the higher level  $h_2$ , vapour formation again continues until the pressure  $p_2$  is attained. This raising of the piston can be continued until all the liquid is evaporated (stroke volume =  $v_1$ ). For all intermediate

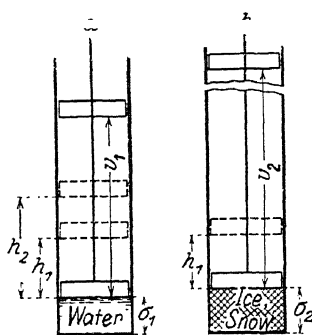


FIG. 225

values of the total water and steam volumes between  $\sigma_1$  and  $\sigma_1 + v_1 = v_s$ , equilibrium therefore occurs, and this is not disturbed if the residual water is sprayed into the vapour space to form wet vapour. The curve I showing these relationships is given in Fig. 226 by plotting the temperatures to a base of pressures (pressures for  $\text{H}_2\text{O}^*$  in inches of mercury and for  $\text{CO}_2^\dagger$  in lb./in.<sup>2</sup> abs.).

**Solid bodies and vapour (Sublimation).** If in place of liquid in the vessel closed by the piston (Fig. 225b), ice exists at, say,  $15^\circ \text{F.}$ , and if this temperature is maintained constant by external cooling, then, on suddenly raising the piston to  $h_1$ , vapour is again directly formed (without liquefaction) from the ice until a definite saturation pressure  $p_{s'}$  reigns in the evacuated space. This pressure is called the ice-vapour or sublimation pressure. Thereafter no further evaporation occurs and equilibrium is established between the ice and vapour. Evaporation only occurs again when the piston is raised to  $h_2$ , say. By raising the piston sufficiently high (in practice, by means of an exhaust pump) the whole of the ice can be converted to vapour, which then occupies the saturation volume  $v_{s'}$ .

\* From Holborn, Scheel, and Henning's heat values. *Phys. Techn. Reichsanstalt.*

† From R. Plank and J. Kubrianoff. Thermal properties of carbon dioxide in the gaseous, liquid and solid state. (Berlin, 1929.)

A state of equilibrium exists for all total volumes lying between  $\sigma_2$  and  $v'_s$ , and this state remains unaltered if the residual ice (which may also be in the form of snow or hoar frost) is mixed with the vapour. The curve II of sublimation pressures is plotted as explained above both for  $\text{CO}_2$  and  $\text{H}_2\text{O}$ , and from the same sources.

This curve cuts the curve I at a point  $P$ , and it is only at this point that liquid, vapour, and solid can exist together in stable equilibrium. For water, this state is found to be at  $32^\circ \text{F.}$  and  $4.579 \text{ mm. Hg}$  pressure, while for  $\text{CO}_2$  it occurs at  $-69.9^\circ \text{F.}$  and  $75.3 \text{ lb./in.}^2 \text{ abs.}$

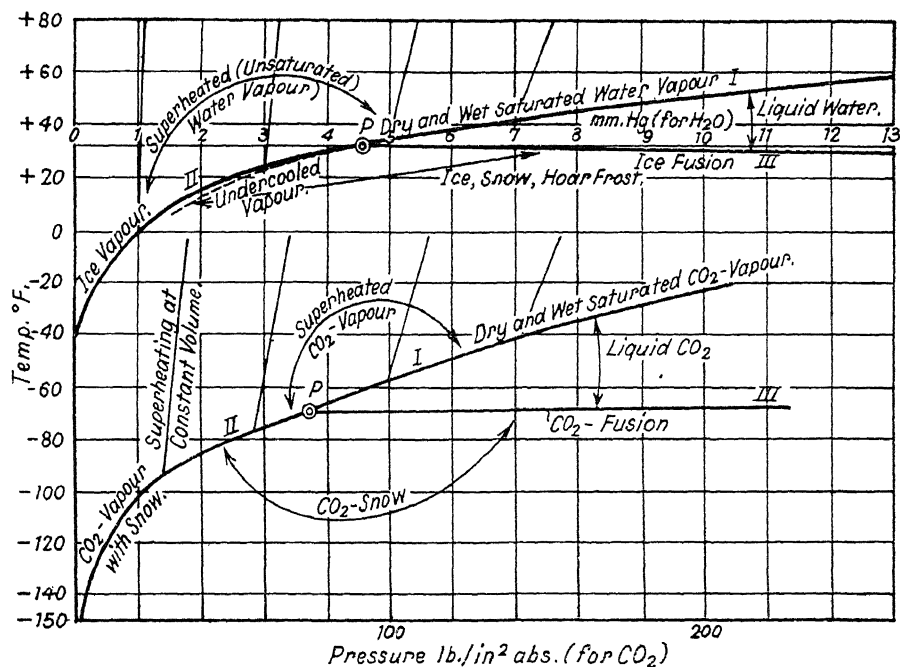


FIG. 226

**Solid and liquid bodies (Melting).** Let the ice contained in the vessel (Fig. 225*b*) be melted. With an external pressure of  $15 \text{ lb./in.}^2 \text{ abs.}$  this is only possible at a temperature of  $32^\circ \text{F.}$ , which remains constant until all the ice is melted. If the ice is at  $30^\circ \text{F.}$ , say, it must be raised to  $32^\circ \text{F.}$  before it can be melted. If, however, the ice, at  $30^\circ \text{F.}$ , be subjected to a pressure of  $1850 \text{ lb./in.}^2 \text{ abs.}$ , melting can occur at this temperature provided heat be supplied and the ice-water mixture maintains this temperature till all the ice be melted. Increased pressures cause a further lowering of the melting point. For other substances, as, for example,  $\text{CO}_2$ , which does not, like ice, suffer a reduction in volume on melting but an increase, a pressure rise causes the melting temperature to be raised. In order to effect an appreciable change in the melting temperature large pressure increases are required, so that the melting curves III in Fig. 226 are almost horizontal.

The melting curve must pass through the point  $P$ , since this point

represents the solid-liquid equilibrium state. The point  $P$  is called the "fundamental" or "triple" point.

The slopes of the three equilibrium curves represented by the quotients  $\frac{dp}{dT}$

all follow from the Clapeyron-Clausius equation, which applies not only to evaporation but also to sublimation and melting, if in place of the latent heat  $L$  the sublimation heat ( $L + s$ ) or the latent heat of fusion  $s$  and the corresponding volumes of the solid state or ice vapour are respectively substituted.

Hence for the equilibrium curves—

I. Liquid-vapour—

$$\frac{dp'}{dT} = 777.8 \frac{L}{T} \frac{1}{v_{s1} - \sigma_1} \quad . \quad . \quad . \quad . \quad . \quad (450)$$

II. Solid-vapour—

$$\frac{dp''}{dT} = 777.8 \frac{L + s}{T} \frac{1}{v_{s2} - \sigma_2} \quad . \quad . \quad . \quad . \quad . \quad (451)$$

III. Solid-liquid—

$$\frac{dp'''}{dT} = 777.8 \frac{s}{T} \frac{1}{\sigma_1 - \sigma_2} \quad . \quad . \quad . \quad . \quad . \quad (452)$$

From equations (450) and (451) it follows that the sublimation curve rises more rapidly with temperature than the evaporation curve, since  $L + s$  is greater than  $L$ . From equation (452) it follows for water that, with rising melting pressures ( $+ dp'''$ ) the melting temperature drops ( $- dT$ ), since  $\sigma_{liq.}$  is less than  $\sigma_{solid}$ , while for  $\text{CO}_2$  with  $\sigma_{liq.} > \sigma_{solid}$  the melting temperature rises with the pressure. In addition, the melting pressure curve is much steeper than the vapour pressure curve, since the volume change  $\sigma_{liq.} - \sigma_{solid}$  is considerably less than  $v_s - \sigma$ .

## ENTROPY AND ENTROPY DIAGRAMS FOR SATURATED VAPOURS

**General relations, hot vapours.** The treatment already given for the entropy and entropy diagrams for water is fundamentally the same for all vapours. In order to draw the liquid limit curve the specific heat  $c_{liq.}$  of the liquid at the boiling state must be known. This then gives, as on page 374,

$$= \frac{c_{liq.} dT}{T}$$

If  $c_{liq.}$  is constant (independent of  $T$ ), then

$$\phi_{liq.} = \bar{T}_n$$

provided the entropy at  $T_0 (= 492)$  is taken as zero. If  $c_{liq.}$  is a function of  $T$ , as is always the case in the critical region, then  $\phi_{liq.}$  is most easily

determined by the graphical method explained on page 377 or page 390. In the  $T\phi$  diagram the subtangent of the liquid curve gives the value of  $c_{liq.}$  (Fig. 66).

The entropy of dry saturated vapour is (page 375)

$$\frac{L}{T}$$

and the saturation limit curve is obtained by adding the values  $\frac{L}{T}$  to the liquid limit curve. In the case of steam the curve thus obtained lies to the right of the vertical through the critical point. The slope of this curve at any point gives the value  $\frac{dL}{dT} - \frac{L}{T}$ . If this value is negative so

that, with increasing temperature ( $+dT$ ) the entropy decreases ( $-d\phi$ ) the saturation curve is of the same nature as that of steam. (Curve I, Fig. 227.) The course of this curve is such that steam, which is adiabatically expanded from a point on it, becomes wet and, if adiabatically compressed, becomes superheated (points *E* and *C*).

It was impossible to decide before the establishment of the second law, whether the one case or the other would occur. The usual assumption made was that the steam remained dry saturated during compression. This question is of considerable importance in the theory and practice of reciprocating steam engines, and has lately been raised in connection with the use of other vapours, such as mercury and phenyl ether, as working substances.

It is conceivable that the upper limit curve for other vapours could be entirely different from that of steam. Differentiating the expression for the entropy of saturated vapour with respect to  $T$ , the slope of the tangent to the upper limit curve is

$$\begin{aligned} \frac{d\phi_s}{dT} &= \frac{d\phi_{liq.}}{dT} + \frac{dL}{dT} \\ &= \frac{c_{liq.}}{T} + \frac{1}{T} \left( \frac{dL}{dT} - \frac{L}{T} \right) \\ &= \frac{1}{T} \left( c_{liq.} + \frac{dL}{dT} - \frac{L}{T} \right) \end{aligned}$$

This value will be negative, zero, or positive accordingly as

$$\frac{L}{T} \begin{matrix} > \\ = \\ < \end{matrix} \frac{dL}{dT} + c_{liq.}$$

$\frac{dL}{dT}$ , which gives the rate of change of  $L$  as  $T$  is increased, is always negative, so that the expression

$$\frac{dL}{dT} - \frac{L}{T}$$

is always negative. The course of the entropy curve (Fig. 227) will thus depend on whether the absolute value of this expression is greater or less than  $c_{liq}$ .

Thus, for steam at 150 lb./in.<sup>2</sup> abs.,  $t_s = 358.5^\circ \text{ F.}$ ,  $L = 862.2 \text{ B.Th.U./lb.}$ , so that  $\frac{L}{T} = 1.055$ . From the steam tables,  $L$  decreases in this region by about 0.875 B.Th.U. per degree temperature rise  $\left( = \frac{dL}{dT} \right)$ .

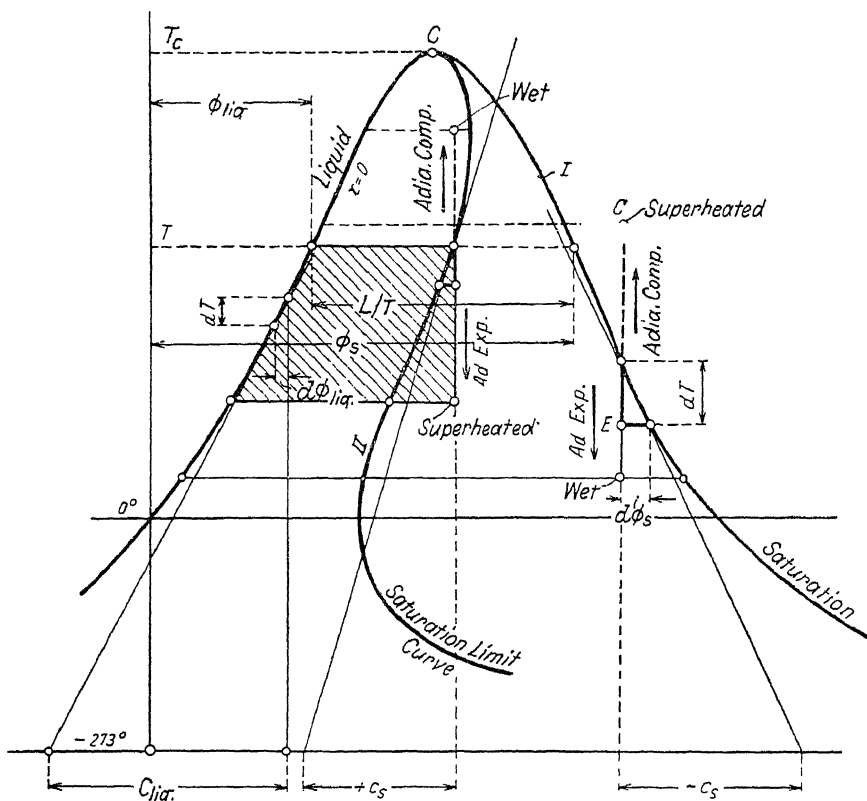


FIG. 227

We thus have

$$\frac{dL}{dT} - \frac{L}{T} = -0.875 - 1.055 = -1.930$$

while  $c_{liq} = 1.06$ .

This gives 
$$\frac{d\phi_s}{dT} = -\frac{0.87}{T}$$

and is thus negative. Again, by writing

$$d\phi_s = \frac{c_s dT}{T} \text{ we have } c_s = -0.87$$

so that  $c_s$ , the specific heat of the saturated steam (along the limit curve) is thus negative. This means that, in order to raise the temperature of the saturated steam by compressing it, so that it remains dry saturated, heat has to be removed. From the above relation we have

$$c_s = c_{liq} + \frac{dL}{dT} - \frac{L}{T}$$

In the case of mercury at  $575^\circ \text{F.}$ , we have, from Fig. 218,  $L = 117 \text{ B.Th.U./lb.}$ , so that  $\frac{L}{T} = 0.113$ . From Fig. 218,  $L$  decreases by about  $2.8 \text{ B.Th.U.}$  for a rise of  $100^\circ \text{F.}$  in this region, so that  $\frac{dL}{dT} = -0.028$ .

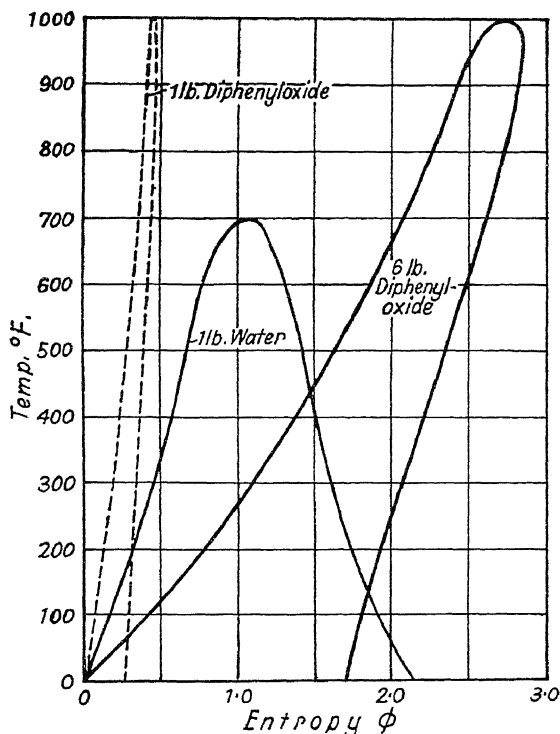


FIG. 228

This gives  $c_s = c_{liq} - 0.028 - 0.113 = c_{liq} - 0.141$   
 and since  $c_{liq} = 0.033$ , we have  
 $c_s = -0.108$

The form of the  $T\phi$  diagram for mercury vapour (Curve I in Fig. 227) is thus similar to that of steam.\*

\* A complete  $T\phi$  diagram for mercury will be found in Stodola's *Steam and Gas Turbines*. (5th Edition, 1922, p. 1090.) This contains lines of constant pressure, constant volume, and constant total heat in the wet and superheat fields up to temperatures of  $600^\circ \text{C.}$  and pressures of 22 atmospheres abs.

For phenylether, at  $t = 575^\circ \text{F.}$ ,  $L = 112.4$ ,  $\frac{dL}{dT} = -\frac{5}{100}$ , and  $c_{liq} = 0.4$ , so that  $c_s = 0.4 - \frac{112.4}{1035} - \frac{6}{100} = 0.4 - 0.169 = +0.331$

As opposed to steam, the specific heat of this vapour is thus positive (along the upper limit curve). If it is to be compressed at constant

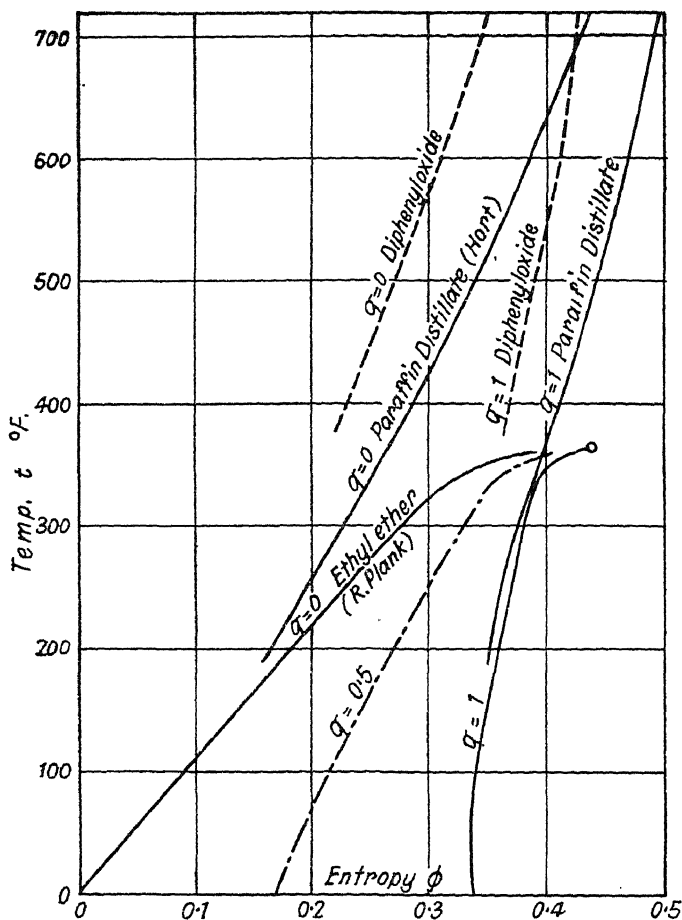


FIG. 229

dryness, heat must be supplied to it. If no heat be supplied (i.e. if it is compressed adiabatically) the vapour becomes wet. Contrary to the case of steam, this vapour becomes superheated when it expands adiabatically, as shown also by the adiabatic in the  $T\phi$  field.

Fig. 228 shows the  $T\phi$  diagrams for phenyl ether and steam,\* while Fig. 229 shows the  $T\phi$  diagram for a paraffin† (distilled between  $350^\circ \text{F.}$  and  $400^\circ \text{F.}$ ) along with that of ethyl ether.‡

\* See Dow, *Power Plant Engineering*, vol. xxx (August, 1926).

† See *Forsch. Arb.* No. 116. H. Hort. "Untersuchung von Flüssigkeiten."

‡ See *Zeitsch. f. Tech. Physik.* No. 1 and 3 (1922). R. Planck.



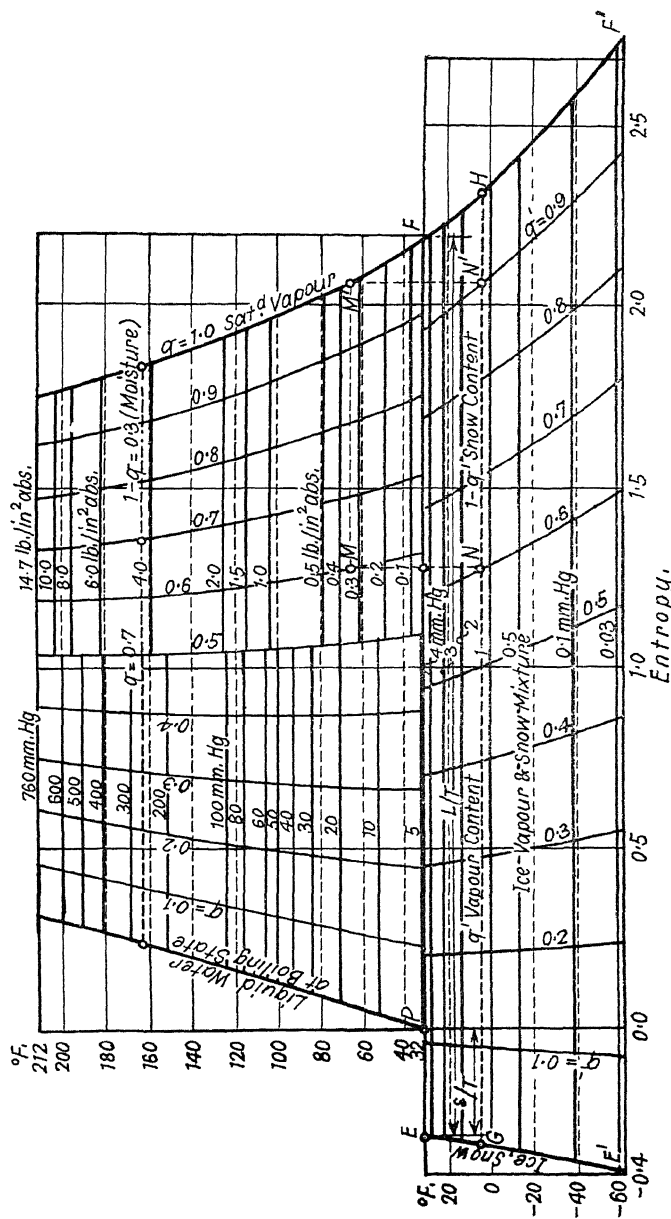


FIG. 230

**Water vapour at low temperatures.** As shown on page 429, water can exist only as a vapour or solid at temperatures below  $32^{\circ}\text{F.}$  and at normal pressures. The regions of the liquid-vapour and solid-vapour states are thus adjacent at  $32^{\circ}\text{F.}$  in the  $T\phi$  field. In Fig. 230 the upper part of the  $T\phi$  diagram has been plotted (above  $32^{\circ}\text{F.}$ ) from the known values of the liquid and vapour entropies. The vapour pressures are marked, in inches of mercury, on the horizontals on the left, and in lb./in.<sup>2</sup> abs. on the right.\*

The vapour pressure of ice or water at  $32^{\circ}\text{F.}$  is  $0.18''\text{ Hg.}$  The point  $P$  is called the "triple point" (see page 429).

If now the latent heat of fusion  $s = 142\text{ B.Th.U./lb.}$  be withdrawn from liquid water at  $32^{\circ}\text{F.}$  (i.e. at the state point  $P$ ) the water freezes, so that the entropy decreases by  $\frac{142}{492}$  entropy units, and the new state point is at  $E$ . If more heat be removed from the ice, its entropy decreases by the amount

$$/492$$

where  $c_{\text{sol}}$  is the specific heat of ice. This specific heat has the value  $0.5$  at  $32^{\circ}\text{F.}$ , but decreases as the temperature is lowered till, at  $-460^{\circ}\text{F.}$ , its value is zero. In this way the ice or snow limit curve between  $32^{\circ}\text{F.}$  and  $-60^{\circ}\text{F.}$  is drawn in Fig. 230. If now the ice is sublimated at  $32^{\circ}\text{F.}$  (i.e. transformed to the vapour state without passing through the liquid state) the entropy increases to the value of the entropy of dry saturated water vapour at  $32^{\circ}\text{F.}$  This value is known from the evaporative process, since the state of dry saturated vapour is exactly the same as that of ice vapour at  $32^{\circ}\text{F.}$  The sublimation heat, added between  $E$  and  $P$ , is equal to the latent heat of fusion, i.e.  $s = 142\text{ B.Th.U./lb.}$ , while the sublimation heat added between  $P$  and  $F$  is equal to the latent heat  $L = 1071\text{ B.Th.U./lb.}$  The entropy change during sublimation is thus

$$\frac{142 + 1071}{492} = \frac{1213}{249}$$

If the ice is sublimated at any lower temperature, such as  $5^{\circ}\text{F.}$ , it can be converted to an ice-vapour mixture, or to a vapour, by a supply of a part, or of the whole, of the sublimation heat. The sublimation heats, however, vary with temperature in the same way as the latent heats. They can be determined from the value at  $32^{\circ}\text{F.}$  if the specific heats, at constant pressure, of ice vapour and of snow are known between  $32^{\circ}\text{F.}$  and  $-t^{\circ}\text{F.}$  They increase slightly between  $32^{\circ}\text{F.}$  and  $-60^{\circ}\text{F.}$ , and then decrease. The average value between  $32^{\circ}\text{F.}$  and  $-60^{\circ}\text{F.}$  can be taken as  $1215\text{ B.Th.U./lb.}$ , so that the change of entropy during sublimation at the temperature  $T$  is  $\frac{1215}{T}$ . The values given by this are

used in constructing the lower portion of Fig. 230. For an ice-vapour-snow mixture, having the vapour content  $q'\text{ lb./lb.}$  and the snow content  $(1 - q')$ , the entropy value is found by dividing the intercepts  $GH$  between

\* From Holborn, Scheel, and Henning. *Wärmetabellen d. Phy. Tech. Reichsanstalt.*

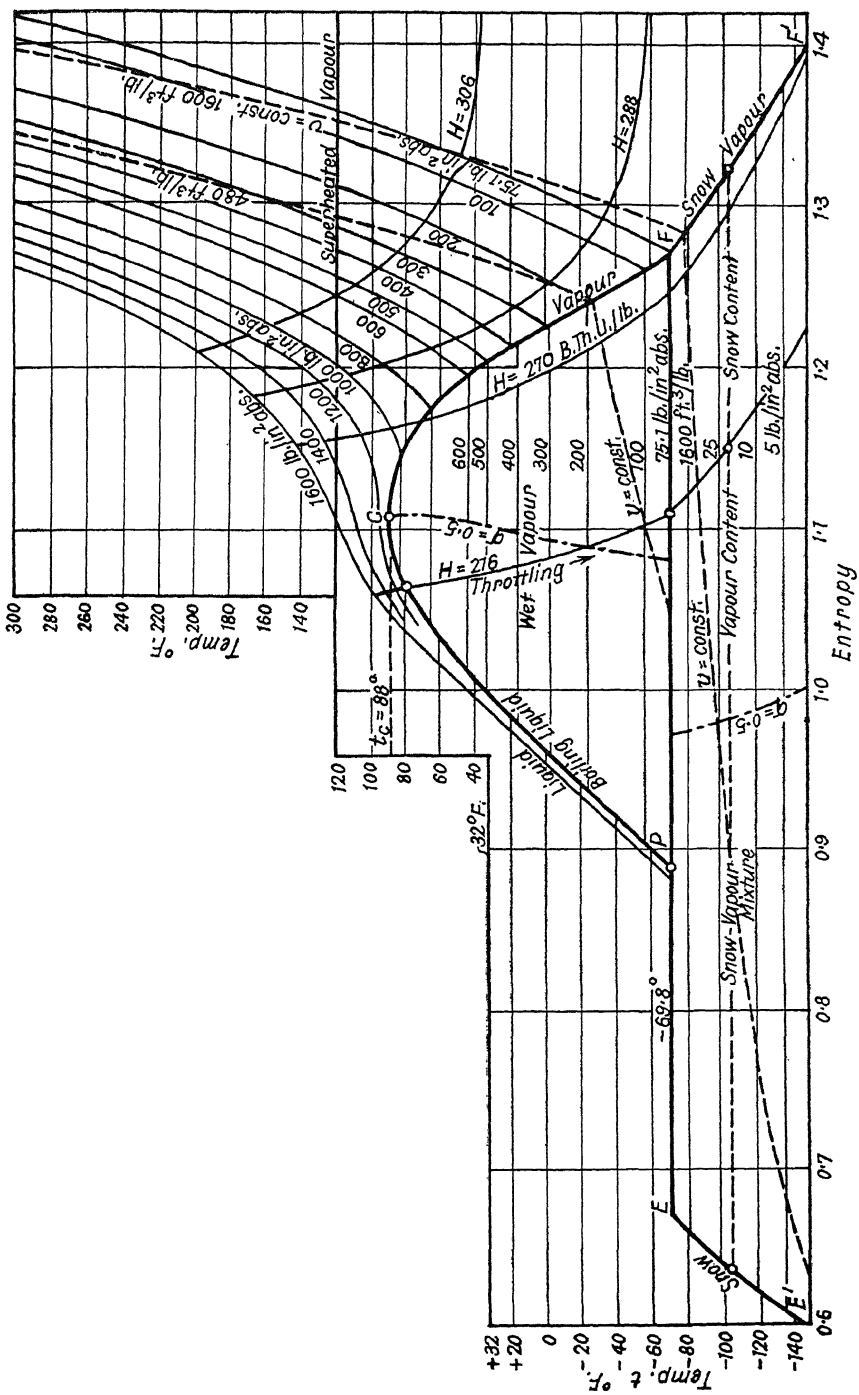


FIG. 231

the ice and vapour curves into equal parts (usually 10 in number), in the same way as is done in the  $T\phi$  field above  $32^\circ\text{F}$ . The vapour pressures on the horizontal temperature lines below  $32^\circ\text{F}$ . have been marked in inches of mercury.

**Example 1.** Moist water vapour at  $70^\circ\text{F}$ ., having a moisture content of 40 per cent expands adiabatically to  $5^\circ\text{F}$ . Find the snow contents at  $32^\circ\text{F}$ . and  $5^\circ\text{F}$ .

The line  $MN$  in Fig. 230 gives  $q' = 0.6$  at  $N$ , so that the snow content at  $5^\circ\text{F}$ . is 40 per cent. Similarly at  $32^\circ\text{F}$ .,  $q' = 0.63$  and  $1 - q' = 0.37$  or 37 per cent.

If, however, the vapour is initially dry,  $M'N'$  represents the adiabatic expansion, and the snow present in the ice vapour after expansion is only 10 per cent.

**Vapours at low temperatures (Carbon dioxide and air).** The entropy diagrams of cold vapours are the same in form as those of steam, but the critical points are found to be at very much lower temperatures. The limiting curve for carbon dioxide is shown by the heavy line in Fig. 231. It includes the snow and ice vapour limit curves as given by R. Planck.\* In Fig. 232 the liquid and vapour limit curves are shown for air. These are taken from *Forschungsarbeiten* (1926), No. 274, H. Hausen. The complete limit curve is situated at very low temperatures, below the critical temperature of  $-220.7^\circ\text{F}$ . The constant pressure lines of the wet mixture are not horizontals in this case, but rise slightly from left to right. This is on account of air being a mixture of oxygen and nitrogen, which show different boiling points for the same pressure.

### SUPERHEATED VAPOURS

**Characteristic equation.** Superheated vapours deviate increasingly in their properties from perfect gases as they approach the saturated state. The nature of this deviation is shown in Fig. 159, and is of the same character for all substances. It is represented by the van der Waal equation,

$$\left(p + \frac{a}{v^2}\right)(v - b) = RT \quad . \quad . \quad . \quad . \quad (453)$$

This equation thus serves as a typical characteristic equation for all superheated vapours.

When considering state regions far removed from the critical state,  $b$  is small compared with  $v$ , so that equation (453) can be written

$$pv \quad . \quad \frac{a}{v} \quad (454)$$

The term  $\frac{a}{v}$  thus represents the deviation from the perfect gas state, as explained on page 322. Writing the equation as

$$v = \frac{RT}{p} + \frac{a}{pv} \quad (455)$$

\* *Z.V.d.I.* (1929), page 221. R. Planck.

it will be seen that, for the same  $p$  and  $T$  values, the volume of superheated steam is always less than that of the perfect gas, the volume of which is

$$v = \frac{RT}{P}$$

Since van der Waal's equation applies to points at, or near, the saturated state, as well as to points far removed from this state, it can

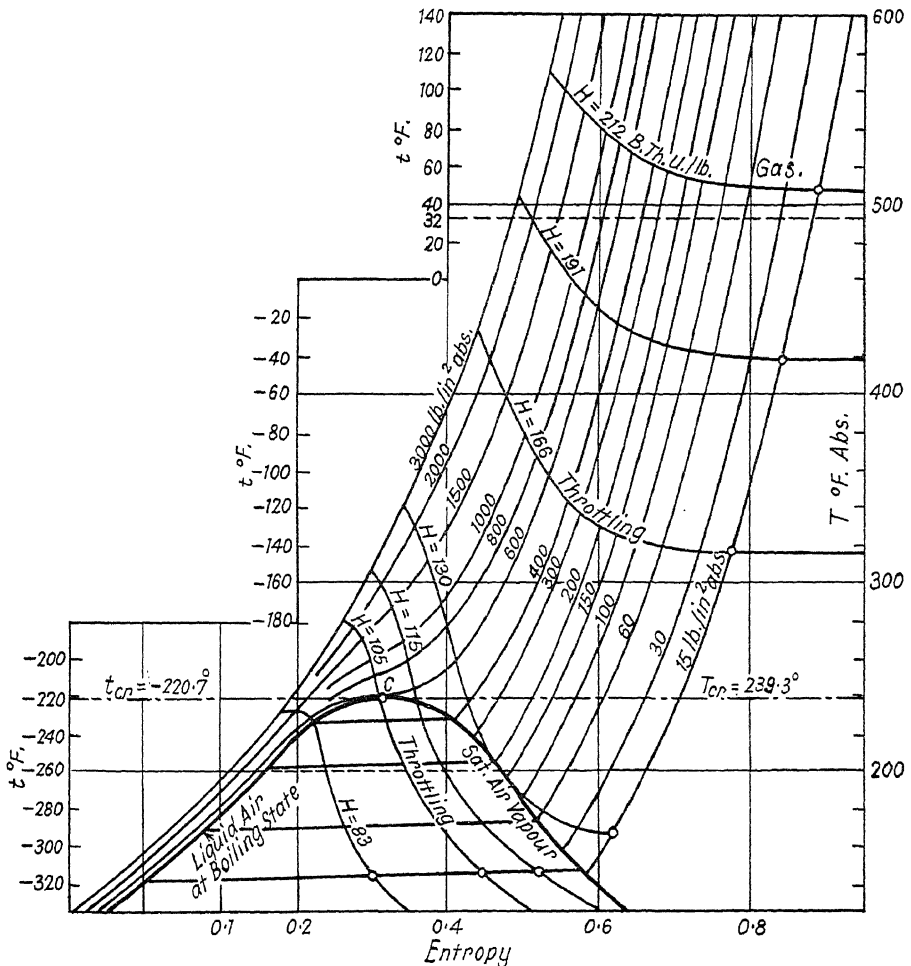


FIG. 232

be used as a fundamental test for the various approximate equations. Thus, the frequently adopted approximate equation for superheated steam,

$$pv = RT - Cp \quad . \quad . \quad . \quad . \quad . \quad . \quad (456)$$

is fundamentally in error, since the correction term is in the form  $Cp$

instead of  $\frac{a}{v}$ . If, in equation (454), we take the volume, in the term  $\frac{v}{a}$ , as approximately equal to  $\frac{RT}{p}$ , the equation becomes

The correction term must, therefore, contain the temperature as well as the pressure. Equation (456) is thus valid only for a mean steam temperature  $T$  and within a limited range.

The variation in the value of the specific heat  $c_p$  has recently been adopted as offering a rigid test for the validity of any characteristic equation in the superheat region. This was first applied, by means of the Munich values, to steam, and was later applied to such vapours as ammonia, carbon dioxide, and air. The Munich values showed an unknown and unexpected rise in the specific heat as the saturated state was approached. This, however, is in accordance with van der Waal's equation,\* although the amount of the actual increase is greater than that found by means of this equation. The considerable rise in specific heat with pressure, when the temperature is kept constant, is also quite closely in accordance with the van der Waal equation. This was confirmed by Holborn and Jakob's tests.† Again, the results shown by throttling of superheated steam and gases agree with the van der Waal equation, as confirmed by later throttling tests, particularly those carried out in the Munich laboratory.‡ There can be no doubt, therefore, of the general validity of van der Waal's equation as applied to the superheat region. It is necessary, however, to replace the constant value  $a$  in equations (453) and (454) by a function of the temperature, as was first shown by Clausius with reference to the deviations of  $\text{CO}_2$  vapour, particularly near the critical point.

Hence with  $a = f(T)$

equation (455) assumes the form

$$v = \frac{RT}{p} + \frac{a_0}{pvT^n}$$

If we take§

$$\text{this becomes } v = \frac{RT}{p} - \frac{a_0}{pvT^n}$$

and if, in the correction term, we take  $pv = RT$ , then

$$v = \frac{RT}{p}.$$

\* See Schüle's *Technische Thermodynamik*, vol. ii. Fourth Edition. Article 6.

† *Forsch. Arb.* 187 and 188.

‡ *Forsch. Arb.*, 274.

§ R. Plank, *Z. Techn. Physik* (1924), No. 9. "On the specific heats of superheated vapours."

This is Callander's characteristic equation of superheated steam, where  $n + 1 = \frac{13}{3}$ , so that  $n = \frac{10}{3}$ . Plank, however, takes  $n + 1 = 4$ ,  $n = 3$ , and finds, in this way, close agreement with the Munich  $c_p$  values.

For ammonia, Plank takes  $n = 3.6$  and finds close agreement with the American measurements of the specific heats.

The characteristic equation, given by Plank,\* for carbon dioxide is

$$v = \left( \frac{RT}{\left( \frac{T}{100} \right)^{10/3}} \right)^{1/4}$$

in lb./ft.<sup>2</sup> abs. and  $T$  in ° F. abs.)

in which  $R = 35.1$ ,  $a = 9.35$ , and  $b = 0.0000695$ . This applies to pressures up to 570 lb./in.<sup>2</sup> abs. and to the saturation limit.

These equations can be modified to suit further extensions in the superheat field by choosing values of  $a$  and  $b$ , which agree quantitatively with test results.† Up till now, no equation of state has been established which represents the whole region of state more closely than that of van der Waal. For technical purposes, however, it is preferable to represent the state conditions graphically (as has been described for steam) since, in this way, account can be taken of the test results at all states.

Recent state diagrams for steam, carbon dioxide,‡ and air, which are more extensive and reliable than for any other substance, have been constructed in the closest possible agreement with test results, combined with a strict adherence to thermodynamic relations.

**Specific heat ( $c_p$ ).** The nature of the changes in the specific heat  $c_p$  of superheated steam, as the saturation curve is approached, are shown by the Munich values. At the saturation curve,  $c_p$  increases with increasing pressures, and at constant pressure decreases as the superheat is increased, until it finally assumes a minimum value and then increases with increasing temperature. At very small pressures  $c_p$  follows the changes corresponding to a gas (see page 48). All the curves of  $c_p$  in

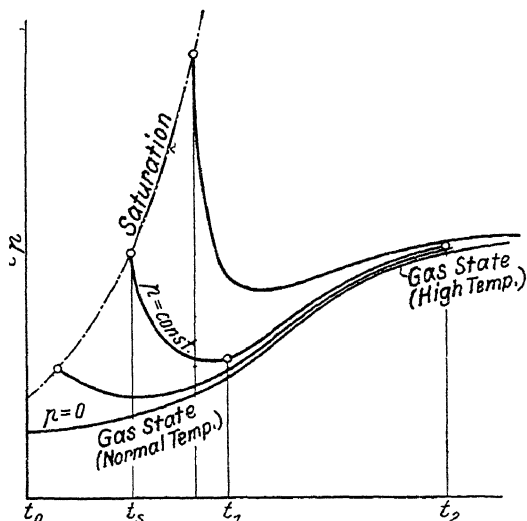


FIG. 233

\* R. Plank and J. Kuprianoff, *Beihefte zur Zeitsch. f. d. ges. Kälteindustrie*. Reihe I. "Thermal properties of CO<sub>2</sub> in the gaseous, liquid, and solid states."

† See, in particular, *Handbuch der Physik*, vol. x, Chap. 3. I. D. van der Waals, Jr.

‡ See first footnote above.

Fig. 233 approach this gas curve, as the temperature is increased, and Fig. 233 serves generally to show the change in the  $c_p$  values for any substance, provided its pressure is below the critical.

At the critical point  $c_p$  becomes infinitely large, as shown also by van der Waal's equation. For supercritical pressures, such as 570 lb./in.<sup>2</sup> for air or 1000 lb./in.<sup>2</sup> for CO<sub>2</sub>, the variation in  $c_p$  is different and is shown in Fig. 234 for air. (See *Forschungsarbeiten* No. 274, Hausen.) According to this,  $c_p$  assumes a maximum value for each pressure, and this value is reduced as the pressure is increased, while it approaches the critical

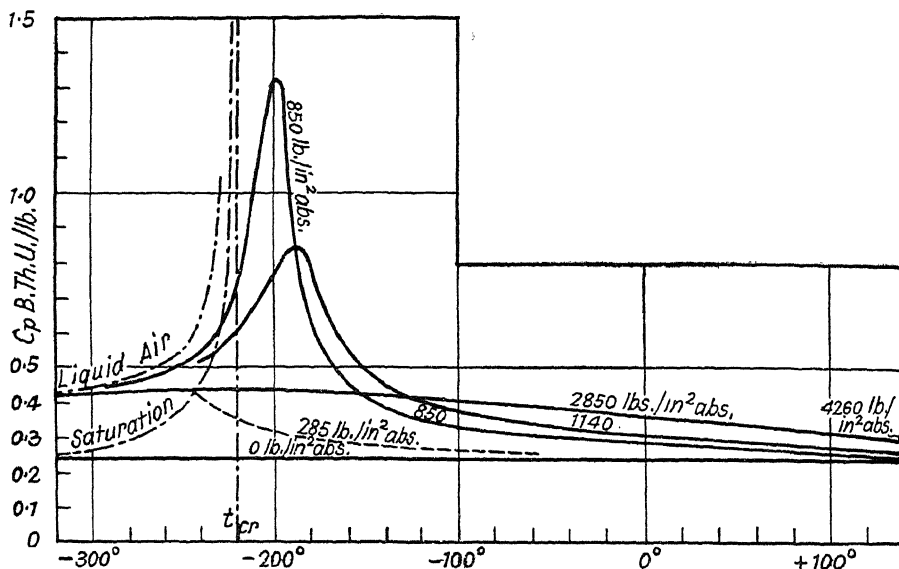


FIG. 234

temperature as the pressure approaches the critical pressure. These relations also follow from van der Waal's equation.

A large amount of experimental work, showing the relation between  $c_p$  and the pressure, at constant temperature, has been carried out by Holborn and Jakob.\* These relations again show agreement with van der Waal's equation. See Schüle's *Technische Thermodynamik*, vol. ii, Art. 146, regarding  $c_p$  values for pressures up to about 3000 lb./in.<sup>2</sup> abs. and temperatures between + 60° C. and - 123° C.

From the above statements it will be clear that, with regard to the total state region of superheated vapours and actual gases,  $c_p$  changes rapidly with pressure and temperature. The laws governing these changes are complicated, but their essential nature can be deduced by means of van der Waal's equation. (See *Zeitschrift für Technische Physik* (1923), page 460, Jakob.)

**State diagrams.** The state of a superheated vapour, or actual gas, is represented by the analytical or graphical relation between the three

\* *Forsch. Arb.* No. 187 and 188 (1916). The specific heat  $C_p$  of air at 60° C. and from 1 to 300 atmospheres.



state magnitudes  $p$ ,  $v$ , and  $T$ , as has already been explained for a perfect gas. A diagram of this type, for example, is shown in Fig. 159, in which the products  $pv$ , for  $T = \text{constant}$ , are plotted to a base of  $p$ . Again, the usual diagrams of isobars, with  $t$  and  $v$  as co-ordinates (such as Fig. 5),

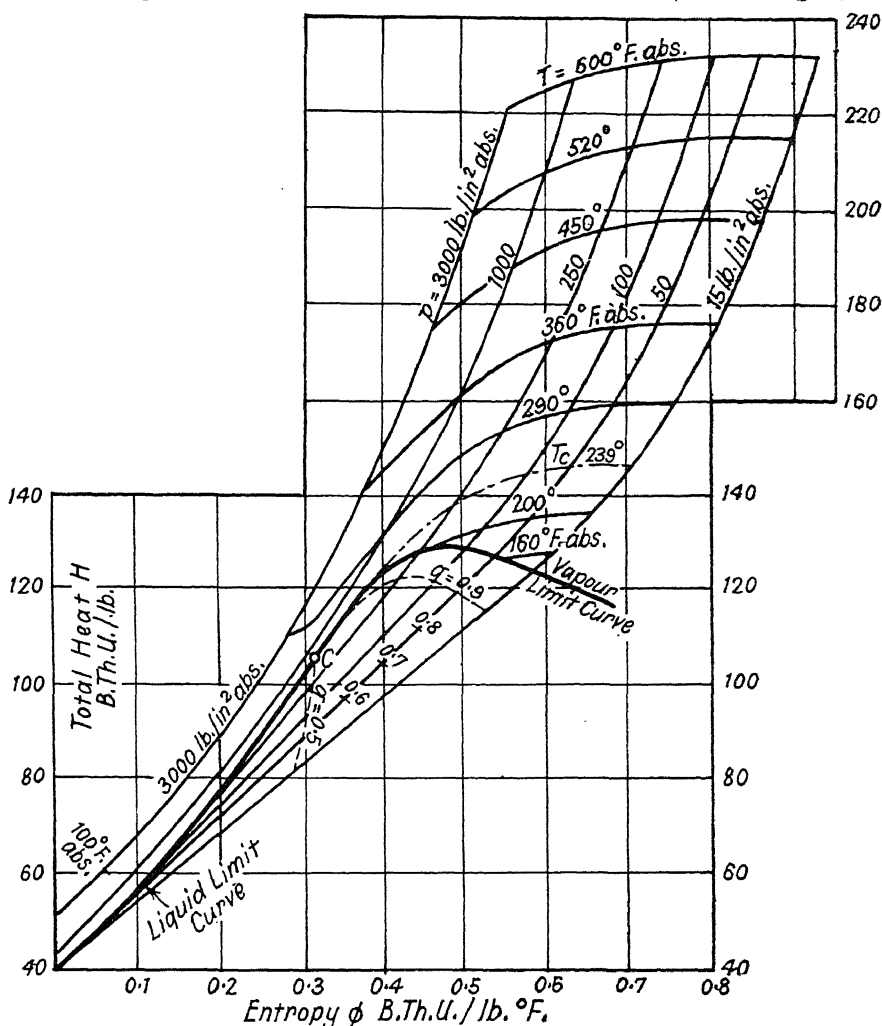


FIG. 235

as well as the Jakob diagram for  $H_2O$ , in which values of  $\frac{pv}{T}$  are plotted to a base of  $p$  values, are state diagrams.

These diagrams, however, do not give the heat quantities contained in gases and vapours. For this, state diagrams in which, for example, the total heat  $H$  is plotted to a base of temperatures at constant pressures (such as the  $H$ - $t$  diagram, for steam, shown in Fig. 215, or the  $H$ - $p$  diagram used by Mollier) are applied and are known as "thermal state diagrams."

A  $\text{CO}_2$  diagram of this type, covering the whole useful field, and drawn to a large scale, has been constructed by Plank and Kuprianoff. (See footnote on page 441.) The usual temperature entropy diagrams also belong to the thermal state diagrams, since heat quantities can be represented on them. Figs. 231 and 232 show  $T\phi$  diagrams for  $\text{CO}_2$  and air. Again, the  $H\phi$  diagrams first applied by Mollier are thermal state diagrams. The first diagram of this type for air, to include the low temperature region down to  $-200^\circ \text{C}$ ., was given by Hausen in *Forschungsarbeiten*, No. 274 (1926). In its general form this diagram (Fig. 235) is similar to the familiar  $H\phi$  diagram for steam.

### THROTTLING OF SATURATED AND SUPERHEATED VAPOURS AND OF ACTUAL GASES

The general conditions relating to throttling have already been dealt with on page 210 *et seq.* It was shown there that the characteristic feature of a throttling action is that, before and after throttling, the thermal potential remains constant, i.e.

$$H_1' = H_2'$$

provided that the approach and leaving velocities are the same, or change but slightly. This relation will now be applied to vapours. Throttling is of particular importance in refrigeration processes and in the liquefaction of gases.

**Wet vapours.** ( $\alpha$ ). Moderately wet vapour. In this case the following relation is almost exact

$$H' = H = h + qL$$

(Total heat, page 361.)

Hence, for the throttling process, we have

$$h_1 + q_1 L_1 = h_2 + q_2 L_2$$

so that

$$\quad \quad \quad . \quad (457)$$

Now, in the case of water,

so that

$$(458)$$

From these relations the quantity  $q_2$ , after throttling, with a given initial quality  $q_1$  and given initial and final pressures, can be found.

In the case of gases, the temperature before and after throttling remains unchanged, but this never occurs with wet vapours, since the temperature decreases with the pressure, so long as the vapour remains wet. The loss of kinetic energy must first be utilized in evaporating the liquid present in the vapour. This is made clear by equation (458). If this is written in the form

$$q_2 = q_1 - q_1 \frac{L_2 - L_1}{r} + \frac{t_1 - t_2}{r}$$

we have

By means of steam tables it is readily found that the second term is always positive, so that  $q_2 > q_1$  and drying occurs. If, for example, throttling takes place from a pressure of  $p_1 = 70 \text{ lb./in.}^2 \text{ abs.}$  to pressures of

then  $p_2 = 60 \quad 40 \quad 30 \quad 20 \text{ lb./in.}^2 \text{ abs.}$

and  $t_1 - t_2 = 9.9^\circ \text{ F.} \quad 35.4^\circ \text{ F.} \quad 52.2^\circ \text{ F.} \quad 74.7^\circ \text{ F.}$

$L_2 - L_1 = 7.5 \quad 25.8 \quad 37.3 \quad 52.7 \text{ B.Th.U./lb.}$

$L_2 - L_1$  is thus always smaller than the corresponding value of  $t_1 - t_2$ , and since  $q_1$  is less than 1, the second term in the numerator is always less

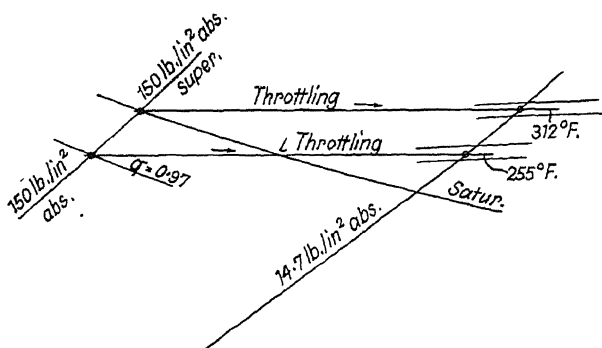


FIG. 236

than the first. The fraction is, therefore, positive, so that  $q_2$  is greater than  $q_1$ .

If wet vapour is to be just dry after throttling, then

$$1 = q_1 \frac{L_1}{L_2} + \frac{t_1 - t_2}{L_2}$$

or 
$$q_1 = \frac{L_2 - (t_1 - t_2)}{L_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad (459)$$

For example, if steam, at  $150 \text{ lb./in.}^2 \text{ abs.}$ , is to be dry saturated, after throttling to  $120 \text{ lb./in.}^2 \text{ abs.}$ , find the least permissible dryness fraction.

With  $L_1 = 862.2$ ,  $L_2 = 877.3$ ,  $t_1 = 358.5$ , and  $t_2 = 341.2$  we have

$$q_1 = \frac{877.3 - (358.5 - 341.2)}{862.2} = \frac{860.0}{862.2}$$

The moisture content is thus

$$\frac{2.0}{862.2}$$

or 
$$\frac{2.0 \times 100}{862.2} = 0.23 \text{ per cent}$$

The drying action due to throttling is, in this case, very small. In the same way it is found that 5.1 per cent mixture can be evaporated in

throttling from 14.7 lb./in.<sup>2</sup> abs. to 1.47 lb./in.<sup>2</sup> abs. (as might occur, for example, in exhausting steam from a cylinder to a condenser).

The  $H\phi$  diagram is particularly suitable for finding the final state in such problems, since throttling is also represented by horizontals in the wet field. Thus, if steam at 150 lb./in.<sup>2</sup> abs. with 7.2 per cent moisture is throttled to 15 lb./in.<sup>2</sup> abs., the final quality is  $q = 0.987$ .

When dry saturated steam is throttled, it becomes superheated, as shown by a glance at the  $H\phi$  chart. Thus, if dry saturated steam is throttled from 150 lb./in.<sup>2</sup> abs. to 14.7 lb./in.<sup>2</sup> abs., its final quality is about 100 F.<sup>o</sup> range of superheat (Fig. 236).

The application of the  $H\phi$  chart is just as simple when wet steam becomes superheated (Fig. 236), whereas the calculative treatment, in this case, is involved.

In order to measure the quality of steam the throttling calorimeter\* is used. Since discharge must take place to the atmosphere, the instrument is only applicable for moderately wet steam (i.e. not more than 5 per cent, depending on the steam pressure). By means of the measured range of superheat  $t_s$  after throttling, along with the initial and final pressures, the initial steam quality is determined. Thus

$$= H_2 + c_p t_s$$

so that

Due to the variation in the value of  $c_p$ , it is better, however, to make use of the  $H\phi$  chart in finding the unknown initial state.

**Excessively wet vapours and pure liquids.** Instead of making use of the relation  $H' - H$  it is necessary, in this case, to introduce the exact value for  $H'$ .

According to definition, this is

$$H' = I + Apv$$

Now, for a wet vapour, we have

$$I = h + q\rho$$

so that  $H' = h + q\rho + Apv$

The total heat, however, is

$$H = h + q\rho + Ap(v - \sigma) \text{ (see page 364)}$$

so that the thermal potential  $H'$  is greater than the total heat by the amount  $Ap\sigma$ . (For water,  $\sigma = 0.016$  ft.<sup>3</sup>/lb.) For the states before and after throttling, we now have

Now, at points which are at some distance from the critical state, we can take  $\sigma_1 \doteq \sigma_2 = \sigma$ , so that

\* A very complete theoretical and practical treatment of the throttling calorimeter will be found in *Forsch. Arb.* No. 98 and 99, by A. Sendtner.

Compared with the corresponding expression for moderately wet steam it is seen that  $q_2$  is now larger by the last term in equation (460), i.e. more evaporation occurs.

When the initial state is all liquid at the saturation temperature we have, with  $q_1 = 0$ ,

$$q_2 = \frac{h_1 - h_2}{L_2} + A \frac{p_1 - p_2}{L_2} \sigma. \quad (461)$$

For water,  $h_1$  and  $h_2$  can be replaced by  $t_1$  and  $t_2$ .

**Example 2.** Liquid water, at 130 lb./in.<sup>2</sup> abs., and at the saturation temperature of 347.4° F., flows into a space which is filled with steam at 14.7 lb./in.<sup>2</sup> abs. Find the quality at discharge.

With  $t_1 = 347.4$ ,  $t_2 = 212$ , and  $L_2 = 971.4$ , we have, after throttling,

$$\begin{aligned} q_2 &= \frac{347.4 - 212}{971.4} + \frac{144}{778} \left( \frac{130 - 14.7}{971.4} \right) 0.016 \\ &= 0.1395 + 0.00035 = 0.1398 \end{aligned}$$

The second term is relatively small and could be neglected.

**Example 3.** Liquid ammonia at 171 lb./in.<sup>2</sup> abs., and at the corresponding saturation temperature of 86° F. is throttled to 41.5 lb./in.<sup>2</sup> abs. Find the resulting percentage evaporation.

The back pressure of 41.5 lb./in.<sup>2</sup> abs. corresponds to a saturation temperature of 14° F., for which the latent heat is  $L_2 = 580$  B.Th.U./lb. The liquid heats at 86° F. and 14° F. are  $h_1 = 51.3$  B.Th.U./lb. and  $h_2 = -15.9$  B.Th.U./lb. respectively, while  $\sigma = 0.0256$  lb./ft.<sup>3</sup>

These now give

$$\begin{aligned} &\frac{51.3 + 15.9}{580} + \frac{144}{778} \left( \frac{171 - 41.5}{580} \right) 0.0256 \\ &= 0.116 + 0.0011 = 0.1171 \end{aligned}$$

Hence 11.71 per cent of the moisture is evaporated by the throttling of the liquid ammonia. Here, again, equation (457) would suffice.

The throttling process on the  $T\phi$  diagram is shown in Fig. 237, by the line  $A_0B_0$  for boiling liquid and  $A_1B_1$  for wet vapour. Point  $B_0$  is located by making the shaded areas below  $GA_0$  and  $C_0B_0$  equal to one another, while  $B_1$  is located by making the shaded areas below  $GA_0A_1$  and  $C_1B_1$  equal to one another. The thermal potentials at  $A_0$  and  $B_0$  are then equal, as also those at  $A_1$  and  $B_1$ .

**Superheated vapours.** The expression for the internal energy  $I$  of these vapours is readily obtained when it is remembered that the whole

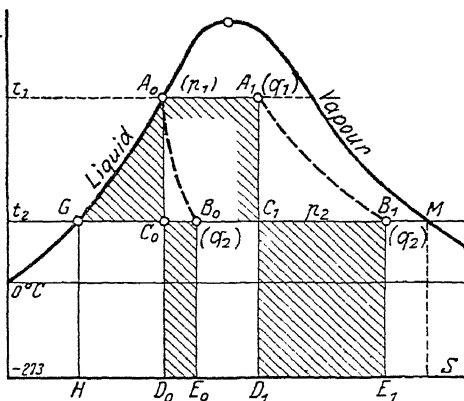


FIG. 237

of the absolute work obtained in adiabatic expansion is derived from the internal energy of the expanding substance. It is now known that the expansion of superheated steam follows the law

$$pv^\gamma = C \text{ (with } \gamma = 1.3 \text{)}$$

The same type of law applies, at least within certain limits, to other vapours (such as ammonia, sulphur dioxide, and carbon dioxide.) The absolute expansion work is given by (see page 108)

$$E = \frac{1}{\gamma - 1} (pv - p_0v_0)$$

where  $pv$  refer to the final state and  $p_0v_0$  refer to the initial state.

$$\text{Hence } I - I_0 = \frac{A}{\gamma - 1} (pv - p_0v_0) \quad . \quad . \quad . \quad . \quad . \quad (462)$$

From this the throttling equation becomes, when the kinetic energy is neglected and the terms  $I_0$  and  $p_0v_0$  are cancelled on both sides

$$\frac{A}{\gamma - 1} (p_1v_1 - p_2v_2) + Ap_1v_1 - Ap_2v_2 = 0$$

$$\text{or } \frac{\gamma}{\gamma - 1} p_1v_1 = \frac{\gamma}{\gamma - 1} p_2v_2$$

$$\text{so that } p_1v_1 = p_2v_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (463)$$

In throttling, therefore, the product of the pressure and specific volume remains constant. The same applies to the throttling of a gas.

For steam, we have approximately

$$p_1v_1 = RT_1 - 0.2563p_1$$

$$\text{and } p_2v_2 = RT_2 - 0.2563p_2 \quad (\text{see page 372})$$

$$\text{Hence } T_1 - T_2 = \frac{0.2563}{R} (p_1 - p_2)$$

$$\text{and with } R = 85.85$$

$$T_1 - T_2 = t_1 - t_2 = \frac{p_1 - p_2}{335} \quad (p \text{ in lb./ft.}^2 \text{ abs.})$$

$$\text{or } t_1 - t_2 = \frac{p_1 - p_2}{2.32} \quad (p \text{ in lb./in.}^2 \text{ abs.})$$

As opposed to gases, a temperature drop thus occurs, due to throttling. Thus, if the pressure drop is 35 lb./in.<sup>2</sup>, the corresponding temperature

$$\text{drop is } t_1 - t_2 = \frac{35}{2.32} = 15.1^\circ \text{ F.}$$

and is thus appreciable.

The entropy diagram enables a closer determination to be made. In the  $T\phi$  diagram II, the change of state during throttling is represented by the curves  $H = \text{constant}$ .

The diagram gives, for example, for initially dry saturated steam at 100 lb./in.<sup>2</sup> abs. and throttled in the steps,

100-70      70-40      40-14.7   lb./in.<sup>2</sup> abs.

a temperature drop of

11°                  11°                  13.5 F°.

If the initial pressure of the dry steam is 200 lb./in.<sup>2</sup> abs., then, for pressure drops of

200-185   185-155   155-128   128-100   100-70   70-40   40-14

the temperature drop is

6° F.    6.7°    7°    8°    8.5°    9.5°    12° F.

If, on the other hand, initially superheated steam, at about 570° F. and 155 lb./in.<sup>2</sup> abs., be throttled, then for the pressure drops

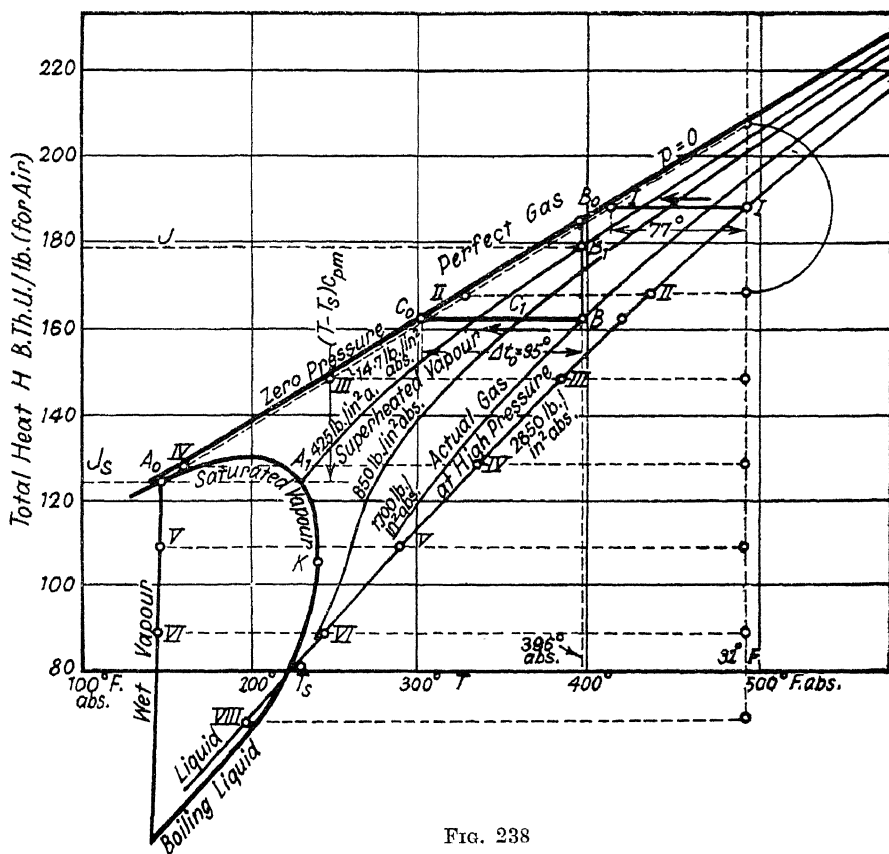
155-128 128-100 100-70 70-40 40-14.7 (lb./in.<sup>2</sup>)
$$t_2 - t_1 = \quad 2.3^\circ \text{ F.} \quad 2.7^\circ \text{ F.} \quad 3.1^\circ \text{ F.} \quad 3.6^\circ \text{ F.} \quad 6^\circ \text{ F.}$$


FIG. 238

Hence, the relations during throttling of steam are much less simple than those given by the Tumlirz-Linde equation, and by the assumption made above on the internal energy. The above simple equation gives temperature drops which are much too high.

The expressions for the internal energy and total heat given above only hold approximately. A closer value for superheated steam which holds also for gases is obtained as follows.

For a perfect gas, which follows the law  $pv = RT$ , the true and mean specific heats at constant pressure ( $c_p$  and  $c_{pm}$ ) are independent of the pressure, and therefore also of the total heat  $H$  measured at any initial temperature  $t_0$ . By plotting the total heats of this gas to a base of temperatures (Fig. 238), a single curve suitable for all pressures ( $p = 0$ ) is obtained (see page 65). For monatomic gases  $c_p$  is independent of the temperature also, and hence the  $H$ - $t$  curve is a straight line; for polyatomic gases, on the other hand,  $c_p$  increases with temperature, and the  $H$ - $t$  curves deviate from the straight line, this deviation increasing as the temperature increases.

For a superheated vapour near the saturation limit,  $c_p$  depends not only on the temperature but also on the pressure, and for the same temperature increases when the pressure is increased. If the total heats are calculated at a definite temperature by adding the extra superheat to the total heats at the saturation states, it will be found that the  $H$  values decrease as the pressure of the superheated steam increases (points  $B_0$ ,  $B_1$ ,  $B$  in Fig. 238). In the expression

$$H = H_s + c_{pm}(t - t_s)$$

$H_s$  increases but slowly as the temperature rises (page 399), and at high pressures actually decreases, while for a given temperature  $t$  the range of superheat  $t - t_s$  decreases as the pressure is increased. Hence  $H$  becomes smaller as the pressure increases in spite of the increase in the value of  $c_{pm}$ .

The change in state caused by throttling is represented on the  $H$ - $t$  diagram by an horizontal straight line  $BC_1C_0$  (Fig. 238), from which it is seen that, as the pressure decreases, the temperature also decreases. Hence the throttling of superheated steam is always accompanied by a cooling effect  $\Delta t$ , which increases as the pressure drop increases.

The tests mentioned on page 405 lead reversely to the determination of the total heats.

If an  $H$ - $t$  diagram is available, such as shown in Fig. 215 for steam or 238 for air,\* the cooling due to throttling can be directly determined. The  $H$ - $p$  diagram, as constructed by Knoblauch, Raisch, and Hausen† for steam, and for  $\text{CO}_2$  by Plank and Kuprianoff,‡ can also be used.

In the  $pv$  diagram, the throttling process is as represented in Fig. 239. It can be considered as made up of two steps—

1. Adiabatic frictionless discharge through the throttled opening down to the back pressure  $p_2$ . This is shown by the line  $AC_1$  (Fig. 239) which, for superheated steam, is given by the law  $pv' = C$ . The energy of the

\* From *F.A.* 274 (Hausen). The first diagram of this type was constructed by Pollitzer (*Z.f.d. ges Kälteindustrie* (1921), page 125).

† Tables and diagrams for steam (Footnote, page 356).

‡ Thermal properties of  $\text{CO}_2$  (Footnote, page 441).



issuing jet is equal to the shaded area  $H_1 - H_2\phi = \Delta H_{ad}$ , and the temperature falls correspondingly to  $t'_2$ . The velocity is  $V'_2$ .

2. Reduction of the kinetic energy to the initial value  $\frac{V_1^2}{2g}$  at constant pressure. Due to this, the steam is re-heated by the amount of the heat drop  $H_1 - H_2\phi$ . This has the same effect as external heating, and in the case of a perfect gas is sufficient to bring the gas temperature back to its original value, and to increase the volume to  $v_2 = \frac{p_2}{p_1} v_1$ .

Any other type of throttling leads to the same final result, e.g. throttling through a porous stopper offering considerable resistance to flow, so that most of the kinetic energy is destroyed, apart from the initial value

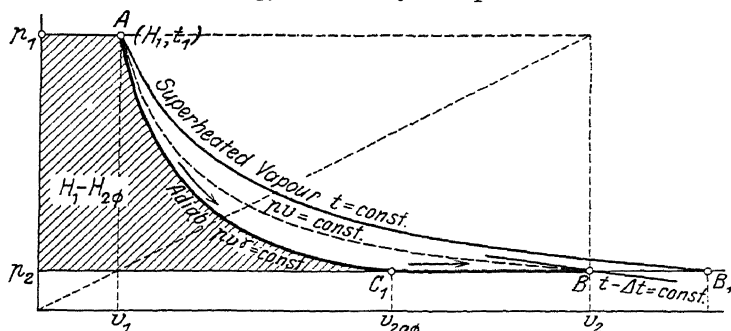


FIG. 239

$\frac{V_1^2}{2g}$ , or with such resistance that the kinetic energy remains constant at  $\frac{V_1^2}{2g}$ . All three processes lead to the same final state  $B$ . In the case of superheated steam the volume is of the same magnitude as for gases, but the temperature at  $B$  is lower than the initial temperature  $t_1$ , since its isotherm (as given by van der Waal's equation, say) lies above the gas isotherm.

In the  $H\phi$  diagram, the three processes are as shown in Fig. 240. They all lead to the same point  $B$ , but since the isotherm in this diagram rises, as the pressure decreases, the temperature at  $B$  at which  $H_1 = H_2$  is lower than at  $A$ . Compare also the diagram drawn to scale for superheated steam in Fig. 236.

H. Hausen has carried out a large amount of experimental work on the Joule-Thomson differential effect for air. The tests extend to pressures of 200 atmospheres and temperatures between  $+10^\circ\text{C.}$  and  $-175^\circ\text{C.}$ \* These tests enabled Hausen to construct a complete state diagram for air as detailed below.

1. Joule-Thomson differential effect for temperatures between  $900$  and  $90^\circ\text{C. abs.}$  ( $+27^\circ\text{C.}$  to  $-183^\circ\text{C.}$ ) and pressures from  $0$  to  $200$  atmospheres as dependent on the temperature, with curves of constant pressure.

\* F.A. 274 (1926). H. Hausen. The Joule-Thomson effect and properties of air. (Comm. of the Lab. Phy. Tech. of Munich Hochschule; and from the Linde Eisenmaschinen A.G., Munich.)

2. The same, with dependence on the pressure and with curves of constant temperature.

3. Dependence of the specific heat  $c_p$  on the temperature, with curves of constant pressure (see Fig. 234).

4.  $HT$  diagram (as in Fig. 238).

5.  $T\phi$  diagram, with curves of constant pressure and constant total heat (throttling curves).

6.  $H\phi$  diagram, with curves of constant pressure, constant temperature, and constant vapour quality (as in Fig. 235).

7.  $pv - p$  diagram (as in Fig. 159).

**Actual gases.** It has been shown on page 212 that when an ideal gas is throttled no change in temperature occurs, even for large pressure drops, apart from the slight cooling caused by an increase in velocity.

Superheated vapours, contrary to this, however, show a decided temperature drop when throttled.

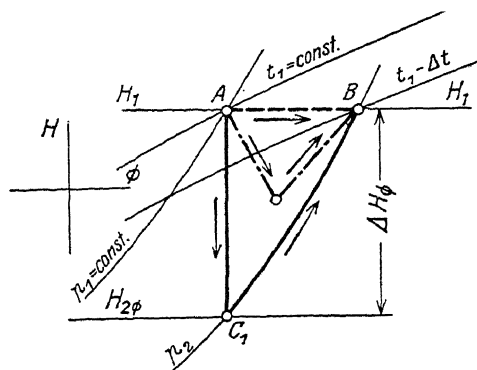


FIG. 240

If now the actual gases are merely vapours far removed from the critical state (highly superheated vapours), it is to be expected that they should also show a temperature drop due to throttling. This cooling effect was established by Thomson and Joule (1853) for pressures up to about 70 lb./in.<sup>2</sup> abs., and temperatures between 0° and 90° C., previous to the

discovery of the critical state by Andrews (1869). They found that, for air,  $\Delta t = 0.034 (p_1 - p_2)$  (F.° and  $p$  in lb./in.<sup>2</sup> abs.)

while, for CO<sub>2</sub>  $\Delta t = 0.171 (p_1 - p_2)$

For steam, it was shown above, that

$$\Delta t = 0.09 (p_1 - p_2) \text{ to over } 0.4 (p_1 - p_2)$$

depending on the pressure and the range of superheat.

Of these three substances, air is the farthest from the critical state, and shows accordingly, for the same pressure drop, a cooling effect 20 per cent of that for CO<sub>2</sub> and from 38 per cent to 9 per cent of that for superheated steam.

The temperature drop for gases depends also on the absolute temperature level. According to Thomson and Joule it is inversely proportional to the square of the absolute temperature. For air, therefore,

$$\Delta t = \quad (464)$$

with  $a = 0.034$  for air and  $0.171$  for CO<sub>2</sub>.

The Joule-Thomson equation (464) shows that the cooling due to throttling depends on the pressure drop  $p_1 - p_2$ , and not on the absolute amount of the pressure. The same cooling effect is, therefore, obtained

between 10 and 8 atmospheres, or 8 and 6, or 6 and 4 atmospheres. If, in these throttling actions, the initial temperature is the same, the cooling effect is also the same. If, however, very large pressure differences are considered, so that a considerable amount of cooling occurs, the temperature drops during throttling, and, due to the dependence on the temperature (as shown in equation (464)), the cooling, for a pressure difference, say, of 100 atmospheres, is not 10 times that for a difference of 10 atmospheres. For an elementary change, equation (464) gives

$$dT = a \frac{492^2}{T^2} dp$$

or  $T^2 dT = a \times 492^2 dp$

Integration gives

$$\frac{1}{3}(T_2^3 - T_1^3) = a \times 492^2 (p_2 - p_1)$$

Hence  $T_2 = \sqrt[3]{T_1^3 - 3 \times a \times 492^2 (p_1 - p_2)}$

so that the cooling effect is

$$t_1 - t_2 = T_1 - T_2 = T_1 - \sqrt[3]{T_1^3 - 3a \times 492^2 (p_1 - p_2)} \quad . \quad (465)$$

This is the total throttling effect, whereas the cooling effect per 1 lb./in.<sup>2</sup> pressure drop is

$$a = \frac{dT}{dp}$$

and is called the differential throttling effect. Its value is, of course, most easily determined from the  $H$ - $t$  or  $H$  $p$  or  $H$  $\phi$  diagrams, when these are available.

From the later tests carried out at Munich,\* at the higher pressures up to 150 atmospheres (as compared with Thomson and Joule's 4.5 atmospheres) it is now known that the cooling effect  $t_1 - t_2$  is not entirely independent of the initial pressure level.

The coefficient  $a$  of equation (464) actually becomes smaller at higher pressures, so that the cooling effect is also reduced. Nöll gives,

for air,

$$a = 0.0339 - 0.00000765p \quad (p \text{ in lb./in.}^2 \text{ abs.})$$

and, for oxygen,

$$a = 0.0396 - 0.00000756p$$

The dependence of the cooling effect on the temperature is also (as shown by equation (464)) to be regarded as valid for not too low temperatures. This is obvious on examining the state diagrams of Hausen (Fig. 238), which also apply to gases. Compare Schüle's, volume ii, for the reversal of the throttling effect, whereby, instead of cooling, heating occurs. For air this occurs at  $+10^\circ \text{C.}$  when  $p = 4500 \text{ lb./in.}^2 \text{ abs.}$

\* Tests by E. Vogel on the temperature change of air and oxygen on being throttled at  $10^\circ \text{C.}$  and from pressures up to 150 atmospheres. *F.A.*, vols. cviii and cix. Also Noell, *Dissert.* (Munich, 1914).

**The Linde liquefaction process.** This process is based on the small cooling effect produced by throttling a permanent gas from a high to a low pressure.

A compressor is used to produce air at a high pressure (1500 lb./in.<sup>2</sup> or more). This compressed air is reduced by means of a throttle valve fitted in the delivery pipe to a lower (but still high) pressure, in the same way as in vapour refrigerating machines. The throttled air is again drawn into the air compressor, recompressed, and again throttled. It thus describes a repeating cycle between the same pressures.

If now the compressed air pressure is 1500 lb./in.<sup>2</sup> abs. and the pressure after throttling 300 lb./in.<sup>2</sup> abs., the temperature drop is about

$$\Delta t = 0.034 \times 1200 = 40.8^\circ \text{F.}$$

If the temperature of the high pressure air is 80° F. say, the temperature of the throttled air is 39.2° F. This colder air can now be returned to the compressor and used on its way to cool the warmer compressed air coming from the compressor. For this purpose, the colder air is led through a pipe placed concentrically in a larger pipe (the outer surface of which is very thoroughly insulated), through which the warmer air flows in the opposite direction. An efficient heat exchange is effected by the use of very long pipes (about 300 ft.). If this heat exchange were complete the throttled air would be heated to the initial temperature of the high pressure air, while this, in turn, would be cooled to the initial temperature of the throttled air, and would in the above case arrive at the throttle valve with a temperature of 39.2° F. In flowing through the valve the temperature drop is again 40.8° F., since the pressure drop is the same. This gives a temperature, after throttling, of 39.2 - 40.8 = -1.6° F. This air is again used to cool the high pressure air, and with perfect heat transmission is heated to the initial temperature of the high pressure air, while this in turn is cooled to -1.6° F. By repeating the cycle many times the throttled air is continuously cooled, and is finally reduced to the state of a vapour. This is gradually condensed to liquid, which is collected in a vessel behind the valve.

By introducing a fresh supply of air into the system it is possible, in spite of the cooling and conversion of part of the air to liquid, to maintain the pressures constant and continuously produce fresh liquid.

There are thus two distinct periods which can be described as the starting period and the steady period. The starting period can extend to several hours, since not only the air but also the pipe system and the insulating material have to be cooled down, and complete insulation is never attained.

An extensive treatment of this and other liquefying systems will be found in Schüle's, volume ii, 4th Edn. (1923), Art. 72, using the van der Waal equation as basis. Hausen has recently given a satisfactory representation of the processes (in the steady period) involved in liquefaction by means of the complete state diagram mentioned above.\*

\* *Zeitsch. f. d. gesamte Kälte-Industrie* (1925), vols. vii and viii. H. Hausen. "On the calculations of air liquefaction plants based on the measurements of the Joule-Thomson effect." Also in the Jubilee Edition, *50 Jahre Kältetechnik* (1879-1929). *History of the Linde Eismaschinen A.G., Wiesbaden*. H. Hausen. "The physical fundamentals of gas liquefaction and rectification."

In the Hausen  $Ht$  diagram the process described above for the starting period, which finally leads to liquefaction, with an initial pressure of 2850 lb./in.<sup>2</sup> abs. and temperature of 32° F., is represented in Fig. 238. All losses of heat have been neglected here. The line  $I-I$  shows the first throttling. Due to this the throttled air is cooled to -45° F., and this is used to cool fresh air at 2850 lb./in.<sup>2</sup> abs. and 32° F. to the point II. This is repeated until, at VII, the pure liquid state is reached.

### CALCULATION OF THE AVAILABLE WORK AND THE OUTLET VELOCITY OF STEAM BY MEANS OF THE HEAT DROP

The available work  $E_0$  due to expansion of 1 lb. of steam from the pressure  $p_1$  to the back pressure  $p_2$  is represented by Fig. 241, in which  $BC$  is the adiabatic pressure-volume curve. This work is the same, whether performed in a reciprocating engine or turbine with complete expansion, or appearing as kinetic energy of a discharged jet. In the latter case the corresponding outlet velocity is (page 222)

$$V =$$

The work area is

$$p_2 v_2 + \text{area } BCC'B'$$

The absolute expansion work  $E_e = BCC'B'$  is, for an adiabatic change of state, equal to the decrease in internal energy ( $I$ ) of the steam between  $B$  and  $C$ , since it comes entirely from this change (page 380).

**Wet steam.** From page 363 at  $B$ ,

$$I_1 = h_1 + q_1 p_1$$

and at  $C$ ,

$$I_2 = h_2 + q_2 p_2$$

where  $q_1$  and  $q_2$  are the dryness fractions of the wet steam at  $B$  and  $C$ .

$$\text{Hence } E_e = h_1 + 1 - (h_2 + 777.8$$

$$\text{so that } AE_0 = \quad + \quad + Ap_1 v_1 - (h_2 + q_2 p_2 + Ap_2 v_2)$$

The value  $h + qp + Apv$  is completely determined from the state of the steam, and is known as the thermal potential at constant pressure ( $H'$ ).

$$\text{With } H' = h + qp + Apv = I + Apv$$

$$\text{this gives } E_0 = 777.8 (H'_1 - H'_2) \text{ and } V = 223.8 \sqrt{H'_1 - H'_2} \quad (466)$$

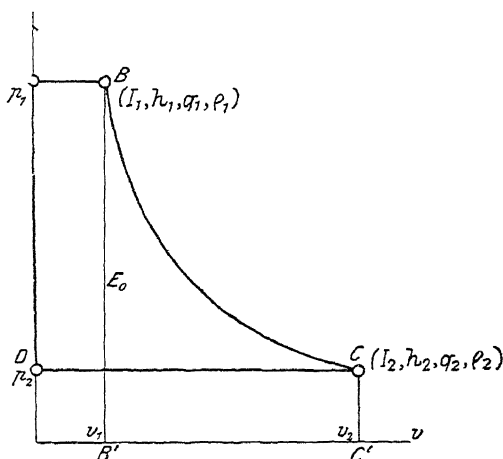


FIG. 241

The value  $H'_1 - H'_2$  is called the "available work," or, more commonly, the "heat drop."

As shown on page 364, the total heat of wet steam is

$$= h + q\rho + \frac{1}{777.8} p(v - 0.016)$$

If the volume 0.016 of the water be regarded as negligible, compared with the volume  $v$ , which, even for very wet steam, is permissible, then

$$H_w = H'$$

i.e. the total heat is the same as the thermal potential at constant pressure. The latter is only greater by the amount  $Ap\sigma$ , so that e.g. at 200 lb./in.<sup>2</sup> abs. the difference is

$$\frac{144 \times 200 \times 0.01839}{777.8} = 0.678 \text{ B.Th.U.}$$

which, compared with the absolute value of between 1000 and 1200 B.Th.U., is negligible.

Hence, very closely,

$$E_0 = 777.8 (H_1 - H_w)$$

It should be carefully noted that, although for initially dry steam  $H_1$  is given by the tables,  $H_w$  cannot be found directly from the tables, since, at the end of adiabatic expansion, the steam is wet. It is necessary to determine  $H_w$  from the relation

$$H_w = h_g + x h_{fg}$$

after the quality  $q_2$  at the end of expansion is determined from the  $T\phi$  diagram as shown on page 379.

See the next paragraph for a simpler method of finding the heat drop by means of the  $H\phi$  chart.

**Superheated steam.** The work area  $AE_0$  (Fig. 241) is given, in general,

$$\text{by } AE_0 = I_1 - I_2 + Ap_1v_1 - Ap_2v_2$$

$$\text{or } AE_0 = H_1 - H_2$$

The entropy diagram is used to determine this value, since  $H_2$  is the total heat at the end of adiabatic expansion. The value of  $E_0$  (Fig. 241) can, however, be determined by calculation, using the equation  $pv^{1.3} = \text{constant}$ , provided the final state is still in the superheat field.

**The heat drop in the entropy diagram.** In the  $T\phi$  diagram (Fig. 242) the adiabatic expansion from  $p_1$  to  $p_2$  is represented by the vertical  $A_1B_1$ , when the steam is initially dry, by  $A_2B_2$  when it is initially superheated and finally wet and by  $A_3B_3$  when it is initially and finally superheated.

At the initial state  $A_1$  the total heat  $H_1$  is, for dry saturated steam, the area below  $OMA_1$  and  $H_2$  is the area below  $ONB_1$ . Hence the adiabatic heat drop  $H_1 - H_2$  is given by the area  $A_1B_1NM$ . For initially wet steam, of quality  $q_1$ , this area is reduced by the rectangle lying to the left of  $A_1B_1$ .



In the diagram (Fig. 243) in which the total heats are plotted to a base of entropy values, an adiabatic expansion is represented by a vertical line. The lines of constant pressure and constant temperature are discussed in the next section. The total heat  $H_1$ , in the initial state, is represented by the ordinate at  $A_1$  for dry steam, and the ordinates such as  $A_2$  and  $A_3$  for superheated steam. The total heats in the final states  $B_1$ ,  $B_2$ ,  $B_3$  are also given by the ordinates at these points. Hence the adiabatic heat drops  $H_1 - H_2$  between the limits  $p_1$  and  $p_2$  are given by the lengths  $A_1B_1$ ,  $A_2B_2$ , and  $A_3B_3$ . They can, therefore, be determined

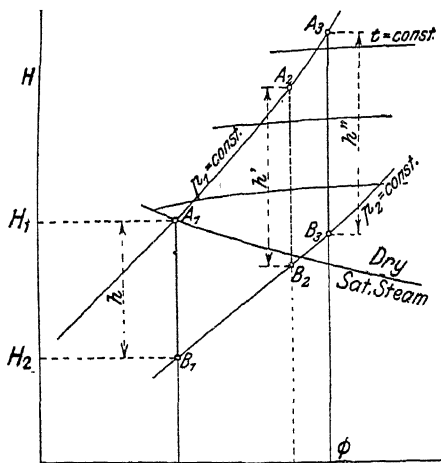


FIG. 243

by using the scale for the ordinates, whereas on the  $T\phi$  diagram a planimeter has to be used to measure the areas corresponding to these heat drops.

This integration can, of course, be avoided in the  $T\phi$  diagram if curves of constant total heat be drawn in, but the heat drops are not given by the lengths between the pressures.

### $H\phi$ DIAGRAM FOR STEAM

Since the introduction of steam turbines, entropy diagrams have been extensively used, mainly because, with their aid, the effect of the

resistance to flow on the changes of state in nozzles and channels is at once made clear. The  $H\phi$  diagram is constructed as follows—

The total heat values for dry saturated steam are first plotted against the corresponding entropy values (Fig. 244), and the pressures marked at suitable intervals. This gives the curve  $AB$ , called the saturation limit curve, for pressures between 250 and 0.5 lb./in.<sup>2</sup> abs. In the same way the liquid limit curve  $A_1B_1$  is drawn between the same pressure limits.

In addition, it is necessary also to be able to represent the more important state changes on the diagram, such as those of constant pressure and constant temperature. The state changes at constant pressure in the wet field are given by straight lines. If, for example, dry steam at 225 lb./in.<sup>2</sup> abs. has heat taken from it at constant pressure, so that it is condensed,  $H$  and  $\phi$  change, as shown by the straight line  $AA'$ . Hence this change of state for different pressures is given by joining the points representing these pressures on the two limit curves by straight lines.

Thus for wet steam the total heat is

$$= h + qp + \quad 1 \quad (\text{page 363})$$

and in this  $v = qv_s + (1 - q) 0.0016$  (page 358)



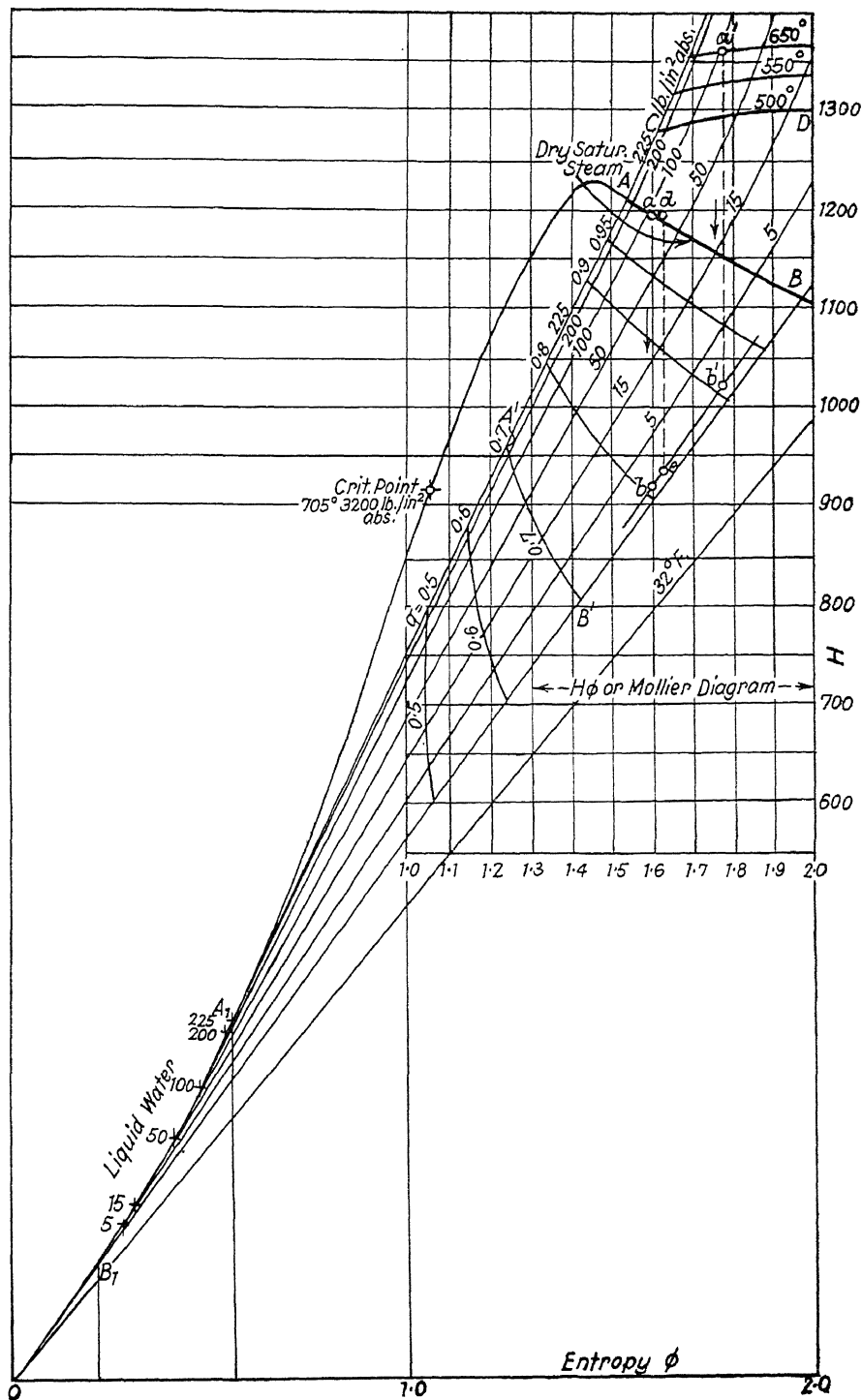


FIG. 244

so that 
$$H = h + qp + \frac{1}{777.8} pq (v_s - 0.0016) + 777.8$$

Again, the entropy is

$$\phi = \phi_i + \frac{qL}{T} \quad (\text{page 375})$$

Now in the expressions for  $H$  and  $\phi$  all the quantities except  $q$  remain constant when the pressure is constant, and by eliminating  $q$  from the two equations, a linear equation connecting  $H$  and  $\phi$  is obtained. This equation, therefore, is represented by a straight line on the  $H\phi$  chart.

The entropy and total heat increases are directly proportional to  $q$ . Hence it is only necessary to divide the distance  $AA_1$  into 10 equal parts, say, to obtain the qualities 0, 0.1, 0.2, 0.3, etc. Thus, at  $A'$  the quality is 0.7, because

In the same way the qualities on the other constant pressure lines are obtained.

The change of state at constant quality is obtained by joining the points of equal quality on the lines  $p = \text{constant}$ , e.g.  $A'B'$  for  $q = 0.7$ .

The lines of constant pressure in the saturation field are also isothermals.

In the superheat field, the curves of constant pressure, which form the continuation of the straight lines of the saturation field, are plotted by determining the entropy values (from the Munich  $c_p$  values).

Since the curves of constant temperature are almost horizontal, the total heats required to produce superheated steam are practically independent of pressure so long as this is not above 300 lb./in.<sup>2</sup> abs.

Thus the total heats for steam at 650° F. and pressures

of	200	150	50	15 lb./in. <sup>2</sup> abs.
are	1352	1355	1362	1364 B.Th.U./lb.

The change of state due to throttling is represented by horizontal lines on the  $H\phi$  diagram, since the total heat remains constant. It is preferable, however, to use the  $H-t$  or  $H-p$  diagrams.

**Example 4.** Find the adiabatic heat drops to 1.5 lb./in.<sup>2</sup> abs. for (a) initially dry saturated steam at 150, 100, 50, and 15 lb./in.<sup>2</sup> abs.; (b) initially superheated steam at the same pressures, and at 650° F.

For saturated steam at 100 lb./in.<sup>2</sup> abs. the heat drop is equal to the distance  $ab$  in Fig. 244. This gives 273 B.Th.U./lb., so that the equivalent work is  $777.8 \times 273 = 212,340$  ft. lb. For superheated steam at 100 lb./in.<sup>2</sup> abs. and 650° F. the heat drop is given by  $a'b' = 337$  B.Th.U. and the corresponding work is  $337 \times 777.8 = 262,118$  ft. lb.

In this way the chart gives, for the pressures,

$p$	= 150	100	50	15 lb./in. <sup>2</sup> abs.
for saturated steam,				
$H_1 - H_2$	= 299	273	226	148 B.Th.U./lb.
and for superheated steam,				
$H_1 - H_2$	= 361	337	292	220 B.Th.U./lb.

**Example 5.** Find the thermal efficiency of a steam turbine operating on an ideal cycle with the temperatures and pressures of the previous example.

The thermal efficiency is the ratio of the heat drop given in Example 4 to the heat supplied (see page 485). The values of the latter, for the different pressures, are as follows for saturated steam—

$H_1 - h =$	1116	1108	1093	1068
$\eta_{th} =$	0.268	0.246	0.206	0.139

and, for superheated steam,

$H_1 - h =$	1272	1275	1279	1281
$\eta_{th} =$	0.284	0.264	0.228	0.17

**Example 6.** Find the discharge velocity when steam at 150 lb./in.<sup>2</sup> abs. and 650° F. is expanded in a suitable nozzle to 1.5 lb./in.<sup>2</sup> abs.

The velocity is given by

$$V =$$

With  $H_2$  as the heat drop

$$E = 777.8 (H_1 - H_2)$$

so that  $V = \sqrt{2g \cdot 777.8 (H_1 - H_2)} = 223.8$

From the  $H\phi$  chart,

$$H_1 - H_2 = 361$$

hence  $V = 223.8 \sqrt{361} = 4240$  ft./sec.

**Example 7.** Find the percentage decrease in available work caused by the throttling of dry saturated steam from 100 lb./in.<sup>2</sup> abs. to 80 lb./in.<sup>2</sup> abs., when the back pressure is 1.5 lb./in.<sup>2</sup> abs.

The throttling from 100 to 80 lb./in.<sup>2</sup> abs. is given by the straight line *ad* in Fig. 244. The heat drop, after throttling, is given by *de*, while before throttling it is given by *ab*. The decrease amounts to 14 B.Th.U.,

hence the loss amounts to  $\frac{14}{272} \times 100 = 5.1$  per cent. The throttled steam is superheated (point *d*). Dry steam at 80 lb./in.<sup>2</sup> abs. gives a heat drop of 254 B.Th.U., and is thus  $272 - 254 = 18$  B.Th.U., less than that for the dry steam at 100 lb./in.<sup>2</sup> abs. Hence most of this is lost, due to throttling.

## REPRESENTATION OF FLOW RESISTANCES IN ENTROPY DIAGRAMS FOR STEAM

The flow of gases and vapours has been considered on page 215 *et seq.* On page 307 the effect of friction on the velocity is dealt with, and the useful and lost flow energy represented on the entropy diagram for gases.

For saturated and superheated steam the representation of the conditions on the entropy diagrams is as follows.

**Saturated steam.** The area  $AB'NM$  in Fig. 245 represents the adiabatic heat drop  $h_{aa'}$ . If *B* represents the actual final state point, from page 309, the rectangle below  $BB'$  gives the loss in kinetic energy and



point  $B_1$  is thus readily located. It is only necessary to measure the distance  $\zeta h_{ad}$  vertically above  $B'_1$  giving  $C_1$ , and then draw an horizontal through  $C_1$  to cut the curve  $p_2 = \text{constant}$ .

**Superheated steam.** In the  $T\phi$  diagram, the adiabatic heat drop is given by the area  $NMA_1A_3B'_3CN$ , while the loss due to friction is given by the area below  $B'_3B_3$ . The latter is the superheat at constant pressure  $p_2$  between the temperatures at  $B'_3$  and  $B_3$ .

Since the mean specific heats are usually reckoned from the saturation curve, it is preferable to employ a graphical solution. This is made much more convenient if lines of constant total heat are drawn on the entropy diagram. It is only necessary then to locate  $B_3$  where the curve  $p_2 = \text{constant}$  intersects the curve of total heat  $H = \text{constant}$ , having a value greater than that at  $B'_3$  by the amount  $\zeta h_{ad}$ . The adiabatic heat drop is similarly determined when the final state point is in the superheat field, by noting the total heat values at  $A_3$  and  $B'_3$  and taking the difference. If  $B'_3$  lies in the wet field it is better to employ the  $H\phi$  chart.

In the  $H\phi$  diagram,  $B'_3C_3$  is again equal to  $\zeta h_{ad}$ , as explained for saturated steam.

**General form of equation of flow** applicable to any fluid and any type of flow without the addition or supply of heat is given from the above as follows—

In the general flow equation, with friction

the loss  $\zeta E_0$  is equal to the difference in total heat at constant pressure between the actual and adiabatic final states,

i.e.

The adiabatic friction drop  $AE_0$  is equal to the difference in total heats before and after adiabatic expansion,

$$\text{or} \quad AE_0 = H_1 - H'_2$$

$$\text{Hence} \quad A \frac{V^2}{2g} = H_1 - H'_2 - (H_2 - H'_2)$$

$$\text{or} \quad A \frac{V^2}{2g} = H_1 - H_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (472)$$

The actual kinetic energy is thus given by the difference in total heat at the initial and final states (distance  $A_3C_3$  in Fig. 246).

If, at the beginning of expansion the initial velocity is  $V_0$ , then

$$\frac{AV^2}{2g} - \frac{AV_0^2}{2g} = \quad (473)$$

and hence, for an elementary change,

$$Ad \frac{V^2}{2g} = -dH \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (474)$$

**Example 8.** Dry saturated steam at 150 lb./in.<sup>2</sup> abs. expands in a nozzle to 1 lb./in.<sup>2</sup> abs. with a frictional loss of 15 per cent.



**Example 9.** Superheated steam at 150 lb./in.<sup>2</sup> abs. and 650° F. expands in a multistage turbine, with a frictional loss of 30 per cent to 1 lb./in.<sup>2</sup> abs.

With frictionless adiabatic flow the expansion is represented by  $A_2B'_2$  (Fig. 247). At  $G$  the steam would be dry saturated. At the final point  $B'_2$  the quality would be

$$\frac{JB'_2}{JB_2} = 0.884$$

The actual final state point is  $H$ . The distance  $B'_2H$  is determined by making the rectangular area below  $B'_2H$  equal to 30 per cent of the adiabatic drop (area  $JMA_1A_2B'_2J$ ). At  $H$  the steam is slightly wet with a quality 0.988. The steam volume is thus increased in the ratio  $\frac{0.988}{0.884} = 1.12$ , while, at the same time, the velocity is decreased in the

ratio  $\sqrt{1 - 0.3} = 0.838$ . The exit area is thus greater than that given by ideal conditions in the ratio  $\frac{1.12}{0.838} = 1.338$ .

If at the end of expansion the steam is still superheated, as shown at  $B_3$  (Fig. 247), the loss area is limited at the upper edge by the curve of constant pressure.

### NOZZLE DESIGN USING THE $H\phi$ CHART

**Example 10.** To determine the areas, and pressure and velocity changes in a nozzle in which steam initially at 200 lb./in.<sup>2</sup> abs. and 600° F. expands to 20 lb./in.<sup>2</sup> abs. (a) without friction, (b) with 6 per cent velocity loss.

(a) From the  $H\phi$  diagram shown on the right of Fig. 248 the heat drop  $AB$  is 202 B.Th.U./lb., giving an outlet velocity of

$$V_1 = 223.8 \sqrt{202} = 3180 \text{ ft./sec.}$$

The steam at outlet is wet (point  $B$ ) having a quality  $q = 0.964$ . Its specific volume is, therefore,

$$v_1 = 0.964 \times 20.06 = 19.35 \text{ ft.}^3/\text{lb.}$$

The exit area required for a mass flow of  $W$  lb./sec. is,

with  $W = \frac{A_1 V_1}{v_1}$

$$A_1 = \frac{W v_1}{V_1} = \frac{W}{164.3} \text{ ft.}^2 =$$

The narrowest cross-sectional area lies where the pressure assumes the critical value (page 242), i.e.  $0.546 \times 200 = 109$  lb./in.<sup>2</sup> abs. Since, as shown in Fig. 248, the steam is still superheated, this determination is correct. In general, however, and in cases where the steam is wet at the critical pressure, the narrowest section and its pressure are determined as follows. A number of pressures are chosen and the corresponding areas determined in the same way as described above for the exit section.

The narrowest section is then readily determined from the graphical variation of the cross-sectional areas. At any section

$$AV = A_1 V_1$$

so that 
$$\frac{A}{A_1} = \frac{V_1}{v_1} \times \frac{v}{V}$$

Thus at the pressure 70 lb./in.<sup>2</sup> abs. the corresponding heat drop is 103 B.Th.U., giving

$$V = 223.8 \sqrt{103} = 2260 \text{ ft./sec.}$$

The corresponding specific volume is found from the  $TV\phi$  diagram by dropping the perpendicular  $A_1 C_1$  (Fig. 248,  $T\phi$  field) corresponding to the adiabatic expansion from 200 lb./in.<sup>2</sup> abs. and 600° F. to 70 lb./in.<sup>2</sup> abs. From  $C_1$  a horizontal is drawn across to the  $TV$  field to cut the constant pressure line of 70 lb./in.<sup>2</sup> abs. at  $C_2$ . The abscissa of this point gives the specific volume  $v = 6.86 \text{ ft.}^3/\text{lb.}$

Hence 
$$\frac{A}{A_1} = 19.35 \times 2260 = 0.50$$

In Fig. 248 the cross-sectional areas have been plotted against the corresponding heat drops. The throat or narrowest section has an area  $0.45 \times$  exit area; the corresponding pressure is 109 lb./in.<sup>2</sup> abs., the specific volume  $4.88 \text{ ft.}^3/\text{lb.}$ , the heat drop 63 B.Th.U./lb., and the velocity 1775 ft./sec. Check

$$\frac{A_{min}}{A_1} = \frac{V_1}{V} \times \frac{v}{v_1} = \frac{3180}{19.35} \times \frac{4.88}{1775} = 0.45$$

In Fig. 248 (centre) the continuous increase in specific volume and reduction in temperature as flow proceeds through the nozzle are shown. At a pressure of 38 lb./in.<sup>2</sup> abs., points  $G$ ,  $G_1$ ,  $G_2$ , the steam enters the wet field; from here onwards the reduction of temperature with volume is much slower.

In Fig. 249 a nozzle section is shown, with a cone angle of 10° in the diverging portion. The throat area is 0.45 times the exit area. The width of the rectangular nozzle section is taken as constant. The upper thick line then represents the nozzle without friction. The pressure curve plotted above this is found by measuring the areas at various sections and reading the corresponding pressures in Fig. 248. The velocity curve is similarly obtained by reading the heat drops in Fig. 248 and calculating the corresponding velocities.

(b) With a 6 per cent velocity loss, i.e.  $\phi = 0.94$ , the exit velocity is

$$V_1' = 0.94 \times 3180 = 2990 \text{ ft./sec.}$$

The loss in kinetic energy is  $\zeta = 1 - \phi^2 = 0.116$ . The final state of the steam is given by point  $E$  on the  $H\phi$  diagram (Fig. 248) on the right.  $DB = 0.116 AB$ , and  $DE$  is at right angles to  $AB$ . The quality is 0.988 (in place of 0.964), the exit area is now

$$A_1' = \frac{W \times 0.988 \times 20.06}{2990} = \frac{W}{150.6} \text{ ft.}^2 =$$





At the narrowest section the relative velocity loss is certainly less than that at the exit section. Between inlet and the throat the nozzle is equivalent to a short, well-rounded orifice, for which the velocity coefficient (page 256) can be taken as

$$\phi = 0.975$$

which gives  $\zeta = 1 - 0.975^2 = 0.05$

The throat area is then found by assuming that the pressure at the throat is practically the same as that given in frictionless flow, i.e. about

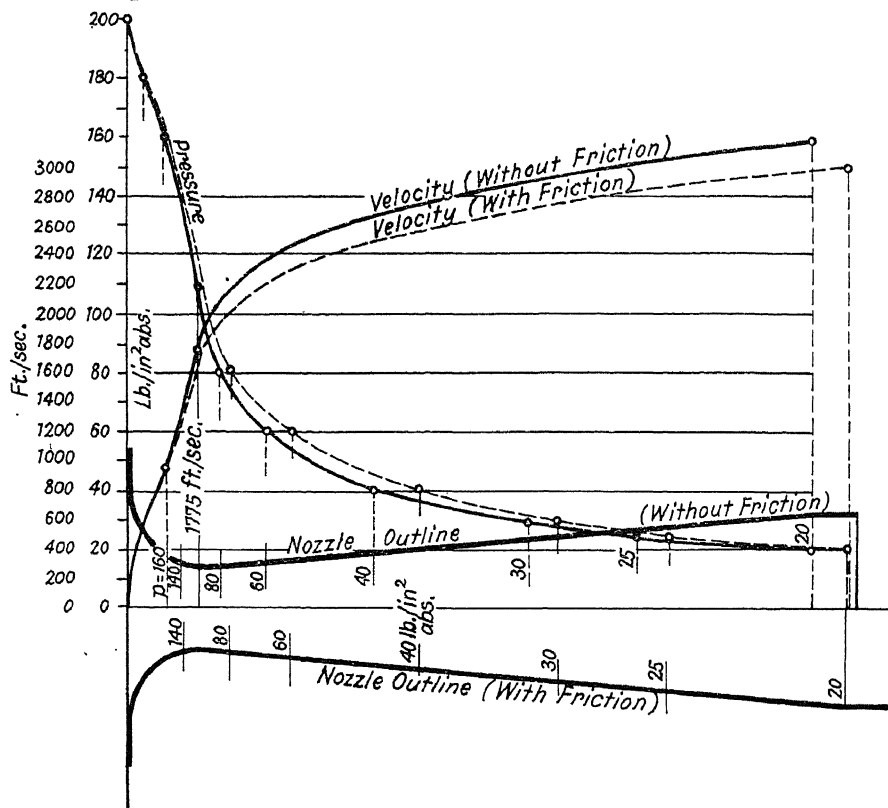


FIG. 249

109 lb./in.<sup>2</sup> abs. From the adiabatic heat drop of 63 B.Th.U./lb. to the throat, the re-heat =  $0.05 \times 65 = 3.25$  has to be subtracted in order to locate the point  $H$  which corresponds to  $H_1$  in the  $T\phi$  and  $H_2$  in the  $TV$  field. With the specific volume  $v' = 4.90$ , and velocity

$$V' = 0.975 \times 1775 = 1730$$

$$\frac{2990}{1730} \times \frac{4.90}{0.988 \times 20.06} = 0.427$$

Hence, when friction is taken into account, the nozzle has to open out more, and, in addition, the cross-sectional areas are greater.





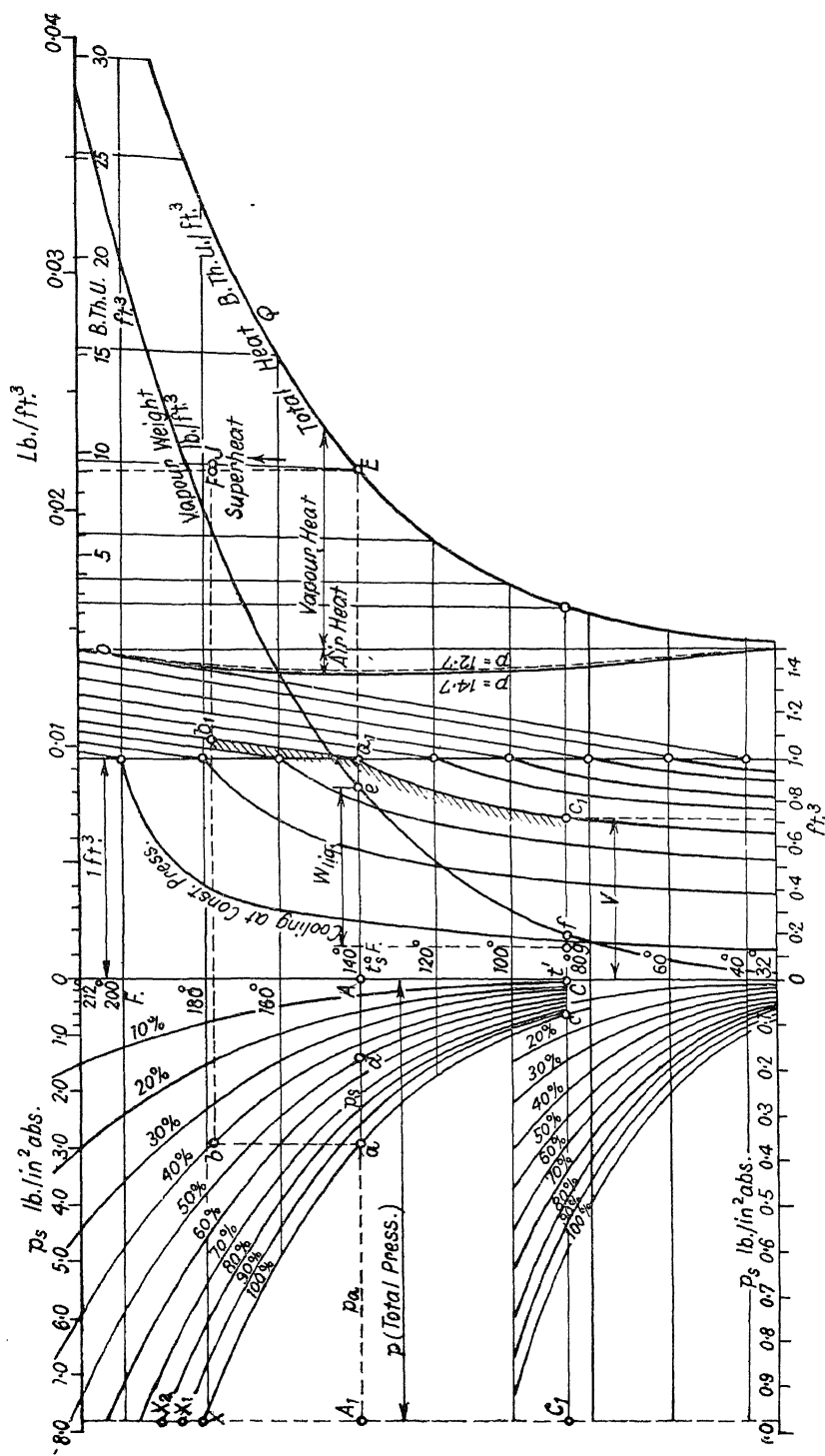


FIG. 250



of the air rises, since the sum of the two pressures remains constant. The partial pressure  $p'$  of the vapour, however, must equal the saturation steam pressure corresponding to the existing temperature, since water is present in the mixture. Hence, as the temperature decreases, condensation occurs and the relation between the temperature and partial pressure is given by the saturation curve  $ac$ .

Starting with saturated air (point  $a$ , say) the total volume  $V$ , say, corresponding to any temperature  $t' < t$  is found as follows. Applying the characteristic equation of a gas to the fractional weight of  $W_a$  of pure air having the partial pressure  $p - p'$  at temperature  $t'$  gives

$$(p - p') V = W_a R_a T'$$

while, in the initial state, with  $V = 1$ ,  $p' = p_s$ ,  $T' = T_s$ ,

$$p - p_s = W_a R_a T_s$$

from which

$$V = \frac{p - p_s}{p - p'} \frac{T'}{T_s} \quad . \quad . \quad . \quad . \quad . \quad . \quad (486)$$

Since  $p'$  is the saturation vapour (steam) pressure (point  $c$ ) corresponding to  $T'$ , the volume  $V$  for any temperature  $T' < T_s$  is calculable.

This volume  $V$  is equal to the total volume  $\bar{V}$  of the mixture. In Fig. 250 (centre) a series of volume values have been calculated in this way, for initial temperatures of 200, 180, 160, . . . to 40° F., and for a total pressure of 14.7 lb./in.<sup>2</sup> abs. The initial volume taken is 1 ft.<sup>3</sup> of saturated air;  $a_1 c_1$ , for example, shows the curve for moist air initially at 140° F. While the total volume of  $A a_1$  (1 ft.<sup>3</sup>) decreases to  $C c_1$  ( $= 0.73$  ft.<sup>3</sup>) the temperature drops from 140° F. to 86° F., and the partial pressure of the vapour (steam) from  $A a$  (2.88 lb./in.<sup>2</sup> abs.) to  $C c$  (0.614 lb./in.<sup>2</sup> abs.). The partial pressure of the air rises, on the other hand, from  $(14.7 - 2.88) = 11.82$  lb./in.<sup>2</sup> abs. to  $(14.7 - 0.61) = 14.09$  lb./in.<sup>2</sup> abs.

The weight of water vapour still present in the space  $V$  at the temperature  $t'$  is  $V d'_t$ , where  $d'_t$  is the density of the saturated vapour. Since the vapour weight initially was  $d_s$  the weight of liquid formed is

$$W_{liq} = d_s - V d'_t = d_s - \frac{p - p_s}{p - p'} \frac{T'}{T_s} d'_t \quad . \quad . \quad . \quad (487)$$

From Fig. 250 the weight  $W_{liq}$  is obtained by subtracting from  $A c = d_s$  the quantity  $V \times \bar{C} f$ .

The determination of the heat removed in cooling unsaturated moist air at constant pressure  $p$  down to the saturation temperature  $t_s$  is effected in exactly the same way as for gas mixtures. If the weight of water vapour in  $W$  lb. of the mixture is  $W_w$  the heat removed in cooling to the dew point is

$$Q = (W - W_w) (c_p)_a (t - t_s) + W_w (c_p)_w (t - t_s)$$

With continued cooling below the dew point conditions are entirely altered, since condensation occurs, causing changes in the partial pressures of air and vapour. The heat removed can, however, in this case, be calculated by taking the sum of the heat quantities which have to be

separately removed from the air and vapour during the different state changes actually experienced by these substances.

The state change experienced by the pure air in the mixture is shown more clearly by plotting the volumes  $V$ , found in Fig. 250, against the partial pressures  $p_a$  of the air (Fig. 251) (for a total pressure of 14.7 lb./in.<sup>2</sup> abs.). The air suffers a considerable amount of compression at the higher initial temperatures. The heat withdrawn from the air is, from the first law in the form given on page 176,

$$Q = W(H_2 - H_1) - A \int_{V_1}^{V_2} V dp \quad . \quad . \quad . \quad (488)$$

or 
$$Q_a = (W - W_w)(c_p)_a(t_s - t') - A \int_1^V V d(p - p') \quad . \quad (489)$$

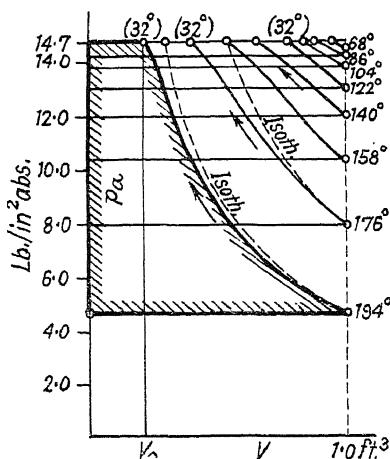


FIG. 251

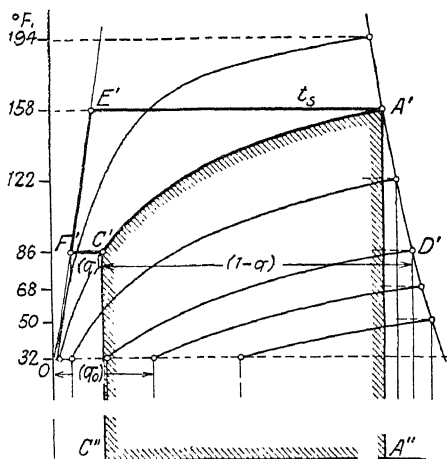


FIG. 252

This heat is thus different from that taken from the air when cooled from  $t_s$  to  $t'$  at constant pressure. It is greater by the amount shown by the second term on the right.

This amount must be added, as the expression itself is negative, i.e. the two terms in equation (489) have the same sign. The difference  $A \int_1^V V d(p - p')$  is given by the area shown shaded in Fig. 251 (in which the initial temperature is 194° F.).

The heat  $\bar{Q}_w$  which has to be removed from the vapour (or steam) is conveniently represented by the heat diagram (Fig. 252). The change of state is completely determined by the lines  $a_1c_1$  and  $ac$  in Fig. 250, since these give the pressure, volume, and temperature, and it is only necessary to transfer the state line  $a_1c_1$  to the heat diagram. The quality of the partly condensed vapour is given by

where  $v'_s$  is the specific volume of the dry saturated steam at the temperature  $t'$  and  $v'$  is the actual volume of the wet steam.



With  $v' = Vv_s$  (since the volume at  $a_1 = 1$ ), we have

$$q = \frac{Vv_s}{v'_s} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (490)$$

In the heat diagram (Fig. 252).

$F'C'$

so that the point can now be transferred. Fig. 252 shows the state curves obtained in cooling saturated moist air at from  $194^{\circ}, 158^{\circ}, 122^{\circ}, 86^{\circ}, 68^{\circ}$ , and  $50^{\circ}$  F. to  $32^{\circ}$  F., when the total pressure is 15 lb./in.<sup>2</sup> abs. The area below the curve  $A'C'$  gives the heat  $Q_w$  taken from the vapour in the air when cooled from the temperature ( $t_s$ ) at  $A$  to the temperature ( $t'$ ) at  $C'$ .

This heat is again different from that abstracted from the vapour when partially condensed at the constant initial pressure to the quality  $q$  and cooled to  $t'$ . The actual heat removed is smaller by the area  $A'E'F'C'$ .

From equation (488) above, the heat removed is

$$Q_w = W_w (H_{t_s} - H'_t) - A \int_{\gamma} V dp' \quad . \quad . \quad . \quad (491)$$

Since now the total heat removed from the mass of 1 ft.<sup>3</sup> of moist saturated air is

$$Q = Q_a + Q_w$$

equations (489) and (491) give

$$Q = (W - W_w) (c_p)_a (t_s - t') - A \int_1^V V d(p - p')$$

$$W_w(H_{ts} - H_{t1}) - A$$

or

$$Q = (W - W_w) (c_p)_a (t_s - t') + W_w(H_{ts} - H'_t) - A \int_1^V V \, dp$$

Now since  $p = \text{constant}$ , it follows that  $dp = 0$ , so that the third term is zero and

$$Q = (W - W_w) (c_p)_a (t_s - t') + W_w (H_{t_s} - H'_{t'}) \quad . \quad (492)$$

and for  $t' = 32^\circ \text{ F.}$ ,

$$Q = (W - W_w) (c_p)_a (t_s - 32) + W_w (H_{ts} - H'_{32}) \quad . \quad (493)$$

Hence the total quantity of heat can be determined as if the pure air had been cooled at constant pressure from  $t_s$  to  $t'$ , and as if the heat taken from the vapour were equal to the difference in total heats before and after cooling. The latter amount is given by the area  $A'E'F'C'C''A''A'$  in Fig. 252.

Equation (493) represents the total heats reckoned from 32° F. The value  $H'_{32}$ , which is the total heat of the vapour in the final state, with the quality  $g$ , at 32° F., is

$$\text{Now} \quad q_{32} = V_{32} \frac{v_s}{v_a} = \frac{p - p_s}{p - p_a} \times \frac{492}{T} \frac{v_s}{v_a} \quad . \quad . \quad . \quad (494)$$

$$\text{hence} \quad H'_{32} = \frac{p - p_s}{p - p_{32}} \frac{v_s}{T_s} \frac{v_s}{v_{s32}} L_{32} \quad . \quad . \quad . \quad (495)$$

As the initial pressure  $p_s$  increases, therefore, the quality at the end decreases, as shown also by Fig. 252. With  $p_s = p$  we have  $q_{32} = 0$  and  $H'_{32} = 0$ , i.e. it is only when vapour alone exists that the complete condensation occurs. Whenever air is present, i.e. when  $p_s < p$ , the fraction  $q_{32}$  remains uncondensed (equation (494)), and the total heat of the moist air is consequently greater than zero.

For an initial volume of 1 ft.<sup>3</sup> we have, with  $W_w = d_s$ ,  $d_s v_s = 1$ ,  $L_{32} = 1070$ ,  $v_{s32} = 3310$ ,  $p = 14.7$ , and  $p_{32} = 0.09$ .

$$W_w H'_{32} = 10.9 \left( \frac{14.7 -}{T} \right)$$

With					
$t_s$	= 200	150	100	50	32° F.
we have	= 0.046	0.184	0.251	0.290	0.302

In Fig. 250 the values of  $Q$  are plotted as abscissae to temperatures as ordinates, so that the total heats  $W_w H_{t_s}$  for the fractional vapour weights in 1 ft.<sup>3</sup> of saturated air (i.e. the amounts  $d_s H_{t_s}$ ) are measured on the right, and the total heats of the air in 1 ft.<sup>3</sup> of mixture, i.e. the amounts  $(W - W_w) c_{pa} t_s$  with  $W - W_w = \frac{2.70(p - p_s)}{T}$  (eqn. 483), are plotted on the left. The horizontal intercepts between the two curves thus represent the total heat in 1 ft.<sup>3</sup> of moist air at the temperature considered. The heat rejected in cooling 1 ft.<sup>3</sup> to 32° F. is smaller than these by the amount  $W_w H'_{32}$ . If the residual vapour at 32° F. be regarded as entirely condensed, this difference disappears and the intercepts between the curves are the same as the heat withdrawn in cooling to 32° F.

In Fig. 250 the superheats of the vapour are represented by the sloping straight lines such as  $EJ$ , for which  $FJ$  is the superheat between  $t_s$  and  $t'$ . The total heats of unsaturated air are not represented. See the next section and Fig. 253 regarding this.

**Total heat diagram for moist air.** In the practical problem of drying damp or wet substances by means of dry or slightly moist air, it is necessary to determine the quantity of vapour which can be taken up by a given amount (say 1 lb.) of air at the existing temperature  $t$  and external pressure  $p$ , in being brought to the saturated condition, and from this to determine the total heat in the saturated air. These values have already been determined in the previous section. In this section a method will be developed which is suitable for some particular drying problems.

The weight  $W_d$  of saturated vapour which can be taken up by 1 lb. of pure air until it becomes saturated is determined by the volume  $v$  of this quantity of air at the partial pressure  $p_a$ , and the temperature  $t$  as contained in the air-vapour mixture at the total pressure  $p$ .

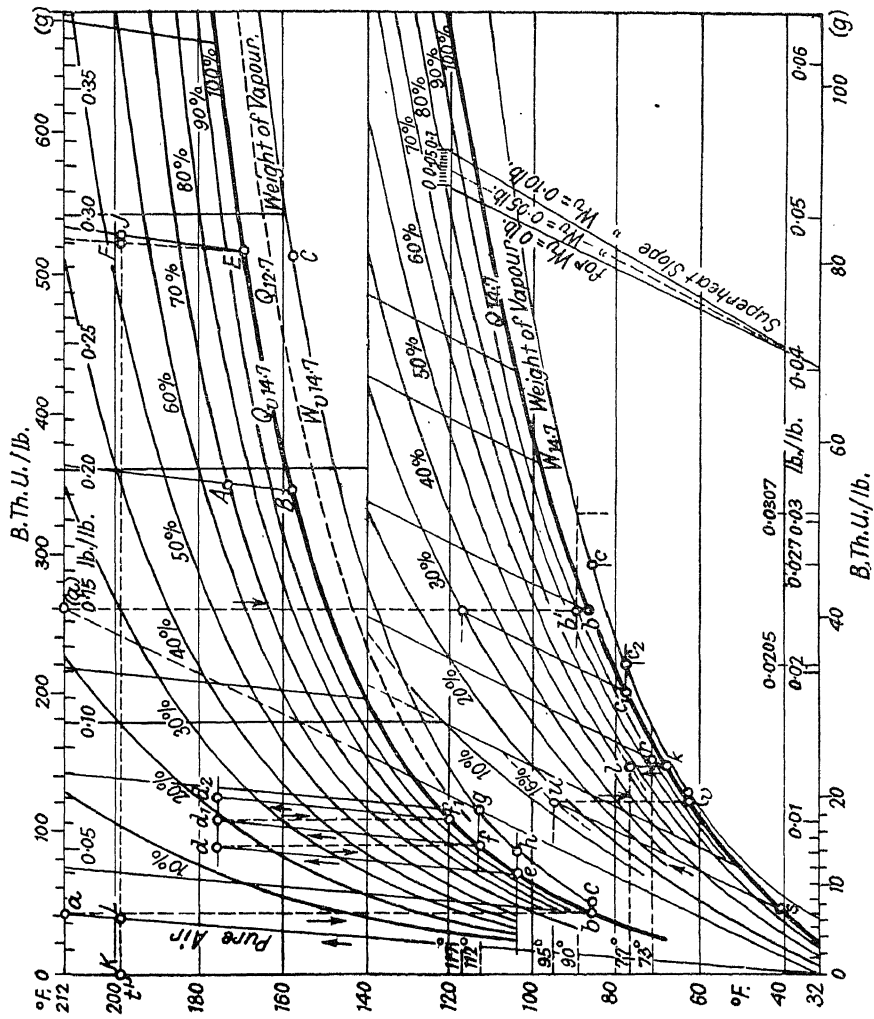


FIG. 253

We have 
$$v = v_0 \frac{p_0}{p_a} \frac{T}{T_0}$$

so that, with  $v_0 = 12.4 \text{ ft.}^3/\text{lb.}$  for  $t_0 = 32$  or  $T_0 = 492$  and  $p_0 = 14.7 \text{ lb./in.}^2 \text{ abs.}$

$$v = 12.4 \times 14.7 (460 + t) \quad . \quad (496)$$

With  $p_s$  as the saturation pressure of the vapour at  $t^\circ$ ,

Hence in the volume  $v$ , the weight of saturated vapour is, with  $d_s$  as the weight of 1  $\text{ft.}^3$  of saturated vapour,

$W_v = v d_s$ , so that with equation 496

$$W_v = \frac{12.4 \times 14.7 (460 + t) d_s}{492 (p - p_s)} \quad . \quad . \quad . \quad (497)$$

This weight is thus dependent on both the temperature and the total external pressure  $p$  of the mixture. For a given quantity of pure air, therefore, and at a given temperature, different barometric pressures produce different vapour quantities.

In Fig. 253 the values of  $W_v$  are plotted for  $p = 14.7 \text{ lb./in.}^2 \text{ abs.}$  against the temperature values. For  $t = 212^\circ$  we have  $W_v = \infty$ , so that the weight curves approach the horizontal through  $212^\circ$  asymptotically.

The heat given to 1 lb. of air, in being raised in temperature from  $32^\circ \text{ F.}$  to  $t^\circ$ , at constant pressure, is

$$W_a = \quad (498)$$

These values are represented in Fig. 253 for temperatures between  $32^\circ \text{ F.}$  and  $212^\circ \text{ F.}$  by the abscissae of the straight line  $Oa$  passing through the origin. The line  $O(a)$  gives the same values to a larger scale and is used for the lower region between  $32^\circ$  and  $140^\circ \text{ F.}$

The total heat reckoned from  $32^\circ \text{ F.}$  contained in the vapour of weight  $W_v$  is

$$Q_v = W_v H$$

so that, with equation (497),

$$= \frac{12.4 \times 14.7 (460 + t) d_s H}{492 (p - p_s)} \text{ B.Th.U.} \quad . \quad (499)$$

These values have been plotted as horizontal additions to the right of the sloping straight line  $Oa$  (for the larger scale, from  $O(a)$ ), so that the abscissae of the resulting curve  $Q_{14.7}$ , measured from the vertical through  $O$  (temperature axis), represent the total heats

of 1 lb. of pure air and  $W_v$  lb. of saturated vapour (i.e.  $(1 + W_v)$  lb. of saturated moist air). These curves also approach the horizontal through  $212^\circ \text{ F.}$  asymptotically. A second heat curve  $Q_{12.7}$  for  $p = 12.7 \text{ lb./in.}^2 \text{ abs.}$

is shown dotted. The values of  $W_v$  and  $Q_v$  for  $p = 14.7$  and  $12.7$  lb./in.<sup>2</sup> abs. are given in the table below. See the previous section for the justification in taking the sum of the total heats in determining the total heat of the moist air.

$t$ °F.	$p_s$ lb/in. <sup>2</sup> abs.	$d_s$ lb./ft. <sup>3</sup>	$W_{v(14.7)}$ lb./lb.	$Q_{v(14.7)}$ B.Th.U.	$W_{v12.7}$ lb./lb.	$Q_{v12.7}$ B.Th.U.
32	0.0885	0.000302	0.00377	4.05	0.0044	4.75
40	0.105	0.000412	0.00524	5.64	0.0061	6.57
60	0.257	0.000834	0.0111	12.06	0.0129	14.00
80	0.510	0.00160	0.0242	26.40	0.0282	30.8
100	0.954	0.00288	0.0434	47.8	0.0509	56.1
120	1.712	0.00498	0.0824	91.5	0.0974	108.1
140	2.930	0.00823	0.1560	174.9	0.1870	209.0
160	4.774	0.01310	0.303	342.0	0.379	427.0
180	7.590	0.02015	0.671	763.0	0.934	1061.0
200	11.50	0.0298	2.280	2610.0	6.08	6970.0
212	14.70	0.0374	$\infty$	$\infty$	—	—

The quantities of heat and vapour weights taken up by 1 lb. of pure air when brought to the saturation condition can be determined graphically in the following simple manner. The moist air in the saturated state, say at  $B$  or at  $E$ , is regarded as being heated at constant pressure. This causes the vapour to be superheated and the humidity decreases. The partial pressures remain unaltered. The water vapour takes up the heat  $W_v c_{pw} (t' - t)$  with  $c_{pw} = 0.465$ . At the same time the air takes up the heat  $0.24 (t' - t)$ . The first quantity is represented in Fig. 253 by the distance  $FJ$ , where  $EJ$  is parallel to  $O(a)$ . The distance  $JL$  thus represents the total heat in the unsaturated vapour at the temperature  $t'$  and this vapour has the same weight as at  $E$ . The increase in the total heat of the air is given by the slope of the line  $Oa$  between the temperatures at  $E$  and  $F$ . Hence the distance  $JK$  gives the total heat of unsaturated moist air at the temperature  $t'$ , and it only remains to determine the humidity corresponding to the point  $J$ .

This is readily obtained, if pressure curves for saturated and unsaturated vapour are first drawn in Fig. 253 similar to those shown on the left of Fig. 250. A line is then drawn from  $J$  to  $E$  (use Fig. 250 in following the method), while an horizontal is drawn from  $E$  to cut the saturation pressure curve at the point  $a$ . A vertical from  $a$  and an horizontal from  $J$  then gives the point  $b$ , which lies on the curve showing the required degree of humidity. By reversing this process, the curves of constant humidity, shown in Fig. 253, are obtained and the original pressure curves can then be dispensed with.

It should be observed that, along a given horizontal line, the horizontal intercepts between these curves increase with the humidity.\* Hence, from Fig. 253, the total heat of a mass of unsaturated air at any humidity and corresponding to 1 lb. of pure air is represented as a distance. The vapour weight per pound of pure air at any state such as  $A$  (Fig. 253) is obtained by dropping from  $A$  to the point  $B$  on the saturation heat curve and

\* *Z.V.d.I.* (1919), page 682. (W. Schüle), and page 821. E. Höhn, *Beitrag z. Theorie des Trocknens und Dörrrens*.

then drawing an horizontal to the point  $C$  on the weight curve, the abscissa of which gives the required vapour weight.

The total heat, as given in Fig. 253, can be found by calculation as follows. The total heat in a saturated air-vapour mixture containing 1 lb. of pure air is, from the above,

$$Q = 0.24(t_s - 32) + HW_v$$

in which  $W_v$  is given by equation (497).

The same mixture, if heated to  $t > t_s$ , contains, in addition, the superheat  $0.465W_v(t - t_s)$ , so that the complete total heat of the unsaturated air with 1 lb. of pure air is

$$Q_\phi = 0.24(t_s - 32) + HW_v + 0.465W_v(t - t_s)$$

If now the temperature  $t$  of the unsaturated air and its partial pressure (or humidity) be given, the dew point  $t_s$  must first be determined (Fig. 250, left). Since no analytical expression gives this, it follows that it is not possible to introduce  $t_s$ , in terms of  $\phi$  and  $t$ , in the above expression for  $Q_\phi$ .

**Applications.** Drying by means of warm air, or the moistening of dry air, are mixing processes accompanied by evaporation in which the external pressure remains constant. By applying the first law in the form shown on page 176,

we have, since  $dp = 0$ ,

$$Q = W(H_2 - H_1)$$

If no heat be supplied to the mixture of air, vapour, and water from without, and only exchange of heat between the constituents occurs, then with  $Q = 0$ ,

i.e. the sum of the total heats of the constituents remains constant.\*

**Example 11.** (Moistening.) Consider a space filled with dry air at  $t_1^\circ$  and 14.7 lb./in.<sup>2</sup> abs. into which sufficient water at  $t_0^\circ$  is sprayed to bring the air to a relative humidity  $\phi \geq 1$ . How much water has to be supplied per pound of pure air, and what is the final temperature  $t_2$ ? The external pressure remains constant.

Let  $W_v$  be the weight of water and  $H_v$  the total heat in 1 lb. of vapour for a relative humidity  $\phi$  at the temperature  $t_2$ . The total heat of air and water before mixing is then

$$H_1 = 0.24(t_1 - 32) + \quad , - 32)$$

and, after mixing,

$$H_2 = 0.24(t_2 - 32) +$$

\* Mollier, therefore, in showing changes of state of moist air, applies diagrams in which the total heats  $H$  of the mixture are plotted against the vapour weights  $W_s$  per pound of pure air (called  $x$  by Mollier. See *Z.V.d.I.* (1929), page 1009, R. Mollier, "The  $i$   $x$  diagram for air-vapour mixtures"; and also Grubenmann, *J x diagram for moist air*. (Berlin. Julius Springer, 1926.)

Therefore, since  $H_1 = H_2$ ,

$$0.24t_1 + W_v(t_0 - 32) = 0.24t_2 + W_vH_v$$

or 
$$0.24t_1 = 0.24t_2 + W_vH_v - W_v(t_0 - 32)$$

For the special case of  $t_0 = 32^\circ \text{ F.}$ , this becomes

$$0.24t_1 = 0.24t_2 + W_vH_v$$

or 
$$0.24(t_1 - 32) = 0.24(t_2 - 32) + W_vH_v$$

i.e. the total heat, reckoned from  $32^\circ \text{ F.}$ , of the dry air in the initial state is equal to that in the final state plus the total heat of the vapour. In the heat diagram (Fig. 253), therefore, the points  $a$  and  $b$ , which represent the initial state of the dry air and the final state of the moist air, are at the same distance from the temperature axis, i.e.  $b$  is vertically below  $a$ . For  $t = 212^\circ \text{ F.}$  and for complete saturation  $t_2 = 86^\circ \text{ F.}$ , while for  $\phi = 30$  per cent,  $t_2 = 117^\circ \text{ F.}$  In the first case the weight of water evaporated is  $W_v = 0.027 \text{ lb.}$  (point  $c$ ). In the second case it is  $W_v = 0.0205 \text{ lb.}$  (point  $c_2$ ).

If the initial water temperature  $t_0$  is above  $32^\circ \text{ F.}$ , then

$$0.24t_1 = 0.24t_2 + \{W_vH_v - W_v(t_0 - 32)\}$$

Hence a short length of the curve of values  $W_vH_v - W_v(t_0 - 32)$  has first to be drawn, which, for  $t_0 = 176^\circ \text{ F.}$ , coincides with the heat curve for  $\phi = 90$  per cent. The final state  $b'$  then lies on this curve and vertically below  $(a)$ . This gives a final temperature of  $90^\circ \text{ F.}$  and a vapour weight of  $0.0307 \text{ lb.}$  for complete saturation.

If the air at the start is moist, having a vapour weight  $W_1$ , the total heat initially is

$$H_1 = 0.24(t_1 - 32) + W_1H_1 + (W_v - W_1)(t_0 - 32)$$

and, finally, 
$$W_2 = 0.24(t_2 - 32) + W_vH_v$$

Hence 
$$0.24t_1 + W_1H_1 = 0.24t_2 + W_vH_v - (W_v - W_1)(t_0 - 32)$$

With  $t_0 = 32$  the change of state, therefore, is again represented by a vertical line. For  $t_0 > 32^\circ \text{ F.}$  a portion of the curve

would have to be drawn first.

**Example 12.** (Evaporating coolers and wet and dry bulb hygrometers.) A stream of dry or unsaturated air at  $t_1^\circ$  and  $14.7 \text{ lb./in.}^2$  abs. is passed through a finely divided quantity of water initially at the same temperature (e.g. through stretched damp cloths or through a water spray). In this way the air becomes saturated. In order to effect this evaporation the heat contained in the air and in the water is used, so that a drop in temperature occurs. After steady conditions are attained, the water temperature remains constant and the whole of the evaporation heat is taken from the air stream.

If  $W_1$  be the weight of vapour in each pound of the unsaturated air in the initial state, and  $W_v$  be the weight in the saturated air finally, the same relations hold as above. The initial and final points in the change of state lie on the same vertical in the heat diagram.

The greatest cooling effect is given by the point on the saturation curve, since evaporation ceases when the air is completely saturated. In determining this limit exactly, however, as in example 11, it should be observed that the initial water temperature is not  $t_0 = 32$ , but  $t_0 = t_1$ .

The wet and dry bulb hygrometer is a well-known application of the above. The air, whose initial degree of humidity is required, is led past two thermometers, one of which has a dry bulb while the other is covered with moistened linen. Due to the evaporative cooling, the wet thermometer shows a lower temperature than the dry, since the flowing air becomes saturated by the water in the linen, which, under steady conditions, assumes the same temperature. For example, if the wet thermometer reads  $68^\circ \text{F.}$  for an external pressure of  $14.7 \text{ lb./in.}^2 \text{ abs.}$  (point  $k$ , Fig. 253) and the dry reads  $77^\circ \text{F.}$  (point  $l$ , vertically above  $k$ ), the air is 66 per cent wet initially.

Another example of evaporative cooling is that in which condenser circulating water is re-cooled. (Cooling towers.)\*

**Example 13.** (Drying by means of warm air.) A wet material can be dried if a stream of dry or unsaturated air flows over, or better, through it (e.g. lightly packed masses of plant leaves or minerals lying on riddles). The problem is to determine how much air, in a given initial state, is required in order to remove 1 lb. of water from the material to be dried, and to find the quantity of heat necessary for this.

As in example 11, the initial and final state points of the drying air lie on the same vertical in Fig. 253. Assume, for example, that the initial temperature is  $176^\circ \text{F.}$  with  $\phi = 15$  per cent (point  $d$ ). The superheat line  $de$  gives an initial vapour content of  $0.049 \text{ lb.}$  per pound of pure air, the perpendicular  $df$  gives the final temperature of the withdrawn air as  $112^\circ \text{F.}$ , and a vapour weight of  $0.066 \text{ lb.}$  Hence 1 lb. of air has taken up  $0.066 - 0.049 = 0.017 \text{ lb.}$  of water, and the weight of pure air required to remove 1 lb. of water from the material to be dried is  $\frac{1}{0.017} = 59 \text{ lb.}$

The heat taken up corresponds to the difference between the total heat in 1 lb. of air at the state  $d$  and the existing state of the external air, which is heated to  $176^\circ \text{F.}$  (and 15 per cent humidity). If this heating takes place at constant pressure, and if the air was initially saturated,  $e$  must represent the initial state ( $104^\circ \text{F.}$ ). From  $e$  to  $d$  the total heat increase is  $92 - 72 = 20 \text{ B.Th.U.}$  per pound of air, so that, per pound of water evaporated, the heat required is  $59 \times 20 = 1180 \text{ B.Th.U.}$  If completely dry air at  $176^\circ \text{F.}$  and at an external temperature of  $68^\circ \text{F.}$  had been used, the final temperature of the withdrawn air would have been  $78^\circ \text{F.}$ , with an evaporated water weight of  $0.021 \text{ lb.}$  and a heat supply of  $24 \text{ B.Th.U.}$  per lb. of air. These correspond to  $\frac{1}{0.021} = 47.6 \text{ lb.}$  of air and a heat supply of  $24 \times 47.6 = 1140 \text{ B.Th.U.}$  per pound of water.

Under actual operating conditions the heat supply is usually effected in steps in such a way that the moistened drying air is re-heated (line  $fd_1$ ) and is then used anew at some other point to dry the material. This causes further moistening as shown by  $d_1f_1$  and so on.

\* See "Condensation," by F. J. Weiss and O. H. Müller, *Z.V.d.I.* (1905), page 5, *et seq.* Also *F.A.* No. 275. Fr. Merkel, "Verdunstungskühlung." Regarding the theory of the August hygrometer, see Mollier, *Z.V.d.I.* (1929), page 1013.



**Example 14.** (Cold drying.) Moist saturated air can be dried by passing it over a cold surface, so that part of its vapour content is condensed on this surface. In this way the absolute moisture content of the air is reduced, while the air itself remains saturated. If now this is again heated, its humidity is decreased and the air is capable of absorbing moisture from other bodies. (In winter this process occurs naturally at low temperatures, since the outer air is dried by cooling, and when this cooled air is heated its humidity can assume a very low value.)

Take, for example, air at  $73^{\circ}$  F. with  $\phi = 90$  per cent (point  $r$ ) and let it be cooled to  $41^{\circ}$  F. (point  $s$ ). The amount of vapour condensed is  $0.0148 - 0.0052 = 0.0096$  lb. per pound of air, and the heat removed is  $25.7 - 7.7 = 18.0$  B.Th.U. In the ensuing heating to  $95^{\circ}$  F. (line  $su$ ) the humidity decreases to 16 per cent and the heat supplied is  $20.5 - 7.7 = 12.8$  B.Th.U. If this air be used for drying (line  $uv$ ) the moisture evaporated down to the saturated state is  $0.0120 - 0.0052 = 0.0068$  lb., with a reduction in temperature to  $63^{\circ}$  F. (point  $v$ ). The heat required is 18 B.Th.U. in the cooling process and 12.8 in the heating process, giving a total of 30.8 B.Th.U. per pound of air. Hence, to evaporate 1 lb. of water from the materials, the weight of air required is  $\frac{1}{0.0068} = 147$  lb., and the heat necessary is  $147 \times 30.8 = 4530$  B.Th.U. If the outer air be used in heating from  $41^{\circ}$  F. to  $73^{\circ}$  F., then only  $18 + 5 = 23$  B.Th.U. are required per pound of air or  $147 \times 23 = 3400$  B.Th.U. per pound of water.

The process can be improved if, in place of cooling the outer air, the discharged air (point  $v$ ) be cooled and again used for drying. The heat required to effect the cooling is then only  $20.7 - 7.7 = 13$  B.Th.U., giving a total of  $13 + 5 = 18$  B.Th.U./lb. of air or  $18 \times 147.1 = 2640$  B.Th.U./lb. of water.\*

\* See *Zeit. f. d. ges. Kälteindustrie* (1919), page 79. M. Hirsch. Cold drving.

## CHAPTER X

### APPLICATIONS

**Steam prime movers.** The term available work of saturated or superheated steam means the mechanical work obtainable from 1 lb. of steam in expanding from an initial pressure  $p_1$  and temperature  $t_1$  to the back pressure  $p_2$  in a suitable engine, unaccompanied by any heat or friction losses.

This work is readily shown by the ideal pressure-volume diagram of a reciprocating engine. (Fig. 254.) Along  $ab$  the steam flows into the cylinder at the same pressure as it possesses in the supply pipe. From  $b$ , where the supply ceases, the steam expands, without any heat exchange

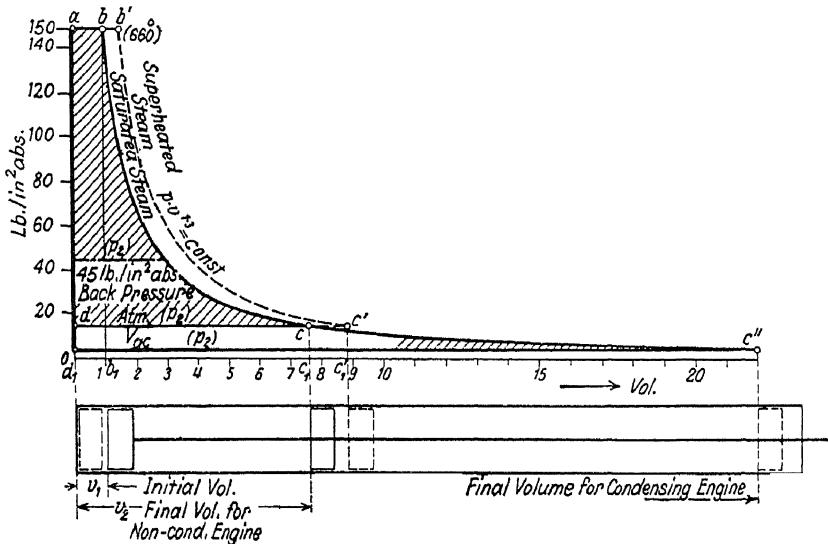


FIG. 254

with the walls, to the back pressure  $p_2$ . At  $c$ , the piston reverses its motion and pushes the steam, having the volume  $dc$ , into the atmosphere, or into an air-free space in which it is condensed by cooling water or into a heating system at or above atmospheric pressure.

The work delivered by the steam in this process is represented by the area  $abcd$  (see page 98). Its value can be determined graphically, or by calculation. In Fig. 254 it is given by

$$\text{Area } abb_1d_1 + \text{area } bcc_1b_1 - \text{area } cdd_1c_1 = \text{area } abcd$$

or

$$- p_2 v_2 - p_2 v_2 =$$

In this the expansion line  $bc$  is assumed to follow the adiabatic law

$$pv^\gamma = \text{constant}$$

This gives  $E_0 = \frac{\gamma}{\gamma-1} (p_1 v_1 - p_2 v_2)$

or, with  $p_1 v_1^\gamma = p_2 v_2^\gamma$

$$E_0 = \frac{\gamma}{\gamma-1} p_1 v_1 \left[ 1 - \left( \frac{p_2}{p_1} \right)^{\frac{\gamma-1}{\gamma}} \right] \quad . \quad . \quad . \quad (500)$$

or, in heat units,

$$AE_0 = \frac{\gamma}{778(\gamma-1)} p_1 v_1 \left[ 1 - \left( \frac{p_2}{p_1} \right)^{\frac{\gamma-1}{\gamma}} \right] \quad . \quad . \quad . \quad (501)$$

This is the required expression for the available work of 1 lb. of steam. It holds for superheated steam with  $\gamma = 1.3$  and for dry saturated steam with  $\gamma = 1.135$ . Thus, for saturated steam with  $p_1 = 170$  lb./in.<sup>2</sup> abs.,  $t_1 = 368.4^\circ$  F.,  $v_1 = 2.662$ , and  $p_2 = 14.7$ , we have

$$AE_0 = 177 \text{ B.Th.U./lb.}$$

For superheated steam at the same pressure, but at  $572^\circ$  F., with

$$\frac{\gamma}{\gamma-1} = \frac{1.3}{0.3} = 4.333$$

$$AE_0 = 202 \text{ B.Th.U./lb.}$$

**Graphical determination.** The diagram area in Fig. 254 can be measured by means of a planimeter, thus giving the work done.

**Determination from the  $H\phi$  diagram.** The available work is also given by the heat drop as shown on page 456, i.e.

$$AE_0 = H_1 - H_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (502)$$

where  $H_1$  is the total heat in 1 lb. of steam in the initial state and  $H_2$  the total heat at the end of the adiabatic expansion (point *c*, Fig. 254).

The values of  $H_1$  and  $H_2$  are found from the  $H\phi$  chart. The value of the available work determined, in general, in this way for steam turbine calculations is the same as given by equation (501).

From the chart the values of the heat drops in the above examples are 178 and 207 B.Th.U. respectively.

**Ideal thermal efficiency.** In order to generate steam, which is to operate in a prime mover, the heat necessary for a feed temperature of  $t_0^\circ$  F. is  $H_1 - h_0$ , where  $h_0$  is the sensible heat at  $t_0$ . Even when all losses are avoided, only the fraction

of the heat supplied is convertible to work. The value given by equation (503) is called the thermal efficiency of the ideal process. It lies between the limits of 0.1 and 0.3, depending on the steam pressure, temperature, and back pressure. On page 461, example 5,  $\eta_{th}$  value for pressures of 170 lb./in.<sup>2</sup> abs. and less have been calculated.

For  $p_1 = 350$  lb./in.<sup>2</sup> abs.,  $t_1 = 660^\circ$  F.,  $p_2 = 0.57$  lb./in.<sup>2</sup> abs. (condenser pressure) the thermal efficiency is  $\eta_{th} = 0.34$ . By increasing the initial pressure to 700 lb./in.<sup>2</sup> abs.,  $\eta_{th}$  increases to 0.37. Fig. 255 shows

the increase in the values of  $H_1$  and  $\eta_{th}$  (on the  $H\phi$  diagram) for steam at 70 lb./in.<sup>2</sup> abs. and 185 lb./in.<sup>2</sup> abs., and ranging from wet steam with 25 per cent moisture to superheated steam at  $800^\circ$  F.

**Thermodynamic efficiency or efficiency ratio.** The value of the ratio of the actual work  $AE_i$  delivered by the piston of the reciprocator or by the rotor of a turbine to the work  $AE_0$  without loss, is called the thermodynamic efficiency or efficiency ratio, i.e.

$$(504)$$

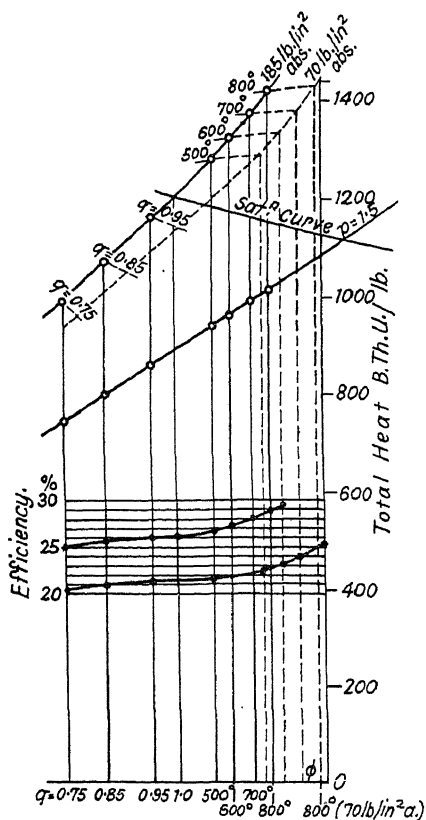


FIG. 255

In all prime movers, internal losses occur which reduce the available work so that  $\eta_o$  is less than unity. The value  $\eta_o$  is therefore a measure of the actual conversion to work of the steam supplied (if the purely mechanical losses in the machine be first neglected), 85 per cent is considered to be a high value for  $\eta_o$  and means that the internal losses amount to 15 per cent of the ideal available work.

In the reciprocating engine this loss is mainly caused by the flow of heat from the steam to the cylinder walls, but losses also occur due to

throttling of the steam at the inlet and exhaust valves and due to incomplete expansion.

In the steam turbine the internal losses are caused by the flow resistance of the steam in the guide channels and moving blades, and also by disk friction.

One of the chief considerations in engine design is to reduce these internal losses to a minimum, i.e. to make  $\eta_o$  as close to 100 per cent as possible.

**Mechanical efficiency.** In the above losses the work necessary to overcome the friction of the machine itself, caused by the motion of the working parts, is not included (piston in the cylinder, piston rod at the stuffing box, cross head in the guides, connecting rod and main bearings, valve gear, and slide valve all cause friction). The mechanical losses in the steam turbine are caused by the shaft bearings, and in the work required for the auxiliaries such as oil pumps, air pumps, and the governor drive.

If now  $AE_e$  represents the work actually delivered by the shaft externally the mechanical efficiency is given by

In the case of reciprocators this value depends chiefly on the load, and for normal loads amounts to 0.8 to 0.93, according to the size and type of engine. At smaller loads the friction does not greatly change, so that the mechanical efficiency decreases until at no load (i.e. for  $AE_e = 0$ ) it becomes zero.

In steam turbines  $\eta_m$  is nearly unity when the auxiliaries are neglected and becomes zero at no load.

**Overall thermal efficiency.** The ratio of the work  $AE_e$  delivered externally to the heat  $H_1 - h_0$  supplied to the live steam gives the overall thermal efficiency of the engine, i.e.

$$\eta_e = \frac{AE_e}{H_1 - h_0} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (506)$$

This value is of fundamental importance in assessing the utilization of heat by the engine. It can be determined by measuring the shaft power and the steam consumption. If  $N_e$  is the shaft horse-power, then the work delivered per hour in heat units is

If the steam consumption per hour is  $W_h$ , the heat supplied per hour is  $W_h(H_1 - h_0)$  B.Th.U.

$$\text{Hence} \quad \eta_e = \frac{2545N_e}{W_h(H_1 - h_0)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (507)$$

The quotient  $\frac{W_h}{N_e} = C$  gives the steam used per horse-power hour.

$$\text{Hence} \quad \eta_e = \frac{2545}{C(H_1 - h_0)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (508)$$

The value

is the heat used per horse-power hour, so that  $\eta_e$  is also given by

In steam turbines the steam consumption and heat used are generally expressed per kW hour instead of per horse-power hour. ( $C_{el}$  and  $Q_{el}$ .) If the efficiency of the electrical generator is  $\eta_{el}$  the kW output at the turbine shaft (at the coupling to the generator) is

$$\text{or, in h.p.} \quad N_e = 1.34 \frac{N_{el}}{\eta_{el}} \text{ (h.p.)}$$

Using these values, we have

$$3412 \quad 1 \quad . \quad (510)$$

$$\text{and} \quad \eta_e = \frac{3412}{Q_{el} \eta_{el}} \quad . \quad (511)$$

In reciprocating engines the steam and heat consumptions are generally measured per indicated horse-power ( $C_i$  and  $Q_i$ ). For a mechanical efficiency of  $\eta_m$

$$C = \frac{C_i}{\eta_m}; \quad Q = \frac{Q_i}{\eta_m}$$

$$\text{and} \quad = \frac{2545 \eta_n}{2545} \quad (512)$$

**Thermal efficiency of the steam plant.** In all steam prime movers, steam must first be generated in a boiler from water and be led by a pipe to the engine. The heat developed in the furnace of the boiler for generating the steam is always greater than the total heat contained in the steam at entrance to the engine. This is due to the unavoidable heat losses in the boiler and supply pipe, and in some cases on account of incomplete combustion. The efficiency of the boiler is the ratio of actual heat  $\dot{H}'_1 - h_0$  contained in the steam as it leaves the boiler to the heat liberated by the complete combustion of the fuel. In measuring these quantities steady conditions have to be maintained.

Thus if  $F$  lb. of fuel, having a calorific value  $H_f$ , be burned per hour, and if  $W_h$  be the weight of steam generated per hour, then the boiler efficiency is

$$\eta_b = \frac{W_h(H'_1 - h_0)}{FH_f} \quad . \quad . \quad . \quad . \quad . \quad . \quad (513)$$

In up-to-date and large boiler plants,  $\eta_b$  is as high as 0.80 to 0.85, and for smaller and less complete plants its value ranges from 0.55 to 0.70. These figures are based on the assumption of normal loads and steady test conditions. At smaller loads or unsteady conditions,  $\eta_b$  is less than the value for normal loads.

A certain fraction of the heat contained in the steam as it leaves the boiler or superheater is always lost in the connecting pipe leading to the engine. The amount of heat lost depends on the length of the pipe, on the temperature of the steam, on the speed of the steam, and on the insulation of the pipe. With an inefficient lay-out it is possible for most of the superheat to be lost. Calling the ratio

the transmission efficiency, the available fraction of the heat liberated by combustion and contained in the steam at entrance to the engine is  $\eta_b \eta_t$ . Hence the shaft work expressed as a fraction of the combustion heat is

This value, which is sometimes called the overall efficiency, is a measure of the utilization of heat in the performance of work. With

$$\eta_e =$$

we have  $\eta_s =$

so that the overall efficiency is the product of the separate efficiencies. Introducing the highest efficiency values, namely,  $\eta_b = 0.87$ ,  $\eta_t = 0.97$ ,  $\eta_{th} = 0.36$ ,  $\eta_g = 0.87$ , and  $\eta_m = 0.94$ , gives

$$\eta_s = 0.87 \times 0.97 \times 0.36 \times 0.87 \times 0.94 = 0.25$$

as the maximum value of heat conversion to work by a steam prime mover.

This corresponds to a heat consumption of

$$= 10200 \text{ B.Th.U./h.p. hour}$$

and, per kW hour,

$$\frac{3412}{0.25} = 13660 \text{ B.Th.U./kW hour}$$

If electrical energy be generated, it is usual to consider the electrical output. Taking  $\eta_{el}$  as the generator efficiency, the overall efficiency of the electrical plant is

In the latest large American power stations the heat consumption amounts to 14,000 B.Th.U. per kW hour. In the Columbia Power Station, where steam at 600 lb./in.<sup>2</sup> abs. and 730° F. is used, the heat consumption is as low as 12,700 B.Th.U./kW hour, which corresponds to an overall efficiency of 27.2 per cent.\*

The boilers of this plant operate on pulverized coal. They are fitted with economizers, air heaters, and air-cooled furnace walls. The steam turbines consist of a high pressure section, in which the steam is expanded in 14 stages to 110 lb./in.<sup>2</sup> abs. From there the steam is returned to the boiler, where it is re-superheated to 730° F. The steam is then supplied to the low pressure section, where it expands, in 12 stages, to the condenser pressure. At the 18th, 22nd, and 24th stages, steam is tapped off and used to heat the feed water (regenerative feed heating).

### STEAM FOR POWER GENERATION AND HEATING

Steam is used in practice for two main purposes—

1. To generate mechanical energy (reciprocating engines and steam turbines).
2. To supply heat for various purposes (heating of buildings, heating of substances, boiling of liquids, and drying).

When used for the sole purpose of generating power there is in all cases a considerable loss of heat, since only a small fraction of from

\* *Elektrizitätswirtschaft* (1927), page 472. "Test results of the Columbia Power Station."

10 per cent to 25 per cent of the heat contained in the steam can be converted to mechanical work. The heat remaining in the steam, which amounts to 90 to 75 per cent, is lost in the exhaust, either to the atmosphere or to the cooling water of the condenser.

When using steam for heating alone, no use is made of the available energy in the steam at all pressures, even when as low as atmospheric. Where there is no need for mechanical energy, as, for example, in the heating of buildings, this presents no great disadvantage, since the steam in being condensed gives up its total heat content (down to the temperature of the condensate) to the substance which is being heated. The sensible heat contained in the condensate can also be saved by feeding this condensate back to the boiler. The thermal efficiency of a heating system can in this way be as high as 90 per cent and, if reckoned on the heat of combustion of the fuel, is nearly as high as that of the boiler (60 to 80 per cent).

**Back pressure machines.** When steam is used in factories it frequently happens that, in addition to generating power, it is also required for heating purposes. This occurs in paper, textile, and chemical factories.

In effecting this it was customary, previously, to obtain the mechanical work ( $AE$ ) by means of a condensing engine or turbine, and to obtain the heating effect ( $W_H$ ) from steam generated in a special low pressure boiler, or from steam throttled down from a high pressure boiler, which served both purposes. The combustion heat required for the steam of the prime mover is then

$$AE$$

where  $\eta_{w1}$  is the thermal efficiency of the condensing engine.

The combustion heat required for the heating steam is

where  $\eta_b$  is the efficiency of the (common) boiler. Hence the total heat required is

$$W = \frac{AE}{\eta_{w1}} + \frac{W_H}{\eta_b} \text{ (B.Th.U.)} \quad . \quad . \quad . \quad (517)$$

The lay-out can, however, be so arranged that the same steam is used both for the power and for the heating. If, for example, heat has to be supplied by steam at about atmospheric pressure, the steam can be generated in a high pressure boiler and be fed to a steam engine or turbine, the back pressure of which is the same as the heating pressure. If the useful work delivered by the steam is  $AE$ , the heat at entrance to the machine is

$$AE$$

where  $\eta_{eII}$  is the thermal efficiency corresponding to the inlet pressure  $p_1$  and the back pressure  $p_2$ . The entering steam may be superheated, even though superheated steam may not be desirable for heating purposes, since





This value is always greater than 1, i.e. the total heat required for the separated power and heating units is always greater than that of the combined power and heating units. Thus, with  $\eta_{eII} = 0.1$ ,  $\eta_{eI} = 0.2$ ,  $\eta_m = 0.85$ ,

$$\frac{W}{W'} = 1 + 0.1 \left( \frac{1}{0.2} - \frac{1}{0.85} \right) = 1.38$$

so that in this case the heat required for the separated arrangement is 38 per cent greater. The combined unit thus shows a definite economic gain, since the fuel consumption, for the same power and heat delivery, is only 72.5 per cent of that for the separate units. In addition, the condensing plant and the necessary cooling water are dispensed with.

Equation (520) shows that the gain is increased as the thermal efficiency  $\eta_{eII}$  of the back pressure engine increases. This efficiency is

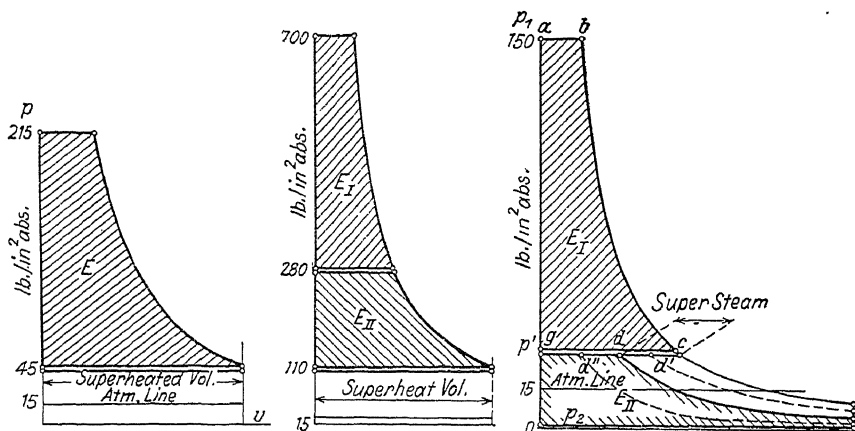


FIG. 256

FIG. 257

FIG. 258

conditional in the first place by the pressure drop in the back pressure engine. As the boiler pressure is increased, therefore (while keeping the back pressure constant), the gain is increased.

Where the heating steam pressures amount to several atmospheres (e.g. 50 to 75 lb./in.<sup>2</sup> abs.) it is necessary to increase the boiler pressure in order to gain the same efficiency, since the ratio of boiler pressure to the back pressure decides the thermal efficiency. Thus, if the boiler pressure is 150 lb./in.<sup>2</sup> abs. when the back pressure is 15 lb./in.<sup>2</sup> abs., an increase in the back pressure to 75 lb./in.<sup>2</sup> abs. means an increase in the boiler pressure to 750 lb./in.<sup>2</sup> abs. Fig. 256 shows the diagram of the back pressure engine when the boiler pressure is 215 lb./in.<sup>2</sup> abs. and heating steam pressure is 45 lb./in.<sup>2</sup> abs., while Fig. 257 shows the diagram for a boiler pressure of 700 lb./in.<sup>2</sup> abs. and heating steam pressure of 110 lb./in.<sup>2</sup> abs.

**Extraction engines or turbines.** It is only in special cases that the amount of steam required for power purposes is the same as that required for heating. If the heating steam quantity is definitely less and the demand variable, the condensing engine with intermediate steam extraction is used in place of the back pressure engine. The steam required for

heating is extracted from the receiver of a compound engine or, in a multistage turbine, at the stage where the pressure is equal to that of the heating steam. The remaining steam in the engine or turbine then expands in the low pressure cylinder or in the low pressure section of the turbine. Fig. 258 shows the ideal pressure-volume diagram. In the high pressure region the steam expands from the boiler pressure  $p_1$  to the heating steam pressure  $p'$ , and enters the receiver at this pressure. The work delivered is  $E_1$ , say. Of the volume  $cg$ , the portion  $cd$  is removed for heating. The remainder,  $dg$ , expands in the low pressure section and delivers the work  $E_{II}$ .

In the same way as in the back pressure machine, the total combustion heat required is less than that necessary when the heating is effected merely by throttling the high pressure steam, since the heating steam delivers work in the high pressure section.

If less steam than that corresponding to the normal extraction  $cd$  is required, a correspondingly smaller volume  $cd'$  is extracted. In the low pressure section the greater quantity  $gd'$  then operates, so that the power output is increased. If now the increase in power corresponding to the reduced heating output be not required, it is necessary to reduce the quantity of steam entering the high pressure section, while maintaining the same pressure in the receiver or intermediate stage. Hence in addition to the ordinary governor of the engine, a pressure regulator for the heating steam must be fitted, which partially closes for a reduction of heating steam and opens for an increase. If the heating be entirely stopped, the engine operates as a normal condensing engine. When the greatest possible amount of heating steam is used, all the steam before the low pressure section is extracted and the engine operates as a back pressure machine with the low pressure section running empty.

**Practical application of steam in producing mechanical work.** Machines used for producing mechanical work from steam are either reciprocators or steam turbines. In the following, the different operations followed by the steam will be discussed on a thermodynamic basis. Since the steam is, in every case, produced from water by the absorption of combustion heat in a furnace, it is necessary to take account also of the generation of the steam so far as this depends on the complete process of obtaining mechanical work from the heat contained in the fuel. This is also necessary for the further reason that in some of the latest arrangements the steam, during its delivery of work, takes up a further supply of heat, either from the furnace or by means of live steam at the boiler pressure, or, on the other hand, gives up heat to the feed water.

**Steam engine.** Live steam is supplied by a boiler and enters the cylinder while the piston moves forward a distance  $s$  (supply period line  $ab$  in the pressure-volume diagram). At  $b$  the supply is cut off by means of a valve. The steam behind the piston expands as the piston moves forward, causing the pressure to drop continuously (expansion period line  $bc$ ). Shortly before the piston comes to the inner dead centre position, the valve opens a passage to the exhaust pipe and the steam first escapes, either to the atmosphere or to a condenser, on account of its excess pressure, and later by the action of the piston on its return stroke. When discharged to a condenser, the steam is condensed by means of cooling water. During this return movement of the piston, the passage to

exhaust remains open till the piston comes to *e*, where the steam pressure is about the same as that of the space into which exhaust occurs. At *e*, the distance of which from the left dead centre position varies, the passage to exhaust is closed, so that the steam remaining in the cylinder is compressed and rises to the pressure shown at *f* (compression period). The piston now moves to the right and the cycle is repeated. The same operations occur on the right-hand side of the piston, except that exhaust and compression occur during the time that supply and expansion occur on the left side of the piston. The operations on the two sides of the piston are completely independent of one another.

A distinction is made between (a) *atmospheric engines*, which exhaust to the atmosphere or to a heating system, and (b) *condensing engines*,

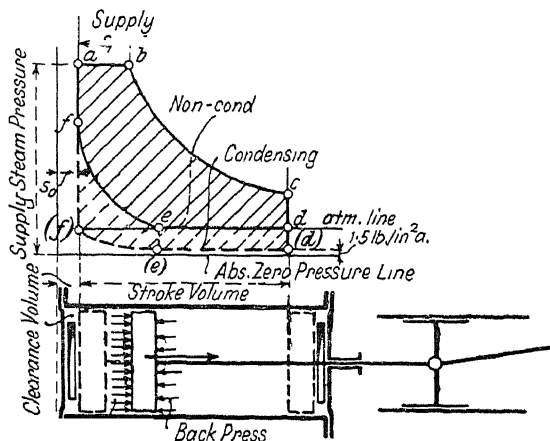


FIG. 259

which exhaust to a low pressure chamber, where the steam is condensed by cooling water. The steam pressure in the condenser cannot be higher than that corresponding to the saturation temperature, as given by the tables, e.g. at 95° F. it is 0.853 lb./in.² abs. The total pressure in the condenser, however, is always greater than the steam pressure, on account of the unavoidable quantity of air present. This air comes in with the steam and also leaks in from the outside. For this reason an air pump is always required in order to maintain the vacuum.

For the same steam supply, the condensing engine delivers more work than the atmospheric engine, as shown by the two diagrams (Fig. 259) *abc(d)(e)(f)* with condenser and *abcdef* without condenser. In the condensing engine, however, the total heat in the exhaust steam is lost, since it is transferred to the cooling water, whereas, in the non-condensing engine, the exhaust total heat, amounting to 80 to 90 per cent of that of the live steam, is still available for heating purposes.

The work done by the steam on one side of the piston is represented by the shaded area. This work is always smaller than that calculated on page 485 for live steam having the same pressure for the following three reasons. Firstly, the steam is in general not expanded to the back pressure, and never in the case of condensing engines. A portion of the

available work is thus lost. The loss may be further increased by compression. Secondly, a pressure loss (throttling) always occurs during supply to the actual engine, and also the discharge pressure in the cylinder is always higher than the back pressure (not shown in Fig. 259, but see Figs. 192, 193, and 194). Thirdly, part of the dry saturated steam entering the cylinder condenses on the colder ports and cylinder walls, so that the available work is again reduced.

The work actually performed by the steam on the piston is given by the area of the indicator diagram taken from the engine, and is thus called the indicated work or power ( $AE_i$  or  $N_i$ ). The ratio of this work to that of the available work, as given on page 485, is called the thermodynamic efficiency  $\eta_\theta$  of the engine. For engines using saturated steam its value lies between 50 and 60 per cent or less, depending on the release pressure, on the amount of clearance, on the efficiency of the valve gear, and on the prevention of steam leakage throughout the system.

### THE STEAM WORK IN THE IDEAL RECIPROCATING ENGINE

In the ideal working process of the steam engine, the clearance volume is assumed to be negligibly small, so that there is no compression period. The form of the diagram then obtained is that shown in Fig. 260 (*abcde*).

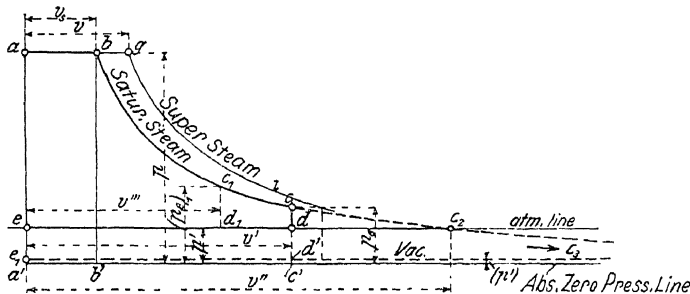


FIG. 260

The actual diagram taken by an indicator can never have the form of the ideal diagram (Fig. 260). The opening and shutting of the inlet and exhaust valves requires a certain period of time, during which these openings gradually increase and then decrease. This causes a rounding of the corners at *b*, *c*, *d* and *e*, as shown by the indicator diagrams of Figs. 267 and 268.\*

In addition, heat exchanges between the steam and cylinder walls have a considerable effect on the work done, and these are not present in the ideal case.

In the following, the amount of mechanical work ideally obtained from unit weight of steam under given pressure and temperature conditions will be determined.

The work performed on one side of the piston during a cycle is represented by the area of the closed pressure volume diagram. This area

\* For further work on the theoretical treatment of the actual flow processes, see *Z.V.d.I.* (1907). W. Schüle, "Zur Dynamik der Dampfströmung in der Kolbendampfmaschine."

(Fig. 260) can have widely different values for the same quantity of steam. If  $ab$  (Fig. 260) is the volume of 1 lb. of steam in the same state as on leaving the boiler, and  $v' = (ed)$  the stroke volume of the cylinder, the work delivered by the steam in the cylinder is  $(abcde)$ .

If the stroke volume is smaller than  $v_1'$ , say  $v''$  the work obtained is only  $abc_1d_1e$ , which is smaller by the amount  $c_1cdd_1$ . The greatest amount of work is obtained when the stroke volume is so large that the release pressure becomes equal to the back pressure. Diagram  $abc_2e$  shows this case for an atmospheric, and  $abc_2c_3e_1$  for a condensing engine.

In an engine having a given stroke volume, and given initial steam pressure, the efficiency of performance is improved as the pressure at the end of expansion decreases, i.e. as the amount of steam supplied per cycle is decreased. A practical limit is finally reached, however (as shown below), when the heat losses to the wall exceed the heat equivalent of the steam work.

By re-drawing the diagrams of Fig. 260 on the basis of a common stroke volume, say  $v'$ , Fig. 261 is obtained. (The release pressures in both figures are about the same.) These diagrams can be regarded as referring to one machine. The larger diagrams, such as  $abcde$ , correspond to full load, and the smaller, such as  $ab_2de$ , to light loads.

Diagrams such as  $ab_3c_3e_1$  are not obtained from single cylinder reciprocating engines because the steam contained in the clearance volume alone has a definite pressure at the end of expansion, and this pressure cannot be decreased. Thus, for a clearance volume of 5 per cent and initial pressure of 180 lb./in.<sup>2</sup> abs., the release pressure is about 8.50 lb./in.<sup>2</sup> abs. (Fig. 261, line  $b_4c_4$ ). A smaller release pressure could, of course, be obtained if the clearance volume were only partially filled, but the initial pressure would then not be the same as in the supply pipe. Thus, if a release pressure of 1.5 lb./in.<sup>2</sup> abs. is required, the clearance volume has to be filled with only sufficient steam to raise the pressure to 30 lb./in.<sup>2</sup> abs. (point  $h$ , Fig. 261). In this case, the inlet valve opens for a very short interval of time. Actual engines with large clearance volumes operate in the same way at light loads, but the high boiler pressure is unnecessary. The points  $h'$  and  $h''$  show the same effect for an engine with 10 per cent clearance and release pressures of 8.5 lb./in.<sup>2</sup> abs. and 15 lb./in.<sup>2</sup> abs. respectively.

Steam turbines are capable of operating with a final pressure considerably below 1.5 lb./in.<sup>2</sup> abs. Hence for turbines the shaded work area  $ab_3tc_3e_1$  is normal. Compared with a reciprocating engine, having a release pressure of 1.5 lb./in.<sup>2</sup> abs. (points  $c_4$  or  $t$ ), the turbine is capable of delivering the extra work shown by the shaded area below  $tc_3$  (Fig. 261). This amount is considerably increased at the lower condenser pressures used in turbine work. Air-free conditions are therefore advantageous for steam turbines.

The work done  $E_0$ , corresponding to the area  $abcde$  of the ideal cycle, is easily determined when the law of expansion is known. Taking the adiabatic exponent as  $\gamma$  gives

$$pv^\gamma = \text{constant}$$

where  $\gamma = 1.135$  for dry saturated steam and  $\gamma = 1.30$  for superheated steam.

The work area  $E_0$  consists of  $abb'a' = pv$  (full pressure work),  $bcc'b' = \frac{1}{\gamma-1} (pv - p_e v')$  (absolute expansion work), and  $-(edc'a') = -p'v'$  (back pressure work).

This gives the work per pound of steam as

$$\begin{aligned} E_0 &= pv + \frac{1}{\gamma-1} (pv - p_e v') - p'v' \\ &= \frac{\gamma}{\gamma-1} pv - \frac{1}{\gamma-1} p_e v \\ &= \frac{\gamma}{\gamma-1} pv \left( 1 - \frac{1}{\gamma} \frac{p_e v'}{pv} - \frac{\gamma-1}{\gamma} \frac{p'v'}{pv} \right) \end{aligned}$$

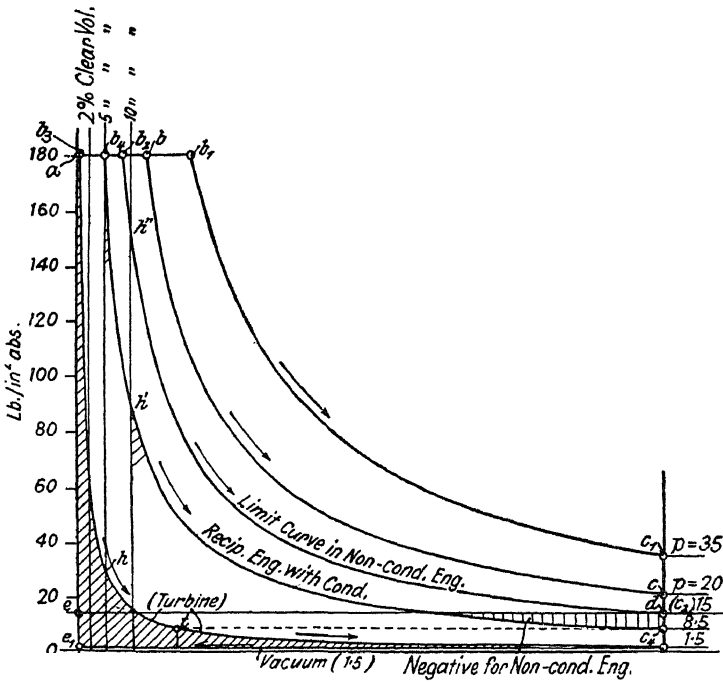


FIG. 261

Now  $\frac{v'}{v}$

so that

$$vp \quad p$$

By writing  $\varepsilon_p = \frac{p}{p_e}$  (pressure expansion ratio)

we have

$$-1 \quad \varepsilon_p^{-\gamma} \quad \frac{\gamma-1}{\gamma} \frac{p'}{p_e} \quad (521)$$





For superheated steam at 650° F. (and 180 to 45 lb./in.<sup>2</sup> abs.) and feed water at 90° F., the value of  $H$ , as given by the Munich specific heats, lies between 1303 and 1313. Hence, for the higher pressures, the mean value (at 650° F.)  $H = 1307$  can be assumed.

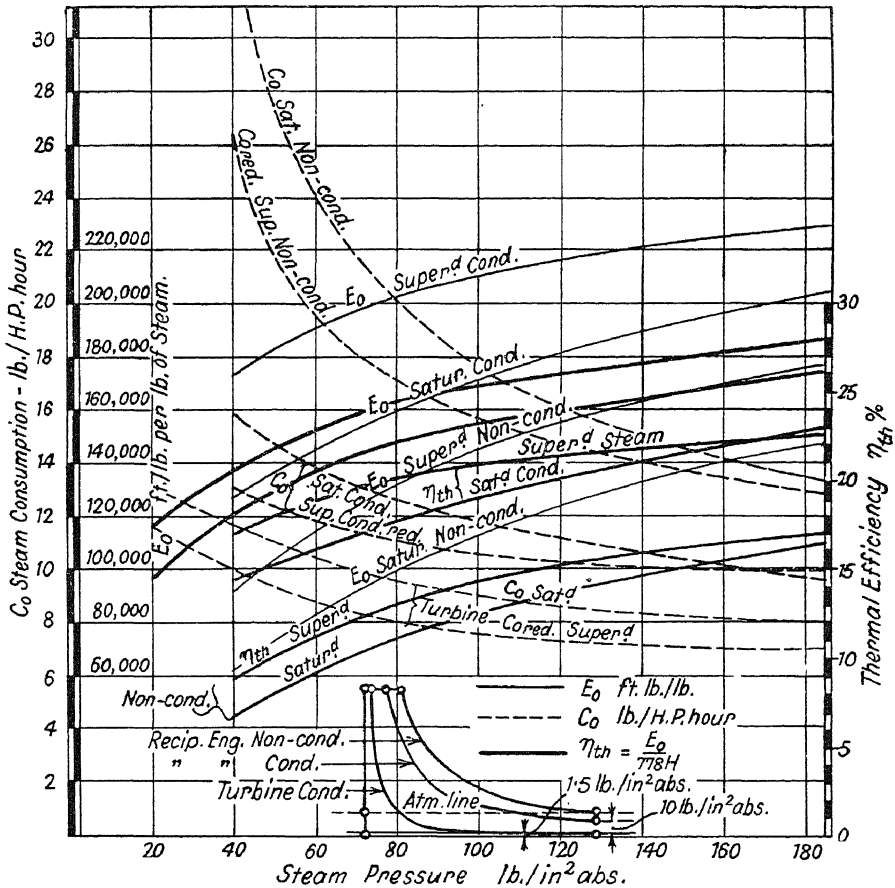


FIG. 262

This gives  $\eta_{th} = 1020000$

or  $\frac{1.95}{C_0}$

**Steam consumption with superheated steam.** A direct comparison of the steam consumptions for superheated and saturated steam does not give a fair measure of performance for the two cases. Account should be taken of the heat used with reference to the fuel, rather than the weight of steam used. If  $H_s$  is the total heat of saturated steam, that of superheated steam, at the same pressure, is  $H_s + c_p t_s$ , where  $t_s$  is the

range of superheat. Hence 1 lb. of superheated steam contains more heat than 1 lb. of saturated steam in the ratio

$$\frac{H_s + c_p t_s}{H_s} = 1 + \frac{c_p t_s}{H_s}$$

The steam consumption of the machine using superheated steam should, therefore, be multiplied by this in order to make it comparable with the consumption of the machine using saturated steam. In the above example this factor is  $\frac{1307}{1143} = 1.142$ . This holds also for the steam consumption of actual engines. Without the introduction of the factor, the steam consumptions of the machines using superheated steam appear unfairly high compared with those using saturated steam.

Since most steam turbines operate with superheated steam at about 570° F., the reduction of the steam consumption to saturated steam conditions at the same pressure is, in general, not necessary. When the steam consumption is given, however, the temperature of the steam should also be given, since the heat value of the superheated steam largely depends on this, while the pressure has only a slight effect on it.

**Explanation of Fig. 262.** This diagram contains the calculated values, according to the above equations (pressures 40 to 180 lb./in.<sup>2</sup>), of—

- (1) the work of 1 lb. steam ( $E_0$  ft. lb.);
- (2) the thermal efficiency ( $\eta_{th}$  per cent);
- (3) the steam consumption per horse-power hour ( $C_0$  lb.)

for atmospheric and condensing engines using saturated and superheated steam (680° F.).

As shown in the included pressure diagram, the assumed release pressure for all cases of atmospheric exhaust is 15 lb./in.<sup>2</sup> abs. and for condensing engines 10 lb./in.<sup>2</sup> abs., as being the smallest value for normal loads in reciprocating engines. Using these values, Fig. 262 gives the ideal limiting values of  $E_0$ ,  $\eta_{th}$ , and  $C_0$ . For superheated steam,  $C_0$  is reduced to the corresponding usual limits of upper and lower steam pressures and ranges of superheat in reciprocators, the remaining curves refer to condensing steam turbines with 1.5 lb./in.<sup>2</sup> abs. vacuum.

Fig. 262 shows first the very considerable advantage of high steam pressures. All the  $E_0$  curves rise with pressure. Since the heat supply  $H$  in the boiler is but slightly increased at higher pressures, the curves of  $\eta_{th}$  also rise with pressure. The fraction of the heat converted to work is increased as the steam pressure is increased.

The effect of condensation is shown by the way in which all the  $E_0$  and  $\eta_{th}$  curves for condensing machines lie above those for non-condensing. With saturated steam the average gain due to condensing for all pressures is about 57,400 ft. lb./lb., and with superheated steam about 65,600 ft. lb./lb. The useful conversion of the heat using saturated steam at 70 lb./in.<sup>2</sup> abs. in a non-condensing engine is 10.3 per cent, and in a condensing engine 17 per cent. Hence the gain is  $\left(\frac{17 - 10.3}{10.3}\right) 100 = 65$  per cent. At 185 lb./in.<sup>2</sup> abs. pressure, the gain is 36.4 per cent.

The use of superheated steam shows a considerable increase in the work ( $E_0$ ) per pound of steam compared with saturated steam, the difference being about 30,000 ft. lb./lb. for high pressures and as much as 40,000 ft. lb./lb. for low pressures. The boilers of plants using superheated steam have, therefore, less steam to evaporate than those using saturated steam for the same power. In non-condensing machines the efficiency for all pressures with superheated steam is higher than with saturated steam, but the difference becomes less as the pressure increases. The same effect is shown in condensing engines at low pressures. Above about 130 lb./in.<sup>2</sup> abs.  $\eta_{th}$  approaches the saturated steam values very quickly until, at about 170 lb./in.<sup>2</sup> abs., it cuts the saturated curve and then apparently runs below it. The values of Fig. 262 are, however, based on equation (521) in which the transition from the superheated to the saturated state is not taken account of. In the case of steam initially at 680° F. and having a final pressure of 10 lb./in.<sup>2</sup> abs., this transition occurs with an initial pressure of 100 lb./in.<sup>2</sup> abs. At higher initial pressures than 100 lb./in.<sup>2</sup> abs. the saturated state is reached before the end of the stroke, and equation (521) gives values of  $E_0$  which are too low, although the difference for pressures near 100 lb./in.<sup>2</sup> abs. is not great. Hence, up to 150 lb./in.<sup>2</sup> abs., the curves for superheated steam in Fig. 262 may be regarded as correct. On the other hand, the intersection of the two curves at about 180 lb./in.<sup>2</sup> abs. does not occur when the transition to the saturated state is taken account of. The two curves, however, lie very close together. Hence, for pressures between 150 and 200 lb./in.<sup>2</sup> abs., the useful conversion of heat to work when using superheated steam between given pressure limits is not greatly different from that obtained with saturated steam. It should be remembered, however, that this applies to the ideal process; actually a definite gain is obtained in using superheated steam on account of the smaller cooling losses. It should be noted that, on account of the corrected comparison of superheated steam with saturated steam, the reduced steam consumptions have been plotted and not the true steam consumptions.

See the following article regarding a more convenient and, in general, more accurate method of determining  $E_0$ ,  $\eta_{th}$ , etc., by means of the entropy charts.

## DETERMINATION OF THE IDEAL USEFUL WORK AND THERMAL EFFICIENCY WITH THE AID OF ENTROPY CHARTS

**Complete expansion.** As shown on page 456, the work represented by the ideal diagram when steam expands down to the back pressure is equal to the difference of the total heats  $H_1$  and  $H_2$  before and after adiabatic expansion. In the case of the reciprocator it is obtained from the  $H\phi$  diagram in exactly the same way as the heat drop is obtained for adiabatic flow, since it is identical to this. Hence, Fig. 241 can also be regarded as the ideal diagram of a reciprocating engine. See examples 4, 5, and 6 on page 460.

Although useful expansion down to the back pressure only occurs in the case of reciprocating engines exhausting to the atmosphere or in the high pressure cylinders of a multiple expansion engine, the ideal diagram with expansion to the back pressure still represents the upper limit of

available work, in spite of the fact that complete expansion to the back pressure in reciprocators is prevented by constructional and thermal reasons. In the case of the steam turbine, however, expansion down to this limit is actually realized, so that the steam leaves the last set of blades at the condenser pressure, and under normal conditions of operation useful work is delivered to these blades.

The comparison of a reciprocating engine with a steam turbine (in so far as the completeness with which both machines make use of steam at given initial and back pressures is concerned) is only possible when based on the diagram with complete expansion.

These conditions hold also when, say, a single cylinder uniflow engine is to be compared with a normal compound engine. The common standard of comparison can only be the ideal diagram with complete expansion.

For these reasons the ideal diagram with expansion down to the back pressure is of considerable importance in the case of reciprocating engines also.

The  $H\phi$  chart gives a useful comprehensive survey of the work value and thermal efficiency of the ideal process between given pressure limits, but with different initial states.

In Fig. 255 an initial pressure of 180 lb./in.<sup>2</sup> abs. and a back pressure of 1.5 lb./in.<sup>2</sup> abs. are assumed. The ordinates of the curve  $p = 185$  lb./in.<sup>2</sup> abs. represent the total heats of steam at this pressure. When below the saturation point, the steam is wet, and when above, superheated. The total heat values increase as the steam becomes drier and then as the range of superheat increases.

The ordinate intercepts between the curves  $p = 185$  and  $p' = 1.5$  lb./in.<sup>2</sup> abs. give the ideal useful heat drops and are equal to the ideal diagram areas with the "toe." These values also increase as the steam becomes drier or more highly superheated. The greatest useful work per pound is obtained at the highest temperature of the superheated steam, and the least work for the steam showing the highest moisture content.

The measure of utilization of the original total heat of the steam (reckoned from 32° F.) is given by the ratio of the ordinate intercepts between the curves  $p = 185$  and  $p = 1.5$  to the total ordinates of the curve  $p = 185$  lb./in.<sup>2</sup> abs.

These ratios, i.e. the thermal efficiencies of the ideal process, have been plotted in Fig. 262. It will be seen that, with increasing dryness and increasing superheat of the steam, the efficiency slowly but continuously increases. Thus, steam at 185 lb./in.<sup>2</sup> abs. and 660° F. gives  $\eta_{th} = 0.28$ , while, with the same pressure 0.95 dry,  $\eta_{th}$  is reduced to 0.26. At 840° F.  $\eta_{th}$  rises to 0.293.

**Incomplete expansion.\*** Entropy diagrams (particularly the  $H\phi$  diagram) are suitable for determining the available work of the steam when used under normal conditions in a reciprocating engine, i.e. with incomplete expansion to the back pressure, provided use is also made of steam tables or the volume portion of the  $TV\phi$  diagram.

Suppose it is required to find the maximum available work delivered by 1 lb. of saturated or superheated steam when the initial pressure is  $p$  lb./in.<sup>2</sup> abs. and initial temperature  $t^\circ$  F., while the back pressure is  $p'$  lb./in.<sup>2</sup> abs. (Fig. 263). The work  $E$  is given by the total shaded area

\* *Z.V.d.I.* (1911), page 1506. W. Schüle. Properties of steam, etc.

$ABCDE$ , and can be divided into two parts  $ABCF$  and  $CDEF$ . The area  $ABCF$  is equal to the heat drop between  $p$  and  $p_e$ , and can therefore be measured on the  $H\phi$  chart as shown in Fig 264, distance  $B'C'$ . The area  $CDEF$  has to be found by calculation. If  $v_e$  is the specific volume of the steam at  $C$ , then

$$CDEF = (p_e - p') v_e$$

The point  $C'$  on the  $H\phi$  diagram, corresponding to  $C$ , usually falls in the wet field, in which case the quality  $q$  is read on the chart. By taking

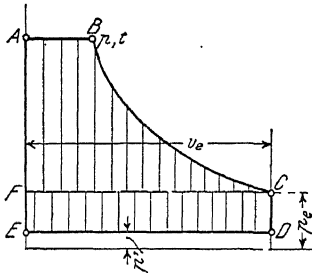


FIG. 263

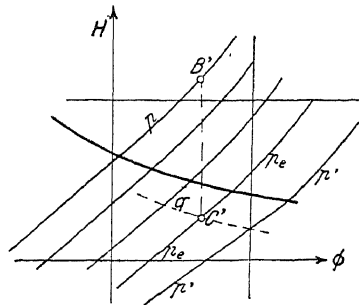


FIG. 264

the volume  $(v_e)_s$  of dry saturated steam at the pressure  $p_e$  from the  $TV\phi$  diagram, or from the tables, we have

For the critical pressure region we have

but this case seldom arises.

If, however,  $C'$  lies in the superheat field, the value of  $v_e$  is found directly from the  $TV\phi$  diagram.

The work in B.Th.U./lb. is now given by

$$AE_0 = (H_1 - H_e) + 144A (p_e - p') v_e$$

With  $A = \frac{1}{778}$ , this gives

$$E_0 = 778(H_1 - H_e) + 144(p_e - p') v_e$$

The thermal efficiency is obtained from this by dividing  $AE_0$  by the heat supplied  $H_1'$  and the mean pressure of the diagram in lb./in.<sup>2</sup> by dividing by  $144 v_e$ ,

$$\text{thus } \eta_{th} = \frac{AE}{H},$$

and

$$\frac{778 (H_1 - H_e)}{144 v_e} +$$

In this way a considerable amount of tedious calculation is saved, and the difficulties and errors involved, due to the transition from the superheated to the saturated state, are avoided. In addition, the method is applicable up to the highest pressures used.

**Example 1.** Steam at 280, 220, 185, 140, 115, and 85 lb./in.<sup>2</sup> abs. is expanded in a reciprocating engine to 10 lb./in.<sup>2</sup> abs. The condenser pressure is 1.5 lb./in.<sup>2</sup> abs. Find the useful adiabatic work  $AE_0$ , the thermal efficiency  $\eta_{th}$ , the steam consumption  $C_0$ , and the mean pressure  $p_m$ —

- (a) For initially dry saturated steam.  
 (b) For initially superheated steam at 650° F.

(a) The  $H\phi$  chart gives for—

$p$	= 280	220	185	140	115	85 lb./in. <sup>2</sup> abs.
$H_1 - H_e$	= 231	219	208	188	174	153 B.Th.U./lb.
$q_e$	= 0.83	0.839	0.848	0.863	0.873	0.889

With  $(v_e)_s = 38.38$  ft.<sup>3</sup>/lb. from the steam tables we have

$v_e$	= 31.8	32.2	32.5	33.1	33.4	34.0 ft. <sup>3</sup> /lb.
-------	--------	------	------	------	------	----------------------------

and with 144  $A(p_e - p') = \frac{144}{778} \times 8.5 = 1.573$

$p'v_e$	= 50.0	50.6	51.1	52.0	52.5	53.5 B.Th.U./lb.
$AE_0$	= 281.0	269.6	259.1	240.0	226.5	206.5 B.Th.U./lb.
$H_1'$	= 1115.6	1113.2	1113.2	1108.6	1106.0	1101.4 B.Th.U./lb.
$\eta_{th}$	= .252	.242	.233	.217	.205	.188
$C_0$	= 9.06	9.55	9.84	10.6	11.25	12.3 lb./h.p. hour
$p_m$	= 49.5	45.2	43.1	39.2	36.7	32.8 lb./in. <sup>2</sup>

(b) $H_1 - H_e$	= 282	268	255	237	223	203 B.Th.U./lb.
	.919	.938	.952	.974	0.99	341.4° F.
	35.2	36.0	36.5	37.4	38.0	39.8 ft. <sup>3</sup> /lb.
	55.3	56.5	57.3	58.8	59.7	62.5 B.Th.U./lb.
$AE_0$	337.3	324.5	312.3	295.8	282.7	265.5 B.Th.U./lb.
$H_1'$	1265	1267	1269	1272	1274	1276 B.Th.U./lb.
$\eta_{th}$	= 0.267	0.256	0.246	0.232	0.222	0.208
$C_0$	= 7.55	7.82	8.14	8.60	8.99	9.58 lb./h.p. hour
$p_m$	= 51.9	48.8	46.4	42.8	40.3	36.0 lb./in. <sup>2</sup>

**Determination of the theoretical steam work, mean pressure, steam consumption, and thermal efficiency of a reciprocating engine with clearance and compression with the aid of the entropy and volume diagrams.** Neglecting losses, the theoretical indicator diagram of an actual engine has the form  $ABCDEF$  shown in Fig. 265. The value of this area is to be found by means of the entropy diagrams in the same way as was done on page 502, where the clearance and compression effects were neglected.

The adiabatic expansion line  $BC$  in Fig. 265 is represented in the  $H\phi$  diagram (Fig. 266) by the vertical  $BC$ . The distance  $BC = h_e$  B.Th.U. on the  $H\phi$  diagram would be equal to the area  $A_0BCC_0$  shown shaded in the pressure diagram if the total weight of steam in the cylinder were 1 lb.

The compression curve  $EF$  in the pressure diagram is represented by the vertical  $EF$  in the  $H\phi$  diagram, in which the point  $E$  (which coincides with  $C'$ ) is found by continuing the line  $BC$  down to the constant pressure curve  $p'$ , where  $p'$  is the back pressure. The state of the residual steam

just at the beginning of compression, i.e. at  $E$  (Fig. 265) is identical to that which the expansion steam would have at the end of expansion if this occurred down to the back pressure (i.e. at  $C'$ ). Immediately after release the steam in the cylinder expands in the same manner as it would if the piston were to continue its motion outwards without exhausting the steam, provided that the influence of the cylinder walls on the residual steam be neglected. Starting with the pressure  $p_c$  at the end of compression, the point  $F$  is found directly on the  $H\phi$  diagram on the curve  $p_c = \text{constant}$ . The distance  $EF = h_c$  B.Th.U. would equal the work area  $F_0FED_0$  of the indicator diagram if the weight of compression steam were 1 lb.

If the live steam supplied to the cylinder per working cycle be  $W$  lb., and if the weight of steam in the cylinder during compression be  $W'$  lb.,

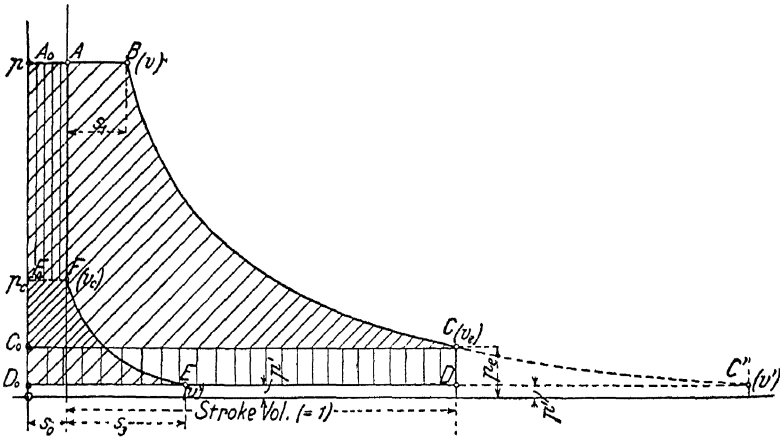


FIG. 265

the weight in the cylinder during expansion is  $(W + W')$  lb. The ratio of  $W'$  to  $W + W'$ , which depends on the fractional clearance volume  $s_0$  and on the compression pressure  $p_c$ , is found as follows—

Since the state of the steam at  $E$  is the same as at  $C'$  (Fig. 266) the weights at these points are proportional to their absolute volumes, thus

$$W + W' = D_0 C'$$

Again, on the compression curve, the absolute volumes are proportional to the specific volumes, i.e.

where  $v'$  is the specific volume corresponding to  $E$  in the  $H\phi$  diagram and  $v_c$  to the point  $F$ . This gives, therefore,





$$AE_0 = \left(\frac{1}{1-\psi}\right) h_e - \left(\frac{\psi}{1-\psi}\right) h_c + \frac{1}{778} \left(\frac{1}{1-\psi}\right) [(p_e - p') v_e - \psi(p - p_c) v_c]$$

or

$$AE_0 = \left[ \left(\frac{1}{1-\psi}\right) (h_e - \psi h_c) + \frac{144}{778} \left(\frac{1}{1-\psi}\right) [(p_e - p') v_e - \psi(p - p_c) v_c] \right] \quad (530)$$

with the pressures in lb./in.<sup>2</sup> abs.

The mean effective pressure  $p_m$  is obtained from this expression by dividing it by the steam volume corresponding to the stroke volume in Fig. 265. The total volume  $C_0C$  is  $\left(\frac{1}{1-\psi}\right) v_e$  and the stroke volume

$$\frac{1}{1+s_0} \times \frac{1}{1-\psi} v_e$$

$$\text{This gives } p_m = (1+s_0)(1-\psi) \frac{AE_0}{Av_e}$$

$$\begin{aligned} \text{or} \quad p_m = (1+s_0) & \left[ \frac{778(h_e - \psi h_c)}{144v_e} \right. \\ & \left. + p_e - p' - \psi(p - p_c) \frac{v_c}{v_e} \right] \text{ lb./in.}^2 \quad (531) \end{aligned}$$

The fractional compression  $s_3$  follows from

$$\frac{s_3 + s_0}{s_0} = \frac{v'}{v_c} \text{ giving } s_3 = \left(\frac{v'}{v_c} - 1\right) s_0 \quad (532)$$

and the supply period from

$$\frac{s_1 + s_0}{1 + s_0} = \frac{v}{v_e} \text{ giving } s_1 = \frac{v}{v_e} (1 + s_0) - s_0 \quad (533)$$

The steam consumption per kW. hour is

$$C_0 = \frac{33000 \times 60}{E_0} \text{ or } \frac{33000 \times 60}{778} \times \frac{1}{AE_0} = \frac{2545}{AE_0}$$

The thermal efficiency is

$$\eta_{th} = \frac{AE_0}{H'_1} \quad (534)$$

where  $H'_1$  is the heat supplied per pound.

**Special cases.** 1. Engine without compression.

In this case  $s_3 = 0$ , hence equation (532) gives  $v_c = v'$ , and, with equation (527)  $\psi = \frac{v_e}{v'} \frac{s_0}{1+s_0}$

Again,  $p_c = p'$  and  $h_c = 0$ , so that equation (530) gives

$$AE_0 = \frac{h_e}{1-\psi} + \frac{144}{778} \frac{v_e}{1-\psi} \left[ (p_e - p') - \frac{s_0}{1+s_0} (p - p') \right] \quad (535)$$

and, from equation (531),

$$p_m = 1 + s_0 \left[ \frac{778 h_e}{144 v_e} + p_e - p' - \frac{s_0}{1 + s_0} (p - p') \right] \quad (536)$$

With  $s_0 = 0$   $\psi = 0$ , the relations shown on page 503, without clearance, are obtained.

2. Engine with a compression pressure equal to that of the entering steam.  $p_c = p$ . In this case the second terms within the square brackets in equations (530) and (531) disappear. In addition, since  $v_c = v$  (initial state)

$$\psi = \frac{v_e}{v} \frac{s_0}{1 + s_0}$$

and  $h_e = h'$ , where  $h'$  = total heat drop (distance  $BC'$ , Fig. 266), equations (530) and (531) give

$$AE_0 = \frac{1}{1 - \psi} (h_e - \psi h') + \frac{144}{778} \frac{(p_e - p') v_e}{1 - \psi} \quad (538)$$

$$p_m = 1 + s_0 \left[ \frac{778 (h_e - \psi h')}{144 v_e} + p_e - p' \right] \quad (539)$$

If expansion takes place down to the back pressure, then, with  $p_e = p'$ ,  $v_e = v'$ ,  $h_e = h'$ , we have

$$AE_0 = h' \quad (540)$$

and

$$\begin{aligned} p_m &= \frac{778 h'}{144 v'} (1 + s_0) (1 - \psi) \\ &= \frac{778 h'}{144 v'} \left[ 1 + s_0 \left( 1 - \frac{v'}{v} \right) \right] \quad (541) \end{aligned}$$

Equation (540) shows that, with simultaneous complete expansion and compression, the clearance volume has no effect on the work obtained, however large the clearance volume may be. If, on the other hand, compression only is complete, the clearance volume does have an effect on the work done and steam consumption, as shown by equation (538), in which  $\psi$  still appears.

**Example 2.** Superheated steam at 185 lb./in.<sup>2</sup> abs. and 570° F. is to be supplied to a reciprocating engine having a release pressure of 10, lb./in.<sup>2</sup> abs. and a back pressure of 1.5 lb./in.<sup>2</sup> abs. Find the useful works, the thermal efficiencies, the steam consumptions, and the mean effective pressures when the compression pressure is 15, 30, 90, 150, and 185 lb./in.<sup>2</sup> abs., and

(a) the clearance volume is 8 per cent of the stroke volume ( $s_0 = 0.08$ )

(b) the clearance volume is 2 per cent of the stroke volume ( $s_0 = 0.02$ )

From the  $H\phi$  diagram,  $q_e = 0.927$ ,  $q' = 0.853$ , and, from the steam tables  $(v_e)_s = 38.38$ ,  $(v')_s = 228$ , hence  $v_e = 0.927 \times 38.38 = 35.5$ ,

$v' = 0.853 \times 228 = 194.5$ . Also from the  $H\phi$  diagram,  $h_e = 240$  B.Th.U. and  $H'_1 = 1,226.6$  B.Th.U./lb. Tabulating—

$p_c$	= 15	30	90	150	185
	= 0.946	.982	$t_s = 95^\circ \text{F.}$	$150^\circ \text{F.}$	$194.6^\circ \text{F.}$
	= 24.8	13.5	5.60	3.70	3.205
$\bar{v}$	= 6.84	13.40	33.8	51.5	59.6
	= 0.547	> 1	—	—	— $s_0 = 0.08$
	= 0.137	.268	.676	1.00	> 1 $s_a = 0.02$

This shows that compression is only possible to about 20 lb./in.<sup>2</sup> abs. when  $s_0 = 0.08$ , and to about 150 lb./in.<sup>2</sup> abs. when  $s_0 = 0.02$ .

(a)  $s_0 = 0.08$ .

$$\psi = \frac{35.5 \times 0.08}{24.8 \times 1.08} = 0.106$$

$$1 = 1.16 \quad h_c = 141 \text{ B.Th.U.} \quad \psi h_c = 15 \text{ B.Th.U.}$$

$$\begin{aligned} v_0 &= 1.16 (240 - 15) + \frac{144}{778} \\ &\times 1.16 [8.5 \times 35.5 - 170 \times .106 \times 24.8] \\ &= 261 - 31.2 = 229.8 \text{ B.Th.U./lb.} \\ &= \frac{2545}{229.8} = 11.1 \text{ lb./h.p. hour} \end{aligned}$$

$$\eta_{th} = \frac{229.8}{1226.6} = 0.187 \quad p_m = 37.1 \text{ lb./in.}^2$$

(b)  $s = 0.02$ .

$p_c$	= 15	30	90	150 lb./in. <sup>2</sup> abs.
$\psi$	= .0281	.0509	0.123	0.186
$h_c$	= 141	190	282	331 B.Th.U./lb.
$\psi h_c$	= 4.0	9.2	34.6	61.5 B.Th.U./lb.
$AE_0$	= 275.1	281	284.1	282.7 B.Th.U./lb.
$C_0$	= 9.25	9.06	8.98	9.02 lb./h.p. hour
$\eta_{th}$	= 0.225	0.229	0.232	0.231
$p_m$	= 41.4	41.4	38.6	35.7 lb./in. <sup>2</sup>

The table shows that the steam consumptions and thermal efficiencies are not greatly affected by the compression pressure when this is greater than 15 lb./in.<sup>2</sup> abs. Without compression,  $AE_0 = 257$  B.Th.U./lb.,  $C_0 = 9.9$ , and  $\eta_{th} = 0.21$ .

On the other hand, the important influence of clearance is seen by comparing the values with those under (a). The steam consumption of 9.25 lb. with 2 per cent clearance, rises to 11.1 lb. when the clearance is raised to 8 per cent, i.e. an increase of 1.85 lb. Without compression, the consumption would be 8.14 lb., as shown on page 504. Hence, due to a

clearance of 2 per cent, the steam consumption is increased  $\frac{9.25}{8.14} =$  times, or by 14 per cent, and, due to a clearance of 8 per cent,  $\frac{11.1}{8.14} = 1.38$  times, e.g. by 38 per cent.

**Example 3.** Superheated steam at 150 lb./in.<sup>2</sup> abs. and 570° F. is supplied to a condensing engine having a release pressure of 15 lb./in.<sup>2</sup> abs. and a back pressure of 3 lb./in.<sup>2</sup> abs. Find the values of  $AE_0$ ,  $C_0$ ,  $\eta_{th}$ , and  $p_m$  for compression pressures of 30, 75, and 120 lb./in.<sup>2</sup> abs. when the fractional clearance is (a) 10 per cent, (b) 2 per cent.

From the  $H\phi$  diagram,  $q_e = .953$ , hence  $v_e = 0.953 \times 26.2 = 25.0$  ft.<sup>3</sup>/lb., and  $q' = .893$ , hence  $v' = 0.893 \times 108.5 = 86.9$  ft.<sup>3</sup>/lb.,  $h_e = 198$  B.Th.U./lb.,  $H'_1 = 1,204$  B.Th.U./lb.

Also it can be shown that, with a fractional clearance volume of 10 per cent, compression pressures of 75 and 120 lb./in.<sup>2</sup> abs. cannot be attained ( $s_3 > 1$ ).

(a)  $s_0 = 0.10$ . With no compression ( $p_c = 3$ ),

$$\psi = \frac{25}{86.9} \times \frac{0.1}{1.1} = 0.0262$$

$$\frac{1}{1 - \psi} = 1.03$$

hence, from equation (536),

$$\begin{aligned} AE_0 &= 1.03 \times 198 + \frac{144}{777.8} \times 1.03 \times 25 \left[ 12 - \frac{0.1}{1.1} \times 147 \right] \\ &= 204 - 6.5 = 197.5 \text{ B.Th.U./lb.} \end{aligned}$$

hence  $C_0 = 12.9$  lb.,  $\eta_{th} = 0.164$

$$p_m = \frac{1.1 \times .974 \times 197.5 \times 777.8}{25 \times 144} = 45.8$$

For a compression pressure of 30 lb./in.<sup>2</sup> abs.  $\psi = 0.182$ ,  $h_c = 154$ ,  $AE_0 = 208$ ,  $C_0 = 12.2$ ,  $\eta_{th} = .173$ . Hence the reduction in steam consumption is  $12.9 - 12.2 = 0.7$  lb.  $s_3 = 0.533$ .

(b)  $s_0 = 0.02$ . Carrying out the calculation in the same way as in example 2 gives—

$$\begin{array}{llllllll} p_c = 30 \text{ lb./in.}^2 \text{ abs.} & AE_0 = 248 & C_0 = 10.28 & \eta_{th} = .21 & p_m = 52.8 & s_3 = .121 \\ p_c = 75 \text{ lb./in.}^2 \text{ abs.} & AE_0 = 248.6 & C_0 = 10.25 & \eta_{th} = .21 & p_m = 50.8 & s_3 = .248 \\ p_c = 120 \text{ lb./in.}^2 \text{ abs.} & AE_0 = 249.0 & C_0 = 10.23 & \eta_{th} = .21 & p_m = 49.2 & s_3 = .372 \end{array}$$

Here, again, the same general characteristics are shown as in example 2, namely, no appreciable improvement with compression pressures above 30 lb./in.<sup>2</sup> abs., but considerable changes accompany a change in the clearance volume. For the same compression pressure the engine, with 2 per cent fractional clearance, shows a steam consumption of 10.28 lb., compared with 12.2 lb. when the clearance is 10 per cent, i.e. 1.92 lb. less.

**Comparison between the actual and ideal indicator diagrams of a steam engine.** The greatest amount of work which 1 lb. of steam can

perform when expanded adiabatically from a given initial state to a given release pressure  $p_e$  when the back pressure is  $p'$  can be calculated as shown on page 495. It is represented by the diagram in Fig. 263.

In actual engines, whether with or without compression of the clearance steam, this maximum work is never realized. The main reasons for this have already been discussed. When saturated or slightly superheated steam is used, part of the steam condenses on entering the cylinder and this part thus delivers no work (or only a slight amount due to re-evaporation). With superheated steam the volume of the steam is reduced, due to the cooling effect of the walls, so that the capacity for work is again reduced. To this has to be added the disadvantageous effect of clearance; but the losses caused by clearance can be partly reduced by compression. The compression also reduces the losses caused

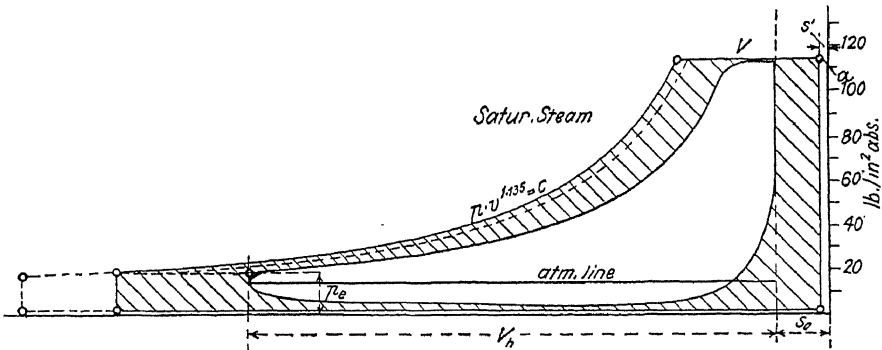


FIG. 267

by the cooling action of the clearance wall and end surfaces, since the mean temperature during a cycle, and consequently the mean wall temperature, is thereby raised. Finally, losses are caused by the pressure drop due to the flow of steam through the restricted valve areas. The initial pressure in the cylinder is less than that in the supply pipe (or in the boiler), while the steam pressure in the cylinder during exhaust is higher than the back pressure. The pressure losses produce a considerable difference between the actual and ideal diagrams (see Figs. 267 and 268). By drawing the ideal diagram over the actual diagram, as given by the boiler pressure  $p$ , the release pressure  $p_e$  and the back pressure  $p'$ , the loss is shown by the difference between the two areas.

Let  $E_0$  be the ideal work and  $E_i$  be the indicated work, the fraction

$$\eta_o = \frac{E_i}{E_0}$$

is a measure of the conversion to work as compared with the ideal conversion.

The ideal diagram corresponding to an actual indicator diagram can only be drawn when the weight of steam used per cycle in the cylinder is known. This weight cannot be determined from the diagram itself, since, with saturated steam, the quality, and, with superheated steam, the temperature at cut-off are not given by the diagram. The weight can only be determined by measuring the condensate formed in the surface

condenser or the feed supplied to the boiler in a given time. In the latter case any condensate formed in the supply pipe has to be deducted.

The following are then represented in Fig. 267.

The shaded area to the right of the perpendicular through the right dead centre position gives the work lost on account of the clearance space.

The shaded area between the actual and ideal expansion line gives the loss due to cooling and the shaded area below the exhaust line gives the loss due to the pressure in the cylinder not being equal to that in the condenser.

The area below the compression curve also represents a certain loss, but cannot be described as a compression loss, since the work of compression is regained during expansion. It is not possible to separate out the various losses definitely. The diagram (Fig. 267) only gives a general survey of these.

The Rankine-Clausius cycle, in which expansion occurs down to the back pressure, can also be taken to represent the ideal diagram. This is necessary when comparisons have to be drawn between turbines and reciprocating engines. In this case the ratio of indicated to ideal work is less, since the loss due to incomplete expansion is then taken account of. See page 484 regarding this.

On page 485 the value  $\eta_{ih}$  gave the fraction of the heat supplied  $H'_1$ , which is converted in the ideal case. Actually, however, the work delivered is that shown by the indicator, the area of which is  $\eta_g$  times that of the theoretical diagram. Hence the fraction of the total heat appearing as indicated work in heat units, or the indicated thermal efficiency is

$$\eta_i = \eta_g \eta_{ih}$$

(indicated thermal efficiency).

As shown on page 487, this value is found from the indicated steam consumption  $C_i$  lb./h.p. hour, since

$$2545$$

A certain fraction of the indicated work is absorbed in internal friction in the engine itself (friction of the piston, bearings, valve gear, stuffing boxes, air pump, etc.). The remaining work, amounting to the fraction  $\eta_m$  of the indicated work, is available as effective work at the engine shaft. The overall thermal efficiency of the engine, based on the heat in the supply steam, is then

(Effective thermal efficiency of the engine.)

Of the heat liberated by combustion in the furnace, the fraction  $\eta_b$  is transmitted in forming the steam, and a further loss occurs in the pipe connecting the boiler and engine, so that the fraction  $\eta_s$  of the heat in the boiler steam is available at entrance to the engine.

Hence the steam engine converts to useful work the fraction

(Overall thermal efficiency of the plant.)

As shown on page 153, this value can be determined from the fuel consumed per horse-power hour.

By combining the maximum values of the efficiencies given on page 489, we have  $\eta_s = 0.25$ .

Up to the year 1921 about the highest values attained were those found by test by Josse, on a steam locomobile engine of 181 h.p. using superheated steam, and by Gutermuth on one of the same type of 105 h.p. normal load. The first (I) operated with slide valve, while the second (II) was fitted with a piston valve. The following table contains the most important data and efficiencies in these tests, along with Heilmann's\* latest results from tests on a Wolf steam locomobile using superheated steam.

Engine	Boiler Pressure lb./in. <sup>2</sup> A.	Steam Temp. before High Pressure Cylr. ° F.	Rev./min.	Weight of Coal of 13,500 B.Th.U./lb. in lb./h.p. hour	Steam lb./h.p. hour	$\eta_s$	$\eta_{b\eta_e}$	$\eta_m$	$\eta_i$
I (Lauz)	185	668	173.5	1.22	10.26	0.154	0.80	0.928	0.208
II (Wolf)	230	624	237	1.09	8.93	0.173	0.78	0.946	0.234
III	215	646	224	0.96	8.41	0.196	—	0.938	0.223
†IV (Wolf)	289	766	190.5	—	7.10	—	—	0.905	—

In the entropy diagram the actual cycle also deviates much from the ideal cycle. Its development on this diagram is involved and takes up considerable time, and cannot be discussed in this volume. (Boulvin's method, See *Z.V.d.I.* (1903), page 1281, M. Schröter and A. Koob.) It adds nothing to the general conclusions already formed, but is useful in depicting the various heat quantities supplied and withdrawn, and is more suitable for a scientific analysis than for practical testing.

**Examples.** Figs. 267 and 268 show the indicator diagrams taken from a single cylinder condensing steam engine. The measurements of the engine, which is fitted with trip valves, are—Cylinder diameter 12.2 in., stroke 16.1 in. Rev./min. for Fig. 267,  $n = 122.2$ , and, for Fig. 268,  $n = 111.7$ . Fractional clearance volume (measured by filling with water) 10 per cent.

(a) Fig. 267. Supply steam dry saturated (with steam jacket). Total steam consumption per hour, excluding condensate in the supply pipe,  $S_i = 1,343$  lb. at 115 lb./in.<sup>2</sup> abs. Load 55.6 h.p.  $C_i = 24.1$  lb. (Dotted expansion line without jacket condensate.)

From Fig. 267, we have  $\eta_g = 0.547$ ., also  $\eta_i = \frac{2545}{24.1 \times 1142} = .093$ , or 9.3 per cent.

The simplest way of determining  $\eta_g$  is from

when  $C_0$  is found from the  $H\phi$  chart, as explained on page 498.

\* Compound steam locomobile with a uniflow low pressure cylinder  $\frac{300 \times 600}{500}$ .

See *Z.V.d.I.* (1930), page 65. Heilmann. Die neuere Entwicklung der Heisedampf-lokomobile.

† Locomobile with triple expansion and re-superheater. Same reference.

‡ Heat consumption 11,400 B.Th.U./h.p. hour.

With  $p = 115 \text{ lb./in.}^2 \text{ abs.}$ ,  $p_e = 19.6 \text{ lb./in.}^2 \text{ abs.}$ , and  $p' = 1.5 \text{ lb./in.}$  abs. (from the diagram), the  $H\phi$  chart gives  $H_1 - H_e = 133 = h_e$  B.Th.U./lb., and  $q_e = 0.9$ , so that  $v_e = q_e(v_e)_s = 0.9 \times 20.1 = 18.1 \text{ ft.}^3/\text{lb.}$

$$\text{Hence} \quad = 133 \quad (19.6 - 1.5) 18.1$$

$$\begin{aligned} &= 133 + 60.5 \\ &= 193.5 \text{ B.Th.U./lb.} \end{aligned}$$

$$\text{so that} \quad C_0 = 2545/193.5 = 12.9$$

$$\text{and} \quad \frac{12.9}{24.1} = 0.536$$

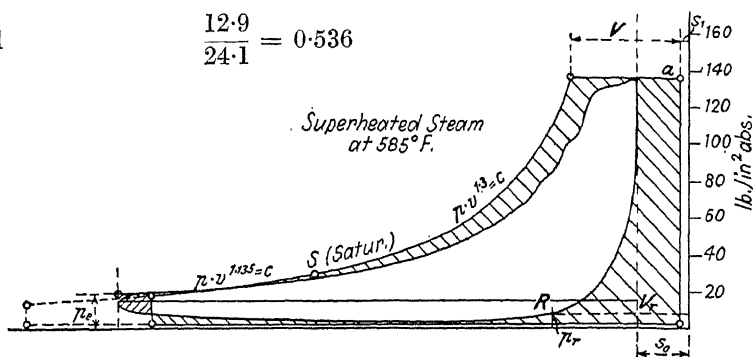


FIG. 268

(b) Fig. 268. Steam supply superheated.  $S_i = 865 \text{ lb.}$  Steam pressure  $135 \text{ lb./in.}^2 \text{ abs.}$  Steam temperature above the valves  $585^\circ \text{ F.}$  Load  $52.4 \text{ h.p.}$   $C_i = 16.54 \text{ lb.}$

Fig. 268 gives

$$= 0.63$$

$$\eta_i = \frac{2545}{16.54 \times 1225} = 0.125$$

From the  $H\phi$  chart, with  $p = 135$ ,  $t_s = 235^\circ \text{ F.}$ , and  $p_e = 20.2 \text{ lb./in.}^2 \text{ abs.}$ , we have  $h_e = 170$  and  $q_e = 0.995$ ,  $v_e = 20.1 \times 0.995 = 20 \text{ ft.}^3/\text{lb.}$  Hence

$$AE_0 = 170 + \frac{144}{777.8} (18.7) 20 = 239.2 \text{ B.Th.U./lb.}$$

$$C_0 = \frac{2545}{239.2} = 10.67$$

$$\frac{10.67}{16.54} = 0.64$$

In both cases the large clearance volume accounts for a considerable fraction of the losses (small  $\eta_g$  value).

The smaller area between the expansion lines in Fig. 268, compared with that in Fig. 267, shows clearly the advantageous influence of superheated steam on the cooling losses.



**Compound expansion.** In compound expansion the steam operates in two cylinders. In the first cylinder, called the high pressure cylinder (I), the steam expands from  $p_1$  to  $p'$ , and, provided no heat or work losses occur, the work delivered is  $E_I$  (Fig. 269). The steam passes from the high pressure cylinder into a receiver, in which the pressure is  $p'$ . From there, steam is supplied to the larger low pressure cylinder (II), in which it expands to the release pressure  $p_3$ , which, in condensing engines, ranges from 6 to 11 lb./in.<sup>2</sup> abs. The exhaust steam from this cylinder flows into the condenser at the pressure  $p_2$ . In the low pressure cylinder the ideal work is  $E_{II}$ . The sum  $E_I + E_{II}$  is the same as the work  $E_0$  calculated on page 485. Hence, in this respect, no gain is apparent when the steam is expanded in two or three cylinders instead of in a single cylinder. The advantage of compounding, however, lies in the fact that the heat losses due to condensation are much smaller, because the port and cylinder surfaces with which the hot steam comes in contact in the high pressure cylinder are much smaller than in the single cylinder engine, which, for the same power, must be equal in size to the low pressure cylinder. In addition, the temperature differences between the entering and leaving steam are much smaller in the two cylinders, so that less condensation occurs. Hence, compounding results in a higher thermodynamic efficiency with a resulting lower steam consumption. The utilization of high pressures in reciprocating engines has only been rendered possible by means of compounding.

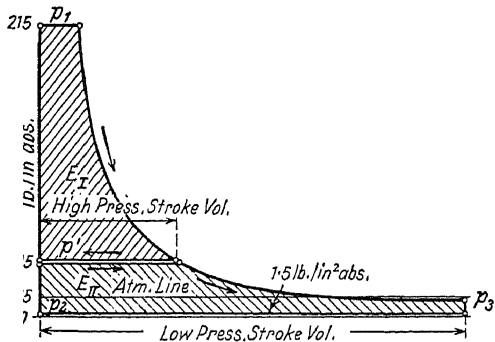


FIG. 269

**Engines using superheated steam.** Before the year 1895 steam engines operated with saturated steam only, at about the same state as it left the boiler, but almost all modern engines, including locomotives, now use superheated steam. The saturated steam coming from the steam space of the boiler is led through a superheater, which is fitted so that the hot furnace gases flow over it (Fig. 187). In flowing through the pipe system which forms the superheater, the saturated steam takes up heat at constant pressure and is delivered to the engine as superheated steam. Superheat temperatures up to 700° F. have been used.

In the cylinders of steam engines and in steam turbines, superheated steam operates in the same way as saturated steam, but the materials used in constructing the cylinders, valves, and stuffing boxes have to be capable of withstanding the higher temperatures, while a different type of lubricating oil must be used in the cylinders.

Since the capacity for work of 1 lb. of superheated steam between given pressures is greater than that of 1 lb. of saturated steam, more work is obtained for the same steam used (i.e. for the same weight of water evaporated in the boiler). It must be remembered, however, that more heat is required to generate 1 lb. of superheated steam than 1 lb. of

saturated steam, so that it is necessary first to find whether a greater fraction of the heat can be converted to work when superheated steam is used and all losses are avoided. Fig. 255 does show that, as the temperature of the steam increases, the thermal efficiency also increases, e.g. for dry saturated steam at 185 lb./in.<sup>2</sup> abs.,  $\eta_{th} = 0.263$ , while at 660°  $\eta_{th} = 0.280$ . This increase, however, is not, in itself, sufficient to justify the use of superheated steam. It is found, however, that with superheated steam the heat losses to the cylinder walls, in spite of the higher temperatures, is relatively smaller than with saturated steam. Condensation only commences after the whole of the superheat is withdrawn, i.e. after the saturation temperature is reached, since, at saturation, the slightest withdrawal of heat effects a consequent condensation. Due to this property of superheated steam, the thermal efficiency of the engine is raised above that of an engine using saturated steam. The two effects thus raise the thermal efficiency appreciably. In the case of the steam turbine the advantage of using superheated steam consists not only in the higher thermal efficiencies obtained, but also because no drops of water are contained in the steam. These drops have a harmful effect on the blades and increase the flow losses. By the time the steam reaches the low pressure cylinder it is generally wet, on account of expansion.

**Variation in steam consumption with superheat temperatures.** Under actual working conditions it frequently happens that a predetermined superheat temperature is not attained on account of various causes. Hence it is important to determine by how much the steam consumption is increased above that with normal superheat. From the results of numerous tests it is found that, for the same engine, the steam consumption per 1° F. rise decreases by about the same amount up to 650° F. (it becomes somewhat greater near the saturation region). In different engines, however, and with different methods of operating the same engine (atmospheric exhaust, condensing, change in load, and jacketing), the steam saved per 1° F. rise varies considerably.

Taking extreme cases, it varies from about 0.013 lb. in efficient compound condensing engines to about 0.055 lb. in single cylinder engines of inefficient design. Hence the gain due to superheat is more marked in the case of engines of low efficiency. For engines of average efficiency, the saving may be taken as 0.03 lb. per 1° F. rise and per horse-power hour.

It should, however, be observed that the saving depends on so many factors that definite conclusions can only be arrived at regarding engines of a known type.\*

A lower limit for the weight of steam saved can be found from the theoretical steam consumptions. As shown on page 504, for example, the steam consumption when the release pressure is 10 lb./in.<sup>2</sup> abs. and the back pressure 1.5 lb./in.<sup>2</sup> abs. is, with

$p_1 =$	280	220	185	140	115	85 lb./in. <sup>2</sup> abs.
$c_i =$	9.06	9.55	9.84	10.60	11.25	12.30 lb. saturated steam
	7.55	7.82	8.14	8.60	8.99	9.58 lb. superheated steam
	1.51	1.73	1.70	2.00	2.26	2.72 lb. saving with superht.

\* See *Zeitschr. Bayer., Rev. Ver.* (1912), Nos. 13–16. *Die Darlegungen von Deinlein*, and also *Zeitschr. Dampfkessel-u. Masch. Betr.* (1914), Nos. 44–46, by V. Kammerer.

Since the ranges of superheat amount to

239.3	260.2	274.6	297.1	312.0	333.6
-------	-------	-------	-------	-------	-------

the saving per 1° F. range of superheat is

0.063	0.066	0.062	0.067	0.072	0.082 lb.
-------	-------	-------	-------	-------	-----------

**Uniflow engines.** In single cylinder and compound engines of normal design the inlet and exhaust passages lie in the wall of the cylinder. Where slide valves are used, the same passages are used for both the supply and exhaust steam. In the latter case, particularly, the port surfaces are considerably cooled by the exhaust steam, so that when the live steam enters on the next cycle it, in turn, is considerably cooled by these surfaces. This also occurs at the cylinder ends and at the walls near the port entrance to the cylinder. In the case of the uniflow engine, however, this is avoided by allowing the steam to exhaust through ports in the centre of the cylinder, which are opened by the movement of the piston when it comes near the end of its stroke (Fig. 270), while the fresh steam enters at the opposite ends. In this way the steam enters at one end of the cylinder and leaves at the centre without changing its direction. Experience and tests have shown that these engines show considerably smaller heat losses than those accompanying single cylinder engines, and are about equal in performance to compound engines.

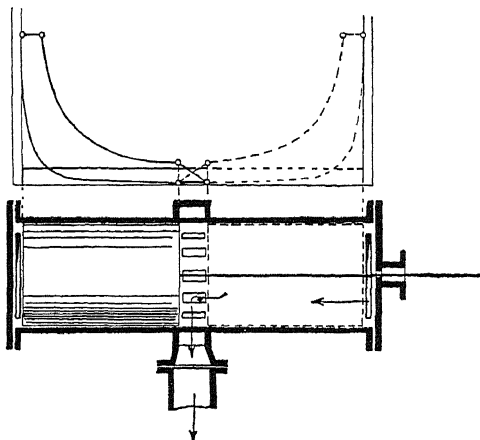


FIG. 270

In the case of condensing uniflow engines, where the pressure during exhaust is about equal to the condenser pressure (1.5 lb./in.<sup>2</sup> abs.), it is necessary, in order to attain the compression pressure required for smooth running, to have a very small clearance space (1.5 to 2 per cent), even with the long period of compression. This small clearance serves, in addition, to reduce the steam consumption. On the other hand, if used as an engine exhausting to atmosphere, this small clearance would produce a dangerously high compression pressure, so that a variable clearance space or an auxiliary exhaust valve would be required.

**Uniflow engine with short stroke.** The placing of the exhaust ports away from the inlet valves, which accounts for the high thermal efficiency in the uniflow engine just described, can also be effected in the manner shown in Fig. 271. In the lower part of this figure the exhaust ports are placed about the middle portion of the stroke. Exhaust must, however, commence near the end of the stroke, so that the opening of these ports is not controlled by the piston, but by a special valve. The shutting of the exhaust ports on the left of the piston occurs when, on the return stroke, the left-hand edge of the piston arrives at the left-hand port

edge, but earlier closing can be effected if necessary by the valve. In this way the compression path is about half as long as in the arrangement shown in Fig. 270.

The upper cylinder with two sets of ports, in Fig. 271, shows how the compression path can be still further reduced.

In both these types of uniflow engines, the steam operates in the same way as in a single cylinder engine. Due, however, to the separated inlet and exhaust passages, the cooling losses are reduced and the thermodynamic efficiency increased, so that the steam consumption for a given power is reduced.

The arrangements shown in Figs. 270 and 271, which are generally applied to single cylinders, have lately been applied to the low pressure cylinder of a compound engine in a steam locomobile (see footnote on page 513).

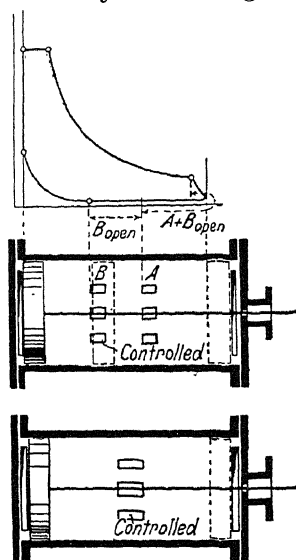


FIG. 271

### PROCESSES INVOLVING THE EXCHANGE OF HEAT BETWEEN THE ENGINE STEAM AND THE STEAM GENERATOR

#### Feed heating by means of exhaust steam.

The oldest and simplest process in which a heat exchange occurs between the working steam and the feed water of the boiler is the feed preheating by means of exhaust steam of the engine or turbine.

In engines which exhaust to the atmosphere, the weight of feed supplied to the boiler must be the same as the weight of steam required by the engine, and the same holds in condensing engines operating with jet condensers. In condensing turbines, which use surface condensers on account of the better vacuum obtained, the condensed steam can, of course, be used directly as feed to the boiler, and the temperature of this feed will be approximately that corresponding to the condenser pressure. If, for example, this pressure is 0.75 lb./in.<sup>2</sup> abs., the condensate temperature is 92° F. Hence the heat required to raise the feed water from the outside temperature of between 50 and 60° F. to 92° F., and amounting to between 32 and 42 B.Th.U./lb., is saved. This gain, however, compared with the total heat of between 1050 and 1250 B.Th.U. of the live steam, is negligible.

The gain is considerably greater in engines operating at atmospheric exhaust, or in back pressure machines, since the temperature of the exhaust steam is then much higher (212 to 288° F.), and the feed water can be raised almost to this temperature, i.e. with atmospheric exhaust to about 195° F., and with a back pressure of 45 lb./in.<sup>2</sup> abs. to about 260° F. The heat necessary for this is between 145 and 200 B.Th.U., and compared with the total heat of live steam, say, at 170 lb./in.<sup>2</sup> abs. and reckoned from 50° F., which amounts to 1177 B.Th.U./lb., the percentage saving is between  $\frac{100 \times 145}{1177} = 12.3$  per cent and  $\frac{100 \times 200}{1177} = 17$  per cent.

This saving is not shown on the pressure-volume diagram but is represented on the  $T\phi$  diagram (Fig. 272). It is given by the difference in sensible heats between  $50^\circ\text{F.}$  and  $195$  or  $260^\circ\text{F.}$  The heat supply without preheating the feed is given by the area  $abc'm'm$ , while with preheating it is  $abc'n'n$ . This raises the thermal efficiency of the process, since the heat supply is now  $H_1 - 160$  or  $H_1 - 228$  instead of  $H_1 - 18$ .

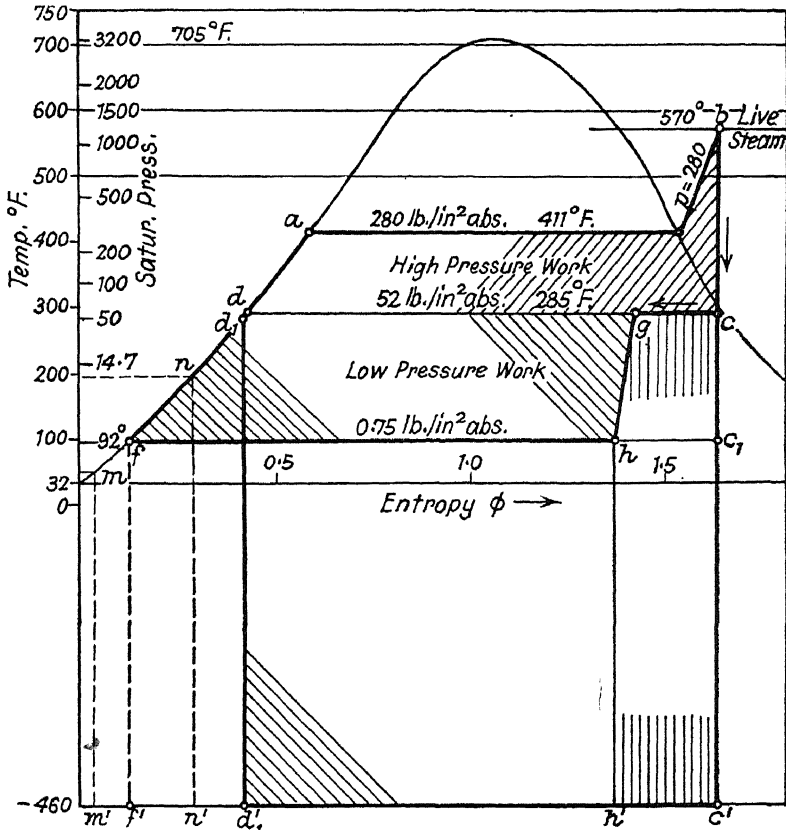


FIG. 272

**The regenerative process.** The normal process of preheating the feed water by means of the exhaust steam represents a partial regain (regeneration) of the otherwise lost heat in this steam. The true regenerative process, however, extends further, in that not only exhaust steam is used, but also steam at one or more points in the course of its expansion through the engine or turbine.

The process and attainable gain will be understood by considering its application to the compound steam engine. Consider such an engine working with superheated steam at  $280\text{ lb./in.}^2\text{ abs.}$  and  $570^\circ\text{F.}$ , and having a condenser pressure of  $0.75\text{ lb./in.}^2\text{ abs.}$

The ideal work available in this case amounts to  $409\text{ B.Th.U./lb.}$  Assuming that, of this,  $150\text{ B.Th.U./lb.}$  represents the work done in the

high pressure cylinder, the work done in the low pressure cylinder is 259 B.Th.U. The high pressure work of 150 B.Th.U. corresponds to a pressure drop from 280 to 52 lb./in.<sup>2</sup> abs., and the exhaust from the high pressure cylinder (or in the receiver) is wet, having the quality 0.973, so that the steam temperature is 285° F. (Compare the  $T\phi$  diagram (Fig. 272) in which  $bc$  represents the adiabatic expansion in the high pressure cylinder.)

At this point, however, the whole of the steam passing from the high pressure cylinder to the receiver does not, as in the case of ordinary compounding, pass to the low pressure cylinder, but a fraction of it is taken from the receiver and used to heat the feed water in a preheater. The heat contained in this "tapped-off" steam is not lost in the condenser, as happens under normal conditions, but is returned to the boiler feed water. In this sense the regenerative process can be regarded as a means of reducing the heat losses in the condenser.

In the above example, let the weight of steam withdrawn from the receiver to heat the feed per pound of steam discharged from the high pressure cylinder be  $z$  lb. The weight of steam entering the low pressure cylinder is now  $(1 - z)$  lb., and this is condensed in the condenser at a pressure of 0.75 lb./in.<sup>2</sup> abs. and a temperature of 92° F. before being returned to the feed pump. On its way to the boiler this  $(1 - z)$  lb. of feed water is led through the preheater, in which it takes up the heat due to the condensing of the  $z$  lb. of tapped-off steam. This  $z$  lb. of condensate then mixes with the feed, and the mixture weighing  $1 - z + z = 1$  lb. is pumped into the boiler. Let the required temperatures of the feed after mixing be 280° F.

The weight of tapped-off steam follows now from the condition that the heat given up by  $z$  lb. of tapped-off wet steam at 52 lb./in.<sup>2</sup> abs. in being condensed and cooled to 280° F. must equal the heat given to the  $(1 - z)$  lb. of feed water in being heated from 92° F. to 280° F.

Hence, if  $y$  is the quality of the tapped-off steam, we have

$$zyL_t + z(283 - 280) = (1 - z)(280 - 92)$$

$$\therefore (0.973 \times 923 + 3)z = 188 - 188z$$

$$1 - z = 0.827 \text{ lb.}$$

The sensible heat in the feed is thus 188 B.Th.U./lb. greater than would be the case without preheating.

The heat which the boiler and superheater give to the steam is now  $1,303 - 249 = 1,054$  B.Th.U./lb. in place of  $1,303 - (92 - 32) = 1,243$  B.Th.U./lb.

On the other hand, the work of the steam in the low pressure cylinder is reduced, since, in place of 1 lb. of steam, the weight is  $1 - z = 0.827$  lb., so that the ideal work is

$$0.827 \times 259 = 214 \text{ B.Th.U./lb.}$$

The total work per pound of live steam in the regenerative process is thus  $150 + 214 = 364$  B.Th.U./lb., while in the non-regenerative process it is 409 B.Th.U.

The percentage of the heat supplied which is converted to work is, in the regenerative process

$$\frac{364 \times 100}{1054} = 0.345$$

while in the non-regenerative process it is

$$\frac{409 \times 100}{1243} = 0.329$$

Hence, in spite of the reduction in work caused by the reduction in the weight of steam operating in the low pressure cylinder, the thermal

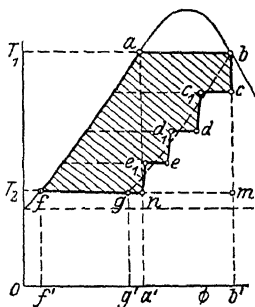


FIG. 273

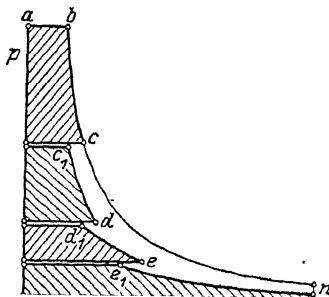


FIG. 274

efficiency in the regenerative process is increased and the increase amounts, in the above example, to

$$100 \quad 0.329 \quad 0.329 \quad 100 = 4.9 \text{ per cent}$$

A simple graphical representation of the process in the  $T\phi$  field is shown in Fig. 272. The work  $E_1$  of the high pressure cylinder is represented by the area  $abcd$  (difference in total heats between  $b$  and  $c$ ). The low pressure work would equal the area  $dcc_1f$  if the weight of steam expanding in the low pressure cylinder still remained equal to 1 lb. Since, however, the weight is only  $(1 - z) = 0.827$  lb., having the state represented by the point  $c$  at entrance to the low pressure cylinder, the work is represented by 0.827 times this area. Hence, if the lengths  $dc$  and  $fc$ , and all the intermediate horizontal intercepts of the low pressure work area are multiplied by 0.827, the resulting line  $gh$ , which is almost straight, found by joining the points thus given, is obtained, and the area  $dghf$  represents the low pressure work in the regenerative process. In addition, the area  $cghh'c'$ , lying below  $cgh$ , is equal to the area of  $fd_1d_1'f'$  lying below  $fd_1$  and represents the heat given to 1 lb. of feed water due to preheating.

The regenerative process was first tried on reciprocating marine engines, but without much success. It has, however, lately been applied successfully to steam turbines, which, on account of the different stages, enable steam to be tapped off at suitable pressures, and thus increase the thermal efficiency.

The number of tap-off points can be increased indefinitely and the feed raised, accordingly, almost to the initial steam temperature, so that an upper limit of thermal efficiency can be deduced. This limit is reached when a very large number of preheaters is used, with a very small extraction of steam from each.

When a limited number of tap-off points is chosen, the step-like limit curve is obtained on the  $T\phi$  and  $pV$  fields (Figs. 273 and 274). If, however, the number is indefinitely increased, so that heat is continuously withdrawn during expansion, the limit curve is continuous, as shown by the dotted curve  $bg$  in Fig. 273. This curve is equidistant horizontally from the liquid line  $af$ . The area  $abgf$  is the work delivered by 1 lb. of steam,

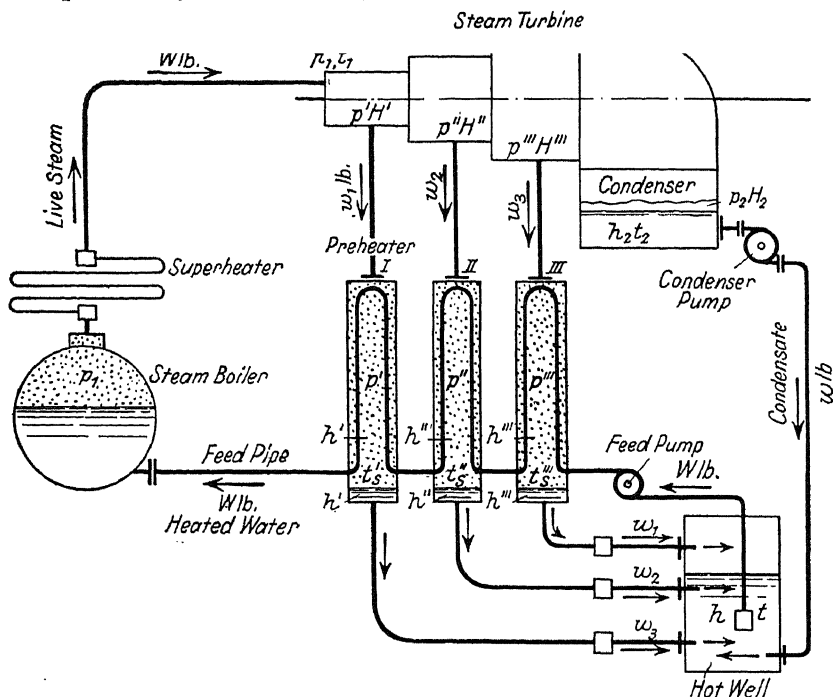


FIG. 275

and the area  $abb'a'$  represents the heat necessary to generate 1 lb. of steam and equals the latent heat, since the preheating is continued until the feed is heated to the boiler temperature (point  $a$ ). The work area  $abgf$  is equal to the rectangle  $abmn$ , so that the thermal efficiency is given by the ratio of this rectangular area to the rectangular area  $abb'a'$ , i.e.

This is the thermal efficiency of the Carnot cycle, which can thus be approached by means of the regenerative process if a sufficient number of preheating steps be adopted.

**Preheating in several stages in steam turbines.** The arrangement is shown in Fig. 275.  $W$  lb. of live steam flow from the boiler and, after



passing through the superheater, enter the turbine at  $p_1$  lb./in.<sup>2</sup> abs. and  $t_1^\circ$  F. The turbine can consist of a high pressure, intermediate pressure, and low pressure section, in which the steam expands down to  $p_2$  and then enters the condenser, where it is condensed to water. The pre-heating steam is tapped-off at three points.  $w_1$  lb. is taken from a stage in the high pressure section at the pressure  $p'$  and led to the preheater I;  $w_2$  lb. from a stage in the intermediate pressure section of the pressure  $p''$  and led to the preheater II; and  $w_3$  lb. from a stage in the low pressure section and led to the preheater III. Hence the weight of steam entering the condenser is  $w = W - w_1 - w_2 - w_3$  lb., which, after being condensed.

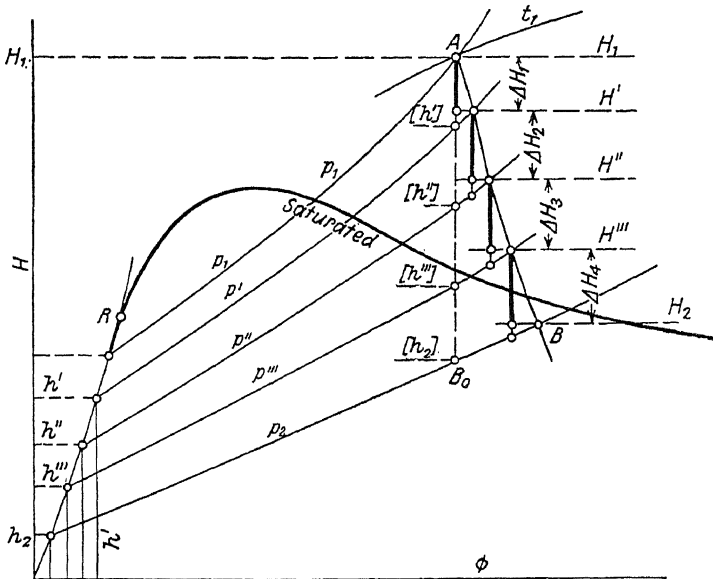


FIG. 276

is led to the hot well and may be at the saturation temperature  $t_2$  corresponding to the condenser pressure  $p_2$ . From there the feed water is returned to the boiler by the feed pump. On its way to the boiler the feed is led through the three preheaters, in which it is heated by the tapped-off steam, so that, in the preheater III it is raised nearly to the saturation temperature  $t_s'''$  corresponding to the pressure  $p'''$ , in II to  $t_s''$ , and in I to  $t_s'$ , the saturation temperature corresponding to the pressure  $p'$ . The feed water enters the boiler with the sensible heat  $h'$  corresponding to this temperature. The tapped-off steam, weighing  $w_1 + w_2 + w_3$  is, at the same time, led to the hot well, so that all the live steam taken from the boiler gathers there as feed water and is returned as such to the boiler.

The process in the steam turbine is conveniently represented on the  $H\phi$  chart (Fig. 276). Expansion begins at the point A, where the pressure is  $p_1$ , the temperature  $t_1$ , and the total heat  $H_1$ . If the flow through the turbine were frictionless, the steam would expand, as shown by  $AB_0$  ( $\phi = \text{constant}$ ). Actually, however, due to the unavoidable friction losses,

the expansion line is  $AB$  (with increasing entropy) (see page 461). The useful work delivered by the steam between  $p_1$  and the first tap-off point is  $W\Delta H_1$ , between the first and second tap-off points  $(W - w_1)\Delta H_2$ , between the second and third tap-off points  $(W - w_1 - w_2)\Delta H_3$ , and between the third tap-off point and the condenser  $(W - w_1 - w_2 - w_3)\Delta H_4$ . The total heats of the tapped-off steam are  $H'$ ,  $H''$ , and  $H'''$ . If no steam be tapped off, the work is  $W(\Delta H_1 + \Delta H_2 + \Delta H_3 + \Delta H_4) = W(H_1 - H_2)$ . Hence, when steam is tapped off the work per pound of live steam is obviously reduced, since the weight of steam operating is reduced after each tap-off point. On the other hand, the heat required to generate the steam is reduced, since the feed water is preheated to  $t'_s$  before entering the boiler.

The following relations, in which  $W = 1$ , serve to determine the quantities  $w_1, w_2, w_3, w, t, h$ , the work  $E$ , and the thermal efficiency.

$$w_1 + w_2 + w_3 + w = 1 \quad . \quad . \quad . \quad . \quad (542)$$

The sensible heat in the feed water tank is given by the sum of the sensible heats contained in the entering water quantities, i.e.

$$w_1 h' + w_2 h'' + w_3 h''' + w h_2 = h \quad . \quad . \quad . \quad . \quad (543)$$

The heat given up by the tapped-off steam on being condensed in a preheater is equal to the heat taken up by the feed water in its passage through the preheater, so that

$$\text{for preheater I} \quad w_1(H' - h') = h' - h'' \quad . \quad . \quad . \quad . \quad (544)$$

$$\text{for preheater II} \quad w_2(H'' - h'') = h'' - h''' \quad . \quad . \quad . \quad . \quad (545)$$

$$\text{and for preheater III} \quad w_3(H''' - h''') = h''' - h \quad . \quad . \quad . \quad . \quad (546)$$

From equation (544) we have

$$w_1 = \frac{h' - h''}{H' - h'} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (547)$$

and from equation (545)

$$w_2 = \frac{h'' - h'''}{H'' - h''} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad .$$

From equation (546), in which the two quantities  $h_3$  and  $h$  are unknown, we have  $h = h''' - w_3(H''' - h''')$  (549)

With equations (542) and (543), this gives

$$H''' - h_2$$

With the values given above for  $w_1$  and  $w_2$  this enables  $w_3$  to be found, and  $w$  is then found from equation (542), giving

$$w = 1 - w_1 - w_2 - w_3 \quad . \quad . \quad . \quad . \quad . \quad (551)$$

as well as  $h$  from equation (549) or equation (543).

The work per pound of live steam is

$$AE =$$

(552)

and is thus calculable when the values of  $\Delta H_1$ ,  $\Delta H_2$ ,  $\Delta H_3$ , and are read from the  $H\phi$  chart for a given case.

The heat supplied by the boiler and superheater per pound of steam is  $Q = H_1 - h'$  (553) and the thermal efficiency based on this heat supply is

$$\eta_{th} = \frac{AE}{Q} \quad (554)$$

**Example 4.**  $p_1 = 400$  lb./in.<sup>2</sup> abs.,  $t_1 = 650^\circ$  F. ( $t_s = 205^\circ$  F.),  $p_2 = 0.5$  lb./in.<sup>2</sup> abs. (Fig. 277). Tap-off at  $p' = 100$ ,  $p'' = 30$ , and  $p''' = 7$  lb./in.<sup>2</sup> abs.

From the  $H\phi$  chart,  $H_1 = 1338$ , and for

$p' = 100$	$p'' = 30$
$p''' = 7$	$p_2 = 0.5$
$h' = 298.6$	$h'' = 219.1$
$h''' = 144.5$	$h_2 = 47.6$

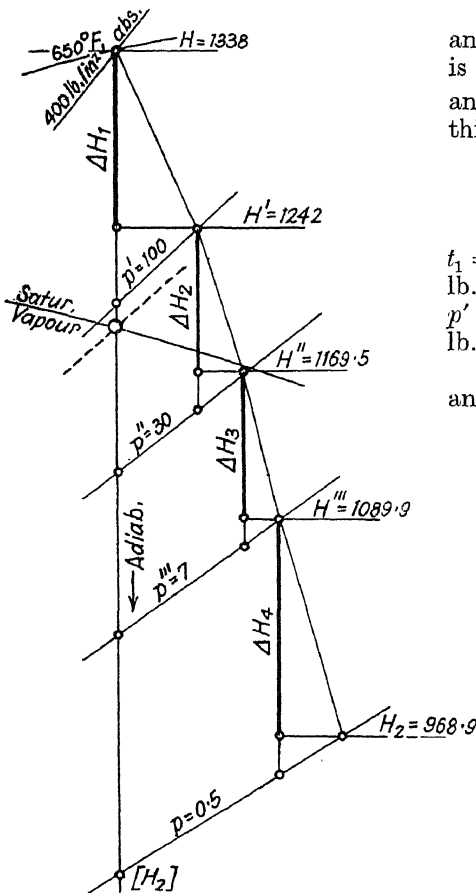


FIG. 277

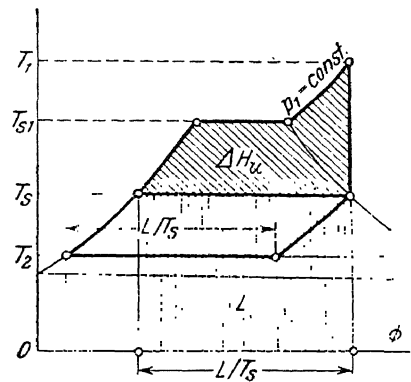


FIG. 278

With frictionless flow the total heats are

$$[H'] = 1,201 \quad [H''] = 1,108 \quad [H'''] = 1,012 \quad [H_2] = 870$$

and the heat drops

$$[\Delta H_1] = 137 \quad [\Delta H_2] = 93 \quad [\Delta H_3] = 96 \quad [\Delta H_4] = 142$$

The assumed stage efficiencies are

$$0.70 \quad 0.78 \quad 0.83 \quad 0.85$$

which give the following total heats at the actual condition points on Fig. 277

$$H' = 1,242 \quad H'' = 1169.5 \quad H''' = 1089.9 \quad H_2 = 968.9$$

and the actual heat drops

$$\Delta H_1 = 96 \quad \Delta H_2 = 72.5 \quad \Delta H_3 = 79.6 \quad \Delta H_4 = 121$$

From the equations (547) to (554) above, we now have, for frictionless flow,

$$\begin{aligned} [w_1] &= 0.0882 & [w_2] &= 0.084 & [w_3] &= 0.0626 & [w] &= 0.7652 & h &= 90.2 \\ AE &= 137 & AE_2 &= 84.7 & AE_3 &= 79.4 & AE_4 &= 119 \\ AE &= 420.1 & Q &= 1039.4 \end{aligned}$$

$$\eta_{th} = 0.405$$

Without regeneration,

$$AE = 468, \quad Q = 1,290.4, \quad \eta_{th} = 0.363$$

Hence, the gain due to regeneration is

$$\frac{(0.405 - 0.363)}{0.363} \times 100 = 11.6 \text{ per cent.}$$

For the actual flow we have

$$\begin{aligned} w_1 &= 0.0844 & w_2 &= 0.0786 & w_3 &= 0.0598 & w &= 0.7772 & h &= 88.0 \\ AE_1 &= 96 & AE_2 &= 66.4 & AE_3 &= 66.7 & AE_4 &= 94 \\ AE &= 323.1 & Q &= 1039.4 \end{aligned}$$

so that  $\eta_{th} = 0.311$

Without regeneration,

$$AE = 369.1, \quad Q = 1,290.4, \quad \eta_{th} = 0.286$$

The gain due to regeneration is thus

$$\left( \frac{0.311 - 0.286}{0.311} \right) 100 = 8.1 \text{ per cent}$$

and is thus smaller than in the theoretical case, since the tapped-off steam in the first preheater has a higher range of superheat.

The highest thermal efficiency when the steam is initially saturated is given by (page 522)

$$\eta_{th \max} = \frac{T_2}{T_1} - 1 - \frac{T_3}{T_1}$$

but requires an infinite number of tap-off points.

If, however, the steam is initially superheated, this equation, which is the same as that given by the Carnot cycle, no longer holds. If, in this case, tap-off first occurs at the stage in which the steam becomes saturated, the useful heat drop to this point is

$$\Delta H_n = H_1 - H_s$$

In the wet field, with continuous preheating of the feed, the necessary

heat supply is equal to the latent heat at the temperature  $T_s$ , since the feed is heated to this temperature. Of this quantity the fraction

$$\frac{T_s - T_2}{T_s}$$

is converted to work, giving

$$\Delta H_s = L \frac{T_s - T_2}{T_s}$$

Since the total heat supplied in generating the superheated steam is  $L + \Delta H_n$  and the total work is

$$\Delta H_n + L \left( 1 - \frac{T_2}{T_1} \right)$$

the thermal efficiency is

$$\begin{aligned} \eta_{th \max} &= \frac{\Delta H_n + L \left( 1 - \frac{T_2}{T_s} \right)}{L + \Delta H_n} \\ &= 1 - \left( \frac{L}{L + \Delta H_n} \right) \frac{T_2}{T_s} \end{aligned}$$

This value is higher than the Carnot efficiency between the temperature limits  $T_s$  and  $T_2$ , but considerably less than the value  $1 - \frac{T_2}{T_1}$ , which gives the Carnot efficiency for  $T_1$  as the upper limit of temperature.

In the above example the steam becomes (with frictionless flow) dry saturated at 88 lb./in.<sup>2</sup> abs. and 319° F., and, giving  $\Delta H_n = 150$  B.Th.U./lb.,  $L = 895$  B.Th.U./lb.

$$\begin{aligned} \text{Hence } \eta_{th \max} &= 1 - \frac{895}{1045} \times \frac{460 + 79.5}{460 + 319} \\ &= 1 - 0.593 = 0.417 \end{aligned}$$

The Carnot efficiency for saturated steam at 88 lb./in.<sup>2</sup> abs. would be

$$1 - \frac{460 + 79.5}{460 + 319} = 1 - 0.693 = 0.317$$

while for saturated steam at 400 lb./in.<sup>2</sup> abs. with  $t_s = 445^\circ$  F., it is

$$\eta_{th} = 1 - \frac{460 + 79.5}{460 + 445} = 1 - 0.596 = 0.404$$

Hence, with superheated steam at 400 lb./in.<sup>2</sup> abs. and 650° F. (preheating to 319° F.), the maximum thermal efficiency is not greatly different from that obtained from dry saturated steam at 400 lb./in.<sup>2</sup> when the feed is preheated to 319° F. (i.e. the saturation temperature, with an infinite number of heaters), but even with only three preheaters raising the feed to 328° F., the value obtained is about as high (0.405)

as shown in the above example. With  $t_1 = 650^\circ \text{F.}$  or  $t_1 = 1110^\circ \text{F. abs.}$  as the upper temperature limit, we have

$$1 - \frac{T_2}{T_1} = 1 - \frac{539.5}{1110} = 0.486$$

which, however, is only attainable with saturated steam at  $2200 \text{ lb./in.}^2 \text{ abs.}$  and not with superheated steam at  $400 \text{ lb./in.}^2 \text{ abs.}$  and  $650^\circ \text{F.}$

**Re-superheating of steam.** Steam which has been expanded in the high pressure cylinder of a compound engine can be returned to a superheater which is heated by the hot flue gases of the boiler or by high pressure steam. The same may occur in the case of multistage turbines, particularly in those in which there are separate high and low pressure sections. The steam, after expanding in one or two stages and before entering the next stage, is led to a superheater.

Due to this arrangement, the work performed in the low pressure section of the engine or turbine is increased, as shown by the pressure

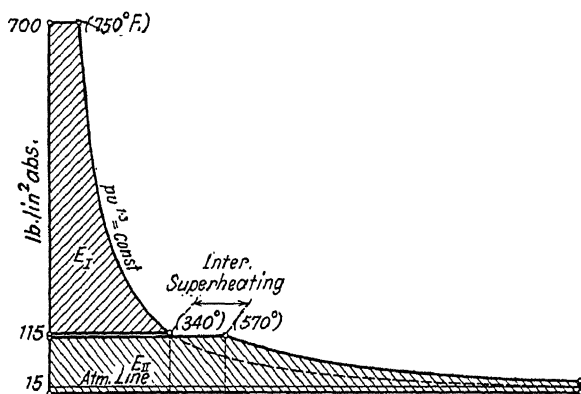


FIG. 279

volume diagram (Fig. 279). At the same time, the thermal efficiency is also increased, since the steam in the low pressure section is now dry, in place of the wet steam obtained when no re-superheating is adopted. Where a high initial pressure combined with a large number of expansions is used, re-superheating of the steam becomes particularly necessary. This process was first applied by W. Schmidt to a high pressure engine ( $850 \text{ lb./in.}^2 \text{ abs.}$  initial pressure and condensing). Modern large turbine installations having high initial pressures are now generally fitted with re-superheaters.

**Preheating of the feed water and combustion air by the heat contained in the boiler flue gases.** The flue gases of the steam boiler leave the heating surfaces of the boiler at a temperature of about  $550^\circ \text{F.}$  Hence they can with advantage be still further cooled before entering the chimney or funnel. Since both the feed water and the combustion air are initially at room temperature, the flue gases can be used to heat these, so that a saving of combustion heat is effected, with a consequent increase in the overall thermal efficiency of the plant. The lower limit of temperature to which the flue gases can be cooled may be taken as  $250^\circ \text{F.}$

The fraction of the combustion heat contained in the flue gases at 550° F. depends (as shown on page 75) mainly on the amount of excess air supplied to the furnace. Assuming an average excess air factor of 1.5, the flue gases contain (as shown on page 78) about 3.3 per cent of the combustion heat per 100° F. temperature rise above room temperature, so that, for a flue gas temperature of 550° F. and room temperature of 60° F., the percentage heat carried away is

$$\frac{-60}{100} \quad 3.3 = 16 \text{ per cent.}$$

If the flue gases are cooled to 250° F. in a feed water economizer or air precooler, the fraction of their heat content which is saved is

$$\frac{550 - 250}{550 - 60} = 0.6 \text{ or } 60 \text{ per cent.}$$

Hence the saving of the combustion heat which can be effected in this way is  $0.6 \times 16 = 10$  per cent.

While economizers have been in general use for some considerable time, it is only within recent years that air preheaters have been adopted. The combustion air in the latter can be heated to 400° F., but, as a rule, a somewhat lower temperature is aimed at, so that an economizer and air preheater can both be employed.

**Application of very high pressure steam.** In the development of the steam engine since the time of Watt, continuous efforts have been made to use higher and higher steam pressures. Up to the beginning of this century the maximum steam pressures in use lay between 170 and 210 lb./in.<sup>2</sup> abs., and during the following 20 years the maximum reached was about 300 lb./in.<sup>2</sup> abs. Apart from special research, the year 1921 may be regarded as the year in which the high pressure steam engine was first introduced. Wilhelm Schmidt, who, 25 years earlier, had succeeded in applying superheated steam to steam engines and turbines, published, in 1921, the results of his comprehensive tests on a quadruple expansion steam engine of 150 h.p., which operated with superheated steam at 815 lb./in.<sup>2</sup> abs. These tests showed that the application of these high steam pressures resulted in a considerable reduction in the heat consumption per unit load, when the provisions necessary for such high pressures are made, such as having more than two expansion stages and making provision for re-superheating the steam. Schmidt showed, at the same time, that boilers for these high pressures could be successfully designed, in spite of the doubts then existing. Although it can scarcely be claimed that such high pressures have become general, it must be admitted that it is due to Schmidt's efforts that pressures of 400 to 600 lb./in.<sup>2</sup> abs. are now found to be necessary in the large steam turbine plants developing 10,000 to 100,000 kW., and that large power stations are now operating with steam pressures as high as 1,200 lb./in.<sup>2</sup> abs.

The higher steam temperatures applied since 1921 result from the use of these higher pressures. These temperatures have risen from about 550° F. to about 750° F., and the necessity for this increase was recognized by Schmidt himself. Since saturated steam at 815 lb./in.<sup>2</sup> abs. has a temperature of about 520° F., the best results, in using this steam, are

only obtained if the steam is superheated through the same range of superheat as is used for a normal pressure of, say, 220 lb./in.<sup>2</sup> abs. Hence, if the latter is superheated through the range 650 - 400 = 250° F., the steam at 815 lb./in.<sup>2</sup> abs. should be heated to 520 + 250 = 770° F. In the tests mentioned above, Schmidt used steam at 870° F.

The success of the Schmidt engine consisted in the saving in heat consumption, which amounted to 30 per cent as compared with a good reciprocating engine, and 23 per cent compared with a compound locomobile.\* The steam heat consumption per 1 h.p. hour amounted to 8,000 B.Th.U., which, with a boiler efficiency of 85 per cent, amounts to a fuel heat consumption of 9,400 B.Th.U. per 1 h.p. hour, or about 9,400/0.9 = 10,460 B.Th.U./b.h.p. hour. The thermal efficiency of the plant is then  $\frac{2545}{10460} \times 100 = 24.4$  per cent. The engine was condensing, the condenser pressure being 0.57 lb./in.<sup>2</sup> abs.

As the steam pressures are raised, both the work per lb. and the thermal efficiency of the ideal cycle increase. Both values are measured in the usual way by means of the  $H\phi$  chart. The following values are thus obtained (for a condenser pressure of 0.75 lb./in.<sup>2</sup> abs.) from the  $H\phi$  chart, for the pressures

$p = 150 \quad 300 \quad 450 \quad 600 \quad 900 \quad 1200 \quad 1500 \quad 1800 \quad 2200$  lb./in.<sup>2</sup> abs.

Saturated steam—

Available work—

330    372    396    413    431    437    442    440    433

Total heat, reckoned from 90° F.—

1140    1151    1154    1154    1149    1140    1127    1108    1076

Thermal efficiency—

0.29    0.323    0.343    0.358    0.376    0.384    0.393    0.397    0.403

Superheated steam at 750° F.—

Pressure—

150    300    450    600    900    1200    1500    1800    2200 lb./in.<sup>2</sup> abs.

Range of superheat—

391    333    294    264    218    183    154    129    100 ° F.

Available work—

422    461    481    493    509    517    520    518    512

Total heat (above 90° F.)—

1347    1340    1333    1325    1310    1293    1276    1256    1225

Thermal efficiency—

0.313    0.344    0.361    0.373    0.389    0.400    0.408    0.413    0.418

These values of  $\eta_{th}$  for both cases are plotted to a base of steam pressures in Fig. 280.

In addition, the values of the thermal efficiency with complete regeneration for saturated steam have been plotted, as well as for superheated

\* See O. H. Hartmann. High Pressure Steam. *V.D.I. Verlag*. (1925).



steam with complete regeneration in the saturation region (page 519). The latter two values give curves which lie very close together. The diagram shows that the increase in thermal efficiency with pressure is much greater than that obtained without regeneration, and amounts (for pressures between 850 and 2,000 lb./in.<sup>2</sup> abs.) to about 4.3 per cent for each 100 lb./in.<sup>2</sup> pressure increase. Without regeneration,  $\eta_{th}$  increases (between 850 and 2,000 lb./in.<sup>2</sup> abs.) by about 3 per cent (absolute) and

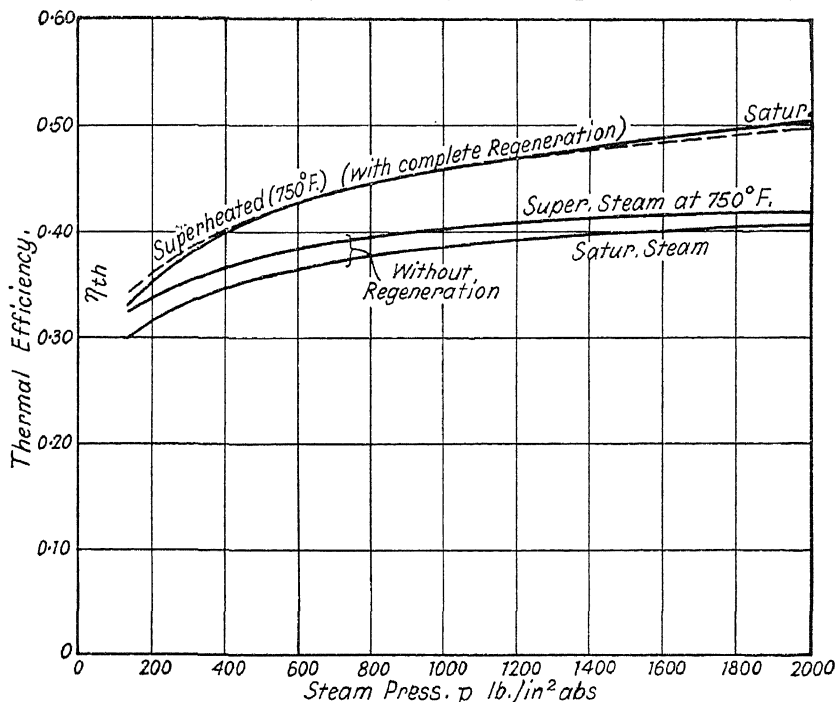


FIG. 280

attains a maximum value of 41 per cent at about 570 lb./in.<sup>2</sup> abs. With regeneration it amounts to 40 per cent at 400 lb./in.<sup>2</sup> abs., and rises to 50 per cent at 2000 lb./in.<sup>2</sup> abs.

In reciprocating engines and turbines the transition from the superheated to the saturated state during expansion is of particular importance and is readily shown by the  $H\phi$  diagram. Thus, in Fig. 281, for steam initially at 850 lb./in.<sup>2</sup> abs. and 825° F., the diagram shows that on adiabatically expanding the steam, it becomes dry saturated at 91 lb./in.<sup>2</sup> abs. (line  $AC_1$ ); steam initially at 1,150 lb./in.<sup>2</sup> abs. and 750° F. becomes dry saturated at 242 lb./in.<sup>2</sup> abs., and steam initially at 1,700 lb./in.<sup>2</sup> abs. and 750° F. becomes dry saturated at 540 lb./in.<sup>2</sup> abs. In the reciprocating engine the expansion is almost adiabatic, but in the turbine the actual expansion curve lies above the adiabatic, due to flow resistances and the deviation from the adiabatic increases with these resistances. In Fig. 281 the states on entering the wet field with a frictional loss of 20 per cent have been plotted (points  $e_2$  for 850 lb./in.<sup>2</sup> abs.,  $d_2$  for 1,150 lb./in.<sup>2</sup> abs.,

and  $c_2$  for 1,700 lb./in.<sup>2</sup> abs.). The transition to the saturated state in the case of reciprocators occurs at considerably higher pressures than in steam turbines.

Now, since wet steam both in reciprocators and turbines lowers the thermodynamic efficiency, it is advantageous to re-superheat the steam when it reaches the saturated state, before further expansion occurs. For this reason, Schmidt fitted two intermediate re-superheaters in his experimental quadruple expansion engine, and in this way obtained a high thermodynamic efficiency even in the low pressure section. The  $H\phi$  diagram (Fig. 281) shows the changes in the steam state for this

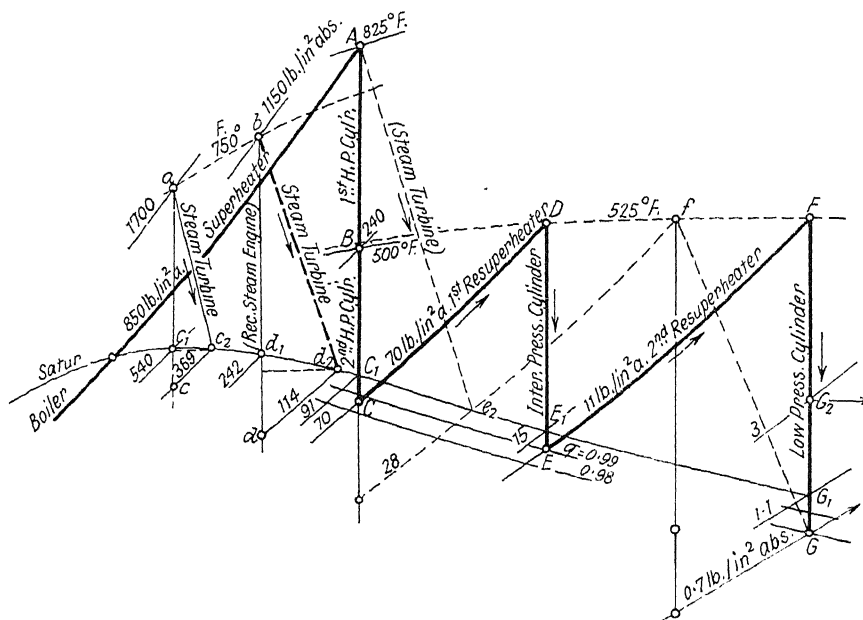


FIG. 281

engine. The steam initially at 850 lb./in.<sup>2</sup> abs. and 825° F. expands in the first high pressure cylinder, whereby its pressure and temperature drop to 240 lb./in.<sup>2</sup> abs. and 500° F. (line  $AB$ ). It now operates in the second high pressure cylinder, in which it expands to 70 lb./in.<sup>2</sup> abs. (line  $BC$ ) and is found to be wet, having entered the wet field at a pressure of 91 lb./in.<sup>2</sup> abs. After leaving this cylinder the steam is superheated in the first intermediate re-superheater, which is heated with saturated steam at the initial pressure of 850 lb./in.<sup>2</sup> abs. This causes the operating steam temperature to rise to 525° F. (line  $CD$ ). The condensate of the heating steam is returned to the boiler. In the intermediate cylinder, into which the superheated steam enters at 70 lb./in.<sup>2</sup> abs., expansion occurs to 11 lb./in.<sup>2</sup> abs., but the steam enters the wet field at 15 lb./in.<sup>2</sup> abs. Hence, before expanding in the low pressure cylinder, the steam is led through a second re-superheater also heated by saturated steam at 850 lb./in.<sup>2</sup> abs., so that its temperature is again raised to 525° F. (line

*EF*). In the low pressure cylinder expansion occurs to 3 lb./in.<sup>2</sup> abs., after which the steam is exhausted to the condenser against the condenser pressure of 0.7 lb./in.<sup>2</sup> abs. In the cylinder itself the steam, after release, enters the wet field when the pressure drops to 1.1 lb./in.<sup>2</sup> abs., but, due to re-heat, it is superheated on entering the condenser.

In the case of a turbine supplied with steam at the same initial condition *A*, and exhausting at the same final state *G*, one re-superheater only would be required, as shown by the dotted line *Ae<sub>2</sub>fG*. In general, therefore, a fewer number of re-superheaters is required in the case of turbines, and when the initial pressure is about 400 lb./in.<sup>2</sup> abs. and the range of superheat sufficiently high, no re-superheater is required.

The importance of utilizing high pressure steam is emphasized by the following remarks\* on the position of turbo-generator design, particularly in America. The normal steam pressures of about 300 lb./in.<sup>2</sup> abs. employed ten years ago rose rapidly to about 700 lb./in.<sup>2</sup> abs., the pressure now used in very large units. Six years ago the first high pressure units using steam at 1,200 lb./in.<sup>2</sup> abs. were built, but only a few attained an output of 10,000 kW. To-day quite a large number of 50,000 kW. machines are in course of construction or in operation, and the total output of these very high pressure turbines amounts to 400,000 kW. Using a fuel with a calorific value of about 12,500 B.Th.U./lb. and a boiler having an efficiency of 85 per cent, the amount of fuel required per kW. hour in these machines is 0.88 lb. (11,000 B.Th.U.) or even less.

**Indirect generation of high pressure steam.** Before the introduction of very high pressure boilers, moderately high pressure steam boilers had already been constructed as water-tube boilers, in which direct heating of the water or steam space was avoided. W. Schmidt had already shown that narrow water-tube boilers could be constructed to withstand pressures up to, and over, 850 lb./in.<sup>2</sup> abs., provided that pure water (i.e. free from impurities and dissolved gases) be supplied as feed. It is not always possible to ensure this, and when impure feed water is supplied, destruction of the directly heated tubes occurs, due to the retardation of heat flow through the walls. This can be avoided when the steam generation, instead of occurring directly by means of hot gases, is effected by means of heated steam at a temperature above that of the high pressure working steam. The arrangement depends on whether saturated or superheated steam be used for heating.

(a) **Saturated heating steam.** (The Schmidt hot steam process.) If the heating steam is saturated, it must be raised to a pressure sufficiently high to ensure that its temperature is above that of the boiler steam saturation temperature, so that a reasonable heat transfer may be effected. Thus, if steam at 850 lb./in.<sup>2</sup> abs. is to be generated, the heating steam has to be raised to between 1,000 and 1,400 lb./in.<sup>2</sup> abs., the corresponding temperatures of which are 545 and 587° F. Since water at 850 lb./in.<sup>2</sup> abs. boils at 525° F., the temperature difference lies between 20° and 60° F., and hence condensation is possible, and the latent heat liberated is transferred to the boiler water. The arrangement is shown in Fig. 282.† A spiral coil is fitted in the water space of the steam boiler

\* *Die Wärme* (1930), page 93; from E. W. Rice in *Gen. El. Review*.

† O. H. Hartmann. Experiences with the Schmidt High Pressure Safety Boiler. *Hochdruckdampf* II, V.D.I. Verlag. (1929.)

*A*, and is filled with saturated steam at 1,400 lb./in.<sup>2</sup> abs., supplied by the steam generator *B*, which is fed with pure water. This part of the system can be used as a water tube boiler having the unheated water and steam collectors *C* and *D*. The heating steam is condensed in the heating coil and the returning hot condensate is cooled to about the boiler feed temperature in the preheater *E*. The saturated steam, at 850 lb./in.<sup>2</sup> abs., generated in the boiler *A*, is led to a superheater *F*, heated to the usual superheat temperature (650 to 750° F.), and then led to the turbine. Since the total heat of steam at 850 lb./in.<sup>2</sup> abs. (reckoned from 90° F.)

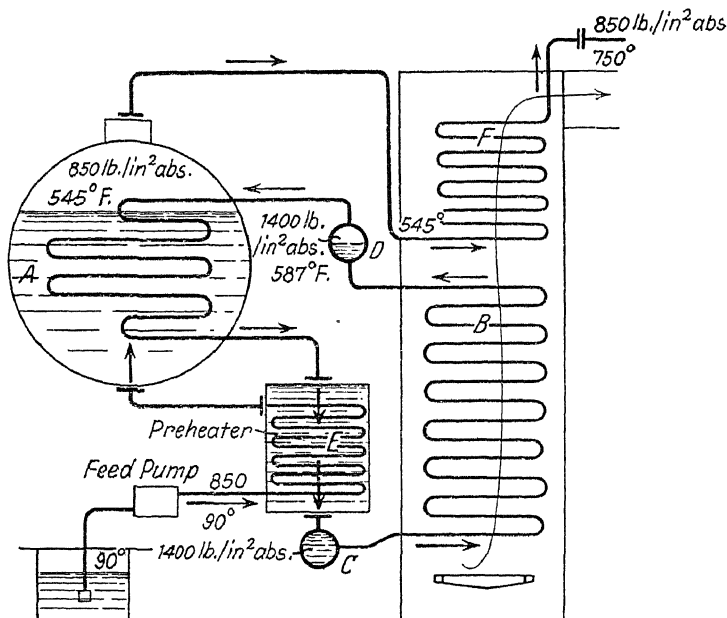


FIG. 282

is 1,151 B.Th.U./lb., and that of steam at 1,400 lb./in.<sup>2</sup> abs. is 1,129 B.Th.U./lb., the ratio of the weight of steam generated to the weight of heating steam is  $\frac{1129}{1151} = 0.98$ .

A high pressure steam locomotive has lately been constructed using the same process and having the closely spaced narrow water tubes of the hot steam generator in the fire space. As a result of tests by the German State Railways the pressure of the boiler steam ranged from 650 to 800 lb./in.<sup>2</sup> abs., and that of the steam in the heating coil from 1,000 to 1,400 lb./in.<sup>2</sup> abs. In addition to the high pressure steam, additional steam was generated at 150 lb./in.<sup>2</sup> abs. by the residual heat in the furnace gases.

(b) **Superheated heating steam.** (The Löffler high pressure transfer process.) In this process, the steam generator, consisting of a cylindrical drum filled about half full of water, is heated by superheated steam, which is led to and mixes with the boiler water. The pressure of this heating steam requires to be only slightly higher than the boiler pressure, but its

temperature must be considerably higher than the boiler saturation temperature. If, for example, steam at 1,400 lb./in.<sup>2</sup> abs., which has a saturation temperature of 587° F., is to be generated, the heating steam temperature must be at least 650° F. to 750° F., since it is only by means of the superheat that excess fresh steam can be generated. The heating steam is not specially produced as in (a), but is generated along with the operating steam and brought to the desired superheat temperature in a fired superheater. The arrangement is shown in Fig. 283. *A* is the steam boiler in which saturated steam at 1,400 lb./in.<sup>2</sup> abs. and 587° F. is generated. This steam is withdrawn from the boiler by the steam pump *B*, discharged to the superheater *C* and returned as superheated steam at 750 to 900° F. to the perforated pipe in the boiler. The steam

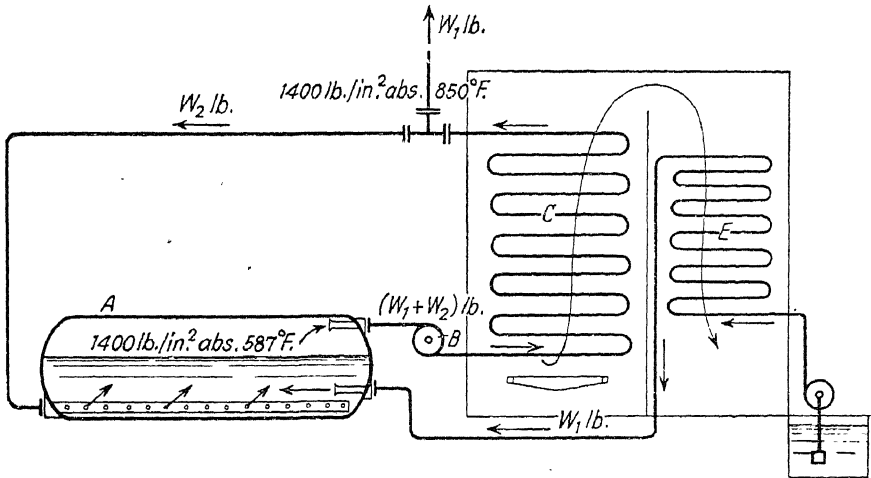


FIG. 283

pump has merely to overcome the pipe resistance, which may amount to several atmospheres, depending on the steam speed and length of pipe. Hence the steam pressure at entrance to the superheater is higher by this amount than in the boiler. At a point in the return pipe the operating steam, amounting to  $W_1$  lb., is led by a branch pipe to the turbine. The remainder, amounting to  $W_2$  lb., returns to the boiler, where its superheat is used to generate the excess weight  $W_1$  lb. of steam. An equal weight  $W_1$  lb. of fresh feed water must be supplied to the steam generator by the feed pump. Before entering the boiler this feed is heated by the products of combustion coming from the superheater, so that the temperature of the feed rises to  $t_1$  and the heat taken up is  $h_1 - h_0$ . The weight  $W_2$  lb. of additional steam which has to be supplied in generating  $W_1$  lb. of operating steam is determined by the condition that the total heat of  $(W_1 + W_2)$  lb. of saturated steam leaving the boiler must equal the total heat of  $W_2$  lb. of return steam and  $W_1$  lb. of feed water entering the boiler, i.e.

$$(W_1 + W_2)H_s = W_2(H_s + h_s) + W_1h_1$$

This gives  $\frac{W_2}{W} = \frac{H_s - h_1}{h_s}$

Hence the return steam weight  $W_2$  is always greater than the weight  $W_1$  of operating steam. Thus with

$p$	= 1800	1500	1200	1000	800	600	400 lb./in. <sup>2</sup> abs.
$H_s$	= 1163	1182	1197	1204	1210	1213	1212 B.Th.U./lb.

and for  $t_2 = 850^\circ \text{ F.}$ , i.e. the temperature of the steam leaving the super-heater,

$H_s + h_s$	= 1389	1402	1415	1422	1430	1438	1445 B.Th.U./lb.
$h_s$	= 226	220	218	218	220	225	233 B.Th.U./lb.

The feed water should be as hot as possible in order to make  $W_2$  as small as possible. In the extreme case the preheating would raise the feed water to the saturation temperature, so that  $H_s - h_1 = L$  (the latent heat). According to Löffler, the preheating should proceed for all pressures, until the sensible heat differs from that corresponding to the boiler pressure by about 90 B.Th.U. This then gives  $H_s - h_1 = L + 90$ . The minimum ratio is

and the value by Löffler

$$\frac{W_2}{W_1} = \frac{L + 90}{h_s}$$

and hence, for

$p$	= 1800	1500	1200	1000	800	600	400
$L$	= 511	568	623	660	699	740	787
$\left(\frac{W_2}{W_1}\right)_{min}$	= 2.26	2.58	2.86	3.03	3.18	3.29	3.38
$\frac{W_2}{W_1}$	= 2.66	2.99	3.28	3.44	3.59	3.69	3.77

The weight of steam dealt with by the steam pump per pound of operating steam is

In finding the work required from the pump, the volume  $V$  of this mass has to be considered and is given by

With  $\Delta p$  as the excess pressure required to overcome the frictional resistances, this work is  $E_R = V \Delta p$ , where (as shown on page 276)

$$\Delta p = \beta \frac{L}{d} \frac{V^2}{V_m}$$

In this,  $V_m$  is the mean specific volume of the steam as it passes from

the steam pump through the superheater to the boiler, and is greater than  $v_s$ . This now gives

$$E_R = \left(1 + \frac{v_s^2}{W_1}\right) \beta \frac{L}{d} \frac{v_s}{v_m} V^2$$

We have  $\frac{v_s}{v_m} = \frac{T_s}{T_m}$

and this value, and thus  $E_R$ , increases as the steam pressure increases. Löffler found in the same way that the steam pump work expressed as a fraction of the available steam work, delivered by 1 lb. of steam, increased

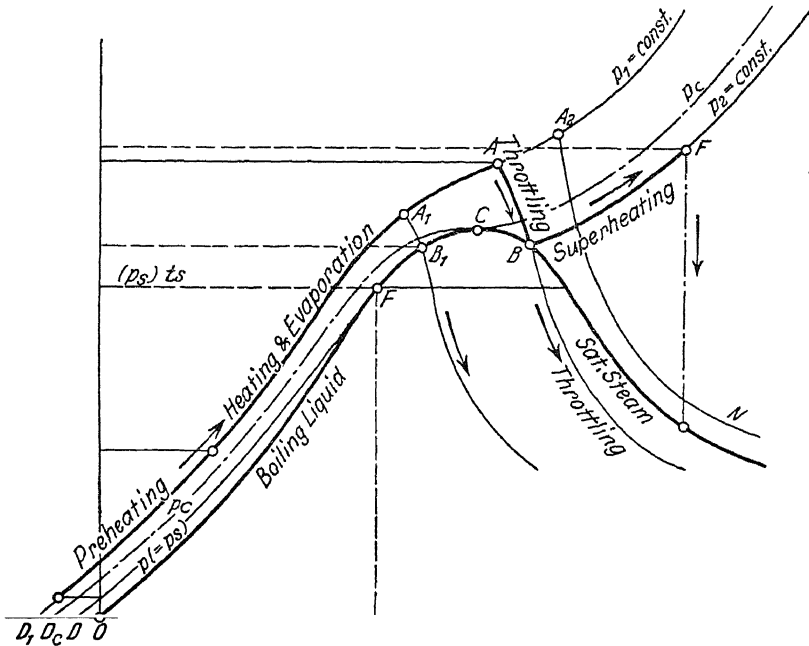


FIG. 284

as the steam pressure decreased. At 280 lb./in.<sup>2</sup> abs. the steam pump work is actually greater than the useful work delivered, and even at 570 lb./in.<sup>2</sup> abs. the pump work amounts to over 20 per cent of the useful work. Hence the arrangement only becomes practicable at pressures above 700 lb./in.<sup>2</sup> abs. At 1,700 lb./in.<sup>2</sup> abs. the pump work amounts to less than 2 per cent of the useful work. The arrangement, therefore, is only suitable for very high pressures.\*

\* The first of these boilers was built by Löffler in the Florisdorf locomotive factory in Vienna (1925). It proved, according to Löffler, that steam could safely be generated at between 1,400 and 1,700 lb./in.<sup>2</sup> abs. and at a temperature of 930° F. The reader should refer to an article in *Hochdruckdampf* II, by Löffler, which refers to a larger plant built at the same place, in which 8 tons/hour were generated at 1,550 lb./in.<sup>2</sup> abs. and 840° F., and also to a still larger boiler generating 15 tons/hour at 1,700 lb. abs. and 930° F.

**Generation of high pressure steam avoiding the saturation region.** (Benson Process.) As shown on page 396, it is possible to produce superheated steam from water without passing the steam through the wet and dry saturated states. In order to do this, the cold liquid water must, when steam is to be formed at constant pressure, be brought up to, or above, the critical pressure, i.e. above 3,200 lb./in.<sup>2</sup> abs. The water at this pressure is then supplied with heat until the temperature rises to the critical value (705° F.). It then changes its state to that of superheated steam.

The changes are best illustrated by means of the  $T\phi$  diagram (Fig. 284). If water be heated from 32° F., so that at every instant the pressure equals the saturation pressure (such as occurs in a closed vessel with a small steam space and no discharge), then, on the  $T\phi$  diagram, the changes occur along the lower limit curve and the heat supplied is given by the area below this curve. The critical state is reached at  $C$ , where the saturation pressure attains its highest value. If now more heat be supplied to the closed vessel, the curve extends beyond the saturation region into the superheat field with a slower rate of pressure increase.

If, on the other hand, the water be first brought to the required steam pressure  $p_s < p_c$ , as always occurs in steam generation, the heating curve is now  $DF$  and only attains the lower limit curve at  $F$  when the temperature of the water rises to  $T_s$ , the saturation temperature corresponding to  $p_s$ . The curve does not now commence at  $O$  but at some point  $D$  to the left of  $O$ . The distance of  $D$  from  $O$  increases as the pressure, to which the cold water is originally subjected, increases, since with isothermal compression of water the entropy decreases. Now, so long as the water pressure  $p_s$  is less than the critical pressure  $p_c$ , the heating curve  $p_s = \text{constant}$  cuts the liquid limit curve (point  $F$ ) between  $O$  and  $C$ , i.e. the water is brought to the boiling point by heating at constant pressure, and a further supply of heat (latent) at constant pressure serves only to generate steam from  $F$  onwards. If the water pressure is equal to the critical, the heating curve at constant pressure touches the limit curve at its highest point (line  $D_cC$ ), and with a further heat supply the water immediately changes to superheated steam, without having to pass previously through a saturated state. If, finally,  $p_s > p_c$ , the heating curve is now  $D_1A_1A_2$ , and the water changes gradually to superheated steam, so that no definite point can be given at which the liquid state ceases and the superheat state commences. Hence, in producing steam from water in this way, no priming occurs, this being one of the difficulties in using high pressure steam. On the other hand, the steam pressure is too high for satisfactorily operating prime movers and, in addition, the steam very rapidly becomes wet when adiabatically expanded from points on curves  $p > p_c$ , unless the steam temperature is considerably greater than the critical. This is a disadvantage in running the engine, but can be avoided in the following way, without having to raise the superheat by an impracticable amount. The steam at any of the points  $A_1$ ,  $A$ , or  $A_2$ , where the pressure is above the critical (3,200), say, 3,500 lb./in.<sup>2</sup> abs., is throttled through a valve in the steam collecting pipe to some suitable operating pressure, e.g. 1,400 to 2,000 lb./in.<sup>2</sup> abs. This means that the total heat remains constant (page 444). In the  $Ht$  diagram (Fig. 285) the state change is represented by a horizontal line. According to the



Josse *Ht* diagram the temperature drops with this change. If, for example, the initial temperature is 842° F., then throttling from 3,270 to 2,130 lb./in.<sup>2</sup> abs. causes a reduction in temperature to 768° F., and throttling to 1,420 lb./in.<sup>2</sup> abs. a reduction in temperature to 703° F. The steam, therefore, remains superheated and cannot (with this initial temperature) be reduced to the saturated state no matter how far throttling proceeds.

On the  $T\phi$  diagram (Fig. 284) the state change, for these conditions, is represented by the curve  $A_2N$ . With a lower initial temperature, such as at  $A$ , the throttling curve cuts the saturation curve at  $B$ , and the steam becomes wet. Starting at a still lower temperature, point  $A_1$  (Fig. 284), the throttling curve  $A_1B_1$  cuts the liquid limit curve and hence effects liquefaction (as, for example, in the Linde air liquefaction process). This latter type of throttling must be avoided at all costs, since dangerous water hammer would occur in the supply pipe and engine.\* The highest temperature in the steam generator must be such that the throttling curves never meet the liquid limit curve, although they may meet the saturation curve beyond the critical point, as shown by the line  $AB$  in Fig. 284. In the latter case the steam must be superheated at constant pressure ( $BF$  from  $B$ ), since saturated steam becomes wet when adiabatically expanded. At  $F$  the steam is now at a suitable state for operating in the engine.

This is the system of steam generation first proposed by Benson, of Siemen's Works.† Fig. 286 shows the working arrangement. The feed pump draws water from the hot well and delivers it at about 3,250 lb./in.<sup>2</sup> abs. to the preheating coil *A*, where it is heated to about 300° F. From there it proceeds through the coil, heated by the radiant heat of the oil or coal dust flame in the fire space, where the water temperature is further raised to about 715° F., which is in the neighbourhood of the critical temperature. It then flows through the superheater coil *C*, at exit from which, where the throttle valve *D* is fitted, the temperature is now at 842° F. The steam leaves the throttle valve at 2,100 lb./in.<sup>2</sup> abs. and 762° F., and proceeds to the superheater *E*, where it is heated to

\* See Fig. 231 regarding the course of the throttling curves on the  $T\phi$  diagram for  $\text{CO}_2$  in this region. The same characteristics are shown by  $\text{H}_2\text{O}$ .

† H. Gleichmann. Benson Process for Generating High Pressure Steam (*Hochdruckdampf, V.D.I. Verlag*). Also there, W. Abendroth. Steam Plant with Benson Boiler in the Siemens's Schuckert Power Station.

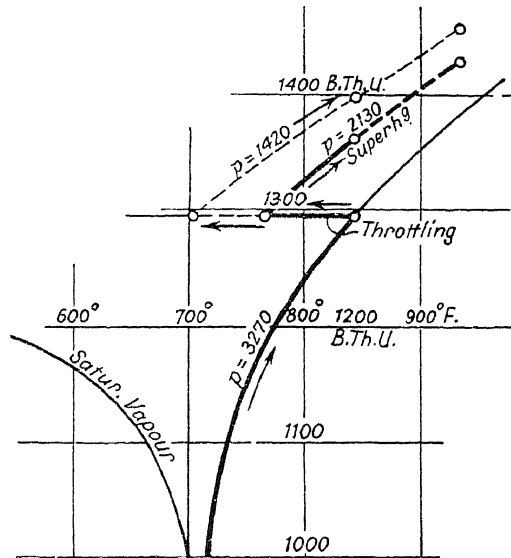


FIG. 285

842° F., after which it is supplied to the turbine.\* Several processes are shown to scale on the  $H\phi$  diagram (Fig. 287), with the same lettering as in Fig. 284.

### STEAM ACCUMULATORS

**General principles.** Operating steam in excess of that required for immediate use in factories cannot be stored in the vapour state, because the tanks required to store this steam would be excessively large, due to

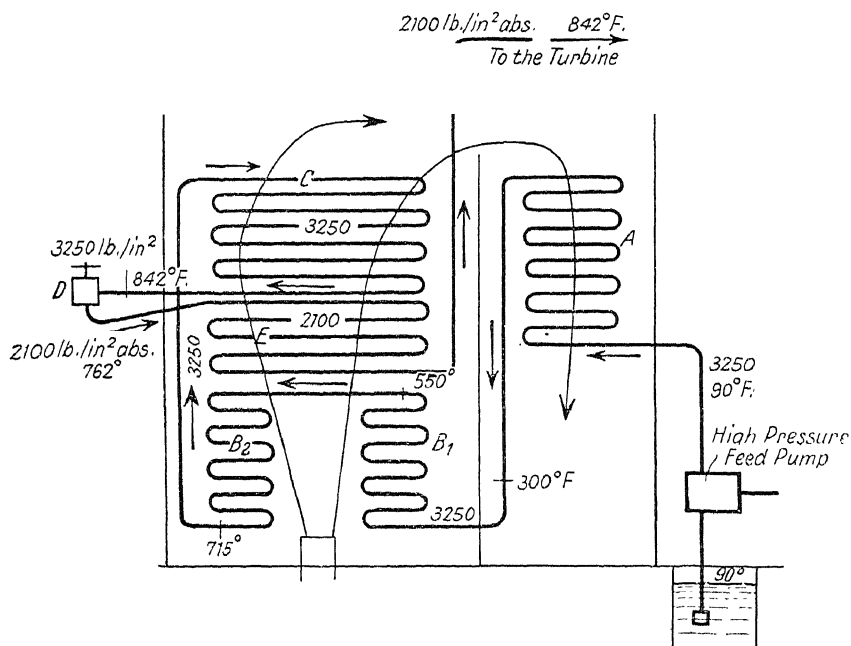


FIG. 286

the large volume occupied by the steam. They would also be excessively heavy, in order to withstand the steam pressure. The steam in excess of that required for immediate use can, however, be stored in a small tank by first condensing it, and can be used later by re-evaporating it. In order to be able to use the steam again, it is essential that only as much cooling water be supplied as is necessary to reduce the steam to water at the saturation temperature, otherwise the change of state is irreversible and accompanied by a consequent loss of pressure and capacity for work. Accumulators, which serve to store the heat only and not the steam energy, are called "warm water accumulators." In the following the steam accumulator alone is considered, and is sometimes known as the "energy" or "power accumulator."

When saturated or superheated steam at the pressure  $p$  is condensed by mixing with cold water, hot water is obtained which can have a

\* See Abendroth and Gleichmann (*Hochdruckdampf* II) regarding tests on the Benson process. Also Josse Tests on the Benson Boiler (*Z.V.d.I.* (1929), No. 51, page 1815.)

maximum temperature  $t_s$ , i.e. the saturation temperature corresponding to  $p$ . The weight  $W_w$  of water at the temperature  $t_0 < t_s$  required to condense 1 lb. of dry saturated steam at the constant pressure  $p$  is calculable, since the heat given up by the steam in condensing it completely to water at  $t_s$ , is the latent heat  $L$ . This heat must be sufficient to raise the water from  $t_0$  to  $t_s$ , so that

$$= W_w(h_s - h_0) \quad . \quad (555)$$

or 
$$W_w = \frac{L}{h_s - h_0} \quad . \quad . \quad . \quad . \quad . \quad . \quad (556)$$

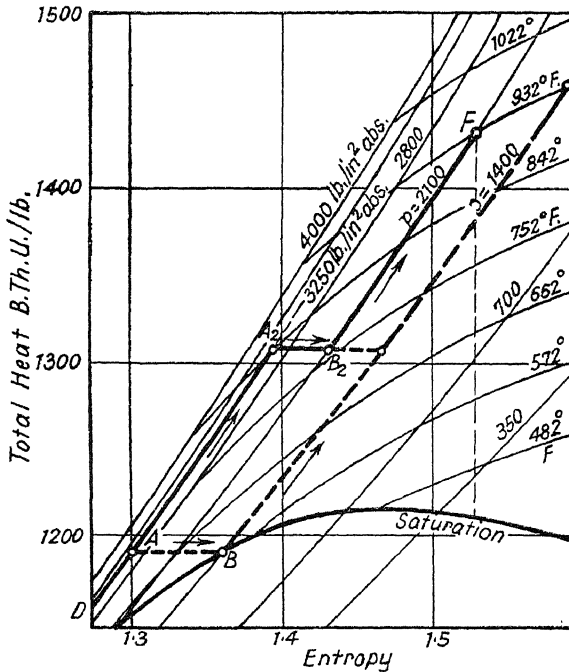


FIG. 287

where  $h_s$  is the sensible heat at the saturation temperature and  $h_0$  at  $t_0$ . For all actual cases it is sufficiently accurate to take

$$W_w = \quad (557)$$

With  $t_0 = 90^\circ \text{ F.}$ ,  $h_0 = 58$ , and with

$p$	15	150	300	750	1500 lb./in. <sup>2</sup> abs.
$t_s$	213.0	358.4	417.4	511.0	596.3° F.
$h_s$	= 181	330.6	394.2	502	567.6 B.Th.U./lb.
$h_s - h_0$	= 123.0	272.6	336.2	444.0	509.6 B.Th.U./lb.
$L$	= 970.2	867.9	815.2	708.9	567.6 B.Th.U./lb.
$W_w$	= 7.88	3.18	2.43	1.60	1.11 lb./lb.

The steam quantities  $W_s$  which can be stored per pound of water at  $90^\circ \text{F.}$ , are with

$$W_s = \frac{1}{W_w}$$

$$W_s = 0.127 \quad 0.315 \quad 0.412 \quad 0.625 \quad 0.9 \text{ lb./lb.}$$

or, per 1,000 lb. of water,

$$1000 W_s = 127 \quad 315 \quad 412 \quad 625 \quad 900 \text{ lb./steam}$$

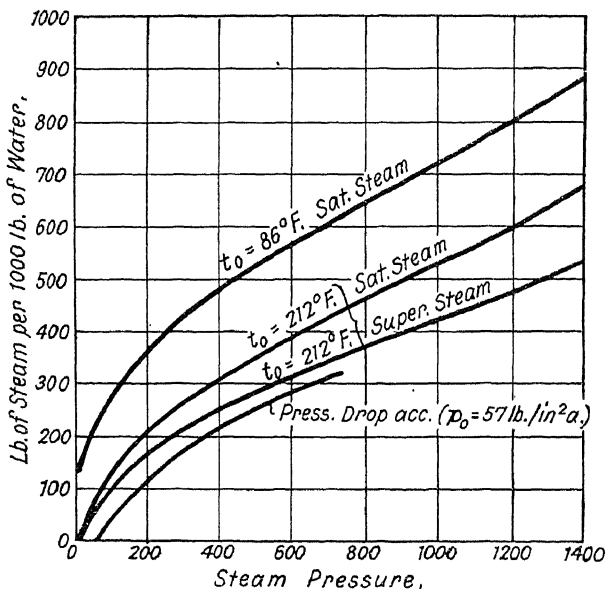


FIG. 288

If the accumulator water is at a higher temperature, as, for example, when it is preheated in an economizer, the water weights  $W_w$ , corresponding to equation (556), are greater and the steam weights  $W_s$  smaller. For  $t_0 = 212^\circ \text{F.}$ , we have, with

$p$	=	150	300	750	1500 lb./in. <sup>2</sup> abs.
$h_s - h_0$	=	150.6	214.2	322	387.6 B.Th.U./lb.
$W_w$	=	5.76	3.80	2.20	1.47 lb.
$1000 W_s$	=	174	264	455	680 lb. steam

In Fig. 288 the values of  $1000 W_s$  have been plotted to a base of pressures between 15 and 1,500 lb./in.<sup>2</sup> abs.

**Superheated steam.** If the steam entering the accumulator is superheated, i.e.  $t_1 < t_s$ , the heat given up in condensing 1 lb. of steam is greater than the latent heat  $L$  by the amount of superheat

$$\text{Hence } L_{su} = W_w(h_s - h_0)$$

so that

$$(558)$$

Since, in tables giving superheated steam values, the total heat  $H_{su}$  is reckoned from 32° F., it is more convenient to replace  $L + h_{su}$  by the equal expression  $H_{su} - h_s$ , so that

$$\frac{H_{\text{eq}} - h_c}{h_s - h_0} \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad (559)$$

$$\text{and} \quad 1000 \, W_s = 1000 \, H_{s'} - h_s \quad (560)$$

Thus, with  $t_1 = 750^\circ \text{ F.}$ ,  $t_0 = 212^\circ \text{ F.}$ ,

	=	15	150	300	750	1500	lb./in. <sup>2</sup> abs.
$H_{su}$	=	1412	1405	1398	1376	1334	B.Th.U./lb.
$h_s$	=	181	330.6	394.2	502	567.6	B.Th.U./lb.
$H_{su} - h_s$	=	1231	1074.4	1003.8	874	766.4	B.Th.U./lb.
1000 $W_c$	=	0	140	214	368	505	lb./lb.

These values, which are smaller than those given with dry saturated steam, have also been plotted in Fig. 288.

There are two definite groups of steam accumulators, called the "pressure drop" accumulator and "constant pressure" accumulator.

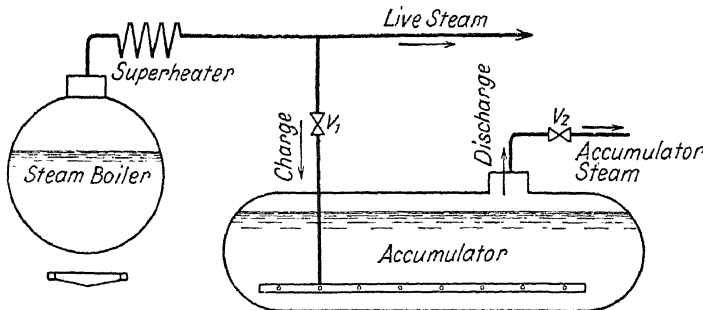


FIG. 289

**Pressure drop accumulator (Ruth's accumulator)** Fig. 289. This consists of a pressure-tight container connected by a pipe on one side to the boiler and on the other side to the steam consumer (engine or heating apparatus). This container is not completely filled with water and the pressure in it is always above atmospheric, even when discharged. When the amount of operating steam required is less than normal, the excess steam from the boiler is delivered through the valve  $V_1$  to the perforated pipe at the foot of the accumulator and condensed as it rises through the water. The steam temperature rises in accordance with the saturation pressure, as more and more steam is supplied, until finally the pressure in the accumulator is the same as in the boiler. The accumulator is then said to be fully charged. Discharging of the accumulator occurs when the amount of operating steam required is above the normal, and is effected by opening the valve  $V_2$ . As more and more steam is withdrawn, the pressure and temperature in the accumulator continuously decreases, due to evaporation, the heat for which comes from the accumulator water.

**Charging of the accumulator.** Let the weight of water contained in the accumulator before charging be  $W_0$  lb., and let the sensible heat be  $h_0$  B.Th.U./lb. Also let the corresponding quantities at the end of charging be  $W_1$  lb. and  $h_1$  B.Th.U./lb., so that the weight of steam introduced and condensed is  $W_s = W_1 - W_0$ . The total liquid heat contained in the accumulator has thus increased by

$$(W_1 h_1 - W_0 h_0) \text{ B.Th.U.}$$

The steam supplied brings with it the total heat, i.e. when dry saturated, the amount

$$(W_1 - W_0) (h_1 + L_1)$$

or, when superheated, the amount

$$(W_1 - W_0) (h_1 + L_1 + h_{su})$$

or

$$(W_1 - W_0) H_{su}$$

Hence, for dry saturated steam,

$$W_1 h_1 - W_0 h_0 = (W_1 - W_0) (h_1 + L_1)$$

or, for superheated steam,

$$W_1 h_1 - W_0 h_0 = (W_1 - W_0) H_{su}$$

These give, for saturated steam,

$$\frac{W_1}{W_0} = 1 - \frac{h_1 - h_0}{L_1} \quad . \quad . \quad . \quad . \quad . \quad .$$

and, for superheated steam,

$$\frac{W_1}{W_0} = \frac{H_{su} - h_0}{H_{su} - h_1} \quad . \quad . \quad . \quad . \quad . \quad .$$

The weight of steam taken up per 1,000 lb. of accumulator water is

$$1000 W_s = 1000 \frac{W_1 - W_0}{W_0}$$

so that, for saturated steam,

$$1000 W_s = 1000 \frac{h_1 - h_0}{L_1}$$

and, for superheated steam,

$$1000 W_s = 1000 \frac{h_1 - h_0}{H_{su} - h_1} = 1000 \frac{h_1 - h_0}{L_1 + h_{su}} \quad . \quad . \quad . \quad (563)$$

These agree with equations (560) and (558) above. The weights of steam absorbed by 1,000 lb. of water are, however, smaller than in the above examples, since the accumulator pressure during discharge cannot fall below the permissible minimum pressure of the steam used in the engine or heater. This pressure amounts to several atmospheres. The accumulator capacity is thus dependent on, and increases with, the

pressure drop. It is also dependent on the boiler pressure and live steam temperature. Thus, with a lowest accumulator pressure of  $p_0 = 60$  lb./in.<sup>2</sup> abs., which gives  $t_0 = 293^\circ \text{F.}$  and  $h_0 = 262$  B.Th.U./lb., and with

$p_1$	=	90	120	200	250	300	750 lb./in. <sup>2</sup> abs.
$h_1$	=	291	313	356	376	394	502 B.Th.U./lb.
$h_1 - h_0$	=	29	51	94	114	132	240 B.Th.U./lb..
$L$	=	895	877	841	822	806	709 B.Th.U./lb.
1000 $W_s$	=	32.4	58.1	112	139	164	337 lb./lb.

These values also have been plotted in Fig. 288.

**Discharging.** During discharge the pressure in the steam space is lower than the saturated steam pressure corresponding to the accumulator temperature. The amount of steam given up by the accumulator per unit time depends on this pressure difference and increases with it. When it is greater than the pressure load  $\rho h$  given by the water depth  $h$ , then complete evaporation of the accumulator contents can occur. During the discharge period the temperature falls, since the latent heat of the steam formed is supplied at the expense of the liquid heat in the accumulator water, giving a corresponding drop in pressure. The temperature drop  $\Delta t$  and the pressure drop  $\Delta p$  are given by the course of the saturation pressure curve. Their relation is also given by the Clapeyron equation (page 427)

$$\frac{\Delta p}{\Delta t} = \frac{777.8L}{T(v_s - \sigma) \times 144} \text{ lb./in.}^2 \text{ } ^\circ \text{F.}$$

For—

$p$	=	60	90	120	200	250	300	750 lb./in. <sup>2</sup> abs.
$t$	=	293	320	341	382	401	417	511 $^\circ \text{F.}$
$v_s - \sigma$	=	7.142	4.862	3.692	2.261	1.817	1.522	0.601 ft. <sup>3</sup> /lb.
$L$	=	916	895	877	841	822	806	709 B.Th.U./lb.
$\Delta p/\Delta t$	=	0.924	1.84	2.31	3.44	4.08	4.70	9.45 lb./in. <sup>2</sup> $^\circ \text{F.}$

For larger pressure drops the steam tables are applied.

Let the weight of water contained in the accumulator at any instant during discharge be  $W$  lb., so that the weight of steam removed between the commencement of discharge and this instant is  $(W_1 - W)$  lb. If now the pressure drops still more, due to the removal of the additional very small amount  $d(W_1 - W) = -dW$ , the heat required to form this weight of steam is  $-LdW$ .

The temperature of the whole accumulator drops by the amount  $dt$  due to this heat removal, and the liquid heat per pound of accumulator water drops by  $dh$ , or by  $Wdh$ , for the total contents.

This gives

$$LdW = Wdh$$

$$\text{or} \quad \frac{dW}{W} = \frac{dh}{L} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (565)$$

For a small measurable pressure drop  $\Delta p$ , therefore,

$$\frac{\Delta W}{W} = \frac{\Delta h}{L}$$

hence, for  $\Delta t = 1$  and  $\Delta h = 1$ ,

$$\frac{\Delta W}{W} = \frac{1}{L}$$

Since  $L$  ranges from 750 to 1,000 B.Th.U./lb., the weight of water for a temperature drop of  $1^\circ \text{F.}$  ranges from  $\frac{1}{750}$  to  $\frac{1}{1000}$  of the total accumulator water.

Integrating equation (565) gives

$$\log_e \frac{W}{W_1} = \int_{t_1}^t \frac{dh}{L} \quad (566)$$

By plotting the values  $\frac{1}{L}$  to a base of liquid heats  $h$ , the value of  $\frac{dh}{L}$  is represented by a narrow strip below the resulting curve. The area between the ordinates at  $h_1$  and  $h$  gives the value  $\log_e \frac{W}{W_1}$ . If this area be expressed as the product of a mean height  $\frac{1}{L_m}$  and the length  $h_1 - h$ , then

$$\log_e \frac{W}{W_1} = - \frac{h_1 - h}{L_m} \quad (567)$$

$$\text{so that} \quad \frac{W}{W_1} = e^{-\frac{h_1 - h}{L_m}} \quad (568)$$

With  $h = h_0$  at the end of discharge,

$$W'_0 = W_1 e^{-\frac{h_1 - h_0}{L_m}} \quad (569)$$

Now it would be expected that at the end of discharge the weight of water contained in the accumulator would be the same as at the beginning of charging period, as given by equation (561), i.e.

$$W_0 = \frac{W_1}{1 + \frac{L_1}{L_m}} \quad (570)$$

Hence,  $W'_0$  should equal  $W_0$ .

From equations (569) and (570), however, these weights are not equal, and their ratio is

$$\frac{W'_0}{W_0} = \frac{1 - \frac{h_1 - h_0}{L_m}}{e^{-\frac{h_1 - h_0}{L_m}}}$$

Since the exponent of  $e = 2.72$  is a relatively small fraction, e.g. with  $t_1 - t_0 = 100$  and  $L_m = 800$ ,

$$\frac{h_1 - h_0}{L_m} = \frac{1}{8},$$



it is sufficient to take, say, the first three terms of the series

$$e^x = 1 + x + \frac{x^2}{2} + \dots,$$

$$\text{where } x = \frac{h_1 - h_0}{L_m}$$

or

$$e^x = 1 + \frac{h_1 - h_0}{L_m} + \frac{1}{2} \left( \frac{h_1 - h_0}{L_m} \right)^2$$

$$\text{from which } \frac{W'_0}{W_0} = \frac{1 + \frac{h_1 - h_0}{L_m}}{1 + \frac{h_1 - h_0}{L_m} + \frac{1}{2} \left( \frac{h_1 - h_0}{L_m} \right)^2}$$

For smaller pressure drops, the series can be stopped at the second term, giving

$$\frac{W'_0}{W_0} = \frac{1 + \frac{h_1 - h_0}{L_1}}{1 + \frac{h_1 - h_0}{L_m}}$$

Since  $L$  rises as the pressure drops,  $L_m > L_1$ , so that

$$\frac{W'_0}{W_0} > 1$$

i.e. at the end of discharge there is more water in the accumulator than at the commencement of charging. This is caused by the fact that the latent heat of the supplied steam is smaller, due to its higher pressure, than the mean latent heat during discharge, which occurs with dropping pressure.

The available work  $\Delta h_1$  per pound of accumulator steam before charging can be determined from the  $H\phi$  diagram when  $p_1$ ,  $t$ , and the back pressure  $p_2$  are known. During discharge the steam pressure drops and the available work per pound of steam decreases until, at the lowest permissible accumulator pressure  $p_0$ , it amounts to  $\Delta h_0$ . The available work of the discharged steam follows from

$$dE = \Delta h \times dW$$

by plotting  $\Delta h$  to a base of  $W$  values and then determining the mean height between the initial and final states.

With initially superheated steam, the available work in the superheated state has to be compared with that in the saturated state, since the accumulator can deliver only saturated steam. This causes the available work to be still further reduced, and an additional disadvantage arises on account of the continuously decreasing pressure of the steam leaving the accumulator and operating in a steam turbine, the thermal efficiency of which is thereby decreased. These conditions have to be considered in a plant used purely for generating power; they do not arise

in heating plants. In combined heating and power units they may be avoided by a suitable switching arrangement.

**Constant pressure accumulator.** In the constant pressure accumulator, the accumulator water is used as feed water for the boiler and not, as in the case of Ruth's accumulator, for the direct generation of steam. The heating of the accumulator water up to the saturation temperature can either be effected by means of heat supplied to the boiler, which is connected to the accumulator (Kiesselbach accumulator), or by heat supplied to the accumulator by excess steam, which condenses on coming in contact with the cold feed water (steam accumulator). Fig. 290 shows the arrangement of the Kiesselbach accumulator.

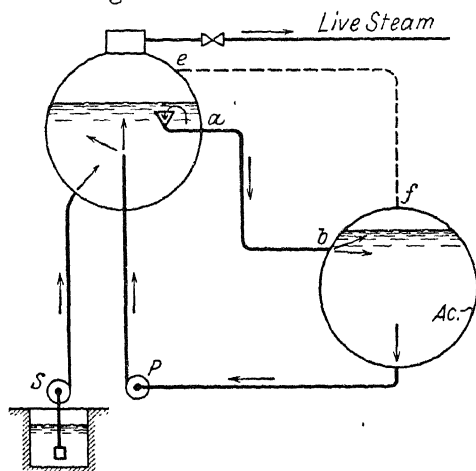


FIG. 290

lies lower than the boiler, and its steam space is kept in free communication with the boiler steam space by the pipe *ef*; its water level is maintained constant by the overflow pipe *ab* from the boiler, to which it is again returned by the pump *P*. In addition, the feed pump *S* supplies cold feed water (or warm feed preheated in an economizer) to the boiler. When the demand for steam is small, the feed pump supplies a large quantity of cold water to the boiler, where it is heated to the boiling point and passed over to the accumulator by means of the pipe *ab*. This process

corresponds exactly to the supply of feed water to a large water space boiler, during a period of reduced steam demand, and assists in cutting down the amount of steam generated. In the Kiesselbach accumulator, the water space can be increased by means of the accumulator volume. When the demand for steam increases the feed pump is stopped and the feed water for the boiler comes entirely from the accumulator, whereby, with the same rate of firing, the amount of steam generated is considerably increased. In this way a uniform rate of firing can be maintained, even with large variations in the steam demands.

**Displacer accumulator.** The arrangement is shown in Fig. 291. The vertical accumulator, in which the water level is maintained constant, contains a steam space above, in which the excess steam, coming from the boiler, is condensed by the colder feed water (preheated in the economizer), so that water at the saturated temperature and at the boiler pressure is present in the upper part of the accumulator. For very high boiler pressures (e.g. 1,500 lb./in.<sup>2</sup> abs.) the steam, before entering the accumulator, is throttled to a lower pressure (e.g. 300 lb./in.<sup>2</sup> abs.). The accumulator is only completely filled with hot water when it is fully charged. When partially charged, more or less of the lower part of the accumulator space is filled with cold water, although a special movable separating wall between the upper hot water and the lower cold water is

not required on account of the difference in density of the water, and on account of the poor conductivity of water. With a normal steam consumption, as much preheated feed water is pumped by the feed pump I and auxiliary pump into the mixing space of the accumulator as is required to maintain the normal steam supply, and as much steam injected as can be converted to boiling water at the boiler temperature by this weight of preheated feed water. The hot water, leaving the accumulator, is pumped continuously to the boiler by the feed pump II. When the steam consumption is reduced below normal and the rate of firing remains unchanged, the weight of steam generated in the boiler is, of course, unchanged. But the accumulator now receives more of this steam, while less passes to the engine or heater. In order to condense the larger steam weight in the accumulator, the auxiliary pump withdraws cold water from the lower part of the accumulator and transfers it to the mixing space, where it condenses the excess steam, and is itself heated to the boiling temperature. This causes the surface separating the hot and cold water in the accumulator to be lowered and the accumulator gradually becomes entirely filled with hot water and the charging process is completed.

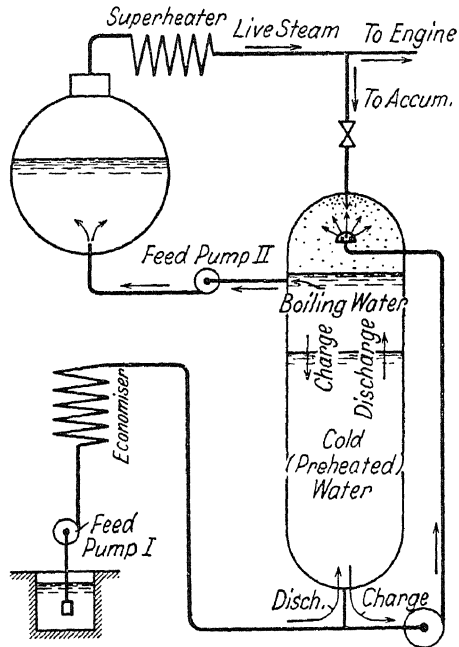


FIG. 201

Discharge occurs when the steam demand increases above the normal, so that less steam enters the mixing space of the accumulator, and hence less spray water is injected. The boiler is then fed solely by the feed pump II, which draws hot water from the accumulator. The feed pump I must, at the same time, deliver a weight of cold water to the lower part of the accumulator equal to the weight of hot water removed from the accumulator. In this way the separating surface in the accumulator rises and the accumulator itself gradually fills with colder water.

The arrangement therefore permits, within certain limits, of a steady rate of steam generation and firing. No loss in the available work of the steam accompanies this type of accumulator.

## REFRIGERATION AND HEAT PUMPS

**Fundamental principles.** In order to lower the temperature of a solid, liquid, or gaseous body weighing  $W$  lb. by  $t^\circ$  F., the heat  $c$  (specific heat) for each degree drop in temperature, and for each pound of the substance must be withdrawn, so that the total heat withdrawn is  $Wct$ .

In order to change a substance from the liquid to the solid state, considerably greater quantities of heat have to be removed after cooling down to the freezing point. The same holds in changing the substance from the vapour to the liquid state. In spite of the very large quantity of heat removed during such changes of state, the temperature remains constant until the last particle has assumed the new state.

Thus, in order to change 1 lb. of water at  $32^\circ\text{F.}$  to ice at  $32^\circ\text{F.}$ , the heat withdrawn (or the cooling effect) amounts to about 144 B.Th.U. Hence, to change 1 lb. of water at  $t^\circ\text{F.}$  to ice at  $t_1^\circ\text{F.}$  the heat which has to be removed is  $Q = (t - 32) + 144 + 0.5(32 - t_1)$ , i.e.  $(t - 32)$  B.Th.U. in cooling from  $t$  to  $32^\circ\text{F.}$ , 144 B.Th.U. to withdraw the latent heat of fusion, and  $0.5(32 - t)$  B.Th.U. in cooling the ice from  $32^\circ\text{F.}$  to  $t_1^\circ\text{F.}$ , since the specific heat of ice is 0.5 B.Th.U./lb. In practice, the round figure 200 B.Th.U., in which an allowance is made for probable losses, can be used tentatively for water at  $70^\circ\text{F.}$  and ice at  $20^\circ\text{F.}$  The theoretical quantity which must be removed is  $38 + 144 + 0.5 \times 12 = 188$  B.Th.U.

The cooling of solid and liquid substances is effected by conducting the heat contained in these bodies to colder bodies. In this way the colder body  $B$  is heated by the warmer body  $A$ . Thus in the artificial manufacture of ice the brine ( $B$ ) cooled below  $32^\circ\text{F.}$  takes up heat from the water ( $A$ ) which has to be frozen, and is thus increased in temperature. The water, or mixture of water and ice, on the other hand, maintains the steady temperature of  $32^\circ\text{F.}$ , in spite of the rapid removal of heat, until all the water has been converted to ice. Thereafter, the ice temperature drops until it is almost equal to that of the brine. The heated brine has to be continuously removed and replaced by fresh cold brine.

The brine itself is brought to the low temperature by means of a still colder substance  $C$  (i.e. the working substance) to which its heat is transferred by conduction, and the warmer brine, coming from the refrigerating chamber, gives up heat continuously to the working substance.

The real refrigerating process is carried out by maintaining this substance ( $C$ ) at a sufficiently low temperature.

Since there are no available colder substances in the surroundings, the heat in the working substance and the heat taken up by it from the brine cannot be removed merely by conduction, so that some other process has to be adopted.

Gases and vapours form convenient working substances, since they can be cooled by converting part of their internal energy to mechanical work, as opposed to the impracticable case of cooling by conduction. The work can either, as in the case of gases, be delivered externally, or, as in the case of vapours, be utilized internally (i.e. for evaporation).

Working substances now used in refrigeration are the vapours of ammonia ( $\text{NH}_3$ ), carbon dioxide ( $\text{CO}_2$ ), and sulphur dioxide ( $\text{SO}_2$ ), and, more recently, methyl chloride ( $\text{CH}_3\text{Cl}$ ) and steam ( $\text{H}_2\text{O}$ ).

When gases are used the necessary low temperatures are only attained by allowing the previously compressed gas (air) to expand adiabatically and deliver work in a cylinder. Wet vapours, on the other hand, attain these low temperatures merely by being throttled from the high to the low pressure, and thus dispense with the expansion cylinder. This means,

of course, that the expansion work which might be used in assisting to drive the compressor is lost. This, however, is relatively unimportant, since the high pressure vapour has always sufficient heat extracted from it to reduce it completely to the liquid state.

After throttling, the liquid is mixed with vapour, but it is only the liquid portion of the mixture which is capable of absorbing heat. While taking up heat from the substance to be cooled, this liquid evaporates

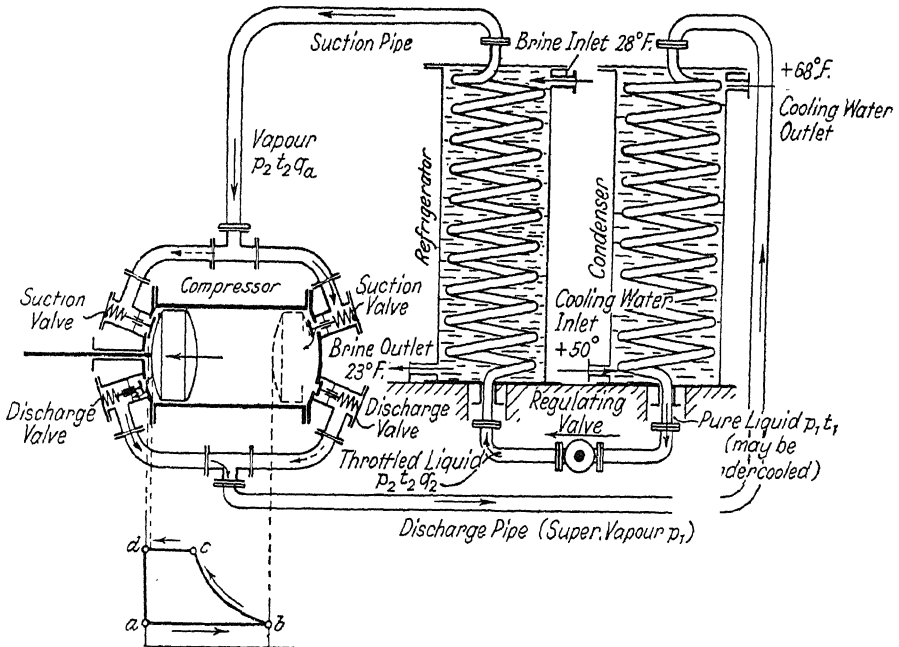


FIG. 292

and maintains its temperature constant until it is completely evaporated. The absorption of heat or refrigerating effect then stops.

Hence, in the case of vapours, the latent heat has an important influence on the refrigerating effect, while, for gases, the specific heat  $c$  is of importance.

**Single stage vapour compression machine.** The compressor draws in almost dry ammonia vapour (or  $\text{SO}_2$  or  $\text{CO}_2$ ) and compresses it to a temperature such that the available cooling water (say between 40 and 70° F.) is capable of reducing it to liquid, while the pressure remains constant. Hence, as shown in Fig. 220, for cooling water at 60° F. the upper pressure limit should be 106 lb./in.<sup>2</sup> abs. The condensation of the vapour is not effected in the cylinder itself but in a coil (or in a system of coils), as shown on the right of Fig. 292. The coil is either immersed in a vessel through which cooling water flows or is open to the air and sprayed with water.

The liquid ammonia now flows from the condenser through a hand-operated valve (regulating or throttling valve) into a second coil connected to the suction pipe of the compressor, so that the pressure in this

coil is equal to the suction pressure. This pressure is conditioned by the temperature in the second coil, and this temperature in turn is conditioned by the requirements of the refrigerating process, i.e. it is predetermined. For machines which are used for cooling rooms or making ice, it ranges from 20 to 10° F. (Normal values of the brine and cooling water have been marked in Fig. 292.) Hence, for ammonia, the suction pressure obtained by throttling is about 40 lb./in.<sup>2</sup> abs.

It is in the second coil that the actual refrigerating process occurs. This coil is immersed in a vessel through which brine flows, and this brine gives up part of its heat to the ammonia flowing through the coil. The ammonia evaporates at constant temperature, while the brine, due to the removal of heat, drops in temperature from about 28° F. to about 23° F. The brine chamber, with evaporator coil, is called the "refrigerator" or "evaporator."

The cold brine is then used for its particular duty (such as the production of ice, cooling of rooms, chilling, etc.) by taking up heat from the substance to be cooled, so that its temperature is increased to 28° F. It then returns to the brine tank at this temperature and is again cooled. The vaporized ammonia is again compressed, condensed, and throttled, so that it also can be re-used in the brine tank as the refrigerating agent.

The process is illustrated diagrammatically in Fig. 292. It commences at the point where the almost dry ammonia vapour leaves the brine tank at the pressure  $p_2$  and the temperature  $t_2$ . The vapour is drawn into the cylinder during the suction stroke and is compressed and discharged on the return stroke. It then flows, as superheated vapour at the pressure  $p_1$ , to the condenser, where it is reduced to liquid ammonia at the temperature  $t_1$  corresponding to the pressure  $p_1$ . In some cases the liquid is cooled below  $t_1$  and is then said to be undercooled. The liquid then proceeds through the regulating valve to the brine tank. Its pressure is reduced by throttling to  $p_2$  and its temperature to  $t_2$  (i.e. the saturation temperature corresponding to  $p_2$ ), and a small fraction of the liquid is evaporated. The brine entering at 28° F. serves to evaporate the ammonia and is thus cooled sufficiently to enable it to be used as a cooling agent.

The work required, neglecting frictional losses, is given by the indicator diagram of the compressor.

The compressor draws in vapour at  $p_2$  lb./in.<sup>2</sup> abs. with a wetness fraction of about 5 per cent (line  $ab$ , Fig. 293). During the subsequent compression (line  $bc$ ) the vapour follows approximately the adiabatic law

$$pv^\gamma = C$$

in which for

Ammonia  $\gamma = 1.30$

Carbon dioxide  $\gamma = 1.30$

Sulphur dioxide  $\gamma = 1.25$

In these it has been assumed that compression occurs in the superheat field, i.e. that the vapour is almost dry at the beginning of compression.

If the condition of the vapour approaches the critical state, as occurs with CO<sub>2</sub>, the adiabatic law of compression deviates considerably from  $pv^\gamma = C$ .



The heat given up by the brine in effecting this change in quality is

$$Q_k = L_2(q_a - q_2)$$

and is the refrigerating effect.

Inserting the value of  $q_2$  gives

$$Q_k = q_a L_2 - h_1 + h_2$$

If it were possible to bring the liquid from the high to the low pressure without partial evaporation ( $q_2 = 0$ ), and if, in the evaporator, all the liquid were converted to vapour ( $q_a = 1$ ), then  $Q_k = L_2$ , i.e. the refrigerating effect would equal the latent heat at the lower temperature limit. Actually, however,  $Q_k < L_2$  and, in the best case, ( $q_a = 1$ )

$$Q_k = L_2 - (h_1 - h_2)$$

i.e. the refrigerating effect is less than the latent heat at the suction pressure by the difference of the upper and lower sensible heats.

Since the refrigerating effect per pound of flow is  $Q_k$  B.Th.U., it follows that the weight of vapour required for a refrigerating effect of 1 B.Th.U. is  $\frac{1}{Q_k}$  lb. The stroke volume of the compressor is conditioned by the volume of vapour drawn in during suction. If now  $v_2$  is the volume of 1 lb. of dry vapour at the suction pressure  $p_2$  (given by tables), the volume occupied by 1 lb. of vapour with  $q_a = 0.95$  is  $0.95v_2$  ft.<sup>3</sup>. Hence, for a refrigerating effect of 1 B.Th.U., the suction volume is

If the refrigerating effect per hour is known this enables the volume dealt with per hour to be determined, and hence also the stroke volume (theoretically necessary) for a given number of rev./min.

**Comparison of NH<sub>3</sub>, SO<sub>2</sub>, and CO<sub>2</sub> machines.** Using the equation

$$Q_k = 0.95L_2 - (h_1 - h_2)$$

for the refrigerating effect per pound we have, with  $t_1 = 18^\circ\text{F.}$  and  $t_2 = 68^\circ\text{F.}$ , from tables,

$$\text{For NH}_3 \quad Q_k = 0.95 \times 578 - (33.6 + 12.7) = 502.7 \text{ B.Th.U./lb.}$$

$$\text{For SO}_2 \quad Q_k = 0.95 \times 167.5 - (12.0 + 4.6) = 142.4 \text{ B.Th.U./lb.}$$

$$\text{For CO}_2 \quad Q_k = 0.95 \times 109 - (23.1 + 7.5) = 73.4 \text{ B.Th.U./lb.}$$

The greatest refrigerating effect for the same weight of vapour is thus obtained when using ammonia.

For the same speed and the same refrigerating effect the ratio of the stroke volumes is given by  $\frac{v_2}{Q_k}$ . These are (for the temperature limits used above)—

$$\text{For} \quad \frac{6.42}{502.7} = \frac{1}{78.3}$$

$$\text{For SO}_2 \quad \frac{1}{142.4} = \frac{1}{29.2}$$

$$\text{For CO}_2 \quad \frac{0.216}{73.4} = \frac{1}{340}$$



The ratio of the stroke volumes is thus

$$\frac{1}{78.3} : \frac{1}{29.2} : \frac{1}{340}$$

or as

$$4.34 (\text{NH}_3) : 11.6 (\text{SO}_2) : 1 (\text{CO}_2)$$

Hence the smallest compressor is that using carbon dioxide, while the largest is that using sulphur dioxide.

The upper and lower pressure limits for the temperatures assumed above are—

$$\text{With } \text{NH}_3 \quad p_1 = 125 \quad p_2 = 45.5 \text{ lb./in.}^2 \text{ abs.}$$

$$\text{With } \text{SO}_2 \quad p_1 = 47.5 \quad p_2 = 16.2 \text{ lb./in.}^2 \text{ abs.}$$

$$\text{With } \text{CO}_2 \quad p_1 = 825 \quad p_2 = 408 \text{ lb./in.}^2 \text{ abs.}$$

Hence by far the highest pressures are required in the  $\text{CO}_2$  machine, while the  $\text{SO}_2$  machine operates with the lowest pressures.

**Refrigerating effect per horse-power hour.** The important quantity in determining the economy of a refrigerating plant is the power required for a given refrigerating effect. In place of the power required to effect a given refrigeration, it is now customary to give the refrigeration effect produced by 1 h.p. acting for an hour. Thus if  $Q$  is the total refrigeration effect for 1 hour, and  $N_i$  is the indicated compressor power, this quantity is given by  $Q/N_i$ .

The theoretical work of compression per pound of vapour is

$$E = \frac{\gamma}{\gamma-1} p_2 v_2 \left[ \left( \frac{p_1}{p_2} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right] \text{ ft. lb.}$$

The theoretical refrigerating effect obtained with this work is

$$Q_k = q_a L_2 - (h_1 - h_2) \text{ B.Th.U.}$$

The refrigerating effect obtained with 1 ft. lb. is thus  $\frac{Q_k}{E}$ .

Since 1 h.p. hour =  $550 \times 3600$  ft. lb., the refrigerating effect from this work is

$$550 \times 3600 \frac{Q_k}{E} \text{ B.Th.U./h.p. hour.}$$

This is the required value  $Q/N_i$ . Inserting the values for  $Q_k$  and  $E$  gives

$$\frac{Q}{N_i} = 550 \times 3600 \frac{q_a L_2 - (h_1 - h_2)}{\frac{\gamma}{\gamma-1} p_2 v_2 \left[ \left( \frac{p_1}{p_2} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right]}$$

Taking the conditions in the examples above, we have, for  $\text{NH}_3$  with

$$p_1 = 144 \times 125, \quad p_2 = 144 \times 45.5, \quad v_2 = 0.95 \times 6.42 \text{ ft.}^3/\text{lb.}$$

$$E = \frac{1.3}{0.3} \times 144 \times 45.5 \times 0.95 \times 6.42 \left[ \left( \frac{125}{45.5} \right)^{0.231} - 1 \right] = 46,000 \text{ ft. lb./lb.}$$

for  $\text{SO}_2$

$$E = \frac{1.25}{0.25} \times 144 \times 16.2 \times 0.95 \times 4.87 \left[ \left( \frac{47.5}{16.2} \right)^{0.200} - 1 \right] = 12,960 \text{ ft. lb./lb.}$$

and for  $\text{CO}_2$

$$E = \frac{1.30}{0.3} \times 144 \times 408 \times 0.95 \times 0.216 \left[ \left( \frac{825}{408} \right)^{0.231} - 1 \right] = 9,310 \text{ ft. lb./lb.}$$

These give, with the above values of  $Q_k$ , for

	$\text{NH}_3$	$\text{SO}_2$	$\text{CO}_2$
$\frac{Q}{\text{h.p.}}$	21,630	21,800	15,600

Other values of the upper and lower temperature limits give other refrigerating effects. The upper limit is conditioned by the cooling water, while the lower limit depends on the duty of the refrigerator. The assumed values of  $68^\circ \text{F.}$  and  $18^\circ \text{F.}$  may be regarded as mean values.

The values calculated for  $\text{CO}_2$  are not so reliable as the others, since the  $\text{CO}_2$  vapour is near the critical state. Under the conditions stated above, the  $\text{CO}_2$  machine shows the smallest refrigerating effect.

The refrigerating effect actually attained, or attainable, in practice is less than the theoretical. Thus, under favourable conditions, the ammonia machine shows a value of 18,000 B.Th.U./h.p. hour.

When efficient machines, using different vapours, are compared it is found that the refrigerating effect does not vary greatly, but other factors have to be taken into account when the relative merits are considered.

The most important causes which render the practical refrigerating effect less than the theoretical are as follows—

1. The unavoidable temperature difference between the working substance and the cooling water, and also the brine.
2. Valve resistances. These are shown by deviations from the ideal indicator diagram.
3. Leaks at the piston and valves.
4. Losses by heat conduction and radiation.

On account of frictional losses at the working parts the power required is higher than that shown by the indicator. The mechanical efficiency lies between 0.85 and 0.95.

**Undercooling.** The vapour temperature  $t_1$  in the condenser cannot be less than that of the supply water. On the other hand, the liquefied ammonia (or  $\text{SO}_2$  or  $\text{CO}_2$ ) can be cooled below the saturation temperature, corresponding to the vapour pressure in the condenser. The temperature  $t'_1$  to which the liquid can be cooled cannot, of course, be less than the lowest temperature of the cooling water. The undercooling  $t_1 - t'_1$  is in this way not greatly different from the water temperature difference.

In the examples given above it has been assumed that no undercooling occurs, so that the refrigerating effect is given by

$$Q_k = q_a L_2 - (h_1 - h_2)$$

where  $h_1$  is the liquid heat corresponding to the upper vapour pressure. Actually, however, this heat may be smaller by the amount  $c(t_1 - t'_1)$

giving  $h'_1 = h_1 - c(t_1 - t'_1)$

in which  $c$  is the specific heat of the liquid under the condenser pressure. This, then, gives, for the refrigerating effect,

$$Q_k = q_a L_2 - (h'_1 - h_2)$$

and is thus greater than without undercooling, so that an improvement is thus effected.

In order to increase the undercooling, special coolers are also sometimes used. The liquid leaving the condenser passes through a second coil before passing to the regulating valve.

Liquid coolers have proved particularly useful in the case of CO<sub>2</sub> machines.

**Representation on the heat diagram.** The process of refrigeration can be shown very clearly by means of the  $T\phi$  diagram (Fig. 295). Starting

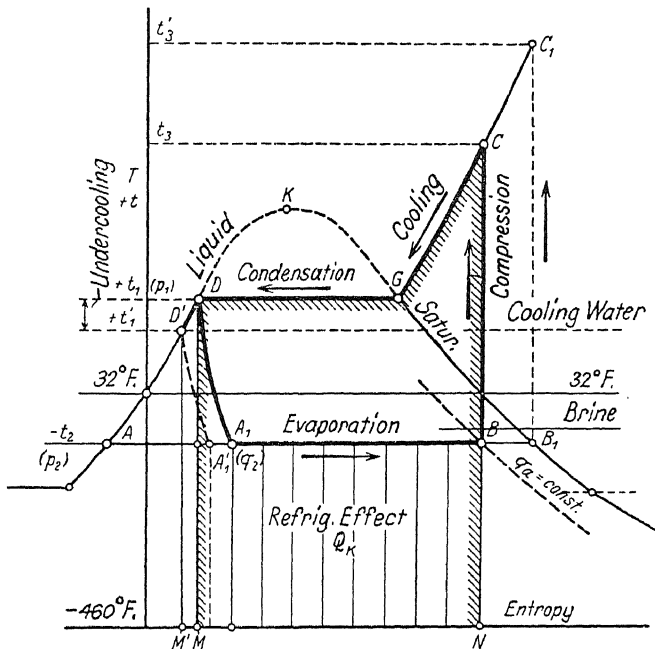


FIG. 295

at the point  $B$ , which represents the state of the vapour at the beginning of compression, where the temperature is  $-t_2$ , the pressure  $p_2$ , and the quality  $q_a$  (almost dry), adiabatic compression is represented by  $BC$ , at the end of which the vapour is superheated to the temperature  $t_3$ . If the vapour is dry at the beginning of compression (point  $B_1$ ), the temperature  $t'_3$  after compression is still higher. In the cooler the vapour is first brought to the saturation limit curve at  $G$ , as shown by  $CG$ . A further abstraction of heat results in condensation at constant pressure  $p_1$ , line  $GD$ . At  $D$  the vapour is completely liquefied at the saturation state. Further cooling (undercooling) follows the liquid limit curve very closely, and leads to  $D'$  at the temperature  $t'_1$ , which is only slightly higher than the cooling water temperature. The cooled liquid is now throttled, as



**Single stage throttling.**  $B$  is the state of the  $\text{CO}_2$  at the beginning of compression and is approximately dry at  $-40^\circ \text{F.}$  and  $140 \text{ lb./in.}^2 \text{ abs.}$  This substance is now compressed adiabatically (line  $BC$ ) to  $425 \text{ lb./in.}^2 \text{ abs.}$  and  $63^\circ \text{F.}$  Now before this superheated vapour is drawn into the second stage compressor, it must be cooled down to about  $21^\circ \text{F.}$  (the saturation temperature corresponding to  $425^\circ \text{F.}$ ). This is not possible with an available water temperature of  $50^\circ \text{F.}$  Hence either the compression pressure in the first stage would have to be increased or the cooling carried down to  $50^\circ \text{F.}$  only. These expedients, however, increase the work of compression, so that most of the advantage of the two stage arrangement is lost. On the other hand, some of the low temperature brine might be used as a cooling agent in place of the available water, but this reduces the refrigerating effect. Assuming that the superheated vapour in the first stage were cooled to  $D$  (the saturation point), it could be drawn into the second cylinder and compressed adiabatically to  $850 \text{ lb./in.}^2 \text{ abs.}$  (line  $DE$ ). The compression temperature rises to  $111^\circ \text{F.}$  The vapour then proceeds to the cooler or condenser, where it is completely liquefied at about  $68^\circ \text{F.}$  (point  $G$ ). Thereafter the liquid at  $850 \text{ lb./in.}^2$  is throttled and passes to the evaporator at a pressure of  $140 \text{ lb./in.}^2 \text{ abs.}$  The quality at entrance to the evaporator is  $q_2 = 0.47$ , and the useful refrigerating effect is given by the area below  $AB$ , or approximately,  $Q_k = (1 - q_2)L_2$ . The work done is given by the area  $BCDEGRB$ .

**Two stage throttling.** In the two stage throttling arrangement, first applied by Linde,\* an intermediate receiver is fitted, into which the vapour from the low pressure cylinder and throttled high pressure liquid enter. This cools the vapour from the low pressure cylinder to the saturation temperature without the use of cooling water, so that compression of the vapour, drawn from the receiver, now follows the line  $DE$  in the high pressure cylinder.

The weight of liquid in the receiver, after the first throttling  $GA'$ , is  $1 - x_1 = 0.69$ , and this liquid is passed from the receiver through a second throttle valve to the evaporator, where its quality on entering is  $q'_2 = 0.22$ , giving a liquid content of  $1 - x'_2 = 0.78$ . This liquid takes up the heat  $(1 - q'_2)L_2$  per pound in the evaporator, but, since its actual weight is  $(1 - x_1) \text{ lb.}$ , the refrigerating effect is

$$\begin{aligned} Q_k &= (1 - x_1) (1 - q'_2)L_2 \\ &= 0.69 \times 0.78 \times L_2 = 0.538L_2 \end{aligned}$$

The refrigerating effect with single stage throttling is

which is nearly the same as with two stage throttling.

The compression work is given by the work  $E_{II}$  done on 1 lb. in the second stage compression in compressing from  $p_3$  to  $p_1$  (line  $DE$ ), together with the first stage compression work  $E_I$ , where the weight is only  $(1 - x_1) \text{ lb.}$  (line  $BC$ ). The value  $E_{II}$  is represented by the area  $DEFGMD$ , while  $E_I$  is given by multiplying the area  $BCDMRB$  by  $1 - x_1$ . The total

\* *Geschichte der Gesellschaft für Lindes Eismaschinen, Wiesbaden (1929), page 69.*

work of compression is thus considerably less than in the single stage process.

Using an ammonia machine, operating on this process, Linde succeeded, in 1898, in maintaining a temperature of  $-49^{\circ}\text{F.}$ , while, in 1928, with a three stage machine, also using ammonia, the temperature produced was  $-103^{\circ}\text{F.}$  Compound machines have now been applied in separating gas mixtures at low temperatures, and they also prove of use on ships plying in the tropics, where the sea water temperature is high.

Three stage compression plants have been used to produce solid carbon dioxide, but so far with only one throttling stage. The carbon dioxide snow is produced by throttling liquid carbon dioxide from 925 lb./in.<sup>2</sup> abs. ( $+77^{\circ}\text{F.}$ ) to 15 lb./in.<sup>2</sup>, whereby the temperature drops to  $-110^{\circ}\text{F.}$  and about 25 per cent of the liquid is converted to snow. (cf. page 437, Fig. 231.) According to Plank, the yield of snow with multistage throttling would be 34 per cent, which could be raised to 40 per cent if use were made of the cold  $\text{CO}_2$  vapour formed along with the snow.

**Tables and diagrams.** Values of the properties of substances used in refrigeration processes, together with useful information on testing refrigerators, will be found in *Regeln für Leistungsversuche an Kältemaschinen und Kühlanlagen nebst Erläuterungen, Tabellen und Diagrammen*. (Rules for Testing Refrigerators and Cold Storage Plants along with Explanations, Tables, and Diagrams.) The following tables for  $\text{NH}_3$ ,  $\text{SO}_2$ ,  $\text{CO}_2$ , and  $\text{CH}_3\text{Cl}$  are contained in the Appendix of this work—Vapour properties for saturated vapour, theoretical refrigeration effect in  $K$  cal. per meter<sup>3</sup> and also per h.p. hour. Tables of the properties of solutions (brine) of sodium chloride, magnesium chloride, and calcium chloride. The following diagrams are also given. Three  $Hp$  diagrams for ammonia, sulphur dioxide, and methylchloride, with logarithmic pressure scale. An  $Hp$  diagram for carbon dioxide (by Plank-Kuprianoff), and the large diagram by the Bureau of Standards showing the properties of ammonia with  $\log p$  as ordinates and  $H$  as abscissae, and containing lines of constant volume, constant quality, and constant entropy.

**Water vapour refrigeration.** Water can be used for refrigerating processes in the same way as the other substances  $\text{NH}_3$ ,  $\text{SO}_2$ , and  $\text{CO}_2$  already discussed. The difficulty of the freezing of water at  $32^{\circ}\text{F.}$  can be overcome by adding a salt such as  $\text{NaCl}$ , and this does not greatly alter the other properties of the water. The special properties of water vapour, however, make its application to refrigerating processes particularly difficult, and it is only within recent times that practicable refrigerators, using water only, have been constructed.

The latent heat of water, which is of importance in determining the refrigerating effect, is  $L = 1070.6$  B.Th.U./lb. at  $32^{\circ}\text{F.}$  At lower temperatures  $L$  is still higher, and the mean value  $L = 1080$  may be used in tentative calculations. If it be assumed that evaporation occurs at  $18^{\circ}\text{F.}$ , that the quality after evaporation is 0.95, and that the water leaves the condenser at  $70^{\circ}\text{F.}$ , the refrigerating effect, per pound of water, is approximately (page 553)

$$\begin{aligned} Q_k &= 0.95 \times 1080 - (70 - 18) \\ &= 974 \text{ B.Th.U./lb.} \end{aligned}$$

and is thus almost twice as large as that of 1 lb. of ammonia.

The volume of 1 lb. of dry saturated steam at 18° F. is 6210 ft.<sup>3</sup> (=  $v_2$ ). The ratio  $v_2/Q_k$ , which gives the suction volume per 1 B.Th.U. refrigerating effect, is thus (for dry vapour)

$$\frac{v_2}{Q_k} = \frac{6210}{974} = \frac{1}{0.157}$$

The corresponding value for ammonia (page 554) is  $\frac{1}{78.3}$  which is  $\frac{74.3}{0.157} = 473$  times smaller. Hence, for the same refrigerating effect, a reciprocating compressor for the water vapour machine would have to have a volume 473 times greater than that of a machine using ammonia. This, therefore, rules out the possibility of using a reciprocating compressor when the working substance is water vapour. In place of the mechanical compression, efforts were previously made to absorb the vapour coming from the evaporator by means of sulphur dioxide, but, in doing this, the chief advantage (i.e. using water alone) was lost. One practicable method of compressing the cold vapour, however, is to use a steam jet pump with steam at any convenient pressure, or waste steam at atmospheric pressure. This process was successfully carried out by Josse and Gensecke.\*

The pressure of the cold vapour, namely, 0.18" Hg at 32° F. and 0.098" Hg at 18° F., necessitates the establishment of high vacua, in place of the super atmospheric pressures employed in other vapour compression machines. The result is that air tends to leak into the system, on account of faulty joints and because of the air held in solution in the water. This air has to be withdrawn continuously from the evaporator, compressed, and discharged to the condenser, from whence it is removed to the atmosphere by a water jet pump. The extra work done by this jet pump is not required in the case of other vapour machines.

The diagrammatic arrangement shown in Fig. 297 will serve to show how the various organs operate. It consists of an evaporator *A* (similar to those employed in other vapour machines) in which the refrigeration is effected, and of a condenser *B* in which the vapour coming from *A* is condensed. The compression of this vapour, from the low pressure existing in *A*, to the higher pressure existing in *B*, is effected by means of a steam jet pump *C*. The discharge of the condensate and air from the condenser to the atmosphere is carried out by the water jet pump *D*, the water of which also serves as cooling water in the condenser.

The machine is started up as follows: A low pressure must first be established in *B* and *A* by the action of the jet pump *D*, after which the jet pump *C* is started so as to reduce the pressure still further in *A*. If the water in *A* has an initial temperature of 60° F. it would commence to boil when the pressure drops to 0.256 lb./in.<sup>2</sup> abs. At this stage the brine (the cooling of which is the main function of the process) can be circulated over the evaporator coils. This brine supplies the heat necessary to evaporate the water and is thus cooled. At the same time the evaporating water, and the water vapour formed, become colder and colder. If it is possible to lower the pressure in *A* to 0.0486 lb./in.<sup>2</sup> abs., and to maintain

\* *Z.V.d.I.* (1911); *Z. des Kälteind* (1911); *Z. ges Turb. wesen* (1913); *Eis-u. Kälteindustrie* (1915).

this by means of the jet pump, the water and vapour temperature can be lowered to 18° F., provided no air enters *A*.

The vapour withdrawn from *A* proceeds to the condenser *B*, where it is condensed by cooling water flowing over the tubes.\* If the tem-

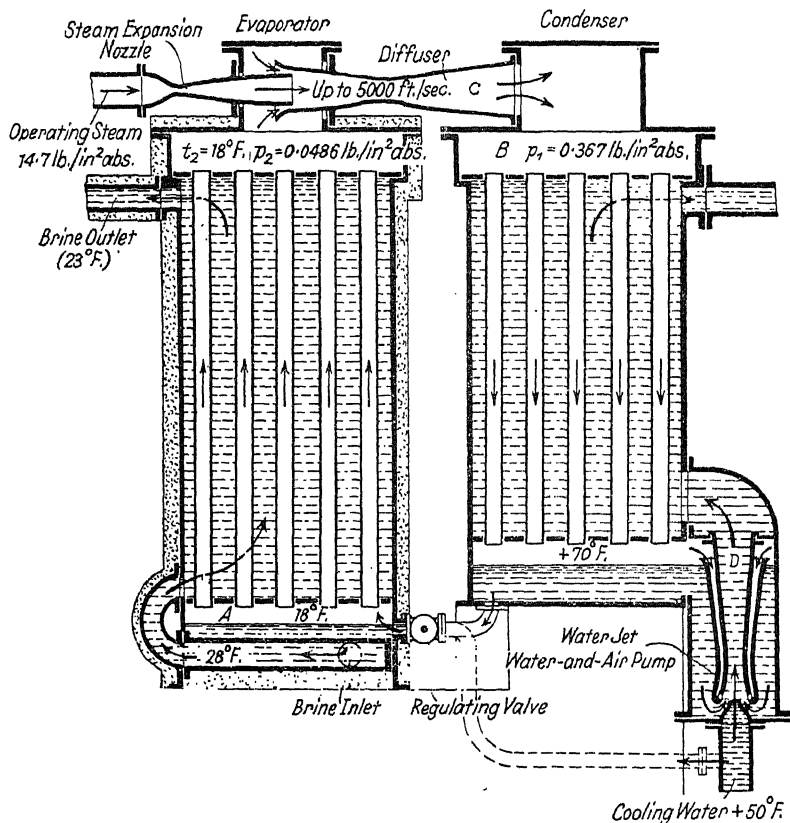


FIG. 297

perature of the condensate, which collects at the bottom, is 70° F., the condenser pressure is 0.367 lb./in.<sup>2</sup> abs., but if any air is present this pressure will be higher. Hence the steam jet pump has to compress and discharge the cold vapour from 0.0486 lb./in.<sup>2</sup> abs. to 0.367 lb./in.<sup>2</sup> abs. (or more).

This enables the theoretical minimum work to be found under air-free conditions. Thus, let the condenser pressure be  $p_1$ , and let the pressure and specific volume in the evaporator be  $p_2$  and  $v_2$ , then the compression work is (page 555)

$$E = -\frac{\gamma}{\gamma-1} p_2 v_2 \left[ \left( \frac{p_1}{p_2} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right] \text{ ft. lb. } \quad (571)$$

\* In the practical arrangement the cooling water generally flows through the tubes.



The refrigerating effect is given closely by (page 554)

$$Q_k = q_a L_2 - (h_1 - h_2) \quad (572)$$

where  $q_a$  is the quality of the water vapour as it leaves the brine tank.

The refrigerating effect per horse-power hour follows from equations (571) and (572), as shown on page 555, giving

$$\frac{Q}{N} = 1,980,000 \frac{Q_k}{N} \quad (573)$$

This value is directly comparable with the values of  $Q/N_i$  obtained on page 556 for  $\text{NH}_3$ ,  $\text{SO}_2$ , and  $\text{CO}_2$ . Taking the temperatures in the evaporator and condenser as given above, and assuming  $\gamma = 1.3$ , we have

$$E = \frac{1.3}{0.3} \times 144 \times 0.0486 \times 0.95 \times 6210$$

$$\left[ \left( \frac{0.367}{0.0486} \right)^{\frac{0.3}{1.3}} - 1 \right] = 106,200 \text{ ft. lb.}$$

With  $Q_k = 974$  we have

$$\frac{Q}{N_i} = \frac{1,980,000 \times 974}{106,200} = 18,740 \text{ B.Th.U./h.p. hour}$$

This value is about 13 per cent less than that given by  $\text{NH}_3$  (21,630) and  $\text{SO}_2$ , but about 20 per cent greater than for  $\text{CO}_2$  between the same temperature limits. Theoretically, therefore, water shows no great disadvantage as compared with  $\text{NH}_3$  and  $\text{SO}_2$ . But it must be remembered that, even if no air is present in the evaporator, the presence of air in the condenser is unavoidable, since the operating steam in the jet pump always carries some air with it. The removal of this air from the condenser involves additional work, which is not required in other vapour machines. The treatment of the not unimportant effect of the presence of air in the evaporator is too involved, however, to be dealt with in this text. It is not absolutely essential that the condensate in the condenser should be passed over to the evaporator through the throttle valve, as is the case in machines using other vapours. The supply to the evaporator can come from the main, which is colder than the condensate. Undercooling, with its accompanying gain, is thus automatically obtained, but the air content of the fresh water is higher, so that additional pumping work is necessary.

The efficiency of a steam jet pump is considerably lower than that of the  $\text{NH}_3$  reciprocating pump, but this disadvantage is of no account when cheap waste steam is available.

The following are the results obtained from a machine designed by Josse and Gensecke\*—

236,000 B.Th.U. per hour refrigerating effect.

890 lb. of steam used per hour at 14 lb./in.<sup>2</sup> abs.

27.9° F. entering temperature of brine at brine tank.

23.5° F. leaving temperature of brine at brine tank.

9.1 h.p. required to operate the two electric motors used in driving the brine and circulating water pumps.

\* *Eis und Kälteindustrie* (1915), page 9. Stetefeld. Neuere Ausführung der Wasserdampfstrahlkältemaschine.

An attempt is now being made to carry out the process in two stages, so as to increase the efficiency.

**The heat pump.** In the application of heating processes, such as the heating of buildings, supplying of hot water, evaporation and distillation of liquids, evaporation of solutions and the drying of solid substances, the heat used must be at a higher temperature  $t_1$  than that ( $t_0$ ) of the surroundings. There are two totally different methods of raising heat to a higher temperature. The first and earliest known method consists in the application of those chemical reactions, usually combustion processes, which are accompanied by the evolution of heat at high temperatures. The second process, which has only recently been applied in practice, is based on the property of gases and vapours of becoming heated when compressed. As shown on page 108, the temperature of a gas increases when compressed without giving up or receiving heat from  $p_0$  to  $p_1$ , in the ratio

$$\frac{T_1}{T_0} = \left( \frac{p_1}{p_0} \right)^{\frac{\gamma-1}{\gamma}} \text{ with } \gamma = 1.4$$

Thus, with

$\frac{p_1}{p_0}$	=	1.2	1.5	2	3	8	12	30
$\frac{T_1}{T_0}$	=	1.054	1.123	1.220	1.369	1.866	2.034	2.642

and with  $t_0 = 70^\circ \text{ F.}$  or  $T_0 = 530$ ,

$t_1 - t_0$	=	29	65	116	195	458	549	$871^\circ \text{ F.}$
-------------	---	----	----	-----	-----	-----	-----	------------------------

If saturated steam be compressed in such a way that it remains saturated, its temperature increases with pressure in accordance with the steam tables, e.g. starting with steam at  $212^\circ \text{ F.}$  and  $14.7 \text{ lb./in.}^2 \text{ abs.}$  and compressing to

$p_1 = 1.2 \times 14.7 \quad 1.5 \times 14.7 \quad 2 \times 14.7 \quad 3 \times 14.7 \quad 8 \times 14.7 \quad 22 \times 14.7 \quad 30 \times 14.7$   
gives

$t_1$	=	219.6	231.3	247.3	271.2	337.3	368.8	$451.2^\circ \text{ F.}$
-------	---	-------	-------	-------	-------	-------	-------	--------------------------

or

		7.6	19.3	35.3	59.2	125.3	156.8	$239.2^\circ \text{ F.}$
--	--	-----	------	------	------	-------	-------	--------------------------

Now when gases or vapours are compressed, mechanical work  $E_1$  must be supplied, and this mechanical work, which is transformed to internal energy in the compressed substance, is the real cause of heating (page 106). This method, therefore, of producing heat at a higher temperature could be described as a mechanical generation of heat as opposed to the usual chemical generation of heat.

Mechanical work could, of course, be used directly in generating heat. This occurs, for example, when solids are subjected to friction or impact, or when liquids or gases are set in turbulent motion. It is known that the generation in this way of 1 B.Th.U. requires an expenditure of 778 ft. lb. of mechanical energy, or, if the latter be expressed in heat units, 1 B.Th.U. of heat is obtained at the expense of 1 B.Th.U. of energy. This is the law of the conservation of energy (page 150).

Obviously this law also holds in the mechanical compression of so that it is impossible to obtain more than 1 B.Th.U. of heat from 1 B.Th.U. of mechanical work. But existing quantities of heat, such as heat contained in our surroundings, which are at a temperature of about  $70^{\circ}\text{F.}$ , say, can be brought to a higher temperature, such as  $200^{\circ}\text{F.}$  or  $2000^{\circ}\text{F.}$ , without an accompanying change in energy, since the energy of 100 B.Th.U. of heat is the same whether this heat has a temperature of  $70^{\circ}\text{F.}$  and is contained, say, in 5 ft.<sup>3</sup> of water or is at  $2000^{\circ}\text{F.}$  and contained in hot gases. But, from the second law, it is impossible to bring an existing quantity of heat to a higher temperature without a definite expenditure of mechanical work. This expenditure increases with the temperature rise, in a similar manner to the increase of work necessary in a water pump when the head is increased, so that arrangements which raise existing quantities of heat from a lower to a higher temperature level are called "heat pumps." The working process and necessary work of such heat pump, using air and saturated steam, will now be determined.

In using air or any other gas let 1 lb. at  $t_0 = 70^{\circ}\text{F.}$  and  $p_0 = 14.7$  lb./in.<sup>2</sup> abs. be compressed adiabatically until the temperature rises to  $t_1 = 115^{\circ}\text{F.}$  The pressure will then be  $p_1 = 19.5$  lb./in.<sup>2</sup> abs. Heat can now be withdrawn at constant pressure from this warm air, the amount being

$$Q_1 = \quad - t_0) = 0.24 \times 45 = 10.8 \text{ B.Th.U.}$$

The work of compression amounts to (page 327)

$$E_1 = 778 c_p(t_1 - t_0) = 778 \times 10.8 \text{ ft. lb.}$$

$$\text{or } AE_1 = 10.8 \text{ B.Th.U.}$$

This shows that, with an expenditure of 10.8 B.Th.U. of work, only 10.8 B.Th.U. of heat have been obtained, and this agrees with the energy law. This result could have been obtained more simply by using a brake on the driving motor of the compressor, thus dispensing with the latter. But it should be noted that the cooled compressed air, which at  $70^{\circ}\text{F.}$  has still a pressure of 19.5 lb./in.<sup>2</sup> abs., is capable of performing mechanical work, due to the excess pressure of  $19.5 - 14.7 = 4.8$  lb./in.<sup>2</sup> If this work be applied in assisting the compressor motor, the actual work required in compressing the air is now only  $E_1 - E_2$ , or the complete external work required in operating the compressor, air motor, and driving motor arrangement, which is called the heat pump, is  $E_1 - E_2$ . Hence, with an expenditure of only  $E = E_1 - E_2$  ft. lb., or  $AE$  B.Th.U. of mechanical work, the same heat  $Q$  is obtained as previously, when no air motor was used. With an expenditure of 1 B.Th.U. of work, the heat quantity

$$\frac{AE_1}{AE_1 - AE_2} \text{ B.Th.U.}$$

is obtained.

This value is always positive and greater than 1, since  $E_2 < E_1$ . It represents the quantity of useful heat delivered externally by the heat pump per 1 B.Th.U. of mechanical work done by the driving motor. We have now to consider why there is an excess of available heat over the heat equivalent of the mechanical work, and to find how much this excess amounts to.

The work  $E_2$  of the air motor with adiabatic expansion is smaller than that required at the compressor in the ratio  $T_0/T_1$ , since the law of expansion is the same as that of compression, and the pressure limits in the two cases are the same, while the initial volume of the compressed air in the motor is smaller than the volume at the end of compression in the compressor, the ratio of these two being  $T_0/T_1$ .

Hence

while

giving  $AE = AE_1 - AE_2 = c_p(T_1 - T_0) \cdot \frac{T_0}{T_1}$

Also  $\frac{Q_1}{AE} = \frac{T_1}{T_1 - T_0}$  B.Th.U.

The excess heat gained for an expenditure of 1 B.Th.U. of work thus amounts to  $T_0/(T_1 - T_0)$ .

With  $t_0 = 70$  or  $T_0 = 530$ , and with

$T_1 - T_0 =$	5	10	20	80	200	500	2000 ° F.
		54	27.5	7.63	3.65	2.06	1.265 B.Th.U.
	106	53	26.5	8.63	2.65	1.06	.265 B.Th.U.

The quantities in the second row now give the total heat delivered with an expenditure of 1 B.Th.U. of work, and they exceed this work by the amounts given in the third row. The latter amounts, therefore, must have existed in the working air before compression, and have been "raised" from  $t_0$  to  $t_1$ , i.e. from 70 to 75°; 80°, etc., by the heat pump.

This will be clearer if the process in the air motor be considered. Due to the adiabatic expansion in this motor, a drop in temperature of the air from  $t_0$  to some value  $t'$  occurs. This drop is obtained as follows: The driving work of the air motor amounts to

$$AE_2 = c_p(T_0 - T') \text{ B.Th.U.}$$

For the same work the expression found above gives

$$AE_2 = c_p \frac{T_0}{T_1} (T_1 - T_0)$$

Equating these two quantities gives

$$T_0 - T' = \frac{T_0}{T_1} (T_1 - T_0)$$

or  $T' = \frac{T_0^2}{T_1}$

Hence, in the air motor the temperature drops below the outer temperature. With  $t_0 = 70$ , and with

$\frac{p_1}{p_0} =$	1.2	1.5	2	3	8	12	30
---------------------	-----	-----	---	---	---	----	----

we have

$$t' = + 44 \quad + 12 \quad - 26 \quad - 72 \quad - 176 \quad - 199 \quad - 260^\circ \text{ F.}$$

The cold exhaust air from the motor must now take up the heat  $Q_2 = c_p(t_0 - t')$  before the process is repeated. This heat is supplied by the surroundings and represents the heat which is "raised" from  $t_0$  to  $t_1$ . Its amount is, with the value for  $t_0 - t'$ ,

$$Q_2 = c_p \frac{T_0}{T_1} (T_1 - T_0) \text{ and } \frac{Q_2}{AE} = \frac{T_0}{T_1 - T_0}$$

No fundamental change is made in the above if, instead of heating the cold exhaust air and then supplying it to the compressor, an entirely new charge of air, already heated to  $t_0$ , is drawn from the inexhaustibly large store in the atmosphere and supplied to the compressor. The same quantity of heat  $Q_2$  would be carried into the compressor with this air as was previously supplied to the cold exhaust air in heating it to  $t_0$ . The only difference is that the exhaust air is heated by the atmosphere from  $t'$  to  $t_0$ , and an equal mass of air already heated at some other place by the atmosphere is now drawn into the compressor.

The quantities  $Q_1$ ,  $Q_2$ , and  $E$  are related by the conservation of energy law as follows,

$$Q_1 - Q_2 = AE_1 - AE_2 = AE$$

as is also confirmed by the relations given above.

By writing this in the form

$$Q_1 = Q_2 + AE$$

it will be seen immediately that the heat taken from the heated compressed air consists of the two parts  $Q_2$  and  $AE$ , of which the first represents the heat supplied by the surroundings, while the second represents the heat due to the driving work. The advantage of the process is based essentially, therefore, on the fact that the existing heat  $Q_2$  of the surroundings has been brought to the higher temperature  $t_1$ . Combined with this, of course, there is the generation of heat  $AE$  at the same temperature, but with moderate heating this is a relatively small amount.

Due to frictional, temperature, and heat losses in the three machines, a considerably greater quantity of work is, of course, necessary than that represented above, in which all losses are neglected.

When saturated steam is employed, the process is as follows: From a large hot-water container with steam space (boiler) let 1 lb. of dry saturated steam at  $t_0^\circ$  and  $p_0$  lb./in.<sup>2</sup> abs. (e.g. at  $212^\circ$  F. and 14.7 lb./in.<sup>2</sup> abs.) be withdrawn and compressed to  $p_1$  lb./in.<sup>2</sup> abs. (e.g. 20 lb./in.<sup>2</sup> abs.), whereby its temperature (disregarding the slight superheat produced) is raised to the corresponding saturation value  $t_1$  ( $= 227.8$ ). The latent heat  $L$  ( $= 961.4$  B.Th.U.) can now be withdrawn from this compressed steam without any accompanying change in pressure and temperature, so that it is converted to water at  $t_1 = 227.8^\circ$  F. The liquid heat above  $212^\circ$  F. can also be withdrawn from this hot water, i.e. the heat  $(t_1 - t_0)$  ( $= 15.8$  B.Th.U.), or, if some other liquid be used,  $c(t_1 - t_0)$  B.Th.U., where  $c$  is the specific heat of the liquid between the temperature  $t_1$  and  $t_0$ . The total heat obtained from the compressed vapour is thus

$$Q_1 = L + c(t_1 - t_0)$$

The expression for the work of compression of 1 lb. of vapour is the same as that used in the compression of air, namely,

$$E = \frac{\gamma}{\gamma - 1} p_0 v_0 \left[ \left( \frac{p_1}{p_0} \right)^{\frac{\gamma - 1}{\gamma}} - 1 \right]$$

in which  $\gamma = 1.3$  for initially dry saturated steam, since the steam becomes superheated when compressed. If, however, the steam is fairly wet initially, then  $\gamma = 1.13$ . For a small pressure increase the work of compression can be taken as

$$E = (p_1 - p_0) v_0 = \Delta p v_0$$

or, in heat units,  $AE = A \Delta p v_0$ , in which  $v_0$  is the mean specific volume between the initial and final states. A gain of work due to expansion, as occurs when air is used, is not practicable. Hence, with an expenditure of 1 B.Th.U. of mechanical work the heat gained is

$$\frac{Q_1}{AE} = \frac{L + c(T_1 - T_0)}{A \Delta p v_0} \text{ B.Th.U.}$$

This value is easily calculated when the values  $L$ ,  $T_1$ , and  $v_0$  are obtained from steam or vapour tables. In the above example, with  $v_0 = 23.2 \text{ ft.}^3/\text{lb.}$  and  $c = 1$ ,

$$\frac{Q_1}{AE} = \frac{(961.4 + 15.8) 778}{144 \times 5.3 \times 23.2} = 43 \text{ B.Th.U.}$$

This is made up of 1 B.Th.U. from the applied work and 42 B.Th.U. contained in the heat stored in the boiler.

The expression for  $\frac{Q}{AE}$  can be obtained approximately by making use of the Clapeyron-Clausius relation

$$\frac{L}{v_s} = AT \frac{\Delta p}{\Delta T}$$

which holds for all vapours. This gives

$$A \Delta p v_s = \frac{L}{T_0}$$

so that, for small values of  $\Delta T = T_1 - T_0$ ,

$$\frac{Q}{AE} = \frac{T_0}{T_1 - T_0} + \frac{c T_0}{L}$$

For the above example, we thus have

$$\frac{Q_1}{AE} = \frac{212 + 460}{15.8} + \frac{212 + 460}{961.4} = 43.3 \text{ B.Th.U.}$$

By comparing the values of  $\frac{Q_1}{AE}$  for steam and air it will be found that the heat gained in the two cases, for an expenditure of 1 B.Th.U. of work, is about the same, provided the temperatures are the same.

The values for steam at 147 lb./in.<sup>2</sup> abs. compared with air at the temperature (60° F.) of the surroundings are, however, higher for the same temperature difference, since the initial steam temperature is  $460 + 212 = 672^\circ$  F. abs. against  $460 + 60 = 520^\circ$  F. abs. for air. The use of steam has the additional advantages that no expansion motor is required and much larger heat quantities are involved for the same size of compressor cylinder. The steam process shows clearly that, of the heat  $Q_1$  delivered by the heat pump, by far the larger portion  $Q_2$  must be supplied by an existing heat source, since each pound of steam drawn into the compressor must first be generated from hot water by supplying latent heat to it. Hence, as a means merely of generating steam, the process is useless, since practically no heat is thus saved and mechanical work has to be applied. But the heat pump operating with steam has lately been applied to evaporative processes in the chemical industries or to drying processes (e.g. such as are required in briquette factories), in which the considerable latent heats are regained, so that the saving of heat necessary for the processes is much reduced. On account of the higher temperature  $t_1$ , it is possible in these processes, or in the production of distilled water, to return the heat  $Q_1$  to the evaporating chamber.

The heat pump can also be used for heating buildings by using the same vapours ( $\text{SO}_2$ ,  $\text{NH}_3$ ) as are used in refrigerators, since these substances evaporate at room temperatures, so that the heat  $Q_2$ , necessary for the process, can be taken from the surroundings, e.g. from cold water at 50–70° F. This heat is then raised by 90 to 110° F. by compression and is then applicable for heating purposes. Economy by this method of heating, as compared with the usual method, is only attained when mechanical work or electrical energy from water power is available and fuels are scarce. Where heat pumps are driven by means of steam or gas engine, the process is uneconomical, since, in driving these engines, the heat supplied amounts to 10–15 times the heat equivalent of the pump work.

As compared with heating by means of an electrical resistance, however, a heat pump, driven by an electric motor, always shows a saving in electrical energy.\*

**Distillation and concentration by means of heat pumps.** The term "distillation" means the evaporation of a liquid followed immediately by condensation of the vapour formed. Water is distilled in order to remove the solids dissolved in it. These solids are not carried over with the vapour during evaporation. By concentration of a solution (e.g. a salt solution) is meant the partial evaporation of the water (or other liquid) in order to obtain a stronger solution. The vapour thus formed can either be condensed, as in distillation, or rejected to the atmosphere.

In both processes, evaporation, involving the supply of sensible and latent heat, is necessary. This heat supply is not, however, lost, as it is contained completely in the vapour, and can be regained during condensation and undercooling. The heat thus regained might be used for other purposes, but it may also be used to generate more vapour in the evaporating vessel. As opposed to steam generation, it will thus be seen that fundamentally distillation involves no supply of heat when the

\* See *Archiv. d. Wärmewirtsch* (1929), page 407. F. Merkel. Das Energieproblem der elektrischen Heizung.

distillate is reduced to the initial temperature of the impure water. In practice, however, the difficulty arises that the temperature of the vapour formed is, at most, equal to that of the evaporating water, while it ought to be several degrees (10 to 20° F.) higher if the heat is to be returned. This difficulty is overcome in the compound evaporator by leading the vapour from the first evaporator to a second evaporator, in which the pressure, and hence also the temperature, is lower than in the first. This means that the incoming vapour can act as a heating agent and is thus condensed in the second evaporator. The condensate so formed is now the distillate of the first evaporator. The weight of vapour formed in the second evaporator equals the weight of distillate from the first evaporator. The newly formed vapour can, in turn, be passed to a third evaporator in which the pressure is still lower, whereby an equal weight of vapour is again formed. With three evaporators in series, therefore, three times as much distillate is formed as that delivered by one evaporator with the same expenditure of heat. If  $H$  is the heat required to produce 1 lb. of distillate, using one evaporator, then  $\frac{H}{3}$  is the heat required

when three evaporators in series are used. By increasing the number, the heat required would be reduced to a negligible amount, but in practice only a few are used in series.

In the first evaporator the pressure might be 30 lb./in.<sup>2</sup> abs., corresponding to a temperature of 250.3° F. In the second evaporator the pressure is reduced to 14.7 lb./in.<sup>2</sup>, so that the temperature is now 212° F. In the third evaporator, for a temperature of 175° F., the pressure would have to be about 7 lb./in.<sup>2</sup> abs., i.e. below that of the atmosphere.

The process involving the use of a heat pump in order to establish the necessary temperature difference, so that a regain of heat may be obtained, is totally different from the above process. The hot vapour at the pressure  $p_1$  and the temperature  $t_{s1}$  is removed by a compressor (for large quantities of vapour a turbo-compressor is preferable) and compressed to a higher pressure  $p_2$ . The temperature is thus raised to  $t_2 > t_{s1}$ . If the vapour is dry saturated before compression, it will be superheated after adiabatic compression, and in this state is led to the heating coils in the evaporator, where it gives up its superheat and latent heat. To effect this, it is essential that the pressure  $p_2$  in the coils should be sufficiently in excess of  $p_1$  to cause the saturation temperature  $t_2$  to be high enough to allow a reasonable rate of heat transmission. The latent heat forms by far the largest portion of the heat transmitted, and, in order to have a reasonable size of heating surface, saturation temperature drops of between 10° and 20° F. are required.

Fig. 298 shows a diagrammatic lay-out of evaporator with heat pump. The water to be distilled, or solution to be concentrated, is first heated in the evaporator by means of steam at the saturation temperature  $t_{s1}$  (e.g. 212°). The compressor is then started and draws in vapour from the evaporator and compresses and delivers it to the heating coils, where its superheat and latent heat are given up. The hot condensate collects in the lower part of the evaporator and flows from there to the preheater, where it gives up its liquid heat to the impure water supplied from a high level tank. This water is thus raised to about  $t_{s1}$ ° F. and, after passing through the preheater, is led to the evaporator. The condensate, on the



other hand, is cooled down to about room temperature and then flows from the preheater to the distilled water tank. In the case of concentration, the process is essentially the same. The liquid in the evaporator, however, becomes continuously richer in the dissolved substances, which do not evaporate. As soon as the desired concentration is attained, the solution is removed to a special vessel.

The direct heat required consists only of that necessary to start the evaporation, and is thus an unimportant factor in the process. The

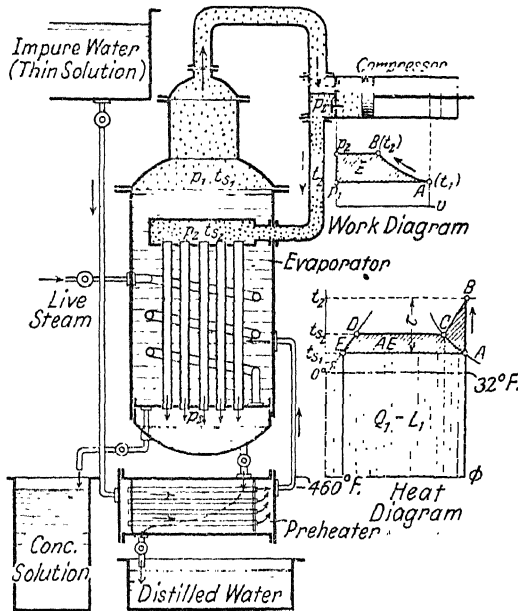


FIG. 298

important item in determining the efficiency is the work required for the compressor. Neglecting all losses, the adiabatic work of compression for 1 lb. of vapour is

$$(574)$$

or, more simply, from the  $H\phi$  diagram for steam,

$$AE = H_2 - H_1$$

i.e. the difference in total heats after and before adiabatic compression.

The compression temperature follows from (page 108)

$$t_{s1} = \tau = T_1 \quad 0.236 \quad (575)$$

or it can be obtained more exactly from the  $T\phi$  diagram for steam.

The saturation temperature drop  $\Delta t = t_{s2} - t_{s1}$  is given by the steam tables.



$\frac{2545}{59.5} = 42.8$  lb. Since the latent heat of steam at 14.7 lb./in.<sup>2</sup> abs. is 971.4 B.Th.U., the heat delivered by an expenditure of 1 B.Th.U. of work is  $\frac{971.4}{59.5} = 16.3$  B.Th.U.

In this way the curves shown in Fig. 300 for saturation temperature differences ranging from 10° to 55° F. have been drawn. The initial

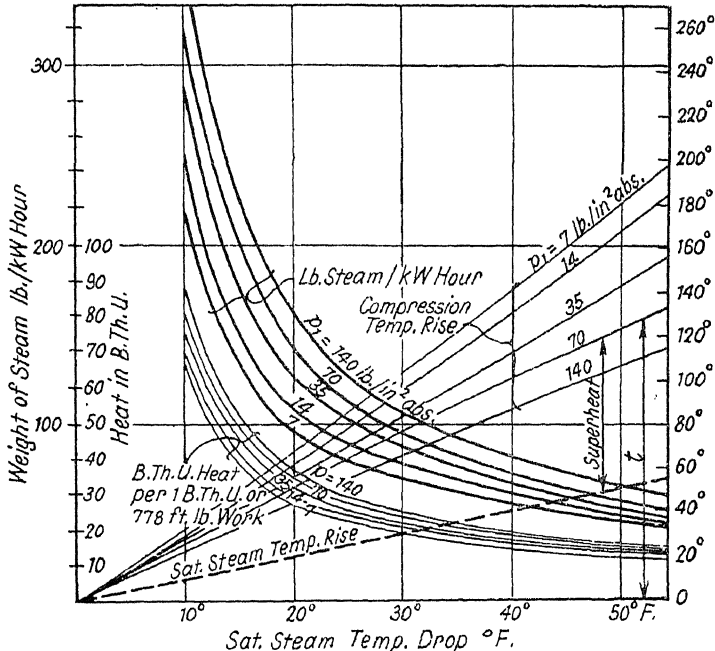


FIG. 300

pressures vary from  $p_1 = 7$  to 140 lb./in.<sup>2</sup> abs. The ordinates give the weight of distilled water obtained per horse-power hour, and the heats delivered per 1 B.Th.U. of work. It will be seen that both values increase as the temperature difference is decreased. On the other hand, an increase in evaporator pressure means an increase in these values, so that the work required for the same temperature drop decreases as the pressure is increased. The total temperature rises have also been plotted in Fig. 300. The ranges of superheat are given by the ordinates between these curves and the dotted line, which gives the saturation temperature values. The range of superheat decreases as the pressure increases for the same value of temperature difference.

In the evaporation of solutions, conditions are changed slightly, since the boiling point of solutions is higher than that of water and increases as the concentration increases. Hence, during evaporation, the saturation temperature difference continuously decreases, so that, in order to have a reasonable final temperature difference, a larger initial temperature difference than that in distillation is necessary.

The actual performances are naturally not so favourable as those shown above for the ideal cases, on account of the unavoidable mechanical losses in the compressor and the heat losses. Thus Ombeck\* found, in the case of a Linde distillation plant with heat pump, having an evaporator pressure of 15.22 lb./in.<sup>2</sup> abs. and a pressure rise of 2.87 lb./in.<sup>2</sup> (corresponding to a temperature rise of 8.6° F.), that the output per kW. hour (at switchboard) amounted to 133 lb. of distillate, whereas, with no loss, it would be about 264 lb. In addition, about 2.6 B.Th.U. of heat had to be supplied per pound of distillate. With a temperature difference of 14° F. the output was 76.5 lb./kW. hour, and no additional heat was required. With no loss the output would have been about 154 lb. of distillate. Stodola, in testing a Wirth common salt concentrator, obtained a mean evaporation of 41.4 lb./kW. hour, and, at the end of the test, 31.3 lb./kW. hour.†

### DYNAMICAL ACTION OF FLOWING GASES AND VAPOURS

**Pressure of deflected free jets (impulse).** An element of a gas moving with uniform speed  $V$  over a curved surface  $AB$  and having a mass  $m$ , exerts a centrifugal force  $C$  on the surface (Fig. 301). The direction of  $C$  is always normal to the surface, and is thus variable; the magnitude of  $C$  is dependent on the instantaneous radius of curvature  $r$ , and is thus also variable, unless the path happens to be circular. Instead of determining the centrifugal pressures from the known equation  $C = \frac{mV^2}{r}$ , it is better, in

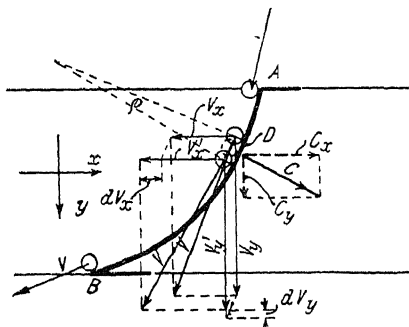


FIG. 301

$x$  and  $y$  at right angles to one another. The centrifugal force  $C$  is caused by the deflection of the mass  $m$  from its straight line motion, which it tends to maintain. Due to the deflection, the components of the velocity  $V$  change in the  $x$  and  $y$  directions. Hence, accelerations and retardations are produced in these directions, and these, by the law of mass acceleration, are proportional to the forces  $C_x$  and  $C_y$ .

At  $D$  let the velocity components be  $V_x$  and  $V_y$ , and at a point close to  $D$  let these change to  $V'_x$  and  $V'_y$  (Fig. 301).

The velocity change in the  $x$  direction is, therefore,

$$V_x = dV_x$$

and, in the  $y$  direction,

\* Z.V.d.I. (1921), page 64. Versuche an Wasserdstillationsanlagen mit Wärmepumpe.

† Z.V.d.I. (1922), page 160. Wirth. Erfahrungen an Eindampfanlagen mit Wärmepumpe.

If  $dt$  is the time taken for the mass to traverse this short distance, the accelerations (or retardations) are  $\frac{dV_x}{dt}$  and  $\frac{dV_y}{dt}$ , so that the corresponding forces are

$$-U_x = m \frac{dV_x}{dt} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (578)$$

[illegible]

The moment due to  $U$ , expressed in terms of the distances from the origin of the co-ordinates, is

$$\Delta M = C_y x - C_x y$$

or, with the values of  $C_x$  and  $C_y$ ,

$$-\Delta M = m \frac{x dV_y - y dV_x}{dt}$$

Now, since  $dx = V_x dt$

and  $dy = V_y dt$

it follows that  $\frac{dx}{V_x} = \frac{dy}{V_y}$

or  $V_y dx - V_x dy = 0$

Adding this to the numerator in the expression for  $\Delta M$  gives

$$- \Delta M = m \frac{xdV_y + V_y dx - (y dV_x + V_x dy)}{dt}$$

With  $x \, dV_y + V_y \, dx = d(x \, V_y)$

$$y \, dV_x + V_x \, dy = d(y V_x)$$

the numerator becomes

$$d(x, V_y) = d(y, V_x)$$

or, more simply,  $d(x \cdot V_q - y \cdot V_s)$

\* But we have  $x \cdot V_y - y \cdot V_x = V\alpha$

(with  $a$  as the perpendicular distance from the origin to the tangent at  $D$ ) since  $V$  is the diagonal in the parallelogram formed by  $V_x$  and  $V_y$ .

$$\text{Hence } -\Delta M = m \frac{d(Va)}{dt} \quad (580)$$

**Pressure and torque exerted by a continuous jet.** By dividing the jet into elements of length  $dl$  (Fig. 302) each element will exert the forces  $C_x$  and  $C_y$ . Let the sum of all the  $x$  components be  $P_x$  and of the  $y$  components  $P_y$ . The resultant of  $P_x$  and  $P_y$  is then the resultant  $P$  of all the centrifugal forces acting on the blade, both in magnitude, direction, and point of application.

Now let  $W_{sec}$  be the weight of the jet passing any cross-section per second. In the time  $dt$  sec., i.e. the time the jet takes to travel over

the path  $db$ , the weight flowing through  $D$  (Fig. 302) is  $W_{sec} dt$ . This weight is also represented by the element lying between  $D$  and  $D'$ , the mass of which was taken as  $m$  above.

$$\text{Hence, } m = \frac{W_{sec} dt}{g}$$

This gives, for the components of the centrifugal pressures,

$$C_x = - \frac{W_{sec} dt}{a} \frac{dV_x}{dt} = - \frac{W_{sec}}{g} dV_x \quad . \quad . \quad . \quad (581)$$

$$\text{and} \quad . \quad . \quad . \quad (582)$$

The torque due to the centrifugal pressure about the origin is

$$\Delta M = - \frac{W_{sec}}{g} d(Va) \quad . \quad . \quad . \quad . \quad . \quad . \quad (583)$$

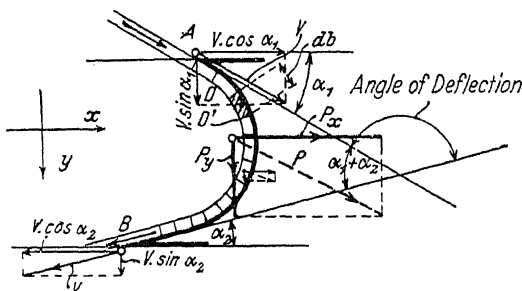


FIG. 302

The sum of all the  $x$  components between  $A$  and  $B$  is, therefore,

$$P_x = \Sigma C_x = - \frac{W_{sec}}{g} \sum_a^B dV_x$$

or, denoting the values of  $V_x$  at  $A$  and  $B$  by  $V_{x1}$  and  $V_{x2}$  (Fig. 302),

$$P_x = - \frac{W_{sec}}{g} [V_{x2} - V_{x1}] \quad . \quad . \quad . \quad (584)$$

and, similarly, the sum of all the  $y$  components gives

$$P_y = - \frac{W_{sec}}{g} [V_{y2} - V_{y1}] \quad . \quad . \quad . \quad (585)$$

and the turning moment of the total blade pressure is

$$M = \frac{W_s}{g} \quad . \quad . \quad . \quad (586)$$

If  $V$  is constant, as assumed here, and if  $a_1$  and  $a_2$  are the shortest distances of the inlet and outlet tangents from the origin, then

$$M = - \frac{W_{sec}}{g} V(a_2 - a_1) \quad . \quad . \quad . \quad . \quad . \quad . \quad (587)$$

If the turning point be considered as situated somewhere on the resultant of all the separate pressures due to the elements, the torque is zero, and therefore  $\alpha_2 = \alpha_1$ , i.e. the resultant  $P$  halves the angle between the inlet and outlet tangents. Its position is thus easily determined, as shown in Fig. 303. In Fig. 302,  $P$  has been drawn at random.

If  $\alpha_1$  and  $\alpha_2$  (Fig. 302) are the angles which the blade at  $A$  and  $B$  makes with the  $x$  direction, then

$$V_{x1} = V \cos \alpha_1, \quad V_{x2} = -V \cos \alpha_2$$

$$\text{and} \quad V_{y1} = V \sin \alpha_1, \quad V_{y2} = V \sin \alpha_2$$

$$\text{and} \quad P_x = \frac{W_{sec}}{g} V (\cos \alpha_2 + \cos \alpha_1) \quad . \quad (588)$$

$$W \quad \frac{v}{g} (\sin \alpha_1 - \sin \alpha_2) \quad . \quad (589)$$

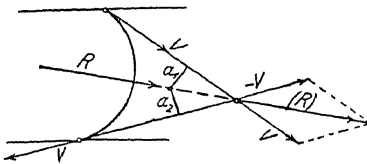


FIG. 303

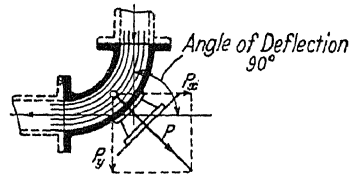


FIG. 304

$\alpha_2$  may be smaller than  $\alpha_1$ , in which case  $P_y$  is directed downwards, since it is positive.  $P_x$  in all cases is directed to the right, even if  $\alpha_1$  is more than  $90^\circ$ , i.e.  $\cos \alpha_1$  is negative because

$$(180 - \alpha_1) > \alpha_2$$

The resultant blade pressure is

$$P = \sqrt{P_x^2 + P_y^2}$$

$$\text{or} \quad P = \frac{W_{sec}}{g} V \sqrt{2 [1 + \cos (\alpha_2 + \alpha_1)]} \quad . \quad (590)$$

The direction of  $P$  with the horizontal is given by

$$\tan \phi = \frac{P_y}{P_x} = \frac{\sin \alpha_1 - \sin \alpha_2}{\cos \alpha_1 + \cos \alpha_2}$$

It should be noted that both the magnitude and direction of the blade pressure are independent of the form of the channel. They are determined by the inlet and outlet angles alone. The position and direction of the resultant force acting on the blades are determined as shown in Fig. 303. See page 593 regarding the work done by impulse forces.

**Example 5.** Pipe bend of  $90^\circ$  (Fig. 304). With  $\alpha_1 + \alpha_2 = 90^\circ$ , the resultant impulse is

$$P = \frac{W_{sec}}{g} V \sqrt{2}$$

**Example 6.** Pipe bend of  $180^\circ$ . With  $\alpha_1 + \alpha_2 = 0$

$$P = 2 \frac{W_{sec} V}{g}$$

Thus, if steam at  $170 \text{ lb./in.}^2$  abs. flows through a pipe of 4 in. diameter with a speed of  $130 \text{ ft./sec.}$ , then, with  $\rho = 0.375 \text{ lb./ft.}^3$ , we have

$$W_{sec} = \frac{\pi}{4} \times \left(\frac{4}{12}\right)^2 \times 130 \times 0.375 = 4.26 \text{ lb./sec.}$$

This gives, for a  $90^\circ$  bend,

$$P = \frac{4.26 \times 130 \times 1.414}{32.2} = 24.3 \text{ lb.}$$

and, for a  $180^\circ$  bend,

$$P = 34.4 \text{ lb.}$$

**Example 7.** Blade at rest, with  $\alpha_1 = 20^\circ$  (entrance angle) and  $\alpha_2 = 20^\circ$  (exit angle), as shown on Fig. 302.

$$\text{Since } \cos \alpha_1 = \cos \alpha_2 = 0.94$$

$$\text{and } \sin \alpha_1 = \sin \alpha_2 = 0.34$$

$$\begin{aligned} \text{we have here } P_x &= \frac{W_{sec}}{g} V (0.94 + 0.94) = 1.88 \frac{W_{sec} V}{g} \\ &= \frac{W_{sec}}{g} V (1 - 0.34) = 0 \end{aligned}$$

The jet pressure in the  $y$  direction is zero. With  $W_{sec} = 1 \text{ lb.}$  and  $V = 3250 \text{ ft./sec.}$ ,

$$P_x = \frac{1.88 \times 3250}{g} = 190 \text{ lb}$$

**Example 8.** A free jet impinges perpendicularly on a flat surface. Experience shows that, provided the surface be large enough, the jet spreads along the surface in all directions.

The pressure of the jet on the plate is the same as if the jet had been deflected through  $90^\circ$ . Choosing the  $x$  direction as coinciding with the jet direction, we have, with equation (584),

$$\text{and } P = \frac{W_{sec} V}{g}$$

This relation is frequently used in the experimental determination of the discharge velocity of steam, since, by measuring  $P$  and  $W_{sec}$ , the value of  $V$  is found.



## REACTION AND WORK OF ACCELERATED GAS AND VAPOUR

**Jets (Wheel).** When a gas or vapour jet emerges from a vessel (Fig. 305) the contents of which are at rest, all the issuing particles flow from certain points, such as  $A$ , and gradually increase in velocity until, at  $B$ , they possess the exit velocity. In general, the paths of the particles are curved ( $AB_1$ ,  $A'B'_1$ ) and the motions are non-uniformly accelerating. In order to produce this motion, forces in the direction and sense of this motion must be applied. If  $m$  is the mass of a jet element (Fig. 305, nozzle I),  $V$  its velocity,  $dt$  the time taken to cover the distance  $ds$  along the direction of motion,  $dV$  the increase in  $V$  during this time, so that  $\frac{dV}{dt}$  is the acceleration, then the driving force is

$$\Delta P = m \cdot \frac{dV}{dt}$$

This force, the magnitude and direction of which continuously change during the discharge of the element, is easily demonstrated experimentally by the

backward opposing force (reaction), without which it cannot be produced. This reaction is transmitted backwards by the gas to the wall of the vessel. The point at which it acts is found by drawing the line of the element backwards (Fig. 305).

Under steady conditions of flow all these driving forces and reactions act simultaneously. In the following, the determination of the resultant both in magnitude and direction will be considered.

**Calculation of the total reaction.** The jet is considered as being split up into a large number of smaller jets, each of which follows a path such as  $AB$ . Let the mass of a very short length of one of these be  $m$ . The force  $\Delta P$  on this acts in the direction of the tangent to the path. It is advisable to split this force into components  $\Delta P_x$  and  $\Delta P_y$ , and the velocity  $V$  into components  $V_x$  and  $V_y$ . We then have (page 575)

$$\Delta P_x = m \cdot \frac{dV_x}{dt}$$

Each separate divided jet supplies a fraction of the total weight  $W_{sec}$  discharged per second, which is calculated as shown on page 240. If  $\Delta W_{sec}$  is this fraction, the weight flowing through the divided jet area in time  $dt$  is  $\Delta W_{sec} dt$ . This weight is equal to the weight of the jet

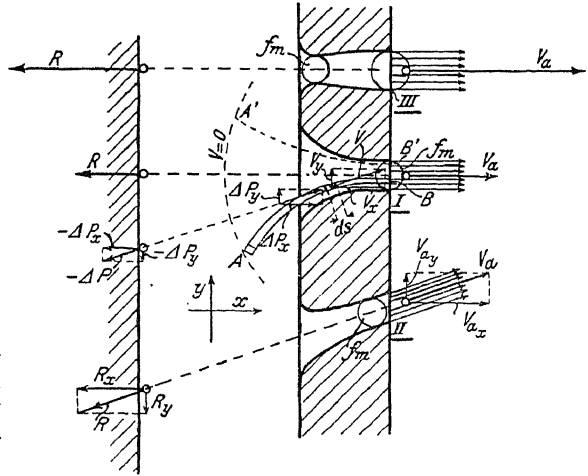


FIG. 305

element itself, since the distance  $ds$  is traversed in time  $dt$  (Fig. 305). This, then, gives

$$m = \frac{\Delta W_{sec} dt}{g}$$

so that 
$$\Delta P_x = m \frac{dV_x}{dt} = \frac{\Delta W_{sec}}{g} dt \frac{dV_x}{dt}$$

or 
$$\Delta P_x = \frac{\Delta W_{sec}}{g} dV_x$$

Similarly 
$$\Delta P_y = \frac{\Delta W_{sec}}{g} dV_y$$

By adding all the vertical and horizontal components of a divided jet, the components of the total reaction of this jet are obtained. Hence, in the expressions for  $\Delta P_x$  and  $\Delta P_y$ , the values  $dV_x$  and  $dV_y$  alone are variable; the required sum is obtained by multiplying  $\frac{\Delta W_{sec}}{g}$  by the total change in the jet velocity components along the path  $AB$ . These are equal to the components  $(V_a)_x$  and  $(V_a)_y$  of  $V_a$ .

Hence 
$$P_x = \frac{\Delta W_{sec}}{g} (V_a)_x$$

$$P_y = \frac{\Delta W_{sec}}{g} (V_a)_y$$

Assuming that all the streams are parallel and have the velocity  $V_a$  at discharge, the components  $R_x$  and  $R_y$  of the total reaction are found from these by writing the whole weight  $W_{sec}$  in place of  $\Delta W_{sec}$  and by changing the sign. We then have

$$R_x = -\frac{W_{sec}}{g} (V_a)_x$$

$$R_y = -\frac{W_{sec}}{g} (V_a)_y$$

The signs are so chosen that the directions of the components of the jet reaction are opposite to the directions of the components of the discharge velocity.

The total reaction is

$$\begin{aligned} R &= \sqrt{R_x^2 + R_y^2} \\ &= \frac{W_{sec}}{g} V_a \end{aligned} \quad . \quad . \quad . \quad . \quad . \quad . \quad (591)$$

**Maximum reaction in convergent nozzles.** (I and II, Fig. 305.) In this case the discharge velocity cannot exceed the acoustic  $V_{max}$ . Hence, for saturated steam, with the values of  $W_{sec}$  and  $V_{max}$  as given on page 246,

$$R_{max} = \frac{3.62 \times 5.84 p_1 a}{g} = 0.678 p_1 a$$



The reaction  $R' = \frac{E}{u}$  is, from this,

$$\left( \frac{v_a}{2} - \frac{u}{2} \right) = W_{sec} \frac{r_a}{2} \quad (593)$$

and is thus smaller than the reaction at rest,  $W_{sec} \frac{r_a}{g}$ .

The useful work  $E$  depends on the velocity  $u$  of the vessel. It attains its maximum value, with  $u = V_a$ , of

$$E_{max} = W_{sec} \frac{V_a^2}{g} \cdot \frac{1}{2g} = W_{sec} \frac{r_a}{2g}$$

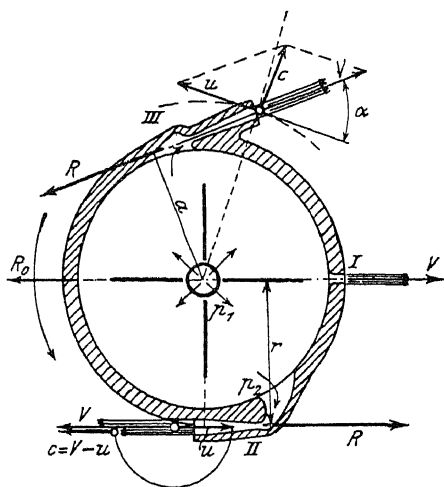


FIG. 306

the total jet energy is then converted to useful work and the absolute velocity of the jet at exit is zero.

For smaller values of  $u$  only the fraction

$$\eta = \frac{E}{2 \frac{u}{2}} \quad (594)$$

is converted to work. For example, with  $u = \frac{1}{2} V_a$ ,  $\eta = 1 - \frac{1}{4} = 0.75$ , while, in the case of the impulse turbine with  $u = \frac{1}{2} V_a$ , the efficiency is  $\eta = 1$ .

For the same efficiency the velocity  $u$  has to be greater in reaction wheels.

**Pure reaction wheel.** Consider a rotating drum supplied with

steam at the centre and fitted with nozzles at the circumference (Fig. 306). The reaction pressures which act in the opposite direction to the issuing steam tend to rotate the wheel in this direction. The maximum torque is obtained, for the same position of the nozzle, from the axis, when the nozzle is tangentially fitted (nozzle II). The torque is reduced for the inclined nozzle III, and becomes zero for the radial nozzle I.

If, with normal steam pressures, the jet is to exert the full reaction, the nozzles must be convergent-divergent in form.

If the drum be allowed to rotate at a uniform speed by the action of the reaction pressure, then useful work can be delivered to a brake or other resistance.

If  $R$  is the reaction under steady running conditions, the useful work with the tangential nozzles (II) is

$$E = Ru$$

and, with inclined (the nozzles III)

$$E = R u \cos \alpha.$$

The value of  $E$ , in particular cases, depends on the absolute velocity  $C$  with which the steam leaves the drum. For nozzle II,  $C = V - u$ , where  $V$  is the jet velocity at exit from the nozzle, and for nozzle III,  $C$  is the diagonal of the parallelogram formed from  $V$  and  $u$ .

For the same values of  $u$  and  $V$ ,  $C$  is reduced as  $\alpha$  is reduced, and hence has its minimum value in the tangential nozzle. This will, therefore, show the maximum work, under otherwise equal conditions, while, in the case of the radial nozzle, no work is performed.

**Useful work of the tangential nozzle.** The steam is supplied through the axis of the wheel and is thrown towards the inner circumference by the centrifugal force. The pressure  $p_2$  at entrance to the nozzle is thus higher than the supply pressure  $p_1$ . In order to compress the steam from  $p_1$  to  $p_2$ , work is done by the centrifugal force acting between the axis and the circumference. This work is equal to  $u^2/2g$  per lb. of steam and equals the compression work between  $p_1$  and  $p_2$ , as shown by the area  $abfe$  in Fig. 307. This work is delivered by the turbine itself.

The discharge velocity  $V$  from the nozzle is conditioned by the adiabatic drop between  $p_2$  and the outer pressure  $p'$ . This drop is given by the area  $efcde$ . If  $E_0$  is the available work at the axis, i.e. the area  $abcd$ , the discharge velocity due to this would be  $V_0$ , which is given by

$$V^2$$

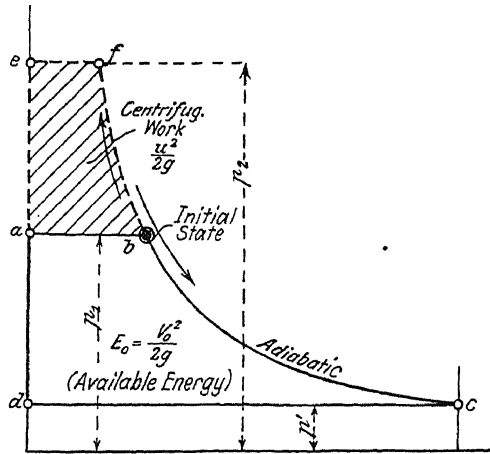


FIG. 307

Due to the centrifugal action, the outlet velocity is  $V > V_0$  and is found from

$$\frac{r}{2g} \quad \frac{u^2}{2g} = \frac{V_0^2}{2g} + \frac{u^2}{2g}$$

For a given circumferential speed  $u$ , steam pressure  $p_1$ , and outer pressure  $p'$ , we then have

$$V = \sqrt{2g E_0}$$

The absolute discharge velocity is now  $V - u$  and its corresponding kinetic energy  $\frac{(V - u)^2}{2g}$ . This gives the lost work, so that the useful work

$$\frac{V^2}{2g} - \frac{(V - u)^2}{2g}$$

Inserting the value for  $V$  gives

$$E = \frac{V_0^2}{2g} - \frac{u^2}{2g}$$

$$= \frac{u}{g} (\sqrt{V_0^2 + u^2} - u)$$

alternatively,  $E = \frac{u}{g} (V - u)$

The efficiency of the wheel is

$$\eta = \frac{E}{E_0} = 1 - \left[ \sqrt{1 + \left( \frac{u}{V_0} \right)^2} - \frac{u}{V_0} \right]^2 \quad . \quad . \quad (595)$$

From this we also have

$$\frac{u}{V_0} = \frac{1}{2} \frac{\eta}{\sqrt{1 - \eta}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (596)$$

**Example 9.** A pure reaction wheel is to operate with saturated steam at 140 lb./in.<sup>2</sup> abs. and with an exit pressure of 1.5 lb./in.<sup>2</sup> abs. Find the circumferential speeds for efficiencies of  $\eta = 0.3$  and  $\eta = 0.7$ , and the speeds in rev./min. for a wheel diameter of 3 ft.

From the  $H\phi$  diagram the heat drop between 140 and 1.5 lb./in.<sup>2</sup> abs. is 293 B.Th.U. Hence (from page 461),

$$V_0 = 223.8 \sqrt{293} = 3820 \text{ ft./sec.}$$

Also, from equation (596),

with  $\eta = 0.3, \quad 0.7$

$$\frac{u}{V_0} = 0.179 \quad 0.639$$

Hence  $u = 684 \quad 2440 \text{ ft./sec.}$

giving  $N = \frac{60u}{\pi D} = 4350 \quad 14,300 \text{ rev./min.}$

The compression head  $\frac{u^2}{2g}$  in heat units is

$$\frac{Au^2}{2g} = 9.35 \quad 119$$

Hence the increased pressure at entrance to the nozzle is

$$p_2 = 155 \quad 470 \text{ lb./in.}^2 \text{ abs.}$$

and the temperature

$$t_2 = 364.6 \quad 505.6^\circ \text{ F.}$$

The above treatment holds also for gases.

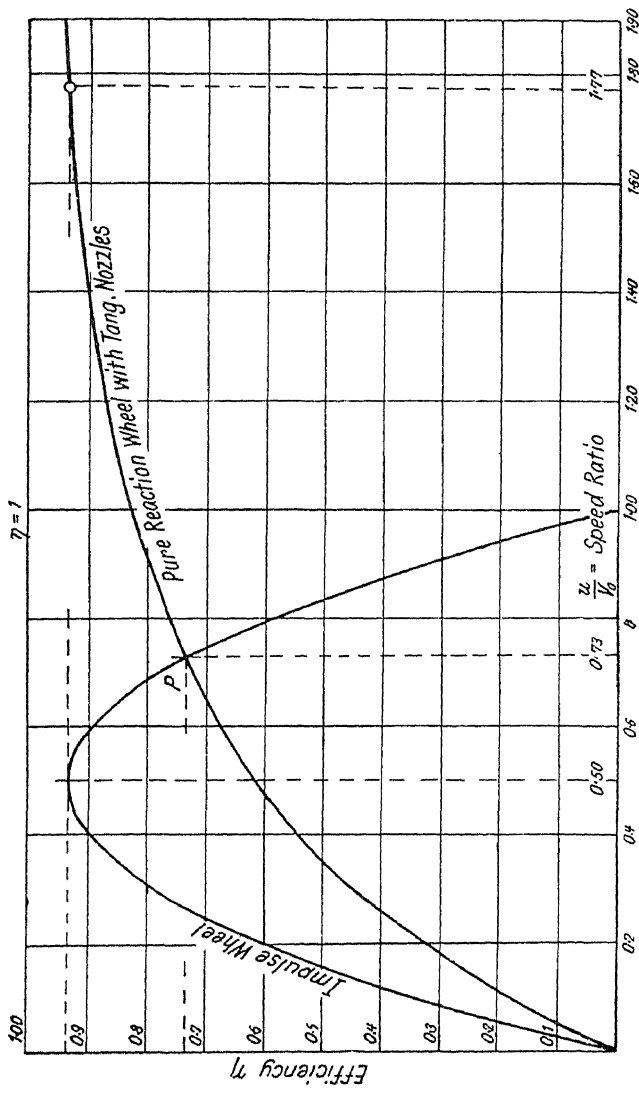


FIG. 308

**Comparison between the pure reaction wheel and the impulse wheel with a jet inclination of  $15^\circ$  as in Fig. 318.** In Fig. 308 the efficiencies of both types have been plotted to a base of  $\frac{u}{V_0}$ , i.e. of speed ratios.

The reaction wheel efficiency approaches unity only when the circumferential velocity is a multiple of the steam speed, and becomes unity when  $\frac{u}{V_0} = \infty$ .

The impulse wheel attains a maximum efficiency of  $\eta = 0.932$  when  $u = 0.5 V_0$ . If the reaction wheel has to have as high an efficiency, it must run with a circumferential speed  $\frac{1.77}{0.5} = 3.54$  times as high, so that, if  $V_0 = 3000$  ft./sec., the circumferential speed is  $0.5 \times 3000 \times 3.54 = 5300$  ft./sec., which is unattainable in practice.

When  $\eta = 0.7$ , the ratio of the circumferential speeds is  $0.65/0.25 = 2.6$ ; for the impulse wheel we would have

$$u = 0.25 \times 3000 = 750 \text{ ft./sec.}$$

and, for the reaction wheel,

$$u = 0.65 \times 3000 = 1950 \text{ ft./sec.}$$

An impulse wheel having a speed ratio of 0.73, which is higher than usual, would show no higher efficiency than a reaction wheel with the same speed ratio (point P).

The frictional flow losses are less in the reaction wheel. By using a large number of stages the speed of rotation can also be reduced.

The pure reaction wheel has never been applied commercially.

#### Combined effect of impulse and reaction

**forces.** If (as in the lower part of Fig. 309) a bend with constant cross-sectional area be fitted to a nozzle having a straight axis, then, in this straight part, the jet will be accelerated. This produces the reaction  $R_r = \frac{W}{g} V_a$ . In the curved portion the jet is merely deflected, and this deflection produces the reaction  $R_A$ , calculated as shown on page 574. Both these forces can be compounded, giving the resultant  $R$ , which, both in magnitude and direction, represents the combined effect of the reaction and impulse forces.

The conditions are similar in a curved channel (upper part of Fig. 309). Each portion of the jet exerts an impulse force, due to the curvature, and a reaction force, due to the accelerations of the jet caused by the change in cross-sectional area. The separate forces  $R_r$  and  $R_A$  are not, however, found directly in this case.

The resulting pressure can, for any case, be found when the inlet and outlet velocities are known in magnitude and direction, since not only the reaction causes acceleration, but also the impulse forces due to the change in direction with accompanying change in velocities, and both forces can be combined and regarded as a single force. The components

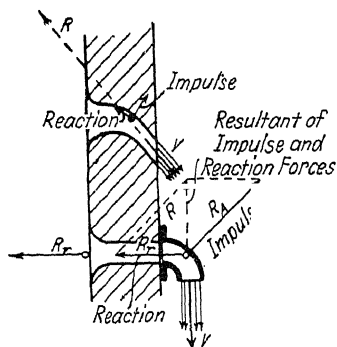


FIG. 309



of the total pressure of an element are proportional to the changes  $dV_x$  and  $dV_y$ , whether these changes are effected by deflection or by acceleration due to changes in area, or by both causes acting simultaneously.

Carrying out the calculation for a curved channel of variable section (Fig. 310) in the same way as for a curved free jet (i.e. impulse, page 574), the following is obtained—

Let the velocity change from  $V_1$  to  $V_2$  as the jet travels through the passage. By splitting  $V_1$  and  $V_2$  into the components  $V_{1x}$ ,  $V_{1y}$ , and  $V_{2x}$ ,  $V_{2y}$ , it will be seen that the velocity in the  $x$  direction increases by the amount  $V_{1x} + V_{2x}$ .

This acceleration corresponds to a force acting on the jet in the  $x$  direction of

$$V_{1x} + V_{2x} \quad . \quad (597)$$

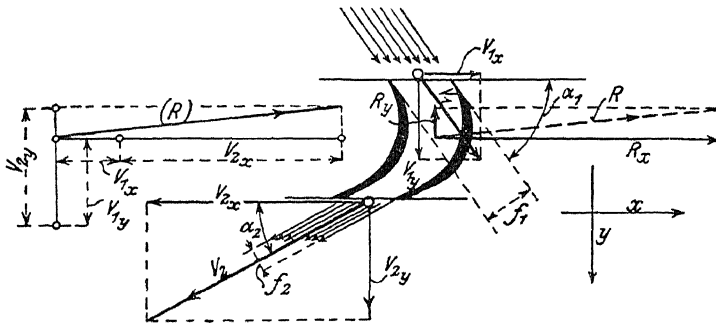


FIG. 310

The opposing force to this, which acts on the blades, is the horizontal component  $R_x$  of the total jet pressure.

In the  $y$  direction, the velocity increases by  $V_{2y} - V_{1y}$ . This acceleration corresponds to a force  $\frac{W}{g} \sec (V_{2y} - V_{1y})$  acting downwards on the jet. The reaction, due to this, gives the force  $R_y$  exerted upwards by the blade walls. If  $V_{1y}$  were greater than  $V_{2y}$ , then  $R_y$  would be directed downwards.

The total jet pressure, in magnitude and direction, is found by measuring the resultant of  $R_x$  and  $R_y$ .

The torque due to the blade pressures about any point is

$$M = -$$

where  $a_1$  and  $a_2$  are the perpendicular distances of the tangents to the inlet and outlet edges of the blade from the point of rotation. For points on the resultant  $M = 0$ , so that  $V_2 a_2 = V_1 a_1$ . In this way the position and direction of the resultants  $P_x$  and  $P_y$  are found simply by treating  $V_1$  and  $-V_2$  as forces. Fig. 311 makes this clear. The same procedure can be followed in obtaining the exact position of  $R$  in Fig. 310, where it has merely been assumed.

Applying this result to the bent nozzle of Fig. 309, it will be seen that the resultant of the impulse force  $R_A$  and of the reaction force  $R_R$ , which can both be independently determined, must be directed vertically, since the horizontal jet velocity, both inside the vessel and at outlet, is zero. The jet thus experiences no change in horizontal velocity, and only the vertical component increases from 0 to  $V_a$ , giving a corresponding vertical jet pressure equal to  $\frac{W}{g} V_a$  lb.

The separate forces are

$$R_A = \frac{W_{sec}}{g} V_a \sqrt{2}$$

(from page 579), acting on the bend at  $45^\circ$  upwards to the right,

and  $R_R = \frac{W_{sec}}{g} V_a$  acting on the vessel.

These give  $R_A = R_R \sqrt{2}$  and  $R = R_R$ .

The triangle formed by  $R_A$  and  $R_R$  and  $R$  is thus right angled.

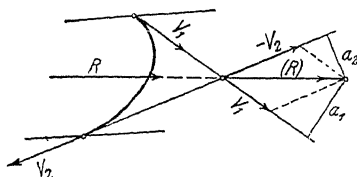


FIG. 311

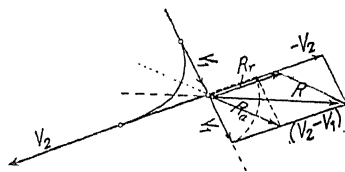


FIG. 312

The total jet pressure, when  $V_1 = 0$ , is thus calculable as if it depended only on the reaction; its direction coincides with, but is opposite to, that of the jet at outlet.

If  $V_1 > 0$ , the direction of the resultant blade pressure is found, as shown in the small accompanying diagram in Fig. 310. It is not now parallel to the jet at outlet.

The separation of the impulse and reaction forces in curved blades is carried out as shown in Fig. 312.  $R_A$  is the diagonal of the rhombus formed with  $V_1$  as the sides.  $R_R$  is proportional to  $V_2 - V_1$ , and acts in the direction of  $V_2$ .  $R$  is the resultant of both, but is also given by the diagonal of the parallelogram formed from  $V_1$  and  $-V_2$ .

## ACTION OF STEAM IN STEAM TURBINES

Consider an arrangement of blades fixed behind one another round the circumference of a circular disk, which is fixed to a shaft, and running in a closed circular space (Fig. 313). This constitutes the blade disk of an axial, simple, or single stage steam turbine. The wheel receives the operating steam from a series of fixed openings (nozzles or channels) placed directly opposite the moving blades. These channels are inclined at the angle  $\alpha_1$  to the direction of motion of the moving blades, which move with the speed  $u$ , so that the angle of approach of the steam relative to the blades is  $\beta$ .

The purpose of the steam turbine is to convert the available work in the steam to useful work. This is effected by allowing the steam to acquire velocity, due to a pressure difference, after which jet pressures are exerted on the moving blades, as shown on page 574. If  $P$  is the component of the jet pressure in the direction of motion of the wheel blades which have a mean speed  $u$ , the work done per second is

The values of  $\alpha_1$ ,  $\beta_1$ , and  $\beta_2$ , and the corresponding areas  $a_0$ ,  $a_1$ , and  $a_2$  are so fixed that the pressure of the steam at the nozzle outlet area  $a_0$ ,

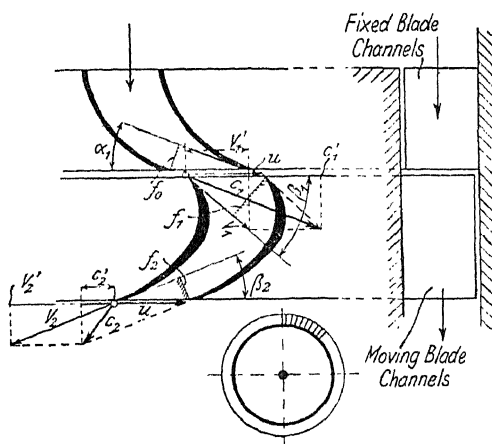


FIG. 313

and in the wheel chamber, is equal to the back or stage pressure. In the former case the velocity  $C_1$ , with which the steam leaves the nozzles, corresponds to the full drop between the initial pressure

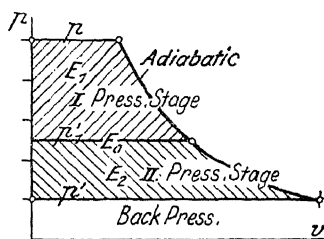


FIG. 314

$p$  and the back pressure  $p'$ , and the nozzles are then convergent-divergent in shape. In the latter case,  $C_1$  is smaller in value and is determined by the heat drop between  $p_1$  and the stage pressure. If this exceeds the critical drop the nozzles must again be convergent-divergent.

**Impulse turbines.** The work which can be obtained from 1 lb. of the steam approaching the wheel is equal to its kinetic energy  $C_1^2/2g$ , which, as shown on page 240, is equal to the area  $E_0$  of the adiabatic  $pV$  diagram (Fig. 314). The steam thus possesses its total kinetic energy at entrance to the wheel, which has to convert as much as possible of this energy to mechanical work at the shaft, so that the ratio  $Pu/E_0$  may be made as high as possible. This is attained by making the absolute velocity  $C_2$ , with which the steam leaves the wheel, as small as possible by a suitable choice of blade angles and blade speed  $u$ . The first turbines constructed on this principle were those of the Swedish engineer, de Laval.

**Velocity and pressure stages.** With normal steam pressures of 150 to 180 lb./in.<sup>2</sup> abs., the discharge velocity from the nozzles is about 3000 ft./sec. when discharging to the atmosphere, and about 4000 ft./sec. when condensers are used. If as much as possible of the energy of the steam is to be utilized by the wheel, the blade speed has to be about half these values (as shown below), i.e.  $u = 1500$  and 2000 ft./sec.

respectively. This means, in the case of a wheel 3 ft. in diameter, that the number of revolutions per minute is  $n = \frac{60 \times 2000}{\pi \times 3} = 12,750$ , and this increases as the diameter is diminished. Apart from the fact that such high speeds produce dangerously high centrifugal stresses, there is the further disadvantage that, for most practical purposes (e.g. electrical or mechanical drives) speeds of over 10,000 rev./min. are unsuitable without the use of reduction gears. Hence it becomes necessary, in single wheel machines, to operate with smaller values of  $u$  and  $n$ , so that the available work per pound of steam is reduced, causing an increase in steam consumption. In order to reduce the blade speeds and still use the steam efficiently, two fundamentally different methods may be used, the one consists in the application of velocity stages, the other in the application of pressure stages. Both methods may also be applied together. Instead of having a single fixed and moving wheel, several of such pairs are arranged in series, so that each succeeding stage takes the steam exhausted from the previous stage and delivers work.

In velocity compounding, steam acquires its full velocity  $C_1$  at entrance to the first set of moving blades, but the speed of these is considerably less than in a single velocity wheel, and the absolute leaving velocity  $C'_2$  from the first wheel is higher. The steam now flows round fixed blades with this velocity, without change in pressure, and is then supplied to the second moving blades, the velocity of which is again  $u$ . The leaving velocity  $C_2$  from this wheel is considerably reduced. The process can be repeated again but, due to frictional losses, it is not profitable to employ more than three stages. The more common arrangement employs two stages only (Curtis wheels).

The steam conditions are somewhat different in pressure compounding. The steam leaves the first set of nozzles or guide blades with a velocity of  $C'_1$ , which corresponds to a fraction only of the total pressure drop, so that the steam pressure  $p'_1$  here and throughout the first moving blades is considerably higher than the back pressure  $p'_1$ . Of the  $pV$  diagram (Fig. 314), only the area  $E_1$  is converted to kinetic energy,

$$\text{i.e.} \quad \frac{C_1^2}{2g} = E_1$$

In order to convert as much of this as possible to useful work, the blading is designed as for a single wheel impulse turbine, but since  $C'_1 < C$ , the necessary blade speed is reduced in the same ratio, so that it amounts to

$$u' = u \frac{C'_1}{C_1}$$

The steam leaving the first stage at a reduced velocity is now used in the second stage. It could be led to a receiver, as in the compound steam engine, but as a rule it is led directly to the second guide blades. If there are only two stages, the pressure in the second moving blades equals the back pressure  $p'$  and the diagram area  $E_2$  (Fig. 314) between  $p_1$  and  $p'$  corresponds to the kinetic energy  $\frac{C_1'^2}{2g} = E_2$ . If  $E_1 = E_2$  then

$C_1'' = C_1'$ , and since  $u$  is common to both wheels, the second set of blades will have the same blade angles as the first.

Instead of two stages, three or more pressure stages can be used. The upper limit of the number of stages is considerably higher than that of a velocity compounded wheel. As the number of pressure stages is increased, the speed of the blades decreases. The  $pV$  diagram then appears as a number of horizontally divided strips (Fig. 315) in which the number of strips equals the number of stages. The pressure at

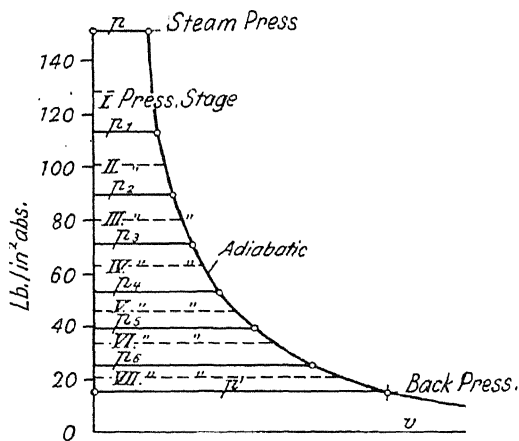


FIG. 315

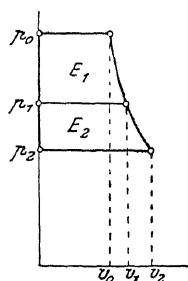


FIG. 316

entrance to the guide channels of the various stages equals the higher pressures of  $p_1$ ,  $p_2$ , etc., of the separate strips.

**Reaction turbines.** This group of turbines is characterized by having a further expansion of steam in the moving blades, so that both the pressure and relative speed change during its passage through the blades (Figs. 310 and 313). In the single stage reaction turbine (in which the pressure at exit from the moving blades is equal to the back pressure), the steam pressure drops from  $p_0$  to  $p_1$  (Fig. 316) in the nozzles or fixed blades, and continues to drop from  $p_1$  to the back pressure  $p_2$  in the moving blades.

In the fixed blades the area  $E_1$  is available for conversion to kinetic energy, while the corresponding area  $E_2$  is available in the moving blades, giving

$$E_1 = \frac{C_1^2}{2g}$$

$$E_2 = \frac{V_2^2}{2g} - \frac{V_1^2}{2g}$$

$$\text{and } E_1 + E_2 = E_0$$

The useful blade pressures are effected here (as in the blading shown in Fig. 310) by the united action of impulse and reaction of the steam.

The ratio  $\frac{E_2}{E_0}$  is called the degree of reaction. Half reaction, i.e.  $E_2 = \frac{1}{2}E_0$  or  $E_1 = E_2$  is frequently adopted. Experience shows that single

stage reaction turbines are unsuitable in practice, not only (as in the case of impulse machines also) on account of the high blade speeds necessary, but also on account of the high leaving losses when normal steam pressures are used. Hence, it is even more necessary in these machines to adopt a series of pressure stages, so that in each stage only a fraction of the pressure drop is converted to work. Even in the early Parson's turbines, therefore, a large number of fixed and moving blade wheels were arranged in series without intermediate receivers, so that moderate blade speeds resulted. This was particularly necessary where the turbines were used in marine work, owing to the low speed of the propeller. The division of the drop is shown again by Fig. 315. The dotted horizontal line divides

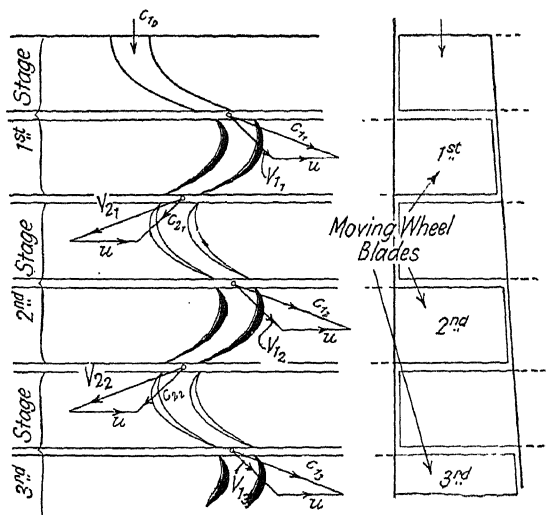


FIG. 317

the work done in each pressure stage, the upper portion showing the pressure drop in the fixed blades and the lower portion the pressure drop through the moving blades. Fig. 317 shows the arrangement of a series of fixed and moving blades in a multistage reaction turbine.

**Work and efficiency of a single stage frictionless impulse turbine. (Laval turbine.)** Steam, issuing from a convergent or (in the case of high pressure drops) convergent-divergent nozzle, flows with the absolute velocity  $C_1$  and is inclined at the angle  $\alpha_0$  to the direction of motion of the blades. These blades, of which one only is shown in Fig. 318, have the velocity  $u$ .

The pressure due to the jet on the blade is calculated in the same way as for the fixed blade. If  $V$  is the constant velocity of the jet, relative to the blade, then, as on page 577,

$$P_x = V (\cos \alpha_1 + \cos \alpha_2)$$

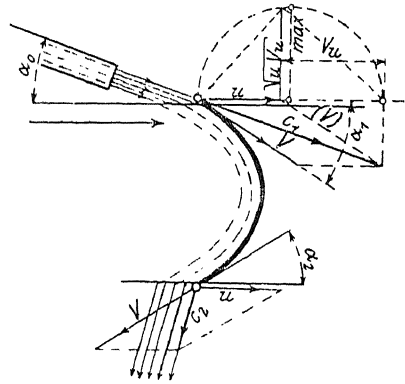
and  $P_y = V (\sin \alpha_1 - \sin \alpha_2)$

The work done per second by the jet is that due to  $P_{\text{jet}}$

i.e. 
$$E = \frac{1}{2} \rho V^2 A \cos^2 \alpha_1$$

or 
$$E = \frac{1}{2} \rho V u (\cos \alpha_1 + \cos \alpha_2)$$

In order that the jet may glide on to the blade without shock, it is necessary that the blade velocity should have a definite value, conditioned by  $C_1$  and the angles  $\alpha_0$  and  $\alpha_1$ . It must equal the component of  $C_1$  and  $V$  in the direction of motion of the blades. If, say, the blade velocity is  $u' < u$ , then the jet causes impact on the blade, due to the velocity difference  $u - u'$ . If on the other hand  $u' > u$ , the jet lags behind the blades and is driven by the next oncoming blade, due to the velocity difference  $u' - u$ . Hence it is necessary that  $u'$  should equal  $u$ , since both effects are harmful. The jet then approaches the blade with a speed  $V$  at the angle  $\alpha_1$ . Expressed more briefly, the value of  $C_1$  must be that given by the diagonal of the parallelogram formed by  $u$  and  $V$ . (Inlet parallelogram.)



Assuming, provisionally, that  $\alpha_1 = \alpha_2$ , then

$$E = 2 \frac{1}{2} \rho V u \cos^2 \alpha_1$$

In turbines, this should be as high as possible, or  $Vu \cos \alpha_1$  should have a maximum value. Calling the projection of  $V$  in the direction of motion of the blades (i.e. the value  $V \cos \alpha_1$ )  $V_u$ , we have to find when  $V_u \cos \alpha_1 = u V_u$  is a maximum.

For a given approach angle  $\alpha_0$  and approach velocity  $C_1$ , the sum  $u + V_u$  has the fixed value  $C_1 \cos \alpha_0$  (Fig. 318). The separate values of  $u$  and  $V_u$  are, however, different, as they depend on the blade angle. Their product has a maximum value if

$$u = V_u = \frac{1}{2} C_1 \cos \alpha_0$$

This follows since, in the right angled triangle, the square of the height is equal to the product of the segments of the hypotenuse (Fig. 318).

In this case the work done is

$$E = \frac{C_1^2}{2g} \cos^2 \alpha_0$$

It attains its maximum value when  $\cos^2 \alpha_0 = 1$ , or  $\alpha_0 = 0$ , giving

i.e. the whole of the kinetic energy of the jet is converted to work on the

blades. In practice, this is not possible, and the best that can be done is to make  $\alpha_0$  as small as possible.

Again, since  $\cos \alpha_0 \doteq 1$ ,

$$u \doteq \frac{1}{2}C_1$$

i.e. the blade speed is half the jet speed. Also, approximately,

$$\alpha_1 = 2\alpha_0$$

In general, if  $u$  deviates from  $\frac{1}{2}C_1$  and  $\alpha_0$  deviates from  $0^\circ$ ,

$$E = \frac{2W_{sec}}{g} u V \cos \alpha$$

which can be written as

$$E = \frac{1}{g} u (C_1 \cos \alpha_0 -$$

since  $u + V \cos \alpha = C_1 \cos \alpha_0$ .

The efficiency is, with

$$= \frac{E}{E_n} = \frac{4u}{C_1^2} (C_1 \cos \alpha_0 - u)$$

or

$$\eta = \frac{4u}{C_1} \left( \frac{C_1 \cos \alpha_0}{C_1} - \frac{u}{C_1} \right)$$

This shows that  $\eta$  depends primarily on the ratio of the blade speed to the steam speed, i.e. on the speed ratio.

This value is a maximum when  $\frac{u}{C_1} = \frac{1}{2} \cos \alpha_0$ , giving

$$\eta_{max} = \cos^2 \alpha_0$$

For all ratios of  $\frac{u}{C_1}$  less than  $\frac{1}{2} \cos \alpha_0$ ,  $\eta$  becomes smaller. The absolute leaving velocity  $C_2$  is found, from

$$\overline{2g} = \overline{2g}$$

and

$$\eta = \frac{E}{E_n} = \frac{C_1^2 - C_2^2}{C_1^2} = 1 - \frac{C_2^2}{C_1^2}$$

to be

$$\sqrt{1 - \eta}$$

In single stage impulse turbines, the steam jet velocity at entrance to the blades must be that corresponding to the full pressure drop. In turbines with condensers having a vacuum of 1.5 lb./in.<sup>2</sup> abs. and operating with supply steam at 150 lb./in.<sup>2</sup> abs., the value of  $C_1$  is 3850 ft./sec. When exhausting to the atmosphere, the value is  $C_1 = 2900$  ft./sec. If the



approach angle is  $\alpha_0 = 18^\circ$ , then  $u = \frac{1}{2}C_1 \cos \alpha_0 = 0.475 C_1$  and  $\eta_{max} = 0.904$ . The blade velocities are then, for condensing machines,

$$0.475 \times 3850 = 1830 \text{ ft./sec.}$$

and for non-condensing machines,

$$0.475 \times 2900 = 1380 \text{ ft./sec.}$$

The leaving velocities are then,  $C_2 = 0.31 C_1$ , giving 1190 and 900 ft./sec. respectively.

**Work and efficiency of the single stage frictionless reaction turbine.** The component  $R_x$  of the blade pressure  $R$  (Fig. 310) is given by the total change in steam velocity in the direction of motion multiplied by  $\frac{W_{sec}}{g}$ . This change can be determined as the difference of the components  $C'_1$  and  $C'_2$  of the absolute leaving velocities  $C_1$  and  $C_2$  from the guide and moving blades, but it is also given by taking the difference of the component relative velocities  $V_1$  and  $V_2$  in the direction of motion, since it will be seen (on referring to the parallelogram in Fig. 313) that the same components are obtained whether  $C_1$  be projected directly or the components of  $V_1$  and  $u$  be projected to the line of motion of the blades.

Fig. 313 shows directly that

$$C'_1 + C'_2 = V'_1 + V'_2 = C'_1 + (V'_2 - u)$$

The quickest way of obtaining the result is to take the component of  $C_1$  at inlet and the components of  $V_2$  and  $u$  at outlet. The former is  $C_1 \cos \alpha_1$ , and the difference of the latter  $V_2 \cos \beta_2 - u$ . The sum of these values

$$C_1 \cos \alpha_1 + V_2 \cos \beta_2 - u$$

gives the total change in velocity in the direction of the blade motion.

Hence the working component for 1 lb. of steam is

$$R_x = \frac{1}{g} (C_1 \cos \alpha_1 + V_2 \cos \beta_2 - u)$$

and the useful work per second performed on the wheel is

$$E = R_x u$$

$$\text{or} \quad E = \frac{u}{g} (C_1 \cos \alpha_1 + V_2 \cos \beta_2 - u) \quad . \quad . \quad . \quad (598)$$

The available work in the steam is

$$E_0 = E_1 + E_2 = E_1 + \left( \frac{\rho}{1-\rho} \right) E_1 = \frac{1}{1-\rho} E_1$$

$$\text{or} \quad E_0 = \frac{1}{1-\rho} \frac{C_1^2}{2g}$$

Of this, the fraction  $\eta_a = \frac{E}{E_0}$  is converted to work (efficiency).

$$\text{Now} \quad \eta_a = 2(1-\rho) \frac{u}{C_1} \left( \cos \alpha_1 + \frac{V_2}{C_1} \cos \beta_2 - \frac{u}{C_1} \right) \quad . \quad . \quad (599)$$

In this,  $u$  and  $\frac{u}{C_1}$  can have arbitrary values, and from these  $C_1$  and  $E_1 = \frac{C_1^2}{2g}$  are established.  $\alpha_1$  can also be chosen at random, and this gives  $V_1$  and  $\beta_1$ . Again,  $\rho$  can have any suitable value and gives the moving wheel drop  $E_2 = \frac{\rho}{1-\rho} E_1$ . The values  $V_2$  and  $\beta_2$  are, however, not arbitrary. Hence, before any general conclusions can be drawn from the above equation,  $V_2$  and  $\beta_2$  must be expressed in terms of the other values.

We have  $V_2^2 - V_1^2 = 2g E_2$

$$\text{and} \quad E_2 = \frac{\rho}{1-\rho} E_1 = \frac{\rho}{1-\rho} \frac{C_1^2}{2g}$$

$$\text{hence} \quad V_2^2 = V_1^2 + \frac{\rho}{1-\rho} C_1^2$$

$V_1$  is found from the velocity triangle at inlet, since

$$\begin{aligned} V_1^2 &= u^2 + C_1^2 - 2u C_1 \cos \alpha_1 \\ &= u^2 \left[ 1 + \left( \frac{C_1}{u} \right)^2 - 2 \frac{C_1}{u} \cos \alpha_1 \right] \end{aligned}$$

$$\begin{aligned} \text{Hence} \quad V_2^2 &= u^2 \left[ 1 + \left( \frac{C_1}{u} \right)^2 - 2 \frac{C_1}{u} \cos \alpha_1 \right] + \frac{\rho}{1-\rho} C_1^2 \\ &= u^2 \left[ 1 + \frac{1}{1-\rho} \left( \frac{C_1}{u} \right)^2 - 2 \frac{C_1}{u} \cos \alpha_1 \right] \end{aligned}$$

Inserting this in equation (599) gives

$$\eta_a = 2(1-\rho) \frac{u}{C_1} \left[ \cos \alpha_1 + \cos \beta_2 \sqrt{\left( \frac{u}{C_1} \right)^2 + \frac{1}{1-\rho} - \frac{2u \cos \alpha_1}{C_1} - \frac{u}{C_1}} \right]. \quad (600)$$

$\cos \beta_2$  should also be expressed in terms of the other values. This, however, would lead to a very involved expression, and since  $\beta_2$  does not vary greatly, even for as widely different degrees of reaction as 1:3 and 2:3, it is permissible to assume the mean value given for  $\rho = 1:2$ , especially since the variation in  $\cos \beta_2$  is still less. Hence, unless an exceedingly wide variation occurs in the value of  $\rho$ , we can assume  $\cos \beta_2 = \cos \alpha_1$ . Alternatively, an arbitrary value of  $\beta_2$  can be assumed for the different values of  $\rho$ , and in this way the necessary exit areas are determined by making use of the equation of continuity

$$\frac{\sin \beta_2}{\sin \alpha_1}$$

In the former case, we have

The corresponding equation for the impulse wheel is found from equation (599) by taking  $V_2 = V_1$ ,  $\beta_2 = \beta_1$ , and  $\rho = 0$ . This gives

$$\eta_a = 2 \frac{u}{C_1} \left( \cos \alpha_1 + \frac{V_1}{C_1} \cos \beta_1 - \frac{u}{C_1} \right)$$

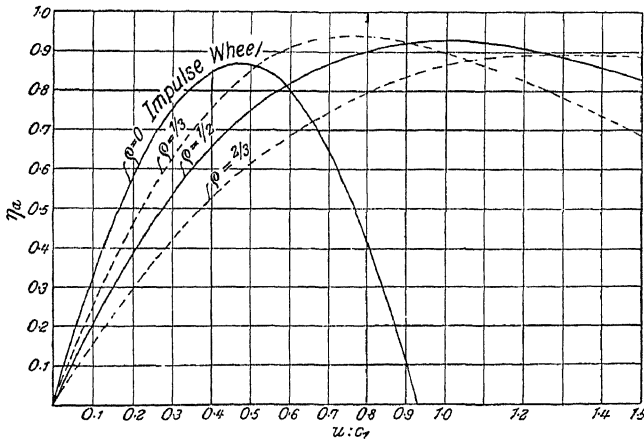


FIG. 319

From the velocity triangle at inlet,

$$V_1 \cos \beta_1 = C_1 \cos \alpha_1 - u$$

Inserting, this gives

$$\eta_a = 4 \frac{u}{C_1} \left( \cos \alpha_1 - \frac{u}{C_1} \right)$$

as on page 594.

In Fig. 319 the values of  $\eta_a$ , as given by these equations, with  $\cos \alpha_1 = 0.93$ ,  $\alpha_1 = 21^\circ 30'$ ,  $\beta_2 = \beta_1$ , and with  $\rho = \frac{1}{2}$ ,  $\frac{1}{3}$ ,  $\frac{2}{3}$ , and 0 have been plotted. The reaction wheels show a maximum value of 0.94, while the impulse only reaches the value 0.87. For the same efficiency, the blade speed of the reaction wheel is always a higher fraction of the steam speed  $C_1$  than in the impulse wheel, sometimes being as much as double, depending on the degree of reaction. For the same total heat drop, however,  $C_1$  is smaller in the reaction wheel. For smaller values of  $u/C_1$  than 0.5, as occur almost always in practice, the impulse wheel shows a higher efficiency. Next to it the wheel with the smallest amount of reaction 1 : 3, shows the highest efficiency.

If, therefore, multistage reaction turbines, having values of  $u/C_1$  considerably less than 0.5, are to show a high efficiency, it is necessary to

choose a small degree of reaction, so that the leaving velocity in any stage may be utilized in the directly following stage.

**Wheel speeds and calculations for the frictionless single stage reaction turbine.** The most important factor in steam turbine design is the blade speed  $u$ . The speed of the turbine is limited by that of the machinery to be driven, unless gearing or some other system of changing the speed be adopted. Turbo-generators run at speeds between 1500 and 3000 rev./min., while, for ships propellers, a speed of 600 rev./min. is considered high. Again, since the diameter  $D$  of the wheels is limited in size, the blade speed, given by

$$u = \pi D \frac{n}{60}$$

is limited to values which are relatively small compared with the steam speeds given by moderately high pressure differences. Thus, with  $D = 3$  ft. and  $n = 1000$ , the value of the blade speed is  $u = 157$  ft./sec. With  $n = 600$  and  $D = 3$  ft.,  $u = 94$  ft./sec., and with  $n = 3000$ ,  $u = 470$  ft./sec.

The strength of the wheels and blade fixing also limits the blade speeds, due to the stresses set up by centrifugal forces. With  $u > 900$  ft./sec., considerable difficulties are met with, due to this; the centrifugal force of a turbine blade rotating in a circle of 3 ft. diameter at 900 ft./sec.

amounts to  $\frac{900^2}{32.2 \times 1.5} = 18650$  times its own weight.

The steam consumption per horse-power hour is another important factor in steam turbine design. Now, as shown below, and also on page 597, the efficiency of the blades depends primarily on the ratio  $u/C_1$ . The steam consumption becomes less (up to a certain limit) as the ratio  $u/C_1$  is increased. Starting with a definite value of blade speed, the steam speed is reduced as  $\frac{u}{C_1}$  is increased. Hence the heat drop for a single wheel turbine is reduced as  $\frac{u}{C_1}$  is increased; this means that the number of wheels necessary for the whole heat drop is increased, and the weight and cost of the turbine increases correspondingly. Hence, on account of economy, mean values of  $\frac{u}{C_1}$  are required, which lie below the best theoretical values. The values of  $\frac{u}{C_1}$  used in reaction turbines vary from 0.5 to 0.3, and for slow speed turbines (such as marine) values of 0.2 and less are chosen.

A third assumption has to be made regarding the angle  $\alpha_1$  at outlet from the guide blades. It is generally chosen between  $20^\circ$  and  $30^\circ$ . A fourth assumption has to be made regarding the angle  $\beta_2$  at the moving blade outlet (frequently made equal to  $\alpha_1$ , which gives half reaction), or the whole drop is assumed, i.e.  $E_0 = E_1 + E_2$ , which, for a known value of  $C_1$ , is the same as assuming a value for  $E_2$  or the degree of reaction.

Due to the second and third assumptions  $\left(\frac{u}{C_1} \text{ and } \alpha_1\right)$ , the inlet angle  $\beta_1$  of the moving blade is determined as in Fig. 320. It becomes

smaller as  $\frac{u}{C_1}$  is reduced. For very small values of  $\frac{u}{C_1}$ , due to this, it is necessary in certain cases to increase the value of  $\alpha_1$ .

The method of calculation now proceeds as follows. The values of  $u$ ,  $u/C_1$ , and  $\alpha_1$  are first assumed. This allows the inlet triangle to be drawn down, thus giving the blade inlet angle  $\beta_1$ . Further,  $C_1 = u \frac{C_1}{u}$ , so that  $V_1$  is given as the third side of the triangle formed with  $u$  and  $C_1$ . Having found  $C_1$ , the work in the guide channels is given by

$$E_1 = \frac{C_1^2}{2g} \text{ and } h_1 = \frac{1}{777.8} \frac{C_1^2}{2g}$$

This gives the pressure drop  $p_0 - p_1$  down to the entrance to the moving blades, either by using the  $H\phi$  chart or by calculation, as on page 219. For small drops it is sufficiently accurate to take

$$AE_1 = h_1 = \frac{144}{777.8} (p_0 - p_1) v_0$$

giving 
$$p_0 - p_1 = \frac{777.8 h_1}{144 v_0} \text{ lb./in.}^2$$

The most convenient final assumption to make is the heat drop  $h_2$  in the moving blades. This can either be taken as equal to  $h_1$  (half reaction),

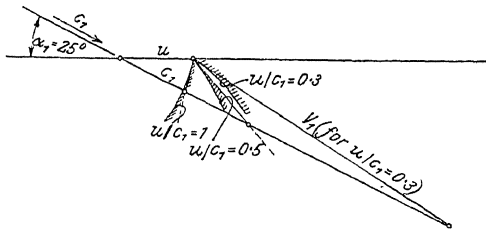


FIG. 320

in which case the angles  $\beta_2$  and  $\alpha_1$  are about equal, or taken greater or less than  $h_1$ , giving  $\beta_2 \gtrless \alpha_1$ . The first assumption is generally made. With  $h_2$  fixed,  $V_2$  is given by

$$\frac{V_2^2}{2g} = \frac{V_1^2}{2g} + 777.8 h_2$$

since 
$$\frac{V_2^2}{2g} - \frac{V_1^2}{2g} = E_2$$

It is necessary that the equation of continuity should be satisfied in the moving blades, i.e.

$$\frac{a_1 V_1}{v_1} = \frac{a_2 V_2}{v_2}$$

giving 
$$\frac{a_2}{a_1} = \frac{V_1 v_2}{V_2 v_1}$$

and, for small drops,

$$\frac{a_2}{a_1} = \frac{V_1}{V_2}$$

$\frac{v_2}{v_1}$  is given after finding the pressure drop in the moving blades from

$$p_1 - p_2 = \frac{777.8 \, h_2}{v_1}$$

either from the adiabatic equation or from the state diagram, which also gives  $v_1$ . For small drops it is sufficiently accurate to proceed as follows:

$$\text{Since} \quad \frac{\Delta p}{\Delta v} = -\gamma \frac{p}{v} \quad (\text{page 107})$$

in the guide blades

$$\Delta v_1 = v_0 - v_1 = \frac{v_0 \Delta p}{\gamma p}$$

$$\therefore \quad v_1 = v_0 \left( 1 + \frac{p_0 - p_1}{\gamma p_0} \right)$$

and, hence, in the moving blades,

$$v_2 = v_1 \left( 1 + \frac{p_1 - p_2}{\gamma p_1} \right)$$

This enables the ratio  $\frac{a_2}{a_1}$  of the areas in the moving blades to be determined. In order to find the blade exit angle, use is made of

$$\begin{aligned} a_0 &= l_0 \sin \beta_0 \\ a_1 &= l_1 \sin \beta_1 \end{aligned}$$

giving  $\sin \beta_2$  directly if the blade height is constant, i.e.  $l_1 = l_2$ ;  $\sin \beta_1$  is found from the velocity triangle at inlet. Unequal blade lengths are only adopted when the leaving velocity  $C_2$  is too high.

Finally, the exit velocity triangle is drawn from the calculated values of  $V_2$  and  $\sin \beta_2$  and the given value of  $u$ . The third side of the triangle gives the absolute leaving velocity  $C_2$  in magnitude and direction. The expression  $\frac{1}{777.8} \frac{C_2^2}{2g}$  gives the leaving loss, so that the useful work on the blades is

$$AE_i = h_1 + h_2 - \frac{1}{777.8} \frac{C_2^2}{2g}$$

The blade efficiency is (neglecting friction)

$$\eta_0 = \frac{AE_i}{h_1 + h_2}$$

**Example 10.** The blade speed of reaction blades is  $u = 230$  ft./sec.,

the speed ratio  $u/C_1 = 0.3$ , the inlet jet angle  $\alpha_1 = 25^\circ$ , and the dry steam pressure at inlet to the guide blades 150 lb./in.<sup>2</sup> abs. Find—

- (1) on the assumption of half reaction,  $\rho = 1:2$
- (2) on the assumption  $\rho = 1:3$ ;
- (3) on the assumption  $\rho = 2:3$

the moving blade inlet angle, the total heat and pressure drops, and the efficiency.

The steam speed is

$$C_1 = \frac{230}{0.3} = 766.7 \text{ ft./sec. and } \frac{C_1^2}{2g} = 9150 \text{ ft. lb.}$$

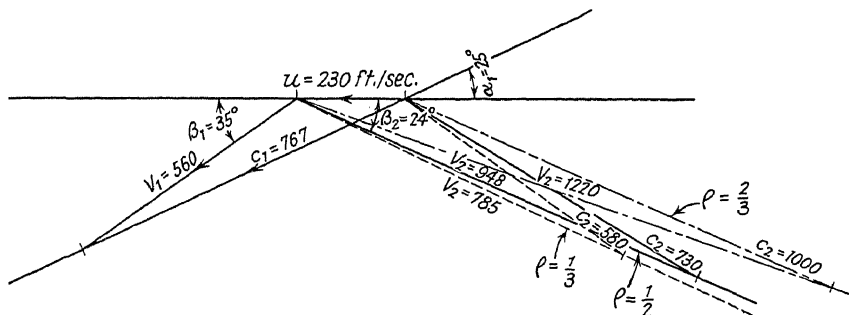


FIG. 321

Hence  $h_1 = 11.76$  and, from the velocity triangle at inlet,  $V_1 = 560$  ft./sec.,  $\beta_1 = 35^\circ$ , and  $\sin \beta_1 = 0.574$ . From steam tables at  $p_0 = 150$ ,  $v_0 = 3 \text{ ft.}^3/\text{lb.}$

hence 
$$p_0 - p_1 = \frac{9150}{144 \times 3} = 21.2 \text{ lb./in.}^2$$

and 
$$v_1 = 3 \left( 1 + \frac{21.2}{1.135 \times 150} \right) = 3 (1 + .125) = 3.38 \text{ ft.}^3/\text{lb.}$$

A closer value of the pressure drop can now be found from the mean volume  $3.19 \text{ ft.}^3/\text{lb.}$ , giving

$$p_0 - p_1 = \frac{9150}{144 \times 3.19} = 19.95 \text{ lb./in.}^2$$

and, correcting for  $v_1$  gives

$$\begin{aligned} v_1 &= 3 \left( 1 + \frac{19.95}{1.135 \times 150} \right) = 3 \times 1.1172 \\ &= 3.35 \text{ ft.}^3/\text{lb.} \end{aligned}$$

With  $\rho = 1:2$ , we have  $E_2 = E_1 = 9,150$ .

Hence 
$$V_2^2 = 560^2 + 64.4 \times 9150$$

or  $= 560^2 + 767^2$ , which gives  $V_2 = 948$ , as given also by drawing a right-angled triangle. Again,

$$v_2 = \frac{9150}{19} = 481.6 \text{ ft./in.}^2$$

and 
$$v_2 = 3.35 \left( 1 + \frac{19}{1.135 \times 130.05} \right) = 3.35 \times 1.121 = 3.78 \text{ ft.}^3/\text{lb.}$$

Taking the mean volume 3.59 gives, more closely

$$p_1 - p_2 = \frac{9150}{144 \times 3.57} = 17.8$$

and 
$$v_2 = 3.35 \left( 1 + \frac{17.8}{1.135 \times 130} \right) = 3.35 \times 1.121 = 3.78 \text{ ft.}^3/\text{lb.}$$

and  $p_2 = 112.2 \text{ lb./in.}^2 \text{ abs.}$

The  $H\phi$  diagram gives, with  $2 \times 11.76 \text{ B.Th.U.}$ ,  $p_2 = 112 \text{ lb./in.}^2 \text{ abs.}$  We now have

$$\frac{2}{a_1} = \frac{560}{948} \times \frac{4.05}{3.35} =$$

with  $h_1 = h_2$ , therefore,

$$\sin \beta_2 = 0.574 \times 0.715 = 0.41 \quad (\beta_2 \doteq 24^\circ)$$

which is nearly the same as  $\beta_1$ .

From the velocity triangle at outlet we now find  $C_2 = 730 \text{ ft./sec.}$ , so that the leaving loss is  $\frac{C_2^2}{2g} = 850 \text{ ft. lb.} = 11.00 \text{ B.Th.U.}$  The heat drop is  $2 \times 11.76 = 23.52$ , hence the efficiency is

$$\frac{23.52 - 11.00}{23.52} = \frac{14.52}{23.52} = 0.533$$

With  $\rho = \frac{1}{3}$ , we have

$$E_2 = \frac{\rho}{1-\rho} E_1 = \frac{1}{2} E_1 = 4575 \text{ ft. lb.} \quad h_2 = 5.89 \text{ B.Th.U.}$$

The inlet values remain unaltered. For exit

$$V_2^2 = 560^2 + 2g \cdot 4757$$

which gives  $V_2 = 785$ . Also  $p_1 - p_2 = 9.19 \text{ lb./in.}^2$ ,  $v_2 = 3.56 \text{ ft.}^3/\text{lb.}$ ,  $p_2 = 120.8 \text{ lb./in.}^2 \text{ abs.}$

$$\frac{a_2}{a_1} = \frac{560}{785} \times \frac{3.56}{3.35} = 0.759$$

With  $h_1 = h_2$ ,  $\sin \beta_2 = 0.574 \times 0.759 = 0.435$ ,  $\beta_2 = 26^\circ$ . The outlet velocity triangle is shown dotted and gives  $C_2 = 580 \text{ ft./sec.}$  and  $\frac{580^2}{2g} = 5220 \text{ ft. lb.} = 6.72 \text{ B.Th.U.}$  The total drop is  $11.76 + 5.89 = 17.65 \text{ B.Th.U.}$  and the useful work  $17.65 - 6.72 = 10.93$ , giving an efficiency  $\eta = \frac{10.93}{17.65} = 0.62$ , which is higher than with half reaction.



With  $\rho = \frac{2}{3}$ , we have  $E_2 = 2E_1 = 18300$ ,  $h_2 = 23.5$  B.Th.U.,  $V_2^2 = 560^2 + 2g \times 18300$ , so that  $V_2 = 1220$  ft./sec.,  $\rho = 97$  lb./in.<sup>2</sup> abs. (from chart).

$$= 23.35 \left( \frac{1.00}{97} \right)$$

$$\frac{2}{1} = \frac{560}{1220} \times \frac{4.34}{3.35} = 0.594$$

$\sin \beta_2 = 0.594 \times 0.574 = 0.341$ ,  $\beta_2 = 20^\circ$ ,  $C_2 = 1000$  ft./sec., leaving loss = 15530 ft. lb.  $\doteq$  20 B.Th.U. Total drop = 11.76 + 23.50 = 35.26 B.Th.U. Useful work = 15.26 B.Th.U.,  $\eta = 0.43$ .

## BINARY VAPOUR AND GAS TURBINES

**Binary vapour turbine.** Up till recently the only vapour used in driving engines and turbines was steam, mainly on account of its cheapness and plentiful supply. Recently, however, engineers have been regarding the possibility of using other vapours, which, for the same pressure and temperature limits, may show higher thermal efficiencies and thus produce more work for the same combustion heat liberated.

The upper temperature limit for working vapours, of any nature, may be taken as about 850° F. Steam at this temperature is superheated, no matter what the pressure may be. Now the thermal efficiency depends on the saturation temperature corresponding to the supply pressure rather than on the superheat temperature, whether or not the process involves partial or complete regeneration. The thermal efficiency increases, therefore, as the saturation temperature is increased. The upper saturation temperature for each particular vapour is limited by its critical temperature, which, for steam, amounts to 705° F. The corresponding steam pressure is 3200 lb./in.<sup>2</sup> abs., which is very high for prime movers. On the other hand, the critical temperature of mercury is considerably higher than 2000° F., and the vapour pressure of mercury at 705° F. is only about 20 lb./in.<sup>2</sup> abs., at 750° F. about 31.3 lb./in.<sup>2</sup> abs., and at 930° F. about 104 lb./in.<sup>2</sup> abs. Hence, mercury vapour at moderate pressures can be used at temperatures which, for steam, would mean a considerable amount of superheat.

In order to show the available work of a mercury vapour turbine, a temperature entropy diagram for mercury has been plotted in Fig. 322. For the purpose of comparison, a  $T\phi$  diagram for steam has also been drawn in this diagram and, in view of what follows, the weights of the two vapours chosen are 1 lb. of H<sub>2</sub>O and 8 lb. of Hg. The specific heat of the liquid mercury is assumed to be 0.033 B.Th.U./lb., and the latent heats are taken from Fig. 218, from which the entropy increase  $L/T$  during evaporation can be calculated. This gives the limiting curves  $AB_1$  and  $C_1D$ . Now the working process cannot be effected with mercury alone, because, even at 430° F., the vapour pressure of the mercury amounts to 32.5 mm. Hg, i.e. 0.625 lb./in.<sup>2</sup> abs., which is the same as that of water vapour at 86° F.; at the latter temperature, which, on account of the air and cooling water temperature, may be regarded as a

lower limit, the vapour pressure of mercury is less than  $\frac{1}{100}$  mm., so that a mercury turbine operating between  $400^{\circ}$  F. and  $86^{\circ}$  F. would demand an exceedingly high vacuum. Hence, the mercury turbine operates between temperature limits of  $750$  and  $428^{\circ}$  F., and below that (i.e. between  $390$  and  $90^{\circ}$  F.) steam is used. In order to effect this, the

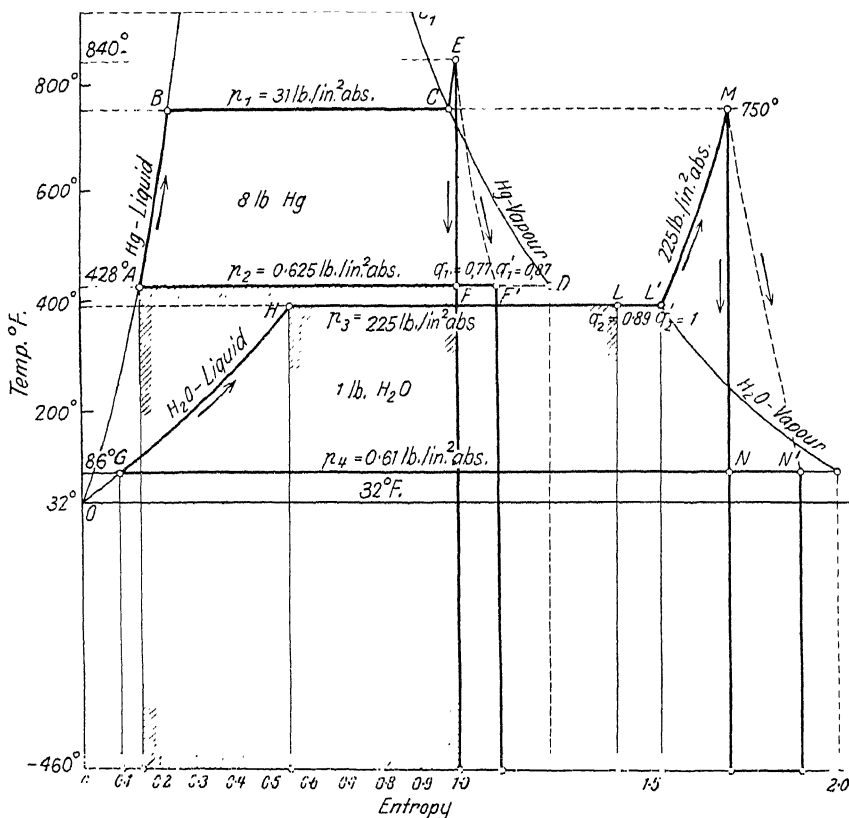


FIG. 322.

mercury vapour, exhausting from the mercury turbine, is condensed in a mercury condenser at  $428^{\circ}$  F. The cooling substance used in the condenser is water at  $390^{\circ}$  F. and  $250$  lb./in.<sup>2</sup> abs., which has been pre-heated in an economizer. The mercury condenser is thus really a steam boiler which is heated by mercury at  $428^{\circ}$  F. The steam generated in this boiler is supplied to a normal turbine with condenser.

These processes are illustrated as follows by means of Fig. 322. The hot gases formed in a normal furnace (oil fired) are first\* used in the mercury boiler, to which the liquid mercury from the mercury condenser is fed at  $428^{\circ}$  F. The liquid is thus heated (line AB) to  $750^{\circ}$  F., which corresponds to a vapour pressure of  $31$  lb./in.<sup>2</sup> abs., and then evaporated

\* The hot gases can also be first used in the steam superheater in order to reduce their temperature before being supplied round the tubes of the mercury boiler.

to dry saturated vapour (line  $BC$ ). The mercury vapour is slightly superheated to  $840^\circ\text{F.}$  after leaving the boiler (line  $CE$ ).\*

At this temperature the mercury vapour enters the turbine, in which it expands adiabatically to  $428^\circ\text{F.}$  and  $0.625\text{ lb./in.}^2\text{ abs.}$  (line  $EF$ ) provided the expansion is frictionless. The ideal work is given by the area  $ABCEEF$ , from which the re-heat due to friction, given by the area below  $FF'$ , has to be subtracted. On leaving the turbine, the mercury vapour is wet, having a quality, with frictionless flow, of  $q_1 = 0.77$ , and with 25 per cent re-heat of  $q'_1 = 0.87$ . This wet mercury vapour, weighing  $W_1\text{ lb.}$ , now gives up its latent heat, represented by the area below  $FA$  and equal to  $W_1 q_1 L_{2Hg}$ , in generating steam in the condenser. Neglecting all losses, the weight of steam, having the quality  $q_2$ , which is generated is  $W_2\text{ lb.}$ , say. The value of  $W_2$  or  $q_2$  is found from

Taking the conditions of the example in Fig. 322, we have  $L_{2Hg} = 120.6$ ,  $L_{2H_2O} = 822\text{ B.Th.U./lb.}$ ,  $W_1/W_2 = 8$ , hence

$$q_2 = q_1 \frac{8 \times 120.6}{822} = 1.173 q_1 = 0.89 \text{ (point } L)$$

With the assumed re-heat, on the other hand,

$$q'_2 = 1.173 \times 0.87 = 1$$

i.e. the weight of dry steam generated by 8 lb. of exhaust from the mercury turbine is 1 lb. (point  $L'$ ). This steam is now superheated to  $750^\circ\text{F.}$  by the furnace gases (point  $M$ ), and in this condition enters the steam turbine, where it expands adiabatically along  $MN$  (frictionless flow), or along  $MN'$  (if 25 per cent frictional re-heat be assumed) to the condenser pressure of  $0.61\text{ lb./in.}^2\text{ abs.}$

The work and heat quantities given by Fig. 322 are as follows (frictionless cases)—

Work—		
Mercury turbine	.	259 B.Th.U.
Steam turbine	.	454 B.Th.U.
Total useful work	.	713 B.Th.U.

Heats supplied—

Heating of 8 lb. of mercury from $430$ to $750^\circ\text{F.}$	$= 8 \times 320 \times 0.033 =$	84.5 B.Th.U.
Evaporation of 8 lb. of mercury at $750^\circ\text{F.}$	$= 8 \times 111.6 =$	893.0 B.Th.U.
Superheating of 8 lb. of mercury from $750$ to $840^\circ\text{F.}$	$= 8 \times 90 \times 0.025 =$	18.0 B.Th.U.
Preheating of 1 lb. of water from $96^\circ$ to $400^\circ\text{F.}$	$=$	304.0 B.Th.U.
Evaporation of $1 - q_2 = 0.1$ lb. of water at $400^\circ\text{F.}$	$=$	82.2 B.Th.U.
Superheating 1 lb. of steam from $400$ to $750^\circ\text{F.}$	$=$	200.0 B.Th.U.

Total heat supplied  $= 1581.7\text{ B.Th.U.}$

Ideal thermal efficiency  $\eta_{th} = 713$

\* The specific heat  $c_p$  of mercury gas can be calculated from the molecular specific heat of monatomic gases  $mc_p = 5$ , giving  $c_p = \frac{5}{199.8} = 0.025$ , since  $m = 199.8$ . As on page 441, however, there can be little doubt that the values of  $c_p$  in the saturation region are considerably greater than 0.025 and vary both with pressure and temperature, but not much seems to be known at present regarding these values.

In order to obtain an equal efficiency, using steam alone, the steam pressure would have to be 850 lb./in.<sup>2</sup> abs., with complete regeneration (see Fig. 280). Under normal conditions, using steam at 1,280 lb./in.<sup>2</sup> abs. the efficiency lies between 0.39 and 0.41 (Fig. 280), and, even with the highest pressures, it does not exceed 0.42. It is thus noteworthy that a combined mercury steam turbine with no regeneration, and with normal pressures, can show an ideal efficiency of 0.45.\* The actual thermal efficiency is, of course, less than this on account of losses in the boiler, condenser, and superheater, as well as in both turbines. Taking the combined efficiency of boiler and superheater as 0.80, and the separate thermodynamic efficiencies of the turbine as 0.75, gives

$$\eta_{th} = 0.8 \times 0.75 \times 0.45 = 0.27 \dagger$$

Turbines of this type (Emmet's) have been built by the General Electric Co. in America, and one, developing 10000 kW., has been installed by this company in the South Meadow Station, Hartford.‡

The large amount of mercury required, and also the poisonous nature of mercury vapour, preclude, however, the general adoption of this type of prime mover, but the machine offers an interesting example of the application of thermodynamic principles.

**The Holzwarth-Schüle combined gas and steam turbine.** The fundamental principles of the compound explosion turbine with high pressure wheel, receiver, and low pressure continuous stream-gas turbine, have been dealt with on page 198. The advisability and need of using steam in this process could only be referred to there, but a more complete discussion of this is given in the following.§

Holzwarth had, in his book published in 1911 (*Hans Holzwarth, Die Gasturbine, Theorie, Konstruktion und Betriebsergebnisse von 2 Ausgeführten Maschinen*), already shown that it was necessary to use the heat in the exhaust gases of the turbine to generate steam, so that this steam could be used to compress the charge.

This was also mentioned by the author in his Volume II (2nd Edition, 1914), and a fuller treatment will be found in the 3rd and 4th Editions (1923). Hence the single stage gas turbine, which, up till then, was the only example of this type, could also be regarded as a turbine operating with gas and steam.

In this type, however, the waste heat of the gas turbine was used to fire a boiler, the steam from which operated in a steam turbine. This process is sometimes adopted in the case of large gas or oil engines.

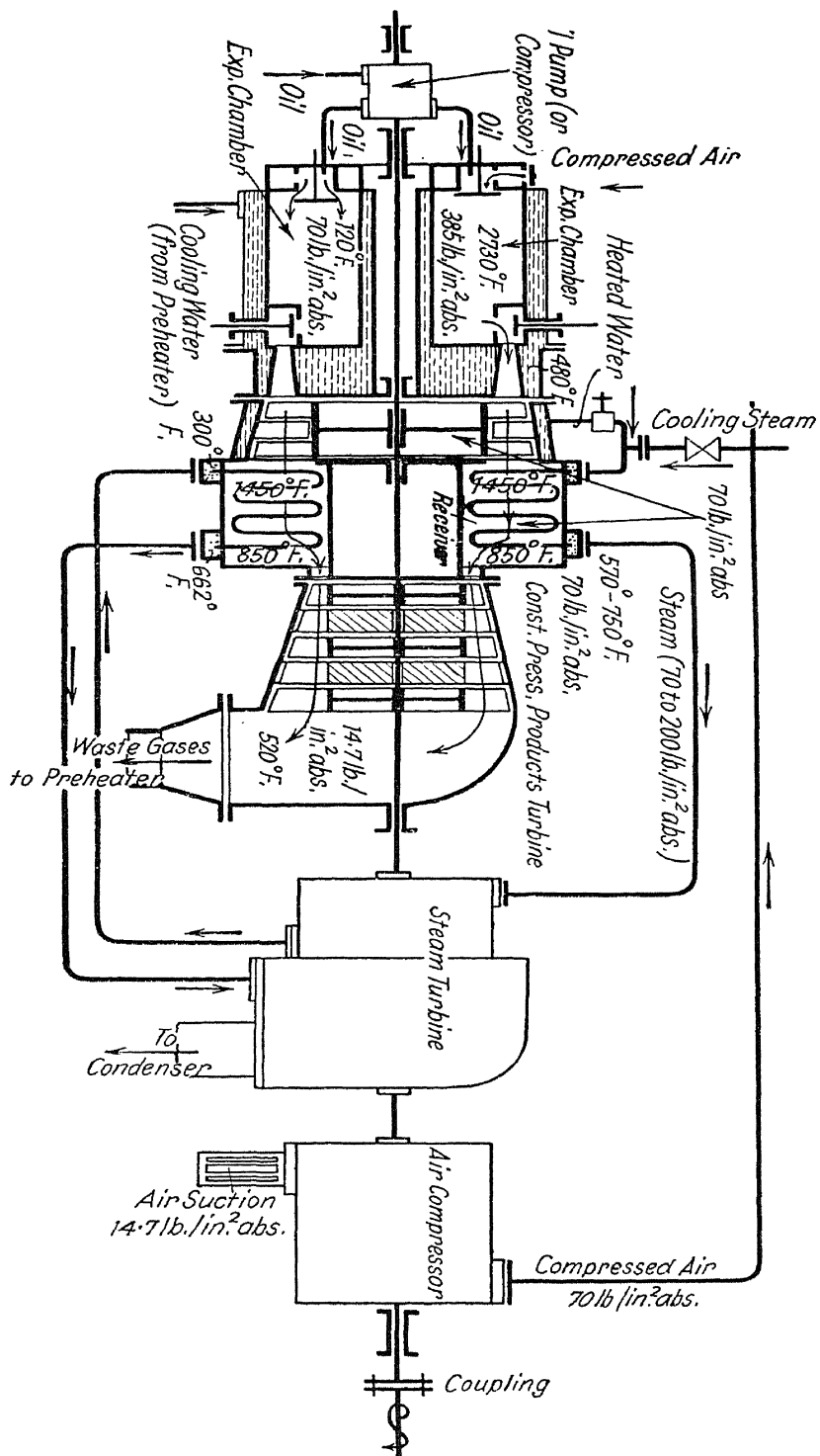
In the later two-stage process of the gas or oil turbine (illustrated diagrammatically in Fig. 323), steam again plays an important part, but in a somewhat different manner from that in the single-stage machine. It is essential to keep the temperature of the hot gases, entering the

\* A contributory factor to this is the favourable shape of the  $T\phi$  diagram for Hg, which is conditioned by the very low specific heat of the liquid mercury, thus approaching the Carnot cycle.

† Stodola also gives  $\eta_{th} = 0.27$ .

‡ From *Power* (1927), No. 22.

§ The process was suggested by the author in 1923 and 1926, while working on the Holzwarth gas turbine. See Report No. 23 of the second World Power Conference, 1930, in which P. Langer reviews the gas turbine as based on this process.



continuous flow second-stage turbine, below 850° F., as is also the case in steam turbines. The temperature of the gases leaving the explosion wheel is, however, much higher and ranges from 1400 to 1800° F. This temperature is, therefore, reduced to 850° F. by means of dry or wet steam by allowing the hot gases to give up part of their heat to a steam superheater or to a combined evaporator and superheater. This, however, reduced the available work of the hot gases in the ratio of their absolute temperatures before and after cooling, i.e. in the ratio of about

$$\frac{850 + 460}{1600 + 460} = 0.64$$

Since the greater load is taken by the low pressure turbine (which can be designed according to the well-established rules used in multistage steam turbines so that high thermal efficiencies may be expected) a loss of this order in the combined explosion turbine is inadmissible. This loss will, however, occur if cooling water be used to remove the heat in the receiver, but if steam be used as the cooling agent the loss in available work of the hot gases is compensated for by the gain in available work of the cooling steam, due to the heat absorbed by it.\*

When a heat exchange occurs between the hot gases and dry or slightly wet steam, and the steam is at the same pressure as the gases in the receiver, not only is there no loss, but, in certain cases, there is actually a gain of available work due to this exchange. Denoting the available work of the steam before and after heating by  $E_I$  and  $E'_I$  and the corresponding quantities of the hot gases in the receiver by  $E_{II}$  and  $E'_{II}$ , the ratio of the total available work after and before the heat exchange is

$$E$$

e.g. for steam and gases at

$$p = 42.5 \quad 85 \quad 140 \text{ lb./in.}^2 \text{ abs.}$$

and with initial and final gas temperatures of 1650 and 840° F., a final steam temperature of 750° F. and condenser vacuum of 0.85 lb./in.<sup>2</sup> abs.,

$$\eta_w = 1.13 \quad 1.052 \quad 1.014$$

Even for wet steam at 85 lb./in.<sup>2</sup> abs. we have, with

$$\begin{array}{cccc} q & = & 0.80 & 0.60 & 0.40 & 0.20 \\ \eta_w & = & 1.0 & 0.976 & 0.968 & 0.956 \end{array}$$

Hence, only a small amount of available work is lost when the cooling steam is very wet.

\* *Zeitschr. f. Techn. Physik* (1930), No. 2. W. Schüle. Maintenance of available work in heat exchange of gaseous bodies. See also page 178. The principle of the conservation of available work of gaseous bodies in heat exchanges was established by Schüle in March, 1926, and was applied to the problem of the gas turbine, as shown by a communication to the Holzwarth Gas Turbine Co., entitled, "Gas Steam Turbine with Heat Exchange between the Gas and Steam."

The cooling steam can be supplied from an existing steam main or can be generated by the heat contained in the gases exhausting from the turbine. In the former case, the steam will be almost or completely dry saturated, but in the latter case it will be wet. In the first case, however, the heat required to generate the steam has to be added to the combustion heat of the gas turbine, whereas, in the latter the heat required comes

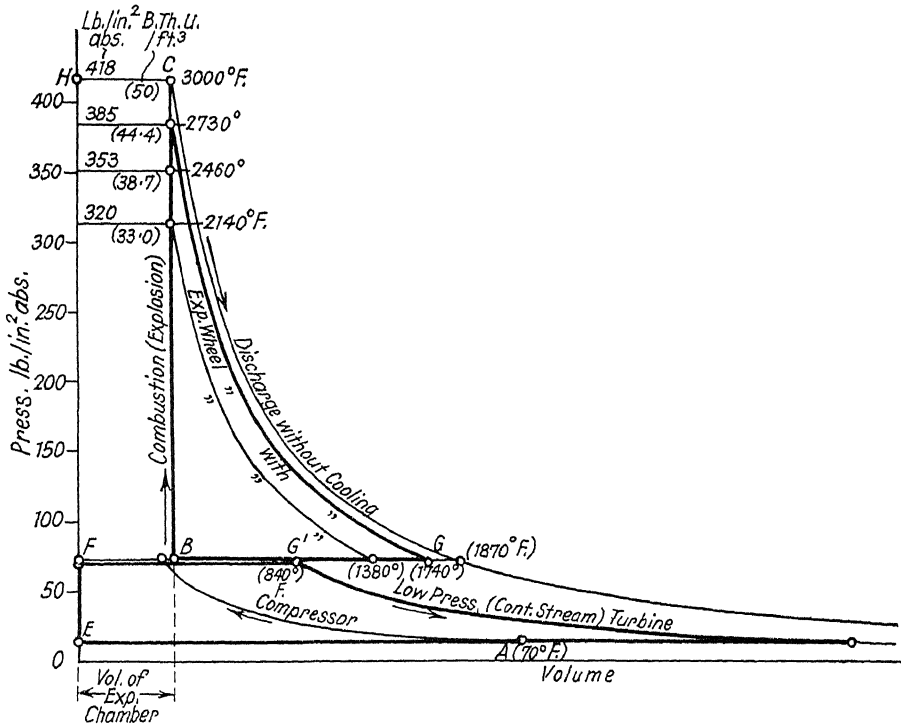


FIG. 324

from the combustion heat of the turbine itself and is thus a direct saving. A closer investigation of this case is given below.

The process is illustrated in Fig. 323 as follows: Air, drawn into a compressor from the atmosphere, is compressed to the pressure  $p_0$  (70 lb./in.² abs.) and then delivered to the explosion chamber. An oil pump serves to deliver the oil to this chamber. Air, and later, oil in the form of a fine spray, enter the chamber at 70 lb./in.² abs., and the air forces the residual gases of the previous cycle through the nozzles which are still open (or through a special exhaust valve) into the receiver. If a gaseous fuel is used, the oil pump is replaced by a gas compressor. The temperature\* of the air is increased in the bomb, due to heat supplied by the walls, and is given as 120° F. in Fig. 323.

Combustion of the charge is now effected by electric ignition. The

\* For the same reason as applies to gas and oil engines, it is necessary that this temperature should be kept low, in order to avoid pre-ignition and to make as much use as possible of the combustion space in the bomb.

value of the explosion pressure depends on the calorific value of the mixture. For a normal mixture, showing a calorific value of 50 B.Th.U./ft.<sup>3</sup> at S.T.P. the combustion temperature rises, as shown by the combustion chart, from 120° to 3000° F., provided no heat is lost to the chamber walls. If, however, 5.6, or 11.3, or 17 B.Th.U./ft.<sup>3</sup> at S.T.P. of the charge is lost to the walls, the corresponding temperature rises are 2730, 2460, and 2140° F. respectively. The combustion pressure, without heat loss, rises to

$$\left( \frac{460 + 3000}{460 + 120} \right)$$

while, with the above assumed losses, the pressures are 385, 353, and 320 lb./in.<sup>2</sup> abs. (Fig. 324).

The controlled nozzle valves are opened as rapidly as possible, so that the completely burned products can flow through the convergent-divergent nozzles to the explosion wheel which is designed generally as a two-row Curtis wheel. Since the pressure in the receiver, which is in free communication with the wheel chamber, is 70 lb./in.<sup>2</sup> abs., the hot gases expand from the explosion pressure (418 to 320 lb./in.<sup>2</sup> abs.) to 70 lb./in.<sup>2</sup> abs. A heat flow occurs to the walls not only during combustion, but also during discharge, and is increased as the time of discharge increases, hence this period of discharge should be as small as possible. In the limiting case of no heat flow, the expansion is adiabatic and (if isentropic) the temperature at discharge from the nozzles is 1870° F., or with heat loss in the bomb between 1740° and 1380° F. (Fig. 324). Actually the temperatures will be lower than these limiting values, according to the measure of heat flow during discharge. The hot gases now act on the explosion wheel at these temperatures, and at a pressure of 70 lb./in.<sup>2</sup> abs. The velocity has a maximum value  $V$  at the first instant of discharge, and if  $H_1$  and  $H_2$  are the total heats of 1 lb. of the hot gases in the bomb before and after adiabatic expansion to the receiver pressure respectively, then

$$V = 223.8 \sqrt{H_1 - H_2}$$

If the velocity is to be found from the total heats  $J$  per ft.<sup>3</sup> at S.T.P., as given by the chart, then, assuming a mean molecular weight of  $m = 30$ , we have

$$V = 774.4 \sqrt{J_1 - J_2}$$

and  $J_1$  and  $J_2$  can be taken from the chart.

We have also, with sufficient accuracy,

$$H_1 - H_2 = A \frac{\gamma}{\gamma - 1} (p_1 v_1 - p_0 v_2) \quad 144$$

i.e. the area  $CGFH$  (Fig. 324).

$$\text{With } p_1 v_1^\gamma = p_0 v_2^\gamma \text{ and } v_1 = \frac{RT_1}{144 p_1}$$

this becomes

$$H_1 - H_2 = ART_1 \frac{\gamma}{\gamma - 1} \left[ 1 - \left( \frac{p_0}{p_1} \right)^{\frac{\gamma - 1}{\gamma}} \right]$$



$$\text{or} \quad J_1 - J_2 = 0.00556 T_1 \frac{\gamma}{\gamma - 1} \left[ 1 - \left( \frac{p_0}{p_1} \right)^{\frac{\gamma - 1}{\gamma}} \right]$$

This gives, with  $\gamma = 1.26$ ,

$$J_1 - J_2 = 29.8 \text{ B.Th.U./ft.}^3 \text{ at S.T.P.}$$

and, from the chart,

$$J_1 - J_2 = 28.7$$

$$\text{Hence } V_{max} = 4,220 \text{ ft./sec.}$$

On the other hand, with the three cases of assumed heat loss,  $J_1 - J_2 = 24.8, 22$ , and  $18.9$ , giving

$$V_{max} = 3850, 3620, \text{ and } 3360 \text{ ft./sec.}$$

During discharge, the velocity continuously decreases, due to the decrease in pressure in the vessel, until it assumes a constant minimum value, due to the fresh charge.

The total available work of the hot gases in being discharged to the wheel at the pressure  $p_0$  ( $= 70 \text{ lb./in.}^2 \text{ abs.}$ ) is represented by the area  $BCG$  (Fig. 324), and is thus smaller than  $H_1 - H_2$  by the area  $CBFH = Av(v_1 - p_0)$ . The available high pressure work is, therefore, per pound,

$$AE_1 = H_1 - H_2 - Av_1(p_1 - p_0)$$

$$\text{or} \quad AE_1 = H_1 - H_2 - ART_1 \left( 1 - \frac{p_0}{p_1} \right)$$

and, for the weight corresponding to  $1 \text{ ft.}^3$  at S.T.P.,

$$AE_1 = J_1 - J_2 - 0.00556 T_1 \left( 1 - \frac{p_0}{p_1} \right)$$

or the combustion heats of

	$Q_1$	= 50.0	44.4	38.7	33.0	B.Th.U./ft. <sup>3</sup> at S.T.P.
we have	$J_1 - J_2$	= 28.7	24.8	22.0	18.9	
and	$AE_1$	= 12.7	10.3	9.0	7.6	

If this work, a certain fraction  $\eta_{uI}$  is usefully converted by the  $l$ , depending on its speed and blade efficiency, the remainder is converted to heat and, as such, passes over to the receiver, so that the temperature there is increased. Assuming  $\eta_{uI} = 0.5$  gives the useful in the four cases as

$$\eta_{uI} AE_1 = 6.35 \quad 5.15 \quad 4.5 \quad 3.8 \text{ B.Th.U./ft.}^3$$

and the same amount appears as frictional re-heat. The temperature in the receiver is thus increased by about

	270°	216°	180°	144° F.
i.e. to	2140°	1960°	1740°	1526° F.

Due to a heat flow to the walls, however, the temperature in the receiver is lowered. Assuming the heat lost in this way to be 10 per cent

B.Th.U./ft.<sup>3</sup>, the temperature in the receiver  
 —, and this amounts, in the four cases, to  
 1890°    1710°    1490°    1276° F.

These temperatures have to be reduced in the receiver by steam cooling to 842° F., i.e. through the ranges

1048°    868°    648°    434° F.

which requires a removal of heat amounting to

24.8    20.5    15.5    10.7 B.Th.U./ft.<sup>3</sup> at S.T.P.

**Low pressure continuous flow gas turbine.** The available work per pound of gas in the low pressure turbine is

$$AE_2 = A \frac{\gamma}{\gamma-1} RT_3 \left[ 1 - \left( \frac{p_2}{p_0} \right)^{\frac{\gamma-1}{\gamma}} \right]$$

or, per ft.<sup>3</sup> at S.T.P.,

$$AE_2 = 0.00556 \frac{\gamma}{\gamma-1} T_3 \left[ 1 - \left( \frac{p_2}{p_0} \right)^{\frac{\gamma-1}{\gamma}} \right]$$

With  $\gamma = 1.3$ ,  $T_3 = (840 + 460)$ , and  $\frac{p_2}{p_0} = \frac{14.7}{70}$ , we have

$$AE_2 = 9.43 \text{ B.Th.U./ft.}^3 \text{ at S.T.P.}$$

If no cooling occurred in the receiver, this work would be increased in the ratio of the absolute temperatures and would then amount to

17.0    15.7    14.1    12.6 B.Th.U.

If the low pressure turbine has a thermodynamic efficiency of 83 per cent, the useful work, without cooling, would be

14.1    13.0    11.7    10.5 B.Th.U.

With cooling, the low pressure work in all four cases is

$$0.83 \times 9.43 = 7.82 \text{ B.Th.U./ft.}^3$$

**Weight of steam generated and work performed by steam.** The work delivered by the steam per ft.<sup>3</sup> of gases is conditioned by the weight of steam which can be generated by means of the heat in the exhaust gases, and by the range of superheat attained by the cooling steam at the receiver. The case in which the steam is generated externally is not considered here.

The available heat in the gases leaving the low pressure turbine is found from the temperature range above about 200° F. Now the temperature of the waste gases is at least

$$T_4 = (840 + 460) \left[ \frac{p_2}{p_0} \right]^{\frac{0.3}{1.3}}$$

Actually, on account of frictional losses, it is somewhat higher.

Thus, with

we have	$p_0$	= 45.0	70	150 lb./in. <sup>2</sup> abs.
	$T_4$	= 1000	905	760 °F. abs.
or	$t_4$	= 540	445	300° F.

In cooling to 200° F. the heat liberated is thus

$$6.86 \quad 4.74 \quad 1.91 \text{ B.Th.U.}$$

Hence, for compression pressures above 70 lb./in.<sup>2</sup> abs., any attempt to save the waste heat in the exhaust gases need scarcely be considered.

The heat transmitted to the cooling water of the explosion chamber in the three cases with 5.6, 11.3, and 17 B.Th.U. heat loss during combustion and 5 B.Th.U. loss during discharge is

$$10.6 \quad 16.3 \quad 22 \text{ B.Th.U.}$$

The first case, in which no heat is lost during combustion, need not be considered here.

It is now necessary to find what quantity of steam can be generated with the aid of these quantities of heat and the condition of this steam on leaving the bomb. In the first place it is assumed that no waste heat boiler is used, but that a waste heat preheater is fitted, in which the cooling water of the explosion chamber is preheated, and that water is only heated (under increased pressure) and not evaporated in the cooling jackets of the bomb. If the pressure of the water here is 550 lb./in.<sup>2</sup> abs., it can be heated to 477° F. In this way the cooling water can take up about 430 B.Th.U./lb. if the supply water is at 60° F. Now, in the three cases given above, the heat available for heating the water at the bomb is

$$10.6 \quad 16.3 \quad 22 \text{ B.Th.U./ft.}^3 \text{ of gases}$$

and at the waste heat preheater

$$4.74 \quad 4.74 \quad 4.74 \text{ B.Th.U.}$$

giving a total of

$$15.34 \quad 21.04 \quad 26.74 \text{ B.Th.U.}$$

Without the waste heat preheater, the weights of water at 550 lb./in.<sup>2</sup> abs. which can be heated to 477° F. are

$$\therefore \frac{22}{430} = 0.0512 \text{ lb.}$$

while, with the waste heat preheater, the weights are

$$0.0357 \text{ lb.}, 0.0490 \text{ lb.}, 0.0622 \text{ lb.}$$

If this hot water be allowed to flow into the superheater tubes in the receiver against a pressure of, say, 200 lb./in.<sup>2</sup> abs., wet steam having the quality of  $q$  is formed. The value of  $q$  is found from

$$h_1 = h_2 + qL_2$$

$$\text{giving} \quad q = \frac{h_1 - h_2}{L_2}$$

With  $h_1 = 460$ ,  $h_2 = 356$ ,  $L_2 = 841$ , this gives

$$q = 0.124, \text{ or } 12.4 \text{ per cent}$$

The total heat before throttling, reckoned from 32° F., is 460 B.Th.U./lb. In order to completely dry this steam, the heat required is  $(1 - q)L_2 = 0.876 \times 841 = 737 \text{ B.Th.U./lb.}$ , so that, for the actual masses

of steam the heats are  $0.0246 \times 737 = 18.1$  B.Th.U.,  $0.0379 \times 737 = 27.9$  B.Th.U., and  $0.0512 \times 737 = 37.8$  B.Th.U., or, with preheating of the water,

$$26.3 \quad 36.1 \quad 45.8 \text{ B.Th.U.}$$

The heats available in the gases are

$$20.5 \quad 15.5 \quad 10.7 \text{ B.Th.U.}$$

It is thus only in the first case, with a heat loss of 5.6 B.Th.U. during combustion, that the available heat in the gases is capable of evaporating the cooling water and superheating the steam thus formed. Since, for each 0.0246 lb. of steam,  $20.5 - 18.1 = 2.4$  B.Th.U. are available for superheating, the heat available per pound is  $\frac{2.4}{0.0246} = 97.5$  B.Th.U.

This means that saturated steam at 200 lb./in.<sup>2</sup> abs. can be superheated to a temperature of 570° F. The weight of steam available for work in the steam turbine for this case is 0.0246 lb. at 200 lb./in.<sup>2</sup> abs., and 570° F., which, with a condenser pressure of 0.57 lb./in.<sup>2</sup> abs., gives a heat drop of 404 B.Th.U./lb., or, for the actual weight,  $0.0246 \times 404 = 10$  B.Th.U. Assuming a thermodynamic efficiency of 0.83 in the steam turbine, the work performed by this unit is  $0.83 \times 10 = 8.3$  B.Th.U./ft.<sup>3</sup> of gases.

The total useful work is now made up as follows—

Gas turbine:			
High pressure wheel	.	.	5.15 B.Th.U./ft. <sup>3</sup> at S.T.P.
Low pressure turbine	.	.	7.82 B.Th.U./ft. <sup>3</sup> at S.T.P.
Steam turbine	.	.	8.30 B.Th.U./ft. <sup>3</sup> at S.T.P.
Total	.	.	21.27 B.Th.U./ft. <sup>3</sup> at S.T.P.

Of this work, part is required to compress the air, amounting to 1 ft.<sup>3</sup> at S.T.P. (in the oil turbine) from 14.7 lb./in.<sup>2</sup> abs. and 70° F., to 70 lb./in.<sup>2</sup> abs. This work is, per pound of air with isothermal compression and no loss,

$$AE_c = Ap_2 V_2 \log_e \frac{p_0}{p_2}$$

or, with  $p_2 V_2 = RT_2$ ,

$$AE_c = ART_2 \log_e \frac{p_0}{p_2}$$

and, for 1 ft.<sup>3</sup> at S.T.P.,

$$AE_c = 0.00556 \times 2.303 T_2 \log \frac{p_0}{p_2}$$

With  $T_2 = 70 + 460 = 530$  and  $\frac{p_0}{p_2} = \frac{70}{14.7} = 4.76$ ,

$$AE_c = 4.6 \text{ B.Th.U./ft.}^3$$

The actual compression work, with an isothermal compression efficiency of 0.63, is

$$\frac{4.6}{0.63} = 7.30 \text{ B.Th.U./ft.}^3$$

This gives the net work as

$$21.27 - 7.30 = 13.97 \text{ B.Th.U./ft.}^3$$

so that the overall thermal efficiency is

$$\frac{13.97}{50} = 0.279$$

In the five other cases, with 11.3 and 17 B.Th.U./ft.<sup>3</sup> loss of heat during combustion, the heat contained in the gases in the receiver is not sufficient to convert the greater quantities of cooling water to steam. On the other hand, the greater amounts of heat in the gases exhausting from the bomb and nozzles can be used to generate wet saturated steam at 200 lb./in.<sup>2</sup> abs., instead of boiling water at 480° F. For this, of course, an evaporator is required, in which the exhaust heat of the gases is transmitted.

Denoting the waste heat by  $Q_w$  and available heat in the receiver by  $Q_R$  per ft.<sup>3</sup> of gases, the weight  $W_s$  of steam formed is given by

and the quality of the steam generated in the evaporator by

Taking the values—

$Q_w =$	10.6	16.3	22	15.34	21.04	26.74 (page 613)
$Q_R =$	20.5	15.5	10.7	20.50	15.50	10.70
or						
$Q_w + Q_R =$	31.1	31.8	32.7	35.84	36.54	37.44

and (for 750° F. and 200 lb./in.<sup>2</sup> abs., and water at 60° F.),

$$H - h_0 = 1403 - 28 = 1,375 \text{ B.Th.U./lb.}$$

$W_s =$	0.0226	0.0231	0.0238	0.0261	0.0266	0.0272 B.Th.U./ft. <sup>3</sup> at S.T.P.
and $q =$	0.168	0.445	0.704	0.308	0.546	0.775

The available work of 1 lb. of steam at 200 lb./in.<sup>2</sup> abs. and 750° F., with a condenser pressure of 0.57 lb./in.<sup>2</sup> abs., is 435 B.Th.U./lb., and hence the work performed by the steam turbine with a thermodynamic efficiency of 83 per cent is

8.15	8.33	8.58	9.42	9.95	10.18 B.Th.U./ft. <sup>3</sup> at S.T.P.
------	------	------	------	------	--

The total work delivered is now made up as follows—

Gas turbine:							
High pressure wheel	5.15	4.50	3.80	5.15	4.50	3.80	B.Th.U./ft. <sup>3</sup> at S.T.P.
Low pressure turbine	7.82	7.82	7.82	7.82	7.82	7.82	
Steam turbine . . .	8.15	8.33	8.58	9.42	9.95	10.18	
Total gross work	21.12	20.65	20.20	22.39	22.27	21.80	
Compressor work.	7.30	7.30	7.30	7.30	7.30	7.30	
Net output . . . . .	13.82	13.35	12.90	15.09	14.97	14.50	B.Th.U./ft. <sup>3</sup> at S.T.P.
Overall thermal efficiency	0.276	0.270	0.258	0.302	0.299	0.290	

A further increase in the thermal efficiency would be possible if the thermodynamic efficiency of 50 per cent for the explosion wheel could be increased. The above values hold only for a compression pressure of

70 lb./in.<sup>2</sup> abs., and a heat of combustion of 50 B.Th.U./ft.<sup>3</sup> at S.T.P. of mixture. Taking other values, still higher thermal efficiencies can be obtained. In addition, there is the possibility of re-superheating the steam by means of the hot gases in the receiver.

The above serves to show that large gas, oil, and pulverized fuel turbines are capable of equalling the performance of large gas and oil engines as well as steam turbines.

# PROPERTIES OF SATURATED STEAM

FROM 0.25 TO 550 lb./in.<sup>2</sup> abs.\*

Pres- sure lb./in. <sup>2</sup> Abs.	Temp. ° F.	HEATS B.Th.U./Lb.			External Latent Heat 144 $Ap$ ( $V_s - \sigma$ )	Internal Latent Heat $\rho$ B.Th.U. per lb.	SPEC. VOL. FT. <sup>3</sup> /LB.	
		Sen- sible $h$	Latent $L$	Total $H$			Steam $V_s$	Water $\sigma$
0.25	59.2	27.3	1059.2	1086.5	57.0	1002.2	1232.0	0.01603
0.50	79.5	47.6	1047.9	1095.5	59.3	988.6	641.0	0.01607
0.75	91.9	59.9	1043.8	1102.7	60.5	983.3	436.0	0.01611
1.0	101.8	69.8	1035.6	1105.4	61.6	974.0	333.0	0.01614
1.5	115.9	83.4	1028.0	1111.9	63.2	964.8	228.0	0.01616
2.0	126.1	94.1	1022.3	1116.4	64.2	958.1	173.4	0.01623
3.0	141.4	109.4	1013.4	1122.8	65.8	947.6	118.5	0.01630
4.0	153.1	121.1	1006.8	1127.9	67.0	939.8	90.5	0.01635
5.0	162.5	130.5	1001.2	1131.7	68.1	933.1	73.6	0.01640
6.0	170.2	138.2	996.5	1134.7	68.9	927.6	62.1	0.01645
7.0	176.5	144.5	992.9	1137.4	69.5	923.4	53.7	0.01649
8.0	182.8	150.8	989.1	1139.9	70.1	919.0	47.34	0.01652
10.0	193.3	161.3	983.0	1144.3	71.0	912.0	38.38	0.01659
12.0	201.9	169.9	977.6	1147.5	71.8	905.8	32.34	0.01665
14.0	209.5	177.5	973.1	1150.6	72.6	900.5	28.04	0.01670
14.7	212.0	180.2	971.4	1151.6	72.8	898.6	26.78	0.01671
16.0	216.1	184.4	968.5	1152.9	73.2	895.3	24.74	0.01674
18.0	222.8	191.2	964.4	1155.6	73.8	890.6	22.17	0.01679
20.0	227.8	196.2	961.4	1157.6	74.2	887.2	20.06	0.01682
25.0	239.9	208.5	953.2	1161.7	75.2	878.0	16.27	0.01691
30.0	250.3	219.1	946.0	1165.1	76.1	869.9	13.73	0.01698
35.0	259.3	228.2	940.2	1168.4	76.8	863.4	11.87	0.01705
40.0	267.1	236.2	934.5	1170.7	77.4	857.1	10.47	0.01711
45.0	274.3	243.5	929.6	1173.1	77.9	851.7	9.37	0.01717
50.0	280.9	250.3	924.6	1174.9	78.4	846.2	8.49	0.01723
55.0	287.2	256.7	920.1	1176.8	78.9	841.2	7.764	0.01728
60.0	292.6	262.3	916.2	1178.5	79.3	836.9	7.159	0.01733
65.0	297.9	267.7	912.0	1179.7	79.6	832.4	6.630	0.01738
70.0	302.5	272.5	908.7	1181.2	79.9	828.8	6.187	0.01743
75.0	307.6	277.8	904.6	1182.4	80.2	824.4	5.792	0.01748
80.0	311.9	282.2	901.1	1183.3	80.5	820.6	5.453	0.01753
85.0	316.4	286.9	897.9	1184.8	80.7	817.2	5.145	0.01758
90.0	320.4	291.1	894.5	1185.6	81.0	813.5	4.880	0.01762
95.0	324.1	294.9	891.5	1186.4	81.2	810.3	4.637	0.01766
100.0	327.7	298.6	888.6	1187.2	81.4	807.2	4.416	0.01770
105.0	331.3	302.4	885.6	1188.0	81.5	804.1	4.212	0.01774
110.0	334.8	306.0	882.7	1188.7	81.7	801.0	4.031	0.01777
115.0	338.0	309.4	880.0	1189.4	81.9	798.1	3.866	0.01781
120.0	341.2	312.7	877.3	1190.0	82.0	795.3	3.710	0.01785
125.0	344.1	315.7	874.8	1190.5	82.2	792.6	3.571	0.01789
130.0	347.4	319.2	871.9	1191.1	82.3	789.6	3.438	0.01793
135.0	350.1	322.0	869.6	1191.6	82.4	787.2	3.315	0.01796
140.0	352.9	325.0	867.0	1192.0	82.6	784.4	3.216	0.01799
145.0	355.6	327.8	864.8	1192.6	82.7	782.1	3.100	0.01803
150.0	358.5	330.9	862.2	1193.1	82.8	779.4	3.002	0.01807
155.0	361.2	333.7	859.9	1193.6	82.9	777.0	2.908	0.01810
160.0	363.7	336.4	857.5	1193.9	83.0	774.5	2.821	0.01813

\* From *Z.V.d.I.* (1911), page 1506. W. Schüle. Die Eigenschaften des Wasserdampfes nach den neusten Versuchen. *Berichtigt nach den Münchener Tabellen* (1923).

PROPERTIES OF SATURATED STEAM (*contd.*)FROM 0.25 TO 550 lb./in.<sup>2</sup> abs.\*

Pres- sure lb./in. <sup>2</sup> Abs.	Temp. ° F.	HEATS B.Th.U./LB.			External Latent Heat 144 $Ap$ ( $V_s - \sigma$ )	Internal Latent Heat $\rho$ B.Th.U. per lb.	SPEC. VOL. FT. <sup>3</sup> /LB.	
		Sen- sible $h$	Latent $L$	Total $H$			Steam $V_s$	Water $\sigma$
165.0	366.1	338.9	855.4	1194.3	83.1	772.3	2.739	0.01817
170.0	368.4	341.3	853.3	1194.6	83.2	770.1	2.662	0.01820
175.0	370.8	343.9	851.0	1194.9	83.3	767.7	2.590	0.01823
180.0	373.1	346.3	849.0	1195.3	83.4	765.6	2.522	0.01826
185.0	375.4	348.8	846.8	1195.6	83.4	763.4	2.454	0.01829
190.0	377.6	351.1	844.7	1195.8	83.5	761.2	2.392	0.01833
195.0	379.9	353.6	842.5	1196.1	83.6	758.9	2.334	0.01836
200.0	381.9	355.7	840.6	1196.3	83.7	756.9	2.279	0.01839
210.0	386.1	360.2	836.5	1196.7	83.8	752.7	2.175	0.01844
220.0	389.8	364.1	833.0	1197.1	83.9	749.1	2.079	0.01849
230.0	393.8	368.5	829.0	1197.5	84.0	745.0	1.992	0.01855
240.0	397.6	372.5	825.4	1197.9	84.1	741.3	1.912	0.01860
250.0	401.2	376.4	821.9	1198.3	84.1	737.8	1.836	0.01865
260.0	404.4	379.9	818.6	1198.5	84.2	734.4	1.768	0.01869
270.0	407.7	383.4	815.4	1198.8	84.3	731.1	1.706	0.01874
280.0	410.7	386.7	812.3	1199.0	84.4	727.9	1.647	0.01879
290.0	414.1	390.3	809.0	1199.3	84.5	724.5	1.593	0.01884
300.0	417.4	393.9	805.6	1199.5	84.5	721.1	1.541	0.01890
325.0	424.9	402.1	797.8	1199.9	84.5	713.3	1.424	0.01901
350.0	432.0	409.8	790.3	1200.1	84.5	705.8	1.323	0.01914
375.0	438.3	416.7	783.5	1200.2	84.5	699.0	1.237	0.01925
400.0	444.6	423.5	776.7	1200.2	84.6	692.1	1.162	0.01937
450.0	456.3	436.4	763.8	1200.2	85.0	678.8	1.041	0.01958
500.0	467.1	448.4	751.6	1200.0	84.0	667.6	0.928	0.01978
550.0	477.0	459.6	740.1	1199.7	84.0	656.1	0.845	0.01998

\* From *Z.V.d.I.* (1911), page 1506. W. Schüle. Die Eigenschaften des Wasserdampfes nach den neusten Versuchen. *Berichtigt nach den Münchener Tabellen* (1923).



## SATURATED STEAM

FROM - 20° C. TO + 9° C.

Temperature <i>t</i> ° C.	PRESSURE		Specific Vol. ft. <sup>3</sup> /lb.
	Mm. Hg.	Lb./in. <sup>2</sup> Abs.	
- 20	0.960	0.01852	15,942
- 19	1.044	0.02014	14,741
- 18	1.135	0.02189	13,588
- 17	1.233	0.02378	12,530
- 16	1.338	0.02581	11,569
- 15	1.451	0.02799	10,687
- 14	1.573	0.03034	9,854
- 13	1.705	0.03289	9,101
- 12	1.846	0.03561	8,428
- 11	1.997	0.03852	7,787
- 10	2.159	0.04165	7,226
- 9	2.335	0.04504	6,698
- 8	2.521	0.04863	6,217
- 7	2.722	0.05251	5,752
- 6	2.937	0.05665	5,320
- 5	3.167	0.06109	4,919
- 4	3.413	0.06584	4,518
- 3	3.677	0.07093	4,198
- 2	3.958	0.07635	3,910
- 1	4.258	0.08214	3,637
0	4.579	0.08833	3,380
+ 1	4.921	0.09493	3,173
+ 2	5.286	0.1020	2,964
+ 3	5.675	0.1095	2,756
+ 4	6.088	0.1174	2,580
+ 5	6.528	0.1259	2,403
+ 6	6.997	0.1350	2,259
+ 7	7.494	0.1446	2,115
+ 8	8.023	0.1548	1,971
+ 9	8.584	0.1656	1,859

## SATURATED STEAM

FROM 0.15 lb./in.<sup>2</sup> Abs TO 3 lb./in.<sup>2</sup> Abs.

Pressure $p_s$		Temp. ° F.	Specific Volume $V$ Ft. <sup>3</sup> /lb.	Density lb./ft. <sup>3</sup>	Latent Heat $L$ B.Th.U. per lb.	Total Heat $H$ B.Th.U. per lb.
Lb./in. <sup>2</sup> Abs.	Mm. Hg.					
0.15	7.77	45.4	2005.0	0.0004988	1063	1077
0.20	10.36	53.1	1520.0	0.0006579	1060	1081
0.25	12.96	59.3	1235.0	0.0008097	1057	1084
0.30	15.55	64.6	1040.0	0.0009615	1054	1087
0.35	18.14	69.0	900.0	0.001111	1051	1089
0.40	20.73	72.8	798.0	0.001253	1049	1091
0.45	23.32	76.3	709.0	0.001410	1047	1092
0.50	25.91	79.6	640.0	0.001563	1046	1094
0.55	28.50	82.5	586.0	0.001707	1044	1095
0.60	31.09	85.3	540.0	0.001852	1043	1096
0.65	33.68	87.8	500.0	0.002000	1041	1097
0.70	36.27	90.1	467.0	0.002141	1040	1099
0.75	38.87	92.2	437.0	0.002288	1039	1100
0.80	41.46	94.3	412.0	0.002427	1038	1100
0.85	44.05	96.3	388.0	0.002577	1037	1101
0.90	46.64	98.2	368.0	0.002717	1036	1102
0.95	49.23	100.0	350.0	0.002857	1035	1103
1.00	51.82	101.7	333.0	0.003003	1034	1104
1.1	57.0	105.0	304.0	0.003290	1033	1105
1.2	62.2	107.9	282.0	0.003546	1031	1107
1.3	67.4	110.6	261.0	0.003831	1030	1108
1.4	72.5	113.3	244.0	0.004098	1028	1109
1.5	77.7	115.7	228.0	0.004386	1027	1110
1.6	82.9	118.0	215.0	0.004651	1025	1111
1.7	88.1	120.2	202.0	0.004951	1024	1112
1.8	93.3	122.3	192.0	0.005208	1023	1113
1.9	98.5	124.3	182.5	0.005480	1022	1114
2.0	103.6	126.2	173.8	0.005754	1021	1115
2.1	108.8	128.1	166.0	0.006024	1020	1116
2.2	114.0	129.8	159.1	0.006285	1019	1116
2.3	119.2	131.4	152.8	0.006545	1018	1117
2.4	124.4	133.0	146.6	0.006821	1018	1118
2.5	129.6	134.5	141.2	0.007082	1017	1118
2.6	134.7	136.0	136.1	0.007348	1016	1119
2.7	139.9	137.4	131.3	0.007616	1015	1120
2.8	145.1	138.8	126.8	0.007886	1014	1121
2.9	150.3	140.2	122.3	0.008177	1013	1121
3.0	155.5	141.6	118.4	0.008446	1012	1122

## SKELETON STEAM TABLES\*

MEAN VALUES AND TOLERANCES FOR THE SATURATED STATE  
PRESSURE AND SPECIFIC VOLUME

Temp. ° C.	Sat. Pres. Kg./cm. <sup>2</sup>		Specific Volume m. <sup>3</sup> /kg.			
	Mean Value	Tolerance ±	Water		Steam	
			Mean Value	Tolerance ±	Mean Value	Tolerance ±
0	0.00623	0.00001	0.00100	0.00000	206.4	0.1
50	0.1258	0.0001	0.00101	0.00000	12.06	0.01
100	1.0333	0.0000	0.00104	0.00000	1.674	0.001
150	4.854	0.002	0.00109	0.00000	0.393	0.001
200	15.85	0.01	0.00116	0.00000	0.1270	0.0006
250	40.60	0.05	0.00126	0.00001	0.0501	0.0002
300	87.7	0.1	0.00142	0.00001	0.0218	0.0005
350	168.7	0.1	0.00179	0.00003	0.0089	0.0002

\* Resulting from the Steam Table Conference held in London, 1929.

MEAN VALUES AND TOLERANCES FOR THE SATURATED STATE  
WATER HEAT AND TOTAL HEAT OF STEAM

Temp. ° C.	Total Heat, International Kilo-calorie/kg.*			
	Water		Steam	
	Mean Value	Tolerance ±	Mean Value	Tolerance ±
0	0	0	595.4	1.2
50	49.95	0.02	618.5	1.0
100	100.04	0.02	639.2	0.5
150	150.93	0.03	656.0	1.5
200	203.60	0.04	667.5	2.0
250	259.35	0.3	670.0	3.0
300	321.8	1.0	657.5	5.0
350	403.7	2.0	617.0	7.0

\* The "International kilo-calorie" is defined as the heat equivalent of the 860th part of an international kilowatt hour.

SKELETON STEAM TABLES (*contd.*)

MEAN VALUES AND TOLERANCES FOR SUPERHEATED STEAM  
SPECIFIC VOLUME

Pressure	1 kg./cm. <sup>2</sup>		5 kg./cm. <sup>2</sup>		10 kg./cm. <sup>2</sup>	
Temp. ° C.	Mean Value m. <sup>3</sup> /kg.	Tolerance ± m. <sup>3</sup> /kg.	Mean Value m. <sup>3</sup> /kg.	Tolerance ± m. <sup>3</sup> /kg.	Mean Value m. <sup>3</sup> /kg.	Tolerance ± m. <sup>3</sup> /kg.
100	1.730	0.003	—	—	—	—
200	2.213	0.004	0.4331	0.0006	0.2100	0.0002
300	2.689	0.005	0.5326	0.0006	0.2631	0.0003
400	3.160	0.005	0.6294	0.0006	0.3125	0.0003
450	3.396	0.005	0.6783	0.0020	0.3367	0.0003
500	3.632	0.005	0.7242	0.0020	0.3604	0.0004
550	3.868	0.005	0.7720	0.0020	0.3846	0.0004

Pressure	25 kg./cm. <sup>2</sup>		50 kg./cm. <sup>2</sup>		100 kg./cm. <sup>2</sup>	
Temp. ° C.	Mean Value m. <sup>3</sup> /kg.	Tolerance ± m. <sup>3</sup> /kg.	Mean Value m. <sup>3</sup> /kg.	Tolerance ± m. <sup>3</sup> /kg.	Mean Value m. <sup>3</sup> /kg.	Tolerance ± m. <sup>3</sup> /kg.
300	0.1011	0.0003	0.0465	0.0004	—	—
400	0.1224	0.0003	0.0591	0.0004	0.0271	0.0004
450	0.1327	0.0003	0.0645	0.0004	0.0305	0.0004
500	0.1426	0.0003	0.0698	0.0004	0.0334	0.0004
550	0.1523	0.0004	0.0749	0.0005	0.0361	0.0005

Pressure	150 kg./cm. <sup>2</sup>		200 kg./cm. <sup>2</sup>		250 kg./cm. <sup>2</sup>	
Temp. ° C.	Mean Value m. <sup>3</sup> /kg.	Tolerance ± m. <sup>3</sup> /kg.	Mean Value m. <sup>3</sup> /kg.	Tolerance ± m. <sup>3</sup> /kg.	Mean Value m. <sup>3</sup> /kg.	Tolerance ± m. <sup>3</sup> /kg.
400	0.0162	0.0003	0.01028	0.00005	0.00636	0.00005
450	0.0190	0.0003	0.01312	0.00025	0.00940	0.00015
500	0.0212	0.0004	0.01512	0.00035	0.01136	0.0003
550	0.0232	0.0005	0.01673	0.00045	0.0130	0.0004

SKELETON STEAM TABLES (*contd.*)

MEAN VALUES AND TOLERANCES FOR SUPERHEATED STEAM

TOTAL HEAT. INTERNATIONAL KILO-CALORIE/KG.

Pressure			5 kg./cm. <sup>2</sup>		10 kg./cm. <sup>2</sup>	
Temp.	Mean Value	Tolerance ±	Mean Value	Tolerance ±	Mean Value	Tolerance ±
100	639.4	0.1				
200	687.5	1.0	682.5	1.5	676.2	1.5
300	735.3	1.5	732.5	2.0	729.5	2.0
400	783.4	1.5	781.7	2.0	780.0	2.0
450	807.7	1.5	807.0	2.0	805.0	2.0
500	832.6	1.5	831.6	2.0	830.2	2.0
550	858.0	2.0	857.5	2.0	856.5	2.0

Pressure	25 kg./cm. <sup>2</sup>		50 kg./cm. <sup>2</sup>		100 kg./cm. <sup>2</sup>	
Temp. ° C.	Mean Value	Tolerance ±	Mean Value	Tolerance ±	Mean Value	Tolerance ±
300	719.8	1.5	700.6	2.5	—	—
400	774.4	1.5	764.0	2.6	741.7	3.0
450	800.5	1.5	793.5	2.0	776.3	3.0
500	827.7	2.0	822.0	3.0	809.5	3.0
550	853.8	2.5	850.0	5.0	840.0	5.0

Pressure	150 kg./cm. <sup>2</sup>		200 kg./cm. <sup>2</sup>		250 kg./cm. <sup>2</sup>	
Temp. ° C.	Mean Value	Tolerance ±	Mean Value	Tolerance ±	Mean Value	Tolerance ±
400	713.0	3.0	676.0	2.5	622.0	4.0
450	757.7	3.0	736.0	4.0	712.0	6.0
500	797.5	4.0	784.0	6.0	769.0	8.0
550	832.0	5.0	823.0	8.0	812.0	10.0



# INDEX

- ADIABATIC** state changes of gases, 105,  
     116, 136  
     steam, 379, 412  
**Air**, 15, 52, 53, 59  
     liquefaction, 454  
     required for complete combustion, 23  
     vapour diagram, 438, 444, 450  
**Atmosphere**, ix, x  
**Atomic weight**, 9  
  
**BAROMETER**, reduction to 0° C., xiv  
**Benson boiler**, 538  
**Binary vapour turbine**, 603  
**Boyle's law**, 1  
  
**CALORIE**, 42  
**Calorific value**, 61  
**Carbon dioxide**—  
     isothermals, 421  
     latent heat, 425  
     specific heat at high temperatures, 53  
     *Tφ* diagram, 438  
     vapour pressure, 419  
**Carnot cycle**, 154  
     in the regenerative process, 522  
     reversed, 159  
     with any working substance, 162  
     with gases having variable specific  
         heat, 159  
**Centigrade heat unit**, 42  
**Characteristic equation**—  
     for an elementary change of state, 101  
     of gases, 6  
         at high pressures, 324  
     of superheated steam, 371, 408, 414  
**Clapeyron-Clausius equation**, 426  
**Cold vapours**, 419, 436  
     as working substance in waste heat  
         engines, 425  
     tables and diagrams, 560  
**Combustion**, 23  
     heat developed in products, 66  
     incomplete, 33  
     temperature—  
         at constant pressure, 69  
         — volume, 80  
**Combustion time**, 74  
     triangle, 32, 36  
**Compound expansions**, 515  
**Compressed air**, 325  
     motors, 335  
     power transmission, 335  
**Compressors**, two-stage, 333  
**Conversion of heat to work**, 93  
**Critical state of gases**, 423  
     pressure ratio, 242  
     pressures and temperatures, 424  
  
**Cyclic processes**, 180  
     elementary, 183  
     reversed (refrigeration), 183  
     thermal efficiency of, 183  
  
**DALTON'S law of partial pressures**, 13  
**De Laval**—  
     nozzles, 246, 465  
     turbine, 592  
**Density**, x  
**Diesel engine**, cyclic process of, 195, 343  
**Diphenyloxide**, 419, 434  
**Discharge**—  
     actual, 252, 255  
     theoretical, from nozzles, 236, 238  
     through de Laval nozzles, 246  
     through measuring nozzles, 256  
     sharp edged diaphragms, 258  
     throttle plates, 265  
**Distillation with the heat pump**, 569  
**Drying**—  
     with cold air, 483  
     — warm air, 482  
  
**EFFICIENCY**, thermal, 153, 487  
**Energy**—  
     equation for gases, 99  
     law, 150  
         internal—  
             of gases and products, 64  
             of gases at high temperatures, 131  
             of steam, 363  
             of substances, 170  
             of superheated vapours, 447  
     units, 150  
**Entropy**—  
     as a magnitude of state, 187  
     diagram for air, CO<sub>2</sub>, H<sub>2</sub>O, and pro-  
         ducts, 142  
         for gases at high temperature, 139  
         for steam. *See steam diagrams.*  
     diagram of cyclic processes, 190  
     in irreversible processes, 204  
     of any substance, 167  
     of gases, 123, 135  
     of saturated steam, 374  
     of saturated vapours, 430  
     of superheated steam, 377  
**Equilibrium between vapour, liquid, and  
     solid state**, 428  
**Ethyl ether**, 434  
**Explosion pressure**, 83  
**Extraction engines or turbines**, 492  
  
**FIRST law of thermodynamics**, 93, 149  
**Flow**—  
     compressive with friction, 311

## Flow—(contd.)

of gases and vapours with compression,  
224, 229

of gases and vapours with expansion,  
222, 228

resistance, 254

resistance, entropy diagram of, 461

resistance in pipes, 274, 282, 283

through nozzles, 235

with back pressures above the critical,  
240

with back pressures below the critical,  
243

with large pressure differences, 238

with small pressure differences, 237

Flue gas analyses, 32

Fuels, 20

gaseous, 22

solid and liquid, 21

GASES, ix

at high pressures, 317

table for, 12

Gas explosions, 78

Gaseous mixtures, 8, 83

Gas—

constant—

before and after combustion, 27

for mixtures, 14

universal, 12

constants, 12

laws, 1

Gas engine—

cycle, 193, 196

Otto cycle, 338

Gas turbine—

Holzwarth cycle, 198

Holzwarth working cycle, 349

Holzwarth-Schüle combined gas and  
steam, 606

Holzwarth-Schüle cycle, 199

Gauge pressures, xii

Gay-Lussac law, 2

$H\phi$  diagram—

for air vapour, 444

for steam, 458

Heat, flue gas, or exhaust loss of, 75

Heat—

and work, 149

consumption in steam engines, 487

diagrams, 123

drop, 455

pump, 564, 569

quantities, 41, 43

High pressure—

steam, 395

indirect generation of (Schmidt), 533

(Löffler), 534

IMPULSE measurements, 260

and reaction forces, 586

jets, 574

Internal combustion engine (Otto), 338

Diesel, 343

turbine, Holzwarth, 349

Holzwarth-Schüle, 199, 606

Internal energy, 64, 131

Irreversible processes, 200, 203, 204

LIQUEFACTION of gases, 424

air, 454

Liquid heat of water, 359

Löffler steam boiler, 534

MANOMETER, xii

Mechanical efficiency, 486

equivalent of heat, 93

Melting points, 166

Mercury, latent heat of, 419

steam turbine (Emmet), 603

temperature entropy diagram for, 604

vapour pressure of, 419, 420

Mixtures, gaseous, 83

Moist air, 16, 469, 476

Moistening of air, 480

Molecular weight, 10

mean or apparent, 9

of gases, 12

specific heat of solids, 44

from the kinetic theory, 131

of gases, 49

variation with temperature, 50, 53,  
57

NOZZLES, 246, 262

graphical calculation for, 465

ORSAT apparatus, 30

PARTIAL pressures, 15

Pipes, pressure loss in, 274

long, 292

rough, 283

smooth, 277

steam, 284

with adiabatic flow, 296

with isothermal flow, 293

Polytropic change of state, 112

Preheating of feed water, 518, 528

combustion air, 528

Products of combustion—

analysis of, 23, 25, 27, 32

combustion heat developed in, 66

dependence on calorific value, 68

specific heat of the, 57

temperature of, 72

volume for different fuels, 72

REACTION—

of jets, 579

wheel, pure, 582

Refrigeration—

by means of cold vapours, 549

by means of water vapour, 560

Carnot process, 159



- Refrigeration—(*contd.*)  
 reversed cyclic process, 183
- Refrigerators, 549  
 compound, 558
- Regenerative process, 519, 522
- Resuperheating of steam, 528
- Reynolds' number, 277
- SECOND law of thermodynamics, 154, 161,  
 170, 187  
 in an irreversible change of state, 203,  
 209
- Sensible heat of water, 359
- Specific—  
 density,  $x$   
 heat, 42  
 volume,  $x$   
   heat, mean and true, 46  
   of gases, 48  
   of solids and liquids, 43  
   ratio  $C_p/C_v$ , 51, 136  
     of  $\text{CH}_4$ ,  $\text{C}_2\text{H}_4$ ,  $\text{C}_2\text{H}_2$ , 56  
     of  $\text{CO}_2$ , 53  
     of  $\text{H}_2\text{O}$ , 54  
     of products, 57
- State diagrams—  
 for gases, 4, 318, 321, 323  
 steam, 412, 415
- Steam—  
 accumulators, 540  
   constant pressure, 548  
   displacer, 548  
   pressure drop (Ruth's), 543  
 at high pressures, 395  
 at low temperatures, 436  
 at normal pressures, 354  
 diagrams,  $T\phi$  and  $TV\phi$ , 390  
   at low temperatures, 436  
      $H\phi$ , 458  
      $Ht$ , 406  
   in high pressure region, 416  
 engine, 493  
 in the products of combustion, 29  
   actual work, 510  
   change of state of steam in the, 383  
   compound, 515  
   Schmidt's high pressure, 529  
   theoretical work of steam in the, 495  
   uniflow, 517  
   uniflow, with short stroke, 517  
   using superheated steam, 515  
   turbine, 588-603
- Superaoustic velocities—  
 in convergent nozzles, 252, 264  
 in nozzles, 246
- Superaoustic velocities—(*contd.*)  
 in turbine guide blades, 252, 270
- Superheated steam, 369  
 characteristic equations for, 371, 408,  
 412  
 specific heat of, 371  
 total heat of, 370
- TEMPERATURE, xiv, 41, 42  
 absolute, 164  
 scale, xiv, 165
- Thermal concentration, 72  
 efficiency, ideal, 485  
 efficiency of cyclic processes, 190  
 efficiency of internal combustion en-  
 gines, 339, 346, 350  
 efficiency of steam prime movers, 485  
 efficiency of steam reciprocating en-  
 gines, 500, 501, 504  
 potential of steam, 362
- Thermodynamic efficiency, 486
- Thomson-Joule cooling effect, 452
- Throttle plates, 265
- Throttling, 210  
 of actual gases, 452  
 of vapours, 444
- Total heat—  
 at constant pressure, 175  
 of gases and products, 64  
 of steam, 363, 397, 405, 458
- Triple point, 430
- UNIFLOW engines, 517
- VACUUM gauge, xii
- Van der Waal's equation, 322, 438
- Vapour—  
 limiting curve, 421  
 saturated and superheated, ix, 438  
 specific heat of, 441  
 state diagrams, 442
- Vapours, general relations, 419
- Viscosity, 274  
 kinematic, 277  
 of air and steam, 281
- WASTE heat engines, 425
- Wet vapour, ix, 363, 368
- Work—  
 available, 177  
 loss due to flow resistance, 306, 461  
 maintenance of, with heat exchange  
   at constant pressure, 178  
 of gases and vapours, 96  
 of vapour, 484, 455



